CS6476 | Computer Vision

Alexis Durocher - MSCS student at Georgia Tech Spring 2018

In [1]:

```
import cv2

from scipy import ndimage
from scipy import misc
import numpy as np
import math as mtugh
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from skimage.feature import peak_local_max
from skimage import io
from skimage.draw import circle, ellipse_perimeter
```

1. Detecting edge

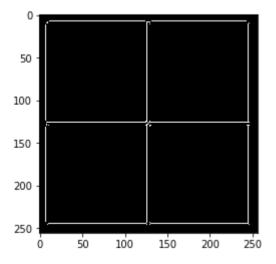
a.

In [2]:

```
psl_input0 = cv2.imread('./psl-input0.png')
# use classic magnitude estimation and Sobel size = 7
psl_input0_edges = cv2.Canny(psl_input0,100,200,1,7,True)

plt.imshow(psl_input0_edges, cmap="gray")
plt.show()

mpimg.imsave('./output/psl-l-a-l.png', psl_input0_edges, cmap="gray")
```



2. Hough method

a.

In [3]:

```
image = ps1_input0 edges
def hough transform(image):
   # Initialize empty accumulator (filled with 0)
   # 2 * diag length : for positive and negative max distance. 0 being diag len
   # 180 because theta from 0 to 180
   width = image.shape[1]
   height = image.shape[0]
   diag len = np.ceil(np.sqrt(width * width + height * height)) # max distance
 for rho = length of diag
   rho range = int(2*diag len)
   hough accu = np.zeros([rho range, 180])
    # get i and j indexes for all indexes
    j indexes, i indexes = np.nonzero(image)
   # Browsing into each pixel of edges picture
    for k in range(len(j indexes)):
        # getting indexes of edge
        i = i indexes[k]
        j = j indexes[k]
        # voting : for each value of theta
        for theta in range(0, 180):
            rho = int(np.round(i * np.cos(np.deg2rad(theta)) + j * np.sin(np.deg
2rad(theta))) + diag len)# positive index for rho
            hough accu[rho, theta] += 1
   return hough accu
hough accu = hough transform(image)
```

In [4]:

```
# Peak finding
def peak finding(hough accu, min distance = 45, max peaks = 6, title = 'Hough Tra
nsform', path = './trash.png'):
    coordinates = peak local max(hough accu, min distance=min distance,
                                 exclude border = False, num peaks =max peaks)
    loca maxs rho = coordinates[:, 0]
    loca_maxs_theta = coordinates[:, 1]
    plt.imshow(hough accu, cmap='gray',aspect='auto')
    plt.title(title)
    # Annotate local maximum
    for i in range(len(loca maxs rho)):
        plt.annotate('X',xy=(loca maxs theta[i],loca maxs rho[i]), arrowprops=di
ct(facecolor='yellow', shrink=0.05),)
    plt.savefig(path)
    plt.show()
    return loca_maxs_rho, loca_maxs_theta
```

In [5]:

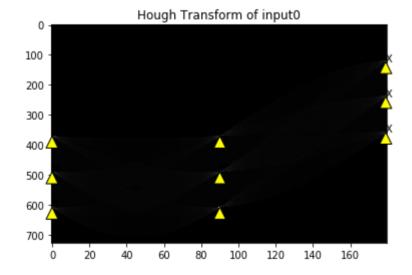
```
def draw line(image, loca maxs rho, loca maxs theta, rgb = (0,255,0)):
    image_copy = image.copy()
    width = image copy.shape[1]
    height = image copy.shape[0]
    diag len = np.ceil(np.sqrt(width * width + height * height))
    for j in range(len(loca maxs rho)):
        rho = loca maxs rho[j] - diag len
        theta = loca maxs theta[j]
        a = np.cos(np.deg2rad(theta))
        b = np.sin(np.deg2rad(theta))
        x1=int(a*rho - diag len*b)
        y1=int(b*rho + diag len*a)
        x2=int(a*rho + diag_len*b)
        y2=int(b*rho - diag len*a)
        #print(x1,y1,x2,y2)
        cv2.line(image\_copy, (x1,y1),(x2,y2), rgb, 3) # green line
        #print('Line {} | rho = {} theta = {}'.format(j,loca_maxs_rho[j], loca_m
axs theta[j]))
        plt.imshow(image_copy)
        plt.title('Detected Line')
    plt.show()
    return image copy
```

In [6]:

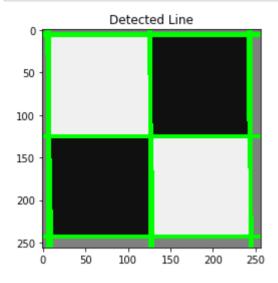
```
hough_accu = hough_transform(image)
```

In [7]:

loca_maxs_rho, loca_maxs_theta = peak_finding(hough_accu, max_peaks=50, title='H
ough Transform of input0', path = './output/ps1 2 a 1.png')



In [8]:



In [9]:

Justification:

The hough accumulator is a 2D array with dimension:

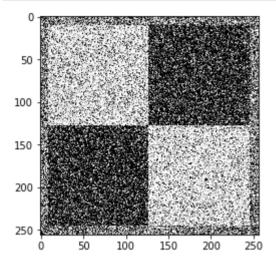
- . width of 180 \mid So we can iterate over each value of theta (it is a line detector so there is no need for iterating over 360 °). We keep the degree value rather than radian for simplying the interpretation of our results.
- . height of 2 times the size of the picture's diagonal | The highest value of rho can be the diagonal size and the lowest is diagnoal_size. Hence, to keep positive index for our accumulator we consider (accu value of rho) = (real value of rho) + diagonal size.

