Solving unfairness in fair allocations - draft

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1 Introduction

Allocating resources in a fair manner is a difficult issue. In the bibliography, we can read about multiple fairness allocation criteria, when an allocation meet a certain criterion we say that it is a fair allocation. There are multiple criteria like envy-freeness and proportionality and some of their variations. In the case of indivisible resources, we can argue that there may not be a way to allocate resources in an ultimate fair way for everyone, since the indivisibility issue does not allow us to divide the items evenly. The fairness criteria can satisfy the agents in a certain way but I believe the fairness issue is not yet resolved.

2 The Setting

2.1 Borda preferences

In my setting, I will examine a case of the popular fair division problem where two conditions are met. First the number of agents is lower than the number of indivisible resources, which is generally true. And that each agents rates each item based on Borda rule. So for example, if we have 5 items, each agent will assign "1", "2", "3", "4", "5" to every resource. This is a brave assumption to make, the agents usually do not have these kind of preferences but we may think an application to it. The main reason, I made that assumption is for research purposes, it is easier to have an equal total number of points for every agent to assign.

2.2 Random normalized preferences

In this setting, each agent has random preferences between the resources and these preferences are normalized so they sum to 1 for each agent.

2.3 Round Robin Protocol

In each round, according to some arbitrarily fixed given ordering, each agent chooses her most valuable good among the available ones (that is, from the set of goods that have not been chosen by the agent's turn). It is not hard to see that Round-Robin leads to an EF1 allocation. [1]

3 Simulations

3.1 Process

I run this protocol for specific preference matrices and counted the average preference satisfied of each agent, unrelated to the ordering. The results indicate that the average difference between the agents is significant. We can still say that every allocation that came out of this protocol is fair based on EF1 but this would make certain agents more rewarded than others on average, which is unfair in a sense. I believe this unfairness originates in the dynamics of preferences declared by the agents. This is true for both settings described above.
If we want to be completely fair, we should not have any way to point an agent that we would prefer to be in his position. [2]

3.2 Result

$$\begin{bmatrix} 7 & 3 & 8 & 6 & 5 & 10 & 2 & 4 & 9 & 1 \\ 9 & 4 & 3 & 2 & 8 & 5 & 7 & 1 & 6 & 10 \\ 4 & 2 & 9 & 7 & 3 & 10 & 8 & 6 & 1 & 5 \\ 1 & 3 & 4 & 9 & 8 & 6 & 5 & 10 & 7 & 2 \\ 1 & 4 & 8 & 10 & 9 & 3 & 6 & 7 & 5 & 2 \end{bmatrix}$$

Figure 1: Preference matrix

Agent	Score
1	16.04299
2	18.11321
3	16.73399
4	16.16819
5	17.15921

Figure 2: Agent scores

This was run for $100,000$ simulations on the same matrix with random ordering each time. This issue remains and in the same quantity, for more simulations or for other random matrices. We can notice that each agent has a significant different average satisfaction and there is clearly an advantage to one of the agents.

4 Resolving the unfairness

4.1 Fair Protocol and Fairness in Expectation

We usually refer the term of "fairness" to allocations, but the observation above leads us to define it for protocol. A fair protocol is a protocol that will satisfy every agent the same on average. We can say that the protocol is fair in expectation. This definition applies only to normalized agent preferences, we generally want the agents to have an equal and same total preference distributed among the resources.

4.2 α-Fair Protocol and Fairness in Expectation

We can define an α-Fair Protocol as a Protocol that will be unfair at most $\alpha\%$. That means that the difference of the best agent and worst agent in expectation will be at most $\alpha\%$ of their average satisfied preference.

4.2.1 The simplest fair protocol

It is not hard to see that the simplest fair protocol would be to roll a n-sided dice and allocate all the resources to one agent, of course this is not fair at all. Using this example protocol, it occurs a philosophical question of fairness. In most cases, a realization of an allocation is unfair but the position of every agent before that is fair as much a fair coin is fair. The question is where do we draw the line of allocating and gambling.

4.3 Abstract Goal

I believe the fairness in expectation can be a useful term that could reinforce allocation's fairness criteria. The goal is not to truly unsolve the unfairness but use the right probabilities to choose a fair allocation between the possible fair allocations. We can think that we will use a fairness criterion to make sure that the allocation is enough fair and then use chance to decide which truly unfair fair allocation we will realize. A fair allocation could be reinforced if it is a product of a fair protocol.

4.4 Problems

It is very hard to prove these definitions, since we will have to ensure that for any preference matrix the protocol remains fair, but we may be able to create a method/network that given any preference matrix we can learn or estimate those probabilities of choosing the right allocation.

5 Bibliography

References

[1] Georgios Amanatidis et al. *Fair Division of Indivisible Goods: A Survey*. 2022. arXiv: 2202.07551 [cs.GT].

[2] Alexis Georgiou. *Fairness Division Experiment*. https://github.com/AlexisGeorgiou/fairness_division_experiment. 2023.