Model v.1

May 7, 2021

1 Gaussian Model

1.1 Swaption pricing

The swaption price at time 0, V_0, is given by the following expression

$$\begin{aligned} V_0 &= \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^{T_0} r(s)\, \mathrm{d}s} Swap(T_0)^+\right] \\ &= P(0,T_0)\ \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^{T_0} x(s)\, \mathrm{d}s}\left(1 - P(T_0,T_N,x(T_0),y(T_0)) - K\sum_{n=0}^{N-1}\tau_n P(T_0,T_{n+1},x(T_0),y(T_0))\right)^+\right] \end{aligned}$$

And

$$\forall n \in [|0,N|], \quad P(T_0,T_n,x,y) = \frac{P(0,T_n)}{P(0,T_0)} e^{-G(T_0,T_N)x - \frac{1}{2}G(T_0,T_N)^2 y}$$

One will use a Monte Carlo estimator of the expectation above by using M independent simulations S_m so that

$$\hat{V_0} = \frac{1}{M}\sum_{m=1}^{M} S_m$$

$$\forall m,\ S_m = e^{I_m}\ \Big(P(0,T_0) - P(0,T_0)P(T_0,T_n,x_m,y_m) - KP(0,T_0)A(T_0,x_m,y_m)\Big)^+$$

Where we have noted x_m, y_m, I_m the m-th Euler simulation of $x(T_0), y(T_0), I(T_0)$

1.2 Euler discretization

We aim to discretize the following differential system with an Euler scheme

$$\begin{cases} \mathrm{d}x(t) &= (y(t) - \chi(t)x(t))\,\mathrm{d}t\ +\ \sigma_r(t)\,\mathrm{d}Wt \\ \mathrm{d}y(t) &= (\sigma_r(t)^2 - 2\chi(t)y(t))\,\mathrm{d}t \end{cases}$$

Constant mean reversion and linear local volatility: First, let for simplicity

$$\chi(t) = \chi \in \mathbb{R}$$

and

$$\sigma_r(t) = \sigma_r(t,x(t),y(t)) = \lambda(a + bx(t))$$

One defines a discretized time interval $0 = t_0 < t_1 < ... < t_N = T_0$ and $\forall i \in [|0,N-1|],\ \Delta_i = t_{i+1} - t_i$

We will simulate $x(T_O)$ and $y(T_O)$ thanks to the scheme:

$$\begin{cases} x_{t_{i+1}} &= x_{t_i} + (y_{t_i} - \chi x_{t_i})\,\Delta_i \;+\; \lambda(a + bx_{t_i})\,\sqrt{\Delta_i} Z_i \qquad , Z_i \sim \mathcal{N}(0,1) \\ y_{t_{i+1}} &= y_{t_i} + (\lambda^2 (a + bx_{t_i})^2 - 2\chi y_{t_i})\,\Delta_i \end{cases}$$

What's more, one has to simulate the variable

$$I(T_0) = -\int_0^{T_0} x(s)\,\mathrm{d}s$$

As $\mathrm{d}I(t) = -x(t)\,\mathrm{d}t$, in line with the results above, one defines

$$I_{t_{i+1}} = I_{t_i} - x_{t_i}\,\Delta_i$$

Remark: $y(t) = \int_0^t e^{-2\chi(t-u)}(a + bx(u))^2\,\mathrm{d}u$ is a path dependent integral and could be calculated with the values $\{x_{t_i},\, t_i < t\}$

1.3 Simulation

In order to price swaptions, one needs the values of the zero coupon bonds for each maturity $\Big(P(0,T_i)\Big)_{i=0,N}$. One can calculate it with

$$P(0,T_i) = e^{-\int_0^{T_i} f(0,u)\,\mathrm{d}u}$$

, but one needs again **the initial forward curve** $t \rightarrow f(0,t)$

To do some calculations and to have a first simple approach, I have chosen to fixe $P(0,T) = e^{-rT}$ even if it's completely wrong in our model because the short rate is stochastic.

[1]:
```
#modules

import numpy as np
import math
from random import *
from time import time
import scipy.stats as stats
import scipy.optimize as opt
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
np.random.seed(10)
```

[7]:
```
# One set the parameters
chi = 1
lmbda = 1
a = 1
b = 0
K = 1
N = 20
M = 10000
```

```
r = 1
maturities = np.arange(0.5,5)      #tenor structure of the underlying swap T0=1,2.
 ↪..6=TN
T0 = maturities[0]
bonds = [np.exp(-r*m) for m in maturities]  #P(0,T_i) fixed to exp(-T_i) for␣
 ↪tests
```

[3]:
```
#Simulation of the diffusions x,y,I
#delta is the time grid of the discretization

def simul(lmbda,a,b,chi,delta):
    N = delta.size
    (x,y,I) =(0,0,0)
    for i in range(N-1):
        delta_i = delta[i+1]-delta[i]
        sigma_i = lmbda*(a+b*x)
        Z = float(np.random.standard_normal(1))
        I = I - x*delta_i
        x = x +(y-chi*x)*delta_i + sigma_i*math.sqrt(delta_i)*Z
        y = y + (sigma_i**2 -2*chi*y)*delta_i
    return [x,y,I]
```

[8]:
```
#test
delta = np.linspace(0,T0,N+1)
s = simul(lmbda,a,b,chi,delta)
(x,y,I) = (s[0],s[1],s[2])
print(x,y,I)
```

```
0.6559682936372841 0.32075703879572887 -0.14451091498710805
```

[9]:
```
#function G
def G(t,T,chi):
    return (1-np.exp(chi*(t-T)))/chi

#payoff of the swaption at T0
def payoff_swaption(maturities,bonds,x,y,chi,K):
    T0 = maturities[0]                                  #maturities = [T0,...,TN]
    nb_maturities = maturities.size
    A = 0
    #calculate the annuity A
    for n in range(nb_maturities-1):
        g = G(T0,maturities[n+1],chi)
        A += (maturities[n+1]-maturities[n])*bonds[n+1]*np.exp(-1*g*x-0.5*y*g**2)
    g = G(T0,maturities[-1],chi)
    swap = bonds[0] - bonds[-1]*np.exp(-1*g*x-0.5*y*g**2) - K*A
    if swap>0:
        return swap
```

```
    else:
        return 0
```

[10]:
```
#test

payoff_swaption(maturities,bonds,x,y,chi,K)
```

[10]: 0.4029888871629175

[11]:
```
#pricing of the swaption by Monte Carlo algorithm
#N is the parameter of discretization in the Euler scheme
#M is the number of simulations in the Monte Carlo estimation
#K is the strike of teh swaption

def swaption(M,N,T0,K,lmbda,a,b,chi,bonds,maturities):
    t1 = time()
    delta = np.linspace(0,T0,N+1)
    Monte_Carlo = 0
    for m in range(M):
        sim = simul(lmbda,a,b,chi,delta)
        (x,y,I) = (sim[0],sim[1],sim[2])
        Monte_Carlo += np.exp(I)*payoff_swaption(maturities,bonds,x,y,chi,K)
    t2 = time()
    print("Execution time: ",t2-t1, "sec")
    return Monte_Carlo/M
```

[12]:
```
#tests
swaption(M,N,T0,K,lmbda,a,b,chi,bonds,maturities)
```

```
Execution time:  4.608523607254028 sec
```