3. Using noised picture

a. Gaussian Filter

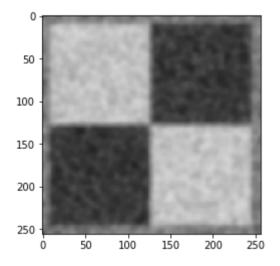
In [10]:

```
psl_input0_noised = cv2.imread('./psl-input0-noise.png')
plt.imshow(psl_input0_noised, cmap ='gray')
plt.show()
```



In [11]:

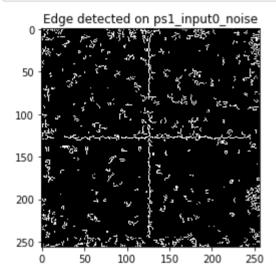
```
ps1_input0_gaussian = ndimage.filters.gaussian_filter(ps1_input0_noised, sigma=3
)
plt.imshow(ps1_input0_gaussian, cmap = 'gray')
plt.show()
misc.imsave('./output/ps1-3-a-1.png', ps1_input0_gaussian)
```

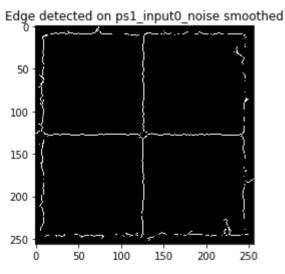


b. Edges detector

In [12]:

```
# to avoid noise, we need to set a treshold high enough.
# But not too high so we keep all the edge lines
ps1_input0_noised_edges = cv2.Canny(ps1_input0 noised,500,1100)
plt.imshow(ps1 input0 noised edges, cmap = 'gray')
plt.title('Edge detected on ps1 input0 noise')
plt.show()
misc.imsave('./output/ps1-3-b-1.png', ps1 input0 noised edges)
# easy to see the cross line ine the midle. But boundaries are hard to detect fr
om noise.
# Now applying on the smoothed version
ps1 input0 gaussian edges = cv2.Canny(ps1 input0 gaussian, 0, 80)
plt.imshow(ps1_input0_gaussian_edges, cmap='gray')
plt.title('Edge detected on ps1 input0 noise smoothed')
plt.show()
misc.imsave('./output/ps1-3-b-2.png', ps1_input0_gaussian_edges)
# no need to set a particular low treshold; noise is already relatively nicely a
# thanks to gaussian filter
```

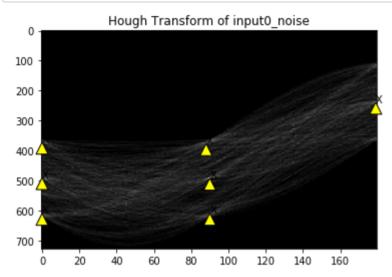


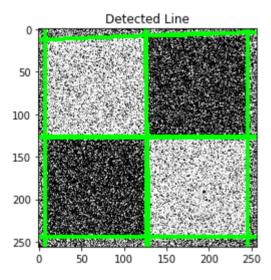


From the two results obtained, it is easy to notice the edges of the cross within the picture. However on the noised picture, boundaries are hard to distinguish from noise. We can expect that the line detector will probably have trouble finding thoses edges.

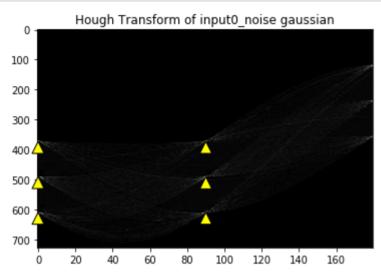
c. Hough Transform

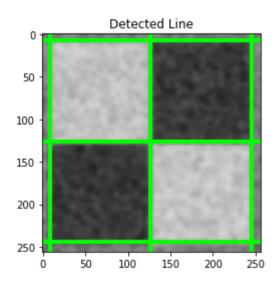
In [13]:





In [14]:





In [15]:

misc.imsave('./output/ps1-3-c-2.png', ps1_3_c_2)

<u>Explication</u> We need to set an appropriate low-treshold and high-treshold so we detect as much as possible the edges while trying to avoid the noise (which corresponds to high peaks in the whole picture)

The hough method is detecting many lines which do not correspond to edges on the original picture but are created because of the noise accumulation which altered the voting phase of the method.

The peak finding use local_maximas to detect peaks in the hough accumulator wich corresponds to the (supposingly) most likely lines on the original picture. But because of the noise, many peaks are considered as local_maximas where they shouldn't.

