Computation of kappa

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```
# Function to compute a_n (cf article)

compute_an <- function(t1,n,K3) {

  build_an_1 <- 2*t1*pi*sqrt(n)/(K3+1)
  build_an_2 <- 16*pi^3*n^2/(K3+1)^4

  return( min(build_an_1,build_an_2)/sqrt(n) )
}
```

```
# Setting parameters
Nrep = 100000

t1 <- 0.64 # universal constant from paper

K4_exp <- 9
K4_gauss <- 3
```

Exponential and normal distributions

```
# Absolute third order moment of exp distr upper bounded while that of
# standard normal distr explicitly computed

K3_exp <- 2.31
K3_gauss <- 2^(3/2)*gamma(2)/sqrt(pi)
```

```
# Compute kappa_n for exp and standard normal

carac_exp_at_an <- 1 / sqrt(1 + compute_an(t1 = t1, n = 100000, K3 = K3_exp)^2)
print(carac_exp_at_an)
```

```
## [1] 0.6355232
```

```
carac_gauss_at_an <- exp( - compute_an(t1 = t1, n = 5000, K3 = K3_gauss)^2/2)
print(carac_gauss_at_an)
```

```
## [1] 0.3012139
```

Student distribution

```
# Function to evaluate the characteristic function of
# the Student distribution at some point of interest (here a_n)
```

```r
stud_carac_fun <- function(t1, n, nu) {

  sigma <- sqrt(nu/(nu-2))
  K4 <- 6/(nu-4)+3
  an <- compute_an(t1=t1, n=n, K3= mean(abs(scale(rt(Nrep, df = nu)))^3) )
  temp1 <- besselK(nu = nu/2, x = sqrt(nu)*abs(an)/sigma)
  temp2 <- (sqrt(nu)*abs(an)/sigma)^(nu/2) / (gamma(nu/2)*2^(nu/2-1))

  return(temp1 * temp2)
}

# Compute kappa_n for the Student distribution

stud_carac_fun(t1 = t1, n = 5000, nu = 8)
```

```
## [1] 0.4274447
```

```r
stud_carac_fun(t1 = t1, n = 5000, nu = 5)
```

```
## [1] 0.5487281
```