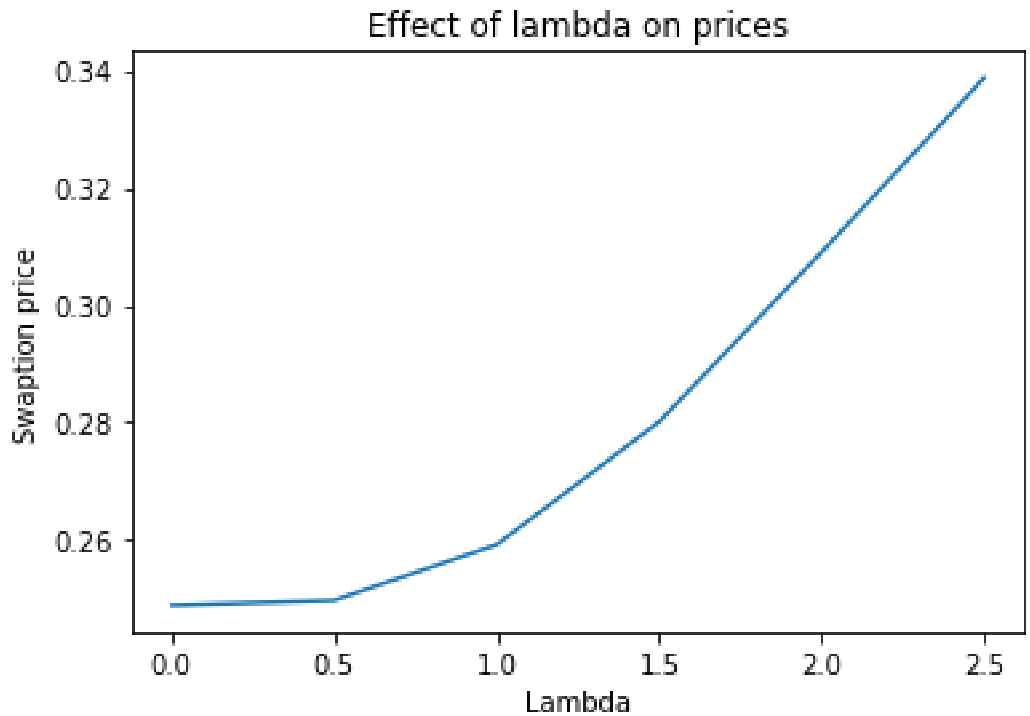
[12]: 0.2590558435912049

1.4 Effect of volatility's parameters

[13]:
```
lmbdas = np.arange(0,3,0.5)

swaptions1 = [swaption(M,N,T0,K,l,a,b,chi,bonds,maturities) for l in lmbdas]
plt.figure()
plt.xlabel('Lambda')
plt.ylabel('Swaption price')
plt.title('Effect of lambda on prices')
plt.plot(lmbdas,swaptions1)
plt.show()
```

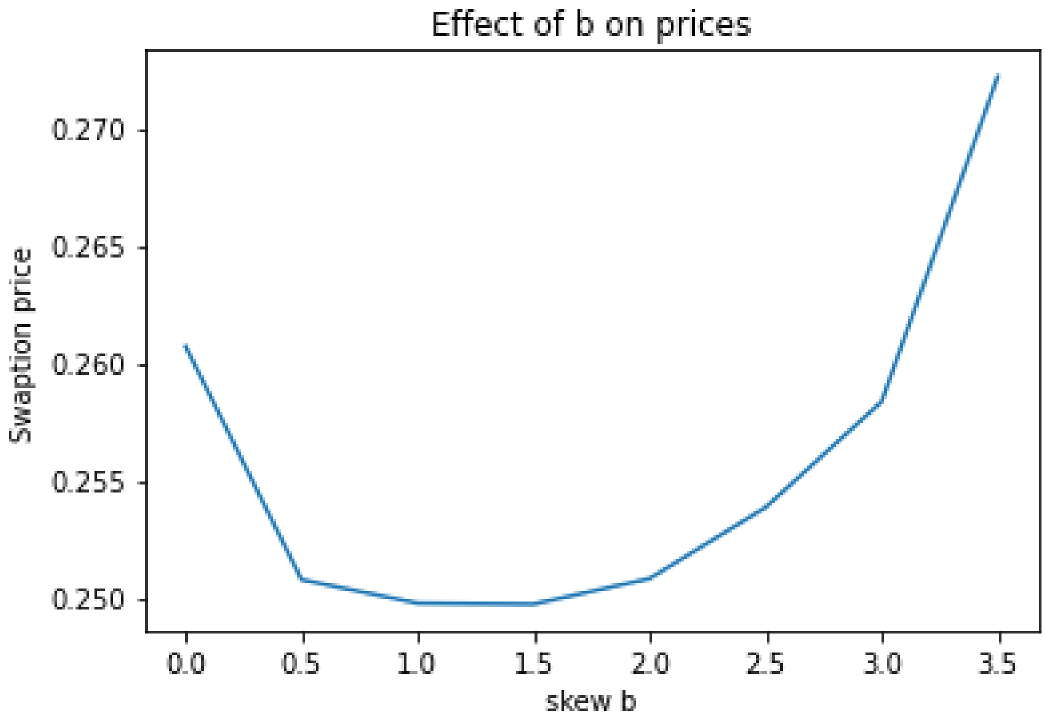
```
Execution time:  4.672131538391113 sec
Execution time:  4.385974645614624 sec
```

```
Execution time:  4.460890769958496 sec
Execution time:  4.1956212520599365 sec
Execution time:  4.113009214401245 sec
Execution time:  4.347233295440674 sec
```



[11]:
```
b_val = np.arange(0,4,0.5)

swaptions2 = [swaption(M,N,T0,K,lmbda,a,b,chi,bonds,maturities) for b in b_val]
plt.figure()
plt.xlabel('skew b')
plt.ylabel('Swaption price')
plt.title('Effect of b on prices')
plt.plot(b_val,swaptions2)
plt.show()
```

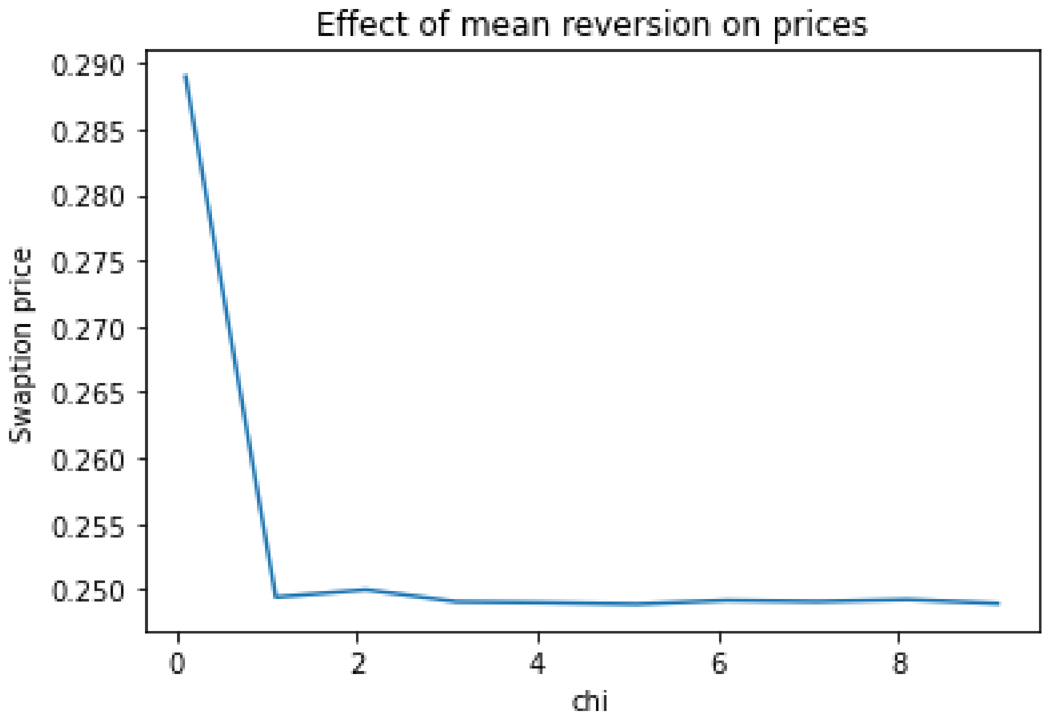
```
Execution time:  3.104661703109741 sec
Execution time:  2.9256393909454346 sec
Execution time:  2.8272581100463867 sec
Execution time:  3.2067902088165283 sec
Execution time:  2.945539951324463 sec
Execution time:  2.7980005741119385 sec
Execution time:  2.7883999347686768 sec
Execution time:  2.8382644653320312 sec
```



Observation: As expected, an increase in λ and b leads to an increase in the short rate volatility and then in the swaption volatility.