As we only want the real edges, we will treshold the peak_finding method by setting a number of peaks maximal so the method only return the higher local_maximas (max peaks = 6 by default)

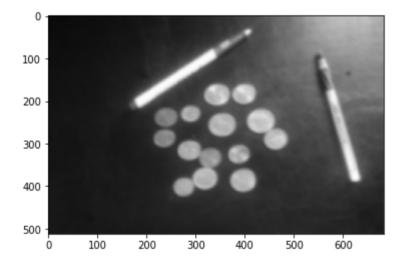
4. Second image: input 1

a. read and smooth

In [16]:

```
input1 = cv2.imread('./ps1-input1.jpg')
input1_gaussian = ndimage.filters.gaussian_filter(input1, sigma=3)
plt.imshow(input1_gaussian, cmap='gray', aspect= 'auto')
plt.show()

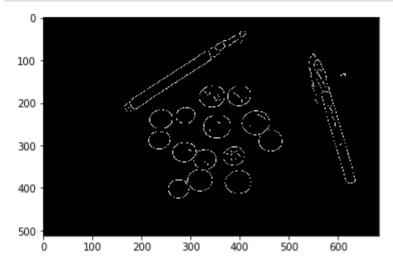
misc.imsave('./output/ps1-4-a-1.png', input1_gaussian)
```



b. edge detector

In [17]:

```
input1_gaussian_edges = cv2.Canny(input1_gaussian,30,40)
plt.imshow(input1_gaussian_edges, cmap = 'gray', aspect= 'auto')
plt.show()
misc.imsave('./output/ps1-4-b-1.png', input1_gaussian_edges)
# Treshold are set low, no noise, so all edges are considered okay.
```



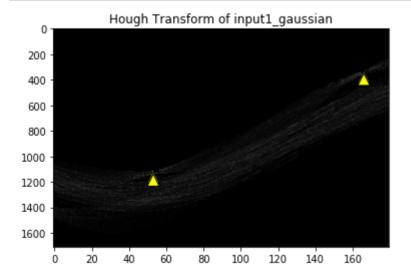
c. Hough Transform

In [18]:

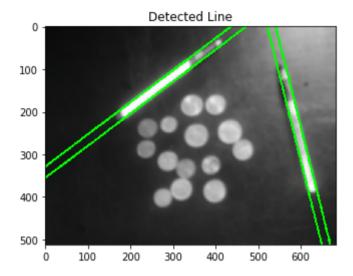
```
hough_accu = hough_transform(input1_gaussian_edges)
```

In [19]:

loca_maxs_rho, loca_maxs_theta= peak_finding(hough_accu, 10, max_peaks = 4, tit
le='Hough Transform of input1 gaussian', path = './output/ps1 4 c 1.png')



In [20]:



In [21]:

Explication

From the hough transform picture (b.) local maximas are very close almost same theta / rho). This is due to the fact that pens' edges are parallel and close lines.

Local maxima can be computed for a certain distance, here it is important to set a small distance for this computation. Indeed we don't want to consider one local maxima for both lines but two actual and distinct peaks. We set the value min distance = 10.

However, setting such a small ditance, to compute local max on, leads to the detection of many particular and unwanted smaller local maximas. We limit at 4 the number of peaks searched so we make sure to detect only the 4 higher peaks.

5. Circles detection

a. input1 circles detection

In [22]:

```
def hough transform circle(image, image edges, max rad):
    image = cv2.cvtColor(image, cv2.COLOR BGR2GRAY)
    # Initialize empty accumulator (filled with 0)
    height = image.shape[0]
    width = image.shape[1]
    hough accu = np.zeros([height, width, max rad])
    # get row and columns indexes for all indexes
    rr indexes, cc indexes = np.nonzero(image edges)
    # compute gradient xx and yy from original picture
    gxx = cv2.Sobel(image,cv2.CV 32FC1,1,0);
    gyy = cv2.Sobel(image,cv2.CV_32FC1,0,1);
    # compute gradient direction for each point
    theta values = cv2.phase(gxx,gyy,angleInDegrees=True);
    # Browsing into each pixel of edges picture
    for k in range(len(rr indexes)):
        # getting indexes of edge
        y = rr_indexes[k]
        x = cc indexes[k]
        theta = np.deg2rad(theta values[y,x])
        for radius in range(0, max_rad):
            a = x + radius * np.cos(theta)
            b = y + radius * np.sin(theta)
            if (b < height) & ( a \ge 0 ) & ( b \ge 0 ) & (a < width) :
                hough accu[b, a, radius] += 1
    return hough accu
```

In [23]:

In [24]:

```
def draw_circle(image, loca_maxs_a, loca_maxs_b, loca_maxs_radius, min_rad = 0,
title = 'Detected Circle'):
    image_copy = image.copy()
    for j in range(len(loca_maxs_a)):
        a = loca_maxs_a[j]
        b = loca_maxs_b[j]
        radius = loca_maxs_radius[j]
        if (radius > min_rad):
            cv2.circle(image_copy, (a, b), radius, (255,0,0), 2)
        #print('Cercle {} | b(row) = {} a(col) = {} radius = {}'.format(j,loca_maxs_b[j], loca_maxs_radius[j]))
        plt.imshow(image_copy)
        plt.title(title)

    plt.show()
    return image_copy
```