[12]:
```
mean_reversion_val = np.arange(0.1,10,1)

swaptions = [swaption(M,N,T0,K,lmbda,a,b,mr,bonds,maturities) for mr in␣
 ↪mean_reversion_val]
plt.figure()
plt.xlabel('chi')
plt.ylabel('Swaption price')
plt.title('Effect of mean reversion on prices')
plt.plot(mean_reversion_val,swaptions)
plt.show()
```

```
Execution time:  2.9679384231567383 sec
Execution time:  2.9655535221099854 sec
Execution time:  2.823681116104126 sec
Execution time:  2.814521312713623 sec
Execution time:  2.8553049564361572 sec
Execution time:  2.8775105476379395 sec
Execution time:  2.8511106967926025 sec
Execution time:  2.885974884033203 sec
Execution time:  2.8101420402526855 sec
Execution time:  3.0466668605804443 sec
```



Remark: One observes that the swaption volatility decreases as the mean reversion increases as it is said p. 553 in Piterbarg. Indeed, the mean reversion parameter tends to keep the factor x around a mean level

1.5 Implied volatility

Assume that the swap rate S is log-normal. We have the following diffusion

$$\mathrm{d}S_{T0,\dots,T_N}(t) = S_{T0,\dots,T_N}(t)\sigma\,\mathrm{d}W_t$$

because it has to be a martingale under $\mathbb{Q}^A$ where

$$\left.\frac{\mathrm{d}\mathbb{Q}^A}{\mathrm{d}\mathbb{Q}}\right|_{\mathcal{F}_t} = \frac{A(t)}{A(0)}e^{-\int_0^t r(s)\,\mathrm{d}s}$$

Hence

$$V_0 \quad = \mathbb{E}_{\mathbb{Q}}\left[e^{-\int_0^t r(s)\,\mathrm{d}s} Swap(T_0)^+\right] = A(0)\mathbb{E}_{\mathbb{Q}^A}\left[(S(T_0)-K)\mathbb{1}_{S(T_0)>K}\right]$$

Let

$$\frac{\mathrm{d}\tilde{\mathbb{Q}}}{\mathrm{d}\mathbb{Q}^A} = \frac{S(T_0)}{\mathbb{E}[S(T_0)]} = e^{-\frac{\sigma^2}{2}T_0+\sigma W_{T_0}}$$

and by Girsanov's Theorem,

$$\tilde{W}_t = W_t^A - \sigma t$$

is a Brownian motion under $\tilde{\mathbb{Q}}$.

It leads to a Black Scholes formula by:

$$\begin{aligned} V_0 &= A(0)S(0)\tilde{\mathbb{Q}}\left(e^{\frac{\sigma^2}{2}T_0+\sigma\tilde{W}_{T_0}} > K\right) \; - KA(0)\mathbb{Q}^A\left(\sigma W_{T_0} > \ln\frac{K}{S(0)} + \frac{\sigma^2}{2}T_0\right) \\ &= \boxed{A(0)S(0)\Phi(d^+) \; - KA(0)\Phi(d^-)} \end{aligned}$$

$$\begin{cases} d^+ &= \frac{\ln\frac{S(0)}{K}+\frac{\sigma^2}{2}T_0}{\sigma\sqrt{T_0}} \\ d^- &= d^+ - \sigma\sqrt{T_0} \end{cases}$$

In particular, $\frac{\partial d^+}{\partial \sigma} = \frac{\partial d^-}{\partial \sigma} + \sqrt{T_0}$ and

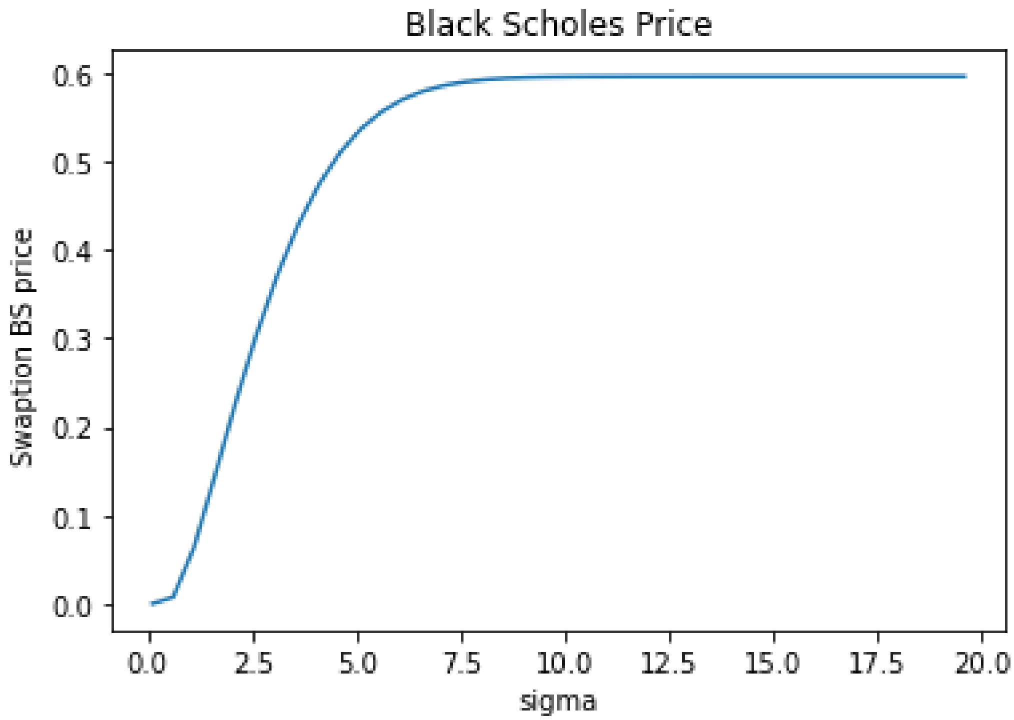
$$vega = \frac{\partial V_0}{\partial \sigma} = A(0)S(0)\frac{\partial d^+}{\partial \sigma}\phi(d^+) \; - KA(0)\frac{\partial d^-}{\partial \sigma}\phi(d^-) = A(0)S(0)\sqrt{\frac{T_0}{2\pi}}e^{-\frac{(d^+)^2}{2}}$$

(Φ is the cdf of the standard normal law and ϕ its density function.)

First view of Black scholes prices to be sure that the pricing formula is right and that the asymptotic behavior is the expected one, i.e $V_0 \to_{\sigma\to\infty} A(0)S(0)$

[14]:
```
#Black Scholes Price

def Price_BS(S0,A0,T0,K,sigma):
    d = (np.log(S0/K) + 0.5*T0*sigma**2)/(sigma*np.sqrt(T0))
    return S0*A0* stats.norm.cdf(d) -K*A0*stats.norm.cdf(d-sigma*np.sqrt(T0))

T0 = maturities[0]
nb_maturities = maturities.size
A0 = sum([(maturities[i+1]-maturities[i])*bonds[i+1] for i in␣
 ↪range(0,nb_maturities-1)])
S0 = (bonds[0]-bonds[-1])/A0
sigma_val = np.arange(0.1,20,0.5)
BS_prices = [Price_BS(S0,A0,T0,2*S0,s) for s in sigma_val]
print('S0*A0=',S0*A0)

plt.figure()
plt.xlabel('sigma')
plt.title('Black Scholes Price')
plt.ylabel('Swaption BS price ')
plt.plot(sigma_val,BS_prices)
plt.show()
```

```
S0*A0= 0.5954216631743912
```



1.5.1 Newton-Raphson algorithm

The simple Newton-Raphson algorithm, applied to find the zero of the function $\sigma \to V_0(\sigma) - \hat{V}$ where $\hat{V}$ is the observed marked to market value of the swaption, is written:

$$\begin{cases} \sigma_0 \quad \text{chosen} \\ \sigma_{n+1} = \sigma_n - \left(\frac{V_0(\sigma_n) - \hat{V}}{\frac{\partial V_0}{\partial \sigma}(\sigma_n)} \right) \end{cases}$$

[15]:
```
def Newton_Raphson(sigma0, nb_it, Mtm, bonds, K, maturities):
    sigma = sigma0
    T0 = maturities[0]
    nb_maturities = maturities.size
    A0 = sum([(maturities[i+1]-maturities[i])*bonds[i+1] for i in␣
 ↪range(0,nb_maturities-1)])
    S0 = (bonds[0]-bonds[-1])/A0

    for i in range(nb_it):
        d = (np.log(S0/K) + 0.5*T0*sigma**2)/(sigma*np.sqrt(T0))
        price_BS = (bonds[0]-bonds[-1])* stats.norm.cdf(d) -K*A0*stats.norm.
 ↪cdf(d-sigma*np.sqrt(T0))
        derivative = (bonds[0]-bonds[-1])*np.sqrt(T0/(2*np.pi))*np.exp(-0.5*d**2)
        sigma = sigma - (price_BS-Mtm)/derivative
```

```
        #print("Sigma, derivative =",sigma, derivative)
    return sigma

Newton_Raphson(0.1,10,0.05,bonds,2,maturities)
```

[15]: 0.5019064713365248

There is some instability with the Newton-Raphson algorithm because the derivative of the Black Scholes price tends to be very small.

1.5.2 Dichotomie

[16]:
```
def bissectrice(nb_it, Mtm, bonds, K, maturities):
    T0 = maturities[0]
    nb_maturities = maturities.size
    A0 = sum([(maturities[i+1]-maturities[i])*bonds[i+1] for i in␣
 ↪range(0,nb_maturities-1)])
    S0 = (bonds[0]-bonds[-1])/A0

    x = 0
    y = 1
    for i in range(nb_it):
        z = (x+y)/2
        sigma = z/(1-z)
        price_BS = Price_BS(S0,A0,T0,K,sigma)

        if(Mtm > price_BS):
            x = z
        else:
            y = z
    z = (x+y)/2
    return z/(1-z)

bissectrice(10,0.05,bonds,2,maturities)
```

[16]: 0.5025678650036683

[17]:
```
# méthode bissectrice de Python

def biss_python(Mtm, bonds, K, maturities):
    T0 = maturities[0]
    nb_maturities = maturities.size
    A0 = sum([(maturities[i+1]-maturities[i])*bonds[i+1] for i in␣
 ↪range(0,nb_maturities-1)])
    S0 = (bonds[0]-bonds[-1])/A0
    return opt.bisect(lambda x: Price_BS(S0,A0,T0,K,x) - Mtm, 0.01, 10)
```

1.5.3 Python's algorithm

[18]:
```
#Brent's method

def Brent(Mtm, bonds, K, maturities):
    T0 = maturities[0]
    nb_maturities = maturities.size
    A0 = sum([(maturities[i+1]-maturities[i])*bonds[i+1] for i in␣
 ↪range(0,nb_maturities-1)])
    S0 = (bonds[0]-bonds[-1])/A0
    f = lambda x: Price_BS(S0,A0,T0,K,x) - Mtm
    return opt.brentq(f, 0.001, 20)
```

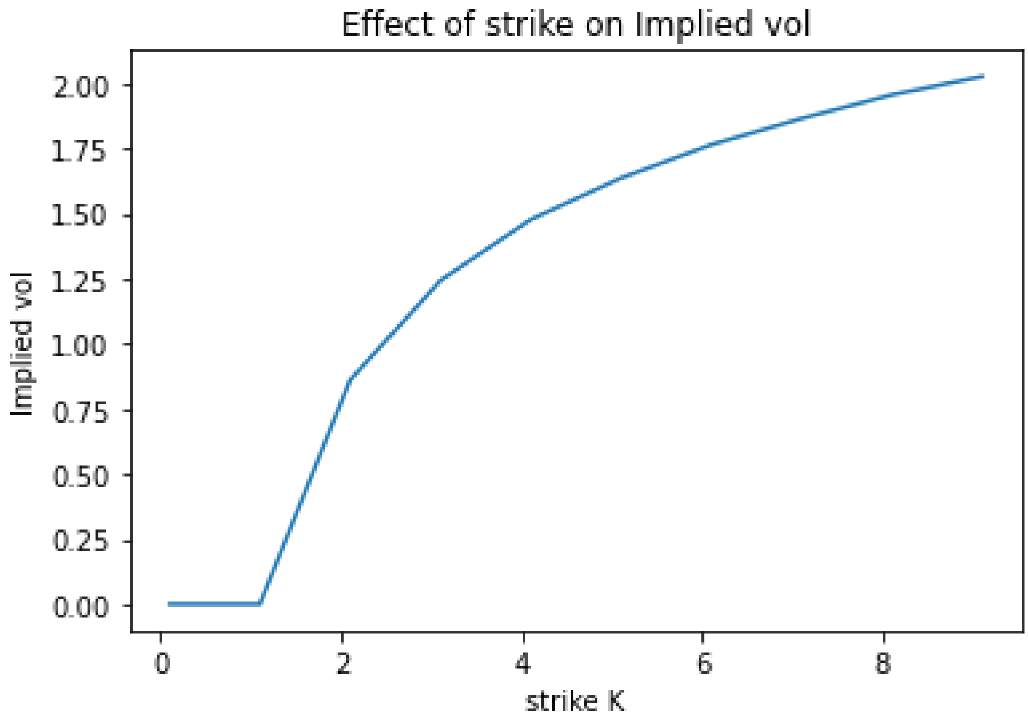
This algorithm is efficient but stops wether $f(a)f(b) > 0$ at one moment

[19]:
```
#Essai sur un prix BS avec sigma connu et comparaison des méthodes

sigma = 0.5
nb_maturities = maturities.size
A0 = sum([(maturities[i+1]-maturities[i])*bonds[i+1] for i in␣
 ↪range(0,nb_maturities-1)])
S0 = (bonds[0]-bonds[-1])/A0
Market_price = Price_BS(S0,A0,T0,2,sigma)

New_vol = Newton_Raphson(0.1, 5, Market_price, bonds, 2, maturities)
bissec_vol = bissectrice(10, Market_price, bonds, 2, maturities)
python_vol = biss_python(Market_price,bonds,2,maturities)
Brent_vol = Brent(Market_price,bonds,2,maturities)
print("Vol BS:", sigma,"Implied vol Newton:",New_vol,"Implied vol Bissec:
 ↪",bissec_vol,"Implied vol python", python_vol,"Implied vol Brent: ",Brent_vol
)
```

```
Vol BS: 0.5 Implied vol Newton: 0.50000042343507 Implied vol Bissec:
0.5003663003663004 Implied vol python 0.5000000000008197 Implied vol Brent:
0.49999999999992195
```