In [25]:

```
hough_accu_circle = hough_transform_circle(input1_gaussian, input1_gaussian_edge
s, 100)
```

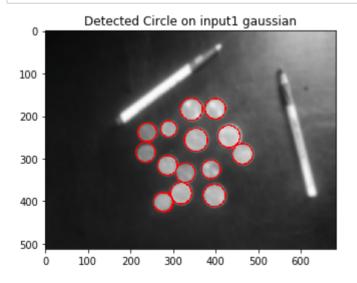
/Users/alexisdurocher/anaconda3/lib/python3.6/site-packages/ipykerne l/__main__.py:28: VisibleDeprecationWarning: using a non-integer num ber instead of an integer will result in an error in the future

In [26]:

```
loca_maxs_a, loca_maxs_b, loca_maxs_radius = peak_finding_circle(
    hough_accu_circle, max_peaks = 15, title='Hough Transform of input1_gaussia
n')
```

In [27]:

ps1_5_a_2 = draw_circle(input1_gaussian, loca_maxs_a, loca_maxs_b, loca_maxs_rad
ius, title = "Detected Circle on input1 gaussian")



In [28]:

```
misc.imsave('./output/ps1-5-a-1.png', input1_gaussian_edges)
misc.imsave('./output/ps1-5-a-2.png', ps1_5_a_2)
```

Explication

The hough method to detect the circles first compute the picture's gradient on x and y before calculating the phase between each gradient value. Hence we have, for each pixel, the corresponding value of theta that we are lookig for to detect the circle center.

This save computation time because there is no more need for iterating through all the theta values.

Rmk: this method includes also the possibility to pass a maximum value for limiting the values of radius tested. We will use it later in this assignment.

Here, local maximas are nicely detected, but we restricted the peaks to the 15 highest in order to see the circles corresponding to the coins detected, and only them.

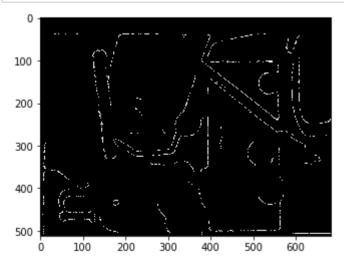
We have used a gaussian filter with a sigma = 3. This helped reducing the noise and we can see that our Canny filter doesn't need very high tresholds to provides a good estimation of the edges on the pictures, which leads to good voting phase in the accumulator and a great detection of our circles parameters.

6. Realistic image - clutter

a. Line Finder

In [29]:

```
input2 = cv2.imread('./ps1-input2.jpg')
input2_gaussian = ndimage.filters.gaussian_filter(input2, sigma = 5)
input2_edges = cv2.Canny(input2_gaussian, 40, 60, 1, 3, True)
plt.imshow(input2_edges, cmap= 'gray')
plt.show()
```

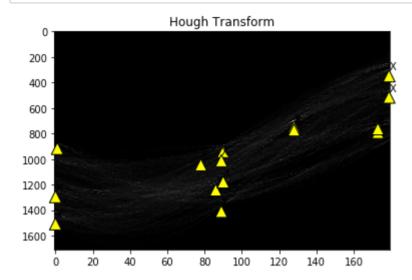


In [30]:

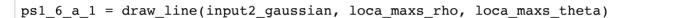
```
hough_accu_6a = hough_transform(input2_edges)
```

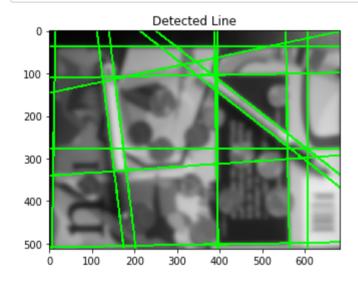
In [31]:

```
loca_maxs_rho, loca_maxs_theta = peak_finding(hough_accu_6a, 20, max_peaks = 15)
```



In [32]:





In [33]:

b. Why other lines?

While looking at the edges detector, we can see that many straight edges are detected. Indeed, in the original picture, some straight line are detected on the 'magazine' where pens are. The Canny edge detector doesn't differentiate the edges of real objects from linear pattern drawn or simply present on the picture.

Here, to emphasize the probleme the number of peaks is not restricted (max = 15) so we can clearly see the problem we face (many lines detected).

c. Solutions

To solve this issue, we can:

- . 1 alter the peak finding or canny edge tresholdso we only detect stronger edges.
- . 2 check for a minium length of the lines
- . 3 check for eventual parallel lines (that)

It turns out that, stronger edges, in our case, doesn't necessarily correspond to the pens' boundaries. So the first solution could not really be used. The second option, won't be of great help neither, because, many 'lines' have almost the same size of our pens (the accumulator doesnt make the difference between many small cut lines or one big lines).