1.5.4 Effect of K

[20]:
```
strikes = np.arange(0.1,10,1)
impli_vol = [bissectrice(10,0.1,bonds,k,maturities) for k in strikes]
plt.figure()
plt.xlabel('strike K')
plt.ylabel('Implied vol')
plt.title('Effect of strike on Implied vol')
plt.plot(strikes,impli_vol)
plt.show()
```



Explanation: One should have an implied volatility surface which is decreasing with respect to strike K, but this phenomenon is justified by the fact that input price is the same for all values of K and is 'randomly' chosen. Indeed Black & Scholes price is decreasing with respect to K:

$$\frac{\partial V_0}{\partial K} = A(0)S(0)\frac{\partial d^+}{\partial K}\phi(d^+) \; - A(0)K\frac{\partial d^-}{\partial K}\phi(d^-) - A(0)\Phi(d^-)$$

And $\frac{\partial d^+}{\partial K} = \frac{\partial d^-}{\partial K} = \frac{-1}{\sigma\sqrt{T_0}K}$ So

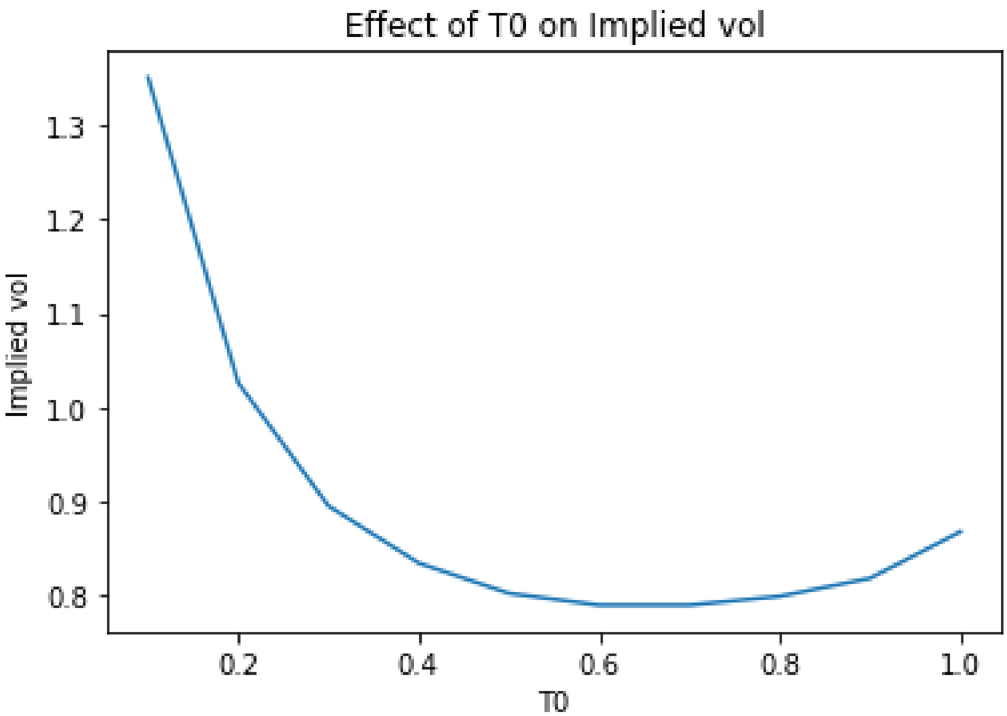
$$\frac{\partial V_0}{\partial K} = \frac{-A(0)}{\sigma\sqrt{T_0}}\Big(\frac{S(0)}{K}\phi(d^+) - \phi(d^+ - \sigma\sqrt{T_0})\Big) \; - A(0)\Phi(d^-) = -A(0)\Phi(d^-) \; \leq 0$$

Hence if $K_1 \leq K_2, \quad \hat{V} = V_0(K_1,\sigma_1) = V_0(K_2,\sigma_2)$ then $\sigma_2 \geq \sigma_1$ which explains the behavior of the curve above.

1.5.5 Effect of T0

[22]:
```
T0s = np.arange(0.1,1.1,0.1)
impli_vol = [bissectrice(10,0.1,[np.exp(-r*m) for m in np.arange(t,5)],2,np.
 ↪arange(t,5)) for t in T0s]
plt.figure()
plt.xlabel('T0')
plt.ylabel('Implied vol')
plt.title('Effect of T0 on Implied vol')
```

```
plt.plot(T0s,impli_vol)
plt.show()
```

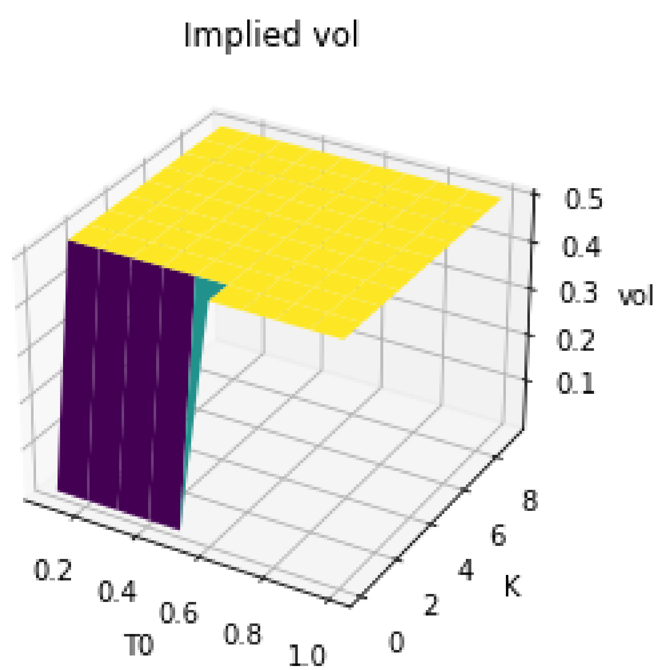


1.5.6 Reliability of bissection method

One will input a swaption price calculated by the Black & Scholes formula for a known volatility. The generated surface is then supposed to be flat and equal to this level of volatility. It is a way to check if the inversion method is unstable somewhere.

[23]:
```
#from a BS price with sigma known --> flat curve is theory

X,Y = np.meshgrid(T0s, strikes)
sigma = 0.5
Z = np.zeros((strikes.size,T0s.size))
i,j= 0,0
for k in strikes:
    for t in T0s:
        maturities = np.arange(t,5)
        bonds = [np.exp(-r*m) for m in maturities]
        nb_maturities = maturities.size
        A0 = sum([(maturities[i+1]-maturities[i])*bonds[i+1] for i in␣
 ↪range(0,nb_maturities-1)])
        S0 = (bonds[0]-bonds[-1])/A0
```

```
        Z[i][j] = bissectrice(10,Price_BS(S0,A0,t,k,sigma), bonds, k, maturities)
        #Z[i][j] = Brent(Price_BS(S0,A0,t,k,sigma), bonds, k, maturities)
        j = (j+1)%10
    i+=1

#Z = np.array([[bissectrice(10,Price_BS(S0,A0,t,k,sigma),[np.exp(-r*m) for m in␣
 ↪np.arange(t,5)],k,np.arange(t,5)) for t in T0s] for k in strikes])
fig = plt.figure()
ax = plt.axes(projection='3d')
ax.plot_surface(X, Y, Z, rstride=1, cstride=1, cmap='viridis', edgecolor='none')
ax.set_xlabel('T0')
ax.set_ylabel('K')
ax.set_zlabel('vol')
ax.set_title('Implied vol');
```

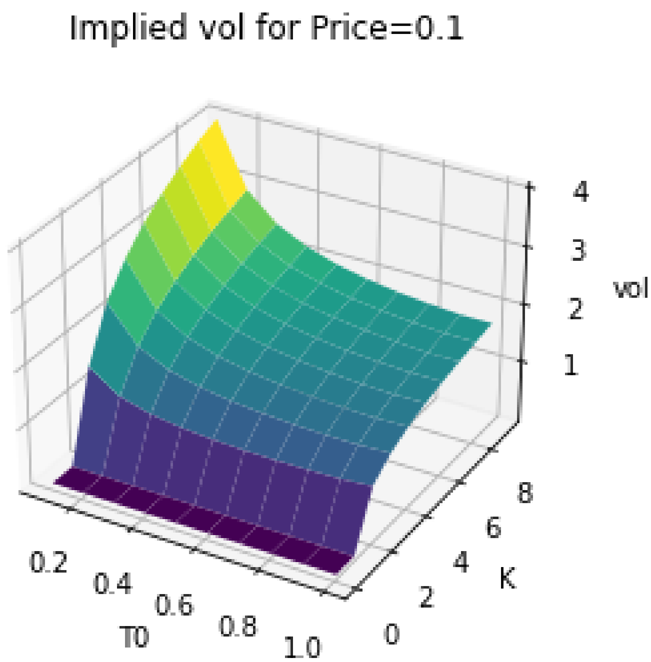


Remark: It can be osbserved that the implied volatility surface is flat almost everywhere. However implied volatility seems to vanish for low levels of K and T_0. The lower the input volatility is, the more important is the unstability.

[24]:
```
#Ploting function of the implied volatility surface

def plot_3D(T0s, strikes, Mtm):
    X,Y = np.meshgrid(T0s, strikes)
    Z = np.array([[bissectrice(10,Mtm, [np.exp(-r*m) for m in np.arange(t,5)],␣
 ↪k, np.arange(t,5)) for t in T0s] for k in strikes])
```

```
    fig = plt.figure()
    ax = plt.axes(projection='3d')
    ax.plot_surface(X, Y, Z, rstride=1, cstride=1, cmap='viridis',␣
↪edgecolor='none')
    ax.set_xlabel('T0')
    ax.set_ylabel('K')
    ax.set_zlabel('vol')
    ax.set_title('Implied vol for Price=' + str(Mtm));
```

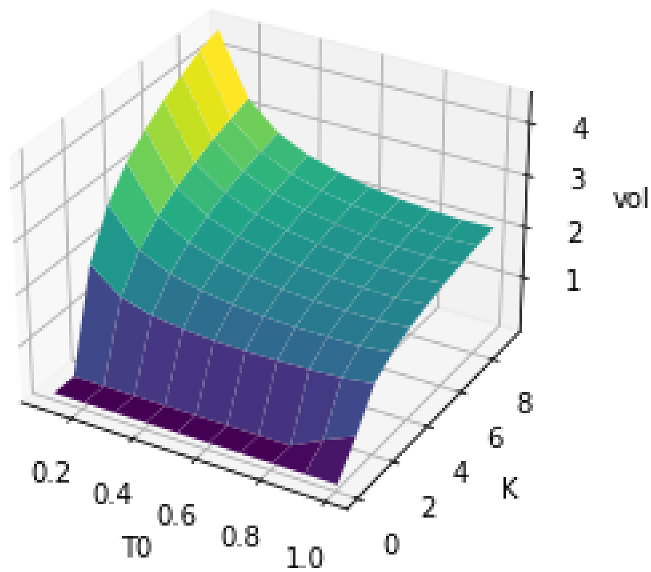
[25]:
```
plot_3D(T0s,strikes,0.1)
```



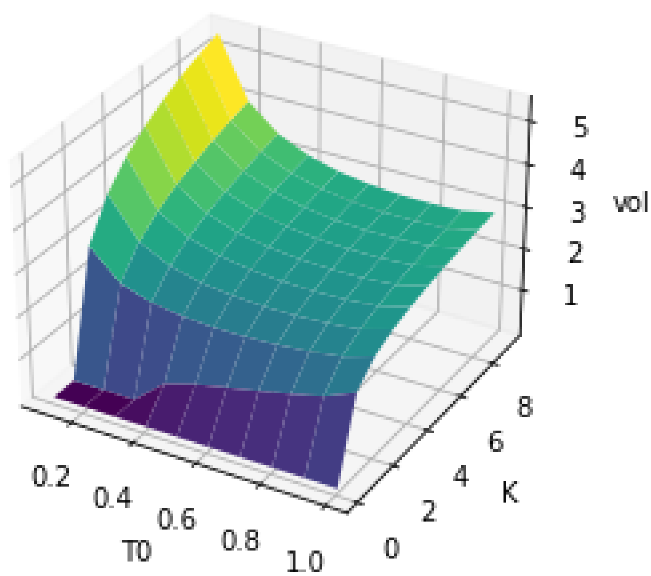
[26]:
```
#Tests with different a
Mtm1 = swaption(M,N,T0,K,lmbda,0,b,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm1)
Mtm2 = swaption(M,N,T0,K,lmbda,3.7,b,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm2)
Mtm3 = swaption(M,N,T0,K,lmbda,4,b,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm3)
```

```
Execution time:  4.506011962890625 sec
Execution time:  3.995288372039795 sec
Execution time:  4.043997049331665 sec
```

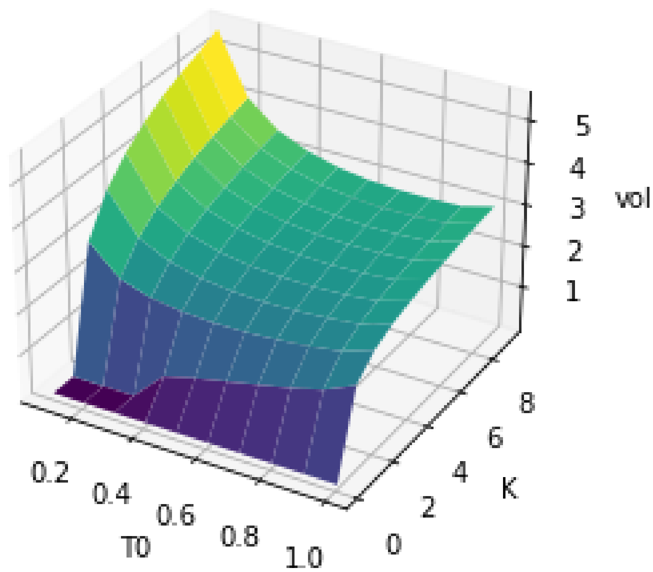
Implied vol for Price=0.14612581178948952