However, the third solution is interesting because, our pens have parallel edges so we could based our choice on that criteria: For each line found, is ther an other line with the same slope (filter on theta relation)?

Also we should consider the 'width' of the pen. Indde, we can notice that lines corresponding of pens' edges are closer than other detected edges. Hence we can also apply an other 'filter' on top of the previous parallel filter that will keep lines parallel within a certain range (filter on rho relation). Rmk: in our case, we don't take into consideration the scale. But the width of pens on the picture depends on the level of zoom we are. This could be solved by considering a factor of zoom as a parameter.

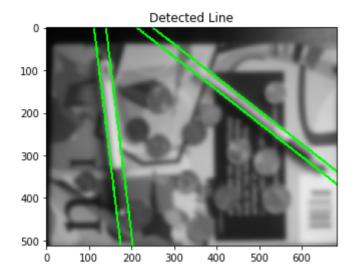
In [34]:

```
def filter parallel lines(loca maxs rho, loca maxs theta):
    new loca maxs_rho = np.array([])
    new loca maxs theta = np.array([])
    for theta, rho in zip(loca maxs theta, loca maxs rho):
            # looking for parallel lines
            (indx,) = np.where((loca maxs theta == theta))
            if (indx.size > 1):
                new loca maxs rho = np.append(new loca maxs rho, rho)
                new loca maxs theta = np.append(new loca maxs theta, theta)
    return new loca maxs rho, new loca maxs theta
def filter close lines(loca max rho, loca maxs theta, max dist):
    new loca maxs rho = np.array([])
    new loca maxs theta = np.array([])
    for theta, rho in zip(loca_maxs_theta, loca_maxs_rho):
            # looking for close line
            (indx,) = np.where((np.abs((loca maxs rho - rho)) < max dist))</pre>
            if (indx.size > 1):
                new loca maxs rho = np.append(new loca maxs rho, rho)
                new loca maxs theta = np.append(new loca maxs theta, theta)
    return new loca maxs rho, new loca maxs theta
```

In [35]:

In [36]:

```
psl_6_c_1 = draw_line(input2_gaussian, loca_maxs_rho, loca_maxs_theta)
```



In [37]:

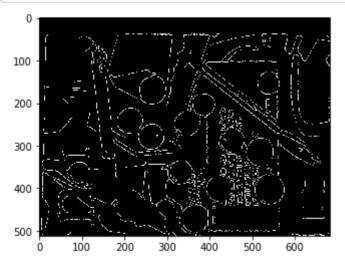
misc.imsave('./output/ps1-6-c-1.png', ps1 6 c 1)

7. Finding circles on realistic images

a. First detection

In [38]:

```
input2 = cv2.imread('./ps1-input2.jpg')
input2_gaussian = ndimage.filters.gaussian_filter(input2, sigma = 2)
input2_edges = cv2.Canny(input2_gaussian, 13000, 19000, 1, 7, True)
plt.imshow(input2_edges, cmap= 'gray')
plt.show()
```



In [39]:

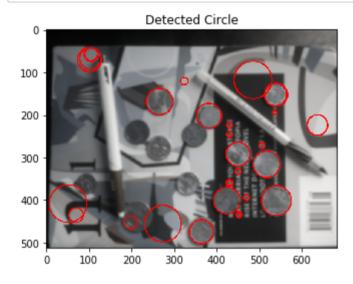
```
hough_accu_6a_circle = hough_transform_circle(input2_gaussian, input2_edges, max
_rad= 100)
```

/Users/alexisdurocher/anaconda3/lib/python3.6/site-packages/ipykerne l/__main__.py:28: VisibleDeprecationWarning: using a non-integer num ber instead of an integer will result in an error in the future

In [40]:

In [41]:

ps1_6_c_0 = draw_circle(input2_gaussian, loca_maxs_a, loca_maxs_b, loca_maxs_rad
ius)



b. False alarms

We can see that there are many false positives and false-negatives. Curved line in the picture are also considered as circles, and some coins are note detected.

By looking at the original edges detector picture, we can notice that some curved line edges appear while some coins' edges are not clearly detected.

1. Constraint on range of radius

We could apply a filter that would keep only the circles within a min_range and max_range. Since the max_range is already a parameter of our hough_accumulator_circle function, we just have to implement the min_range filter.