Implied vol for Price=0.24960160825544112

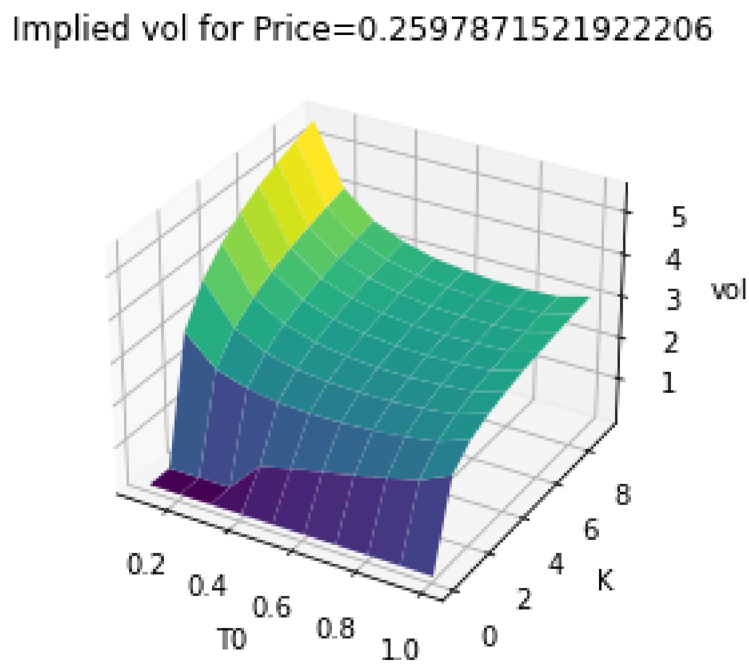


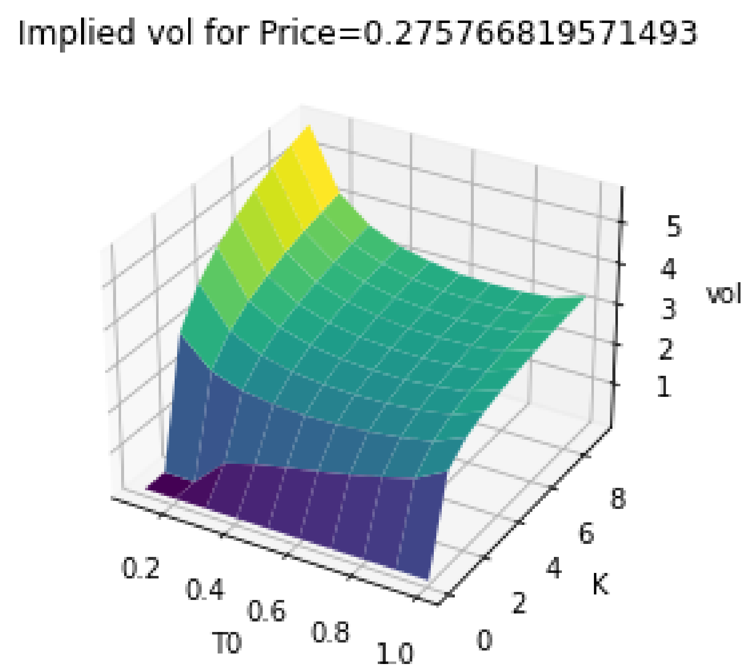
Implied vol for Price=0.2566837355284929

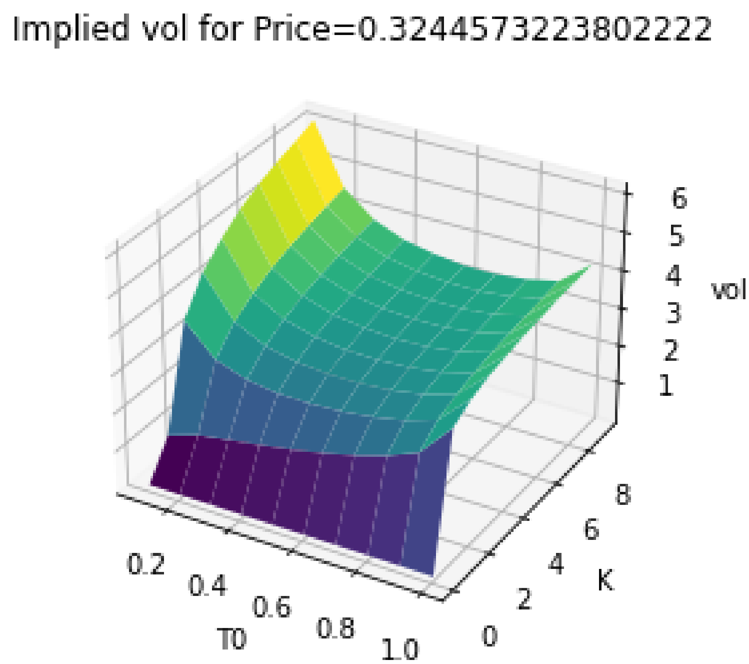


[135]:
```
#Tests with different b
Mtm1 = swaption(M,N,T0,K,lmbda,a,0,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm1)
Mtm2 = swaption(M,N,T0,K,lmbda,a,5,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm2)
Mtm3 = swaption(M,N,T0,K,lmbda,a,8,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm3)
```

```
Execution time:  4.59572696685791 sec
Execution time:  4.397596597671509 sec
Execution time:  4.308629751205444 sec
```

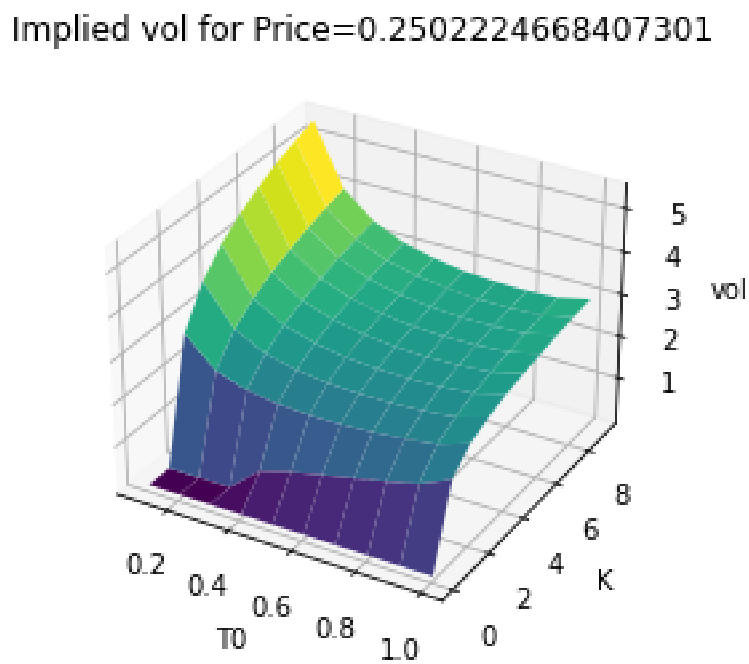

Implied vol for Price=0.2597871521922206
vol
K
T0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
1
2
3
4
5

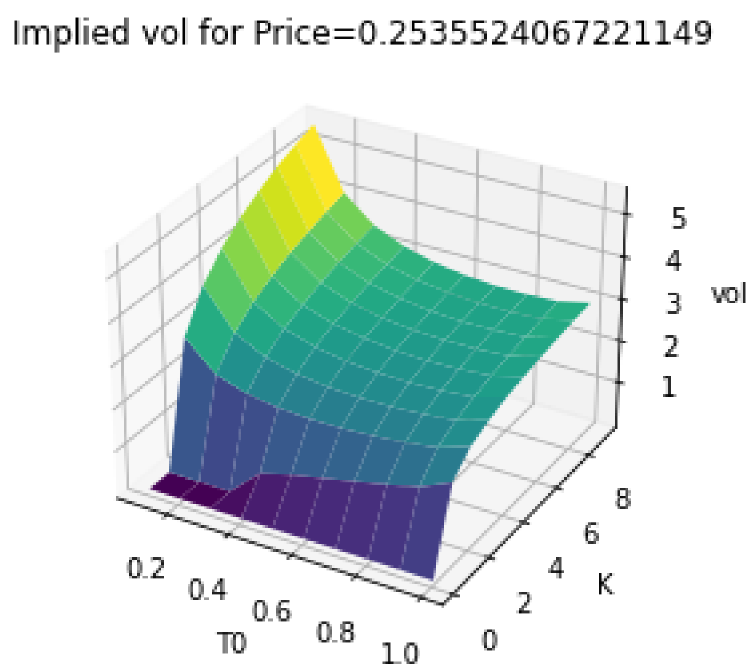

Implied vol for Price=0.275766819571493
vol
K
T0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
1
2
3
4
5