In [42]:

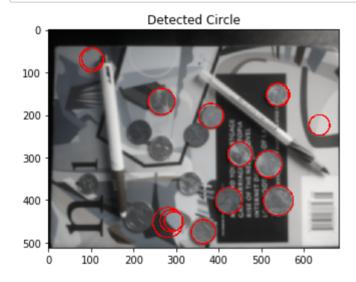
```
hough_accu_6a_circle = hough_transform_circle(input2_gaussian, input2_edges, max
_rad= 40)
```

/Users/alexisdurocher/anaconda3/lib/python3.6/site-packages/ipykerne l/__main__.py:28: VisibleDeprecationWarning: using a non-integer num ber instead of an integer will result in an error in the future

In [43]:

In [44]:

```
ps1_6_c_1 = draw_circle(input2_gaussian, loca_maxs_a, loca_maxs_b, loca_maxs_rad
ius, min_rad = 20)
```

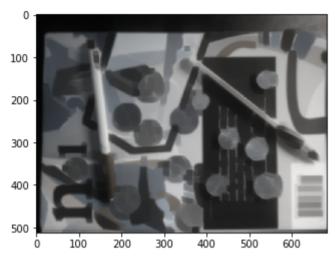


2. Constraint on color variance within each circle

We can also notice that we are here, trying to detect coins that all have consistent (constant) colored pixels within their edges. Hence we could use this criteria to remove non-coins detected circles. We will treshold the standard deviation of all the pixels values within each circle.

In [45]:

```
kernel = np.ones((5,5), np.uint8)
# applying erosion to flat the color to detect coins with similar color.
input2_li_erode = cv2.erode(input2_gaussian, kernel, iterations = 2)
plt.imshow(input2_li_erode)
plt.show()
```



In [46]:

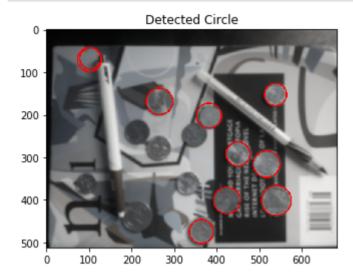
```
def filter_homogenous_color_circle(img, loca_maxs_a, loca_maxs_b, loca_maxs_radi
us, std_treshold):
    new_as, new_bs, new_rads = np.array([], dtype = int), np.array([], dtype = i
nt), np.array([], dtype = int)
    for a, b, radius in zip(loca_maxs_a, loca_maxs_b, loca_maxs_radius):
        rr, cc = circle(b, a, radius)
        stds = np.std(img[rr,cc])
        if (stds < std_treshold):
            new_as = np.append(new_as, int(a))
            new_bs = np.append(new_bs, int(b))
            new_rads = np.append(new_rads, int(radius))
    return new_as, new_bs, new_rads</pre>
```

In [47]:

loca_maxs_a_ho, loca_maxs_b_ho, loca_maxs_radius_ho = filter_homogenous_color_ci
rcle(input2_li_erode,loca_maxs_a, loca_maxs_b, loca_maxs_radius, std_treshold =
30)

In [48]:

ps1_7_a_1 = draw_circle(input2_gaussian, loca_maxs_a_ho, loca_maxs_b_ho, loca_ma
xs_radius_ho, min_rad = 20)



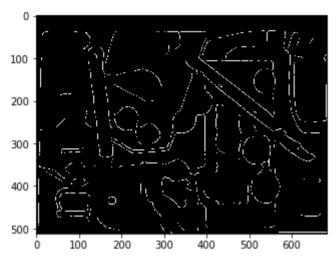
We can see that the bottom circle with a variance of color relatively high (from black to white) has been filtered. Unfortunately it didnt remove the circle detected at the far right of the picture as the variance of the pixel values remains really low.

3. Constraint on line influence

Finally, the edge detector makes us realize that some circles are detected because of curved lines. We will try to remove the edges corresponding to lines (by using the hough method for detecting lines) and see if it helps keeping only the circles.

In [49]:

```
input2 = cv2.imread('./ps1-input2.jpg')
input2_gaussian = ndimage.filters.gaussian_filter(input2, sigma = 5)
# aaplying dilation here to exagerate the shapes of the line
input2_li_dilate = cv2.dilate(input2_gaussian, kernel, iterations = 1)
input2_edges_dil = cv2.Canny(input2_li_dilate, 8000, 10000, 1, 7, True)
plt.imshow(input2_edges_dil, cmap= 'gray')
plt.show()
```

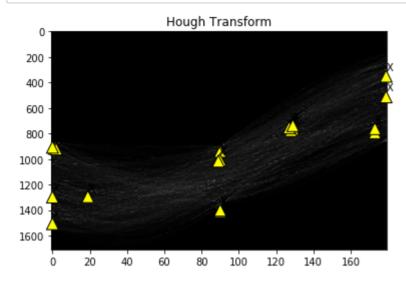


In [50]:

hough_accu_7 = hough_transform(input2_edges_dil)

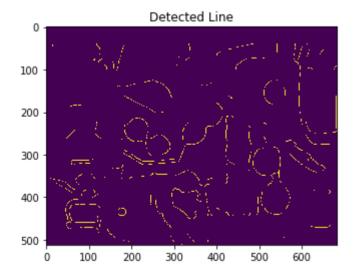
In [51]:

loca_maxs_rho, loca_maxs_theta = peak_finding(hough_accu_7, 5, max_peaks = 20)



In [52]:

 $input2_edges_no_lines = draw_line(input2_edges_dil, loca_maxs_rho, loca_maxs_the ta, rgb = (0,0,0))$



In [53]:

lets try now on this new edges

In [54]:

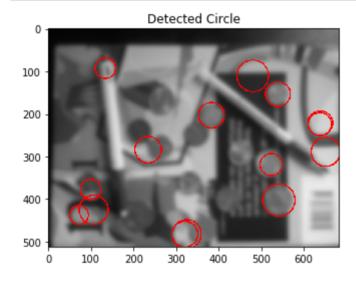
```
hough_accu_7_circle_nol = hough_transform_circle(input2_gaussian, input2_edges_n
o_lines, max_rad= 40)
```

/Users/alexisdurocher/anaconda3/lib/python3.6/site-packages/ipykerne l/__main__.py:28: VisibleDeprecationWarning: using a non-integer num ber instead of an integer will result in an error in the future

In [55]:

In [56]:

```
no_lines_input2_circles = draw_circle(input2_gaussian, loca_maxs_a_nol, loca_max
s_b_nol, loca_maxs_radius_nol, min_rad = 20)
```



We can see that circles in the far right are still detected because of the curved line. Hence this method didn't help, here.

We will keep the 2 methods (constraint on radius and color invariance) above, considering that there none of them provides 100% of accuracy or detection (false positives and false negatives).

```
In [57]:
```

```
misc.imsave('./output/ps1-7-a-1.png', ps1_7_a_1)
```

In [58]:

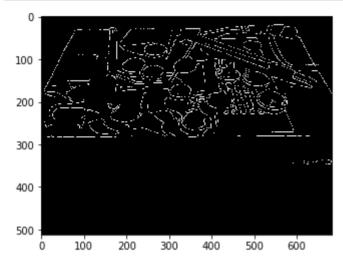
```
# imsave homogenous circles
```

8. Distortion sensitivity

a. Lines & circles detection

In [59]:

```
input3 = cv2.imread('ps1-input3.jpg')
input3_gaussian = ndimage.filters.gaussian_filter(input3, sigma = 2)
input3_edges = cv2.Canny(input3_gaussian, 10, 60, 1, 3, True)
plt.imshow(input3_edges, cmap = 'gray')
plt.show()
```

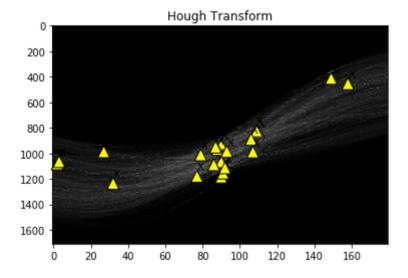


In [60]:

```
hough accu 8a = hough transform(input3 edges)
```

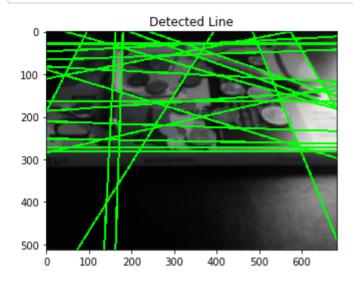
In [62]:

loca_maxs_rho_8, loca_maxs_theta_8 = peak_finding(hough_accu_8a, min_distance= 1
0, max peaks=22)



In [63]:

input3_gaussian_with_line = draw_line(input3_gaussian, loca_maxs_rho_8, loca_max
s_theta_8)



As we can see, to detect the 4 lines edges of our 2 pens, we had to detect 22 lines in total. Many lines have been detected for the same reasons as before (lines on the picture, or edges of magazines).

Also, because of the distortion (angle of picture is inclined), many edges appear to be on the same line because the picture is taken on the side.

In [64]:

hough_accu_8a_circle = hough_transform_circle(input3_gaussian, input3_edges, max
_rad= 40)

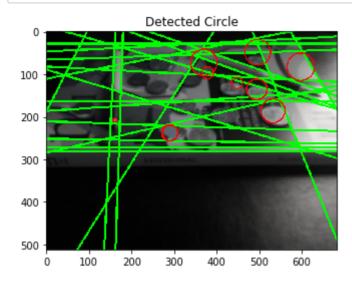
/Users/alexisdurocher/anaconda3/lib/python3.6/site-packages/ipykerne l/__main__.py:28: VisibleDeprecationWarning: using a non-integer num ber instead of an integer will result in an error in the future

In [65]:

loca_maxs_a_8, loca_maxs_b_8, loca_maxs_radius_8 = peak_finding_circle(hough_acc
u_8a_circle,
 min_dist = 20)

In [66]:

ps1_8_a_1 = draw_circle(input3_gaussian_with_line, loca_maxs_a_8, loca_maxs_b_8,
loca_maxs_radius_8)



In [67]:

misc.imsave('./output/ps1-8-a-1.png', ps1_8_a_1)

b. Ellipse?

Circles are detected but doesn't correspond to the coins on the pictures. Because of the distortion circle shapes have become ellipse shapes. A solution would be to build an hough transform that detects ellipse rather than circles.

This is a much more complexe alogithm as it now have to find 5 parameters insted of 3 (for circles) and 2(for lines).

We will use the algorithm discovered by: Xie, Yonghong, and Qiang Ji. "A new efficient ellipse detection method." Pattern Recognition, 2002. Proceedings. 16th International Conference on. Vol. 2. IEEE, 2002.