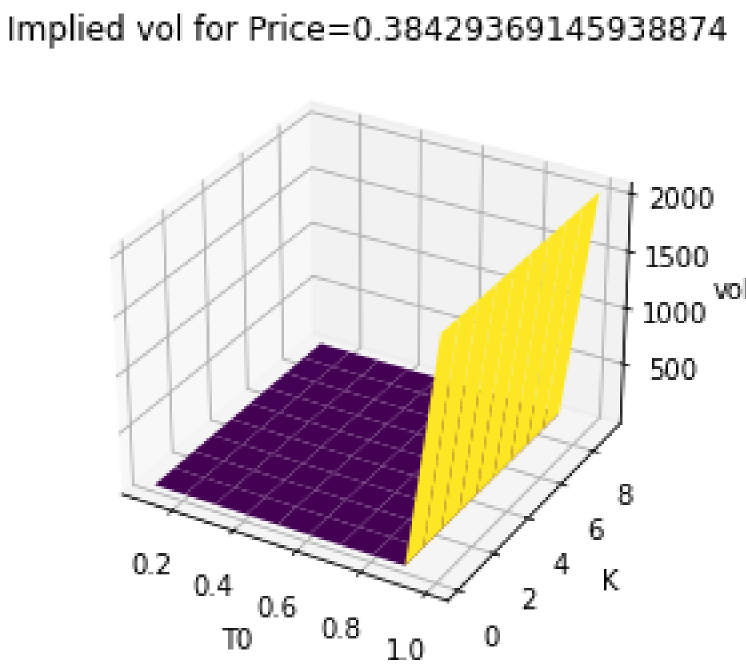


[147]:
```
#Tests with different lambda
Mtm1 = swaption(M,N,T0,K,1,a,b,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm1)
Mtm2 = swaption(M,N,T0,K,2,a,b,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm2)
Mtm3 = swaption(M,N,T0,K,8,a,b,chi,bonds,maturities)
plot_3D(T0s,strikes,Mtm3)
```

```
Execution time:  4.951383113861084 sec
Execution time:  4.460316181182861 sec
Execution time:  4.8040900230407715 sec
```


Implied vol for Price=0.2502224668407301
vol
5
4
3
2
1
K
8
6
4
2
0
T0
0.2
0.4
0.6
0.8
1.0


Implied vol for Price=0.2535524067221149
vol
5
4
3
2
1
K
8
6
4
2
0
T0
0.2
0.4
0.6
0.8
1.0

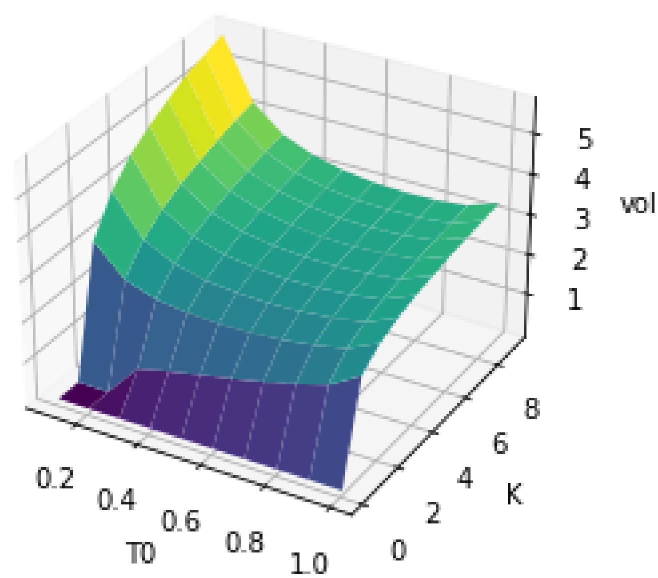


[154]:

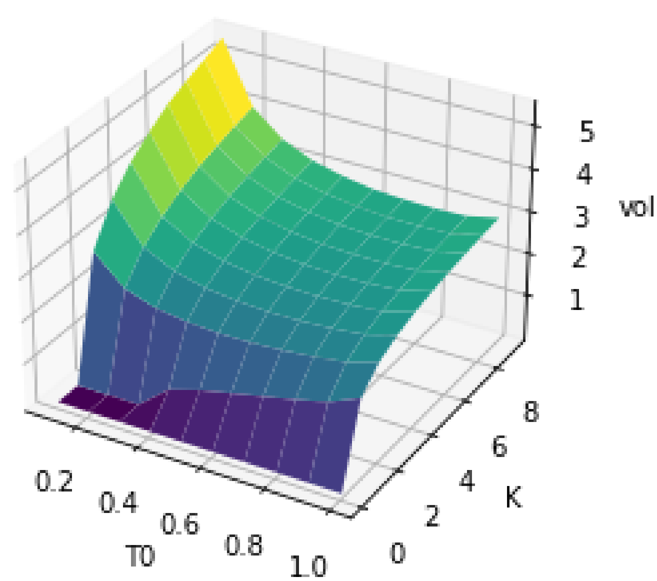
```
#Tests with different chi
Mtm1 = swaption(M,N,T0,K,lmbda,a,b,0.1,bonds,maturities)
plot_3D(T0s,strikes,Mtm1)
Mtm2 = swaption(M,N,T0,K,lmbda,a,b,3,bonds,maturities)
plot_3D(T0s,strikes,Mtm2)
Mtm3 = swaption(M,N,T0,K,lmbda,a,b,10,bonds,maturities)
plot_3D(T0s,strikes,Mtm3)
```

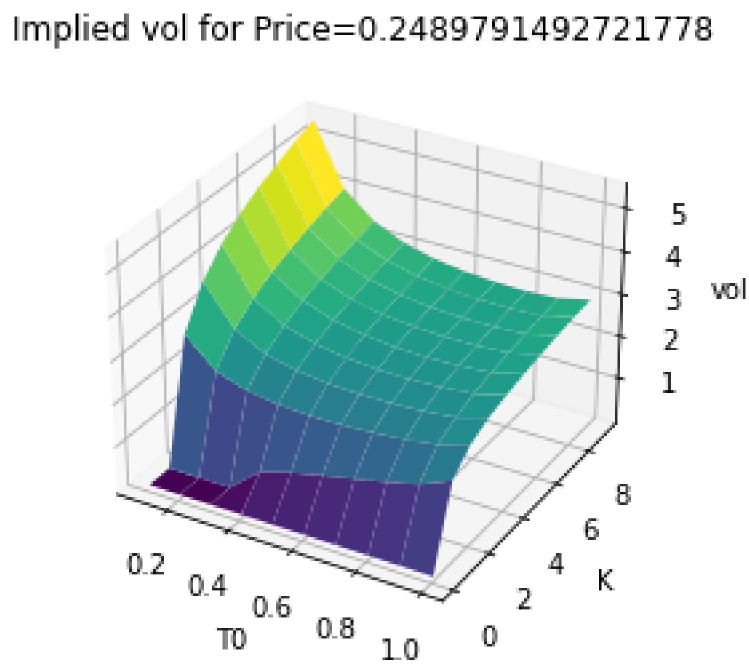
```
Execution time:  4.356647729873657 sec
Execution time:  4.189858913421631 sec
Execution time:  4.336628437042236 sec
```

Implied vol for Price=0.2835435018912898



Implied vol for Price=0.248053506952873





1.6 Pricing by Gaussian swap rate approximation (p.422 10.1.3.2)

$$V_0 \approx A(0)\Big[\big(S(0)-K\big)\Phi(d) \; + \; \sqrt{v}\phi(d)\Big] d = \frac{S(0)-K}{\sqrt{v}} \quad v = \int_0^{T_0} q(t,\bar{x}(t))^2\, \sigma_r(t)^2\, \mathrm{d}t$$

Where

$$q(t,x) = -\frac{P(t,T_0,x)G(t,T_0) - P(t,T_N,x)G(t,T_N)}{A(t,x)} \; + \; \frac{S(t,x)}{A(t,x)} \sum_{i=0}^{N-1} \tau_i P(t,T_{i+1},x)G(t,T_{i+1})$$

Remark: (To do later) Find numerical methods in order to approximate v

```
[ ]:
```