Parameters are: (x0, y0) for the center, alpha for the orientation, (a,b) for the major and minor axes.

In [68]:

```
def compute ellipse params(x1, y1, x2, y2):
    x0 = (x1 + x2)/2
    y0 = (y1 + y2)/2
    alpha = mt.atan((y2 - y1)/(x2 - x1))
    return int(x0), int(y0), int(alpha)
def compute ellipse minor axis(a, d, f):
    theta = mt.acos(
        (mt.pow(a,2) + mt.pow(d,2) - mt.pow(f,2)) /
        (2 * a * d)
    b = mt.sqrt(
        (mt.pow(a,2) * mt.pow(d,2) * mt.pow(mt.sin(theta),2)) /
        (mt.pow(a,2) - mt.pow(d,2) * mt.pow(mt.cos(theta),2))
    return int(b)
def hough_transform_ellispe(image, min_pair_dist = 5, min_treshold = 1):
    # empty 1D accumulator
    width = image.shape[1]
    height = image.shape[0]
    diag len = np.ceil(np.sqrt(width * width + height * height)) # max distance
 for rho = length of diag
    ellipses = np.array([])
    hough accu = np.zeros([diag len])
    # get i and j indexes for all indexes
    y indexes, x indexes = np.nonzero(image)
    # for each 1st pair (x1, y1)
    for x1 in x_indexes:
        for y1 in y indexes:
            if (image[y1, x1] > 0):
                # for each 2nd pair (x2, y2):
                for x2 in x indexes:
                    for y2 in y_indexes:
                        if (image[y2, x2] > 0):
                            if ((x2 != x1) & (y2 != y1)):
                                # euclidian distance between pair1 and pair2
                                dist p1 p2 = np.linalg.norm(
                                    np.array((x1, y1)) - np.array((x2, y2)))
                                if (dist_p1_p2 > min_pair_dist) :
                                    x0, y0, alpha = compute_ellipse_params(x1, y
1, x2, y2)
                                    a = int(dist p1 p2/2) # major axis a
                                    # for each 3rd pair (x, y):
                                    for x in x indexes:
                                         for y in y_indexes:
                                             if (image[y, x] > 0):
                                                 if (((x2 != x) \& (y2 != y)) \&
                                                     ((x1 != x) & (y1 != y))):
                                                     # euclidian distance between
pair1 and pair2
                                                     dist p0 p3 = np.linalg.norm(
                                                         np.array((x0, y0)) - np.
```

```
array((x, y)))
                                                     dist p0 p1 = np.linalg.norm(
                                                          np.array((x0, y0)) - np.
array((x1, y1)))
                                                      dist p0 p2 = np.linalg.norm(
                                                          np.array((x0, y0)) - np.
array((x2, y2)))
                                                      if (
                                                          (dist p0 p3 > min pair d
ist) &
                                                          ((dist p0 p3 < dist p0 p
1)
                                                           (dist p0 p3 < dist p0 p
2))):
                                                          d = dist p0 p3
                                                          f = dist p0 p2
                                                          b = compute ellipse mino
r axis(a, d, f)
                                                          ## increment hough accu
for this b
                                                          hough accu[b] += 1
                                     # find max value in houg accu
                                     max b = np.argmax(hough accu)
                                     votes for b = hough accu[max b]
                                     if (votes for b > min treshold):
                                         major axis = 2*a
                                         minor_axis = max_b
                                         ellipses = np.append(ellipses,
                                                               [x0,
                                                                y0,
                                                                major axis,
                                                                minor axis,
                                                                alpha])
                                         # remove pixels of ellipse in the edges
picture
                                         rr, cc = ellipse perimeter(y0,
                                                                     int(minor axi
s/2),
                                                                     int(major axi
s/2),
                                                                     orientation =
alpha)
                                         y_indexes, x_indexes = np.nonzero(image)
                                         print(len(y indexes))
                                         image[rr, cc] = 0
                                         y indexes, x indexes = np.nonzero(image)
                                         print(len(y indexes))
                                         cv2.ellipse(input3_gaussian_cop,
                                                  (ellipse[0], ellipse[1]),
                                                  (ellipse[2], ellipse[3]),
                                                 ellipse[4],0,360,255,1)
                                         plt.imshow(input3 gaussian cop, cmap= 'g
ray')
                                         plt.show()
                                     hough_accu = np.zeros([diag_len])
    return ellipses
```

For computation complexity reason, it was impossible (too long) to see the result on the input3. The algorithm can be tested on targeted picture.

In []:

```
input3 = cv2.imread('ellipse.png')
input3_gaussian = ndimage.filters.gaussian_filter(input3, sigma = 2)
input3_gaussian_cop = input3_gaussian.copy()
input3_edges = cv2.Canny(input3_gaussian, 10, 60, 1, 3, True)
#plt.imshow(input3_edges, cmap = 'gray')
#plt.show()
```

In []:

```
#ellipses = hough_transform_ellispe(input3_edges, min_pair_dist= 49, min_treshol
d= 500)
```