

Alexis Langlois-Rémillard

alexis.langloisremillard@ugent.be
alexisl-r.github.io

Office: Krijgslaan 281, S9
9000 Ghent, Belgium

Research interests

Algebra, mathematical physics, representation theory of algebras, diagrammatic algebras, cellular algebras, Lie (super)algebras, Dunkl operators, Temperley-Lieb algebras, and complexity theory.

Education

- 2019–
exp. **Ph. D. in Mathematics**, Universiteit Gent
2023 Thesis subject: *The representation theory of the total angular momentum algebra*
2023 Advisors: Profs Joris Van der Jeugt and Hendrik De Bie
- 2017–
2019 **M. Sc. in Mathematics**, Université de Montréal
2019 Thesis: *Sur la structure cellulaire et la théorie de la représentation des algèbres de Temperley-Lieb à couture*
Advisor: Prof. Yvan Saint-Aubin
- 2014–
2017 **B.Sc in Mathematics**, Université de Montréal
2017 *Excellence mention*

Scholarships

- 2019–2023 **FRQNT¹** Doctoral Research Scholarship
2019–2023 **FWO²** EOS PRIMA PhD scholarship
2017–2019 **FRQNT** Master's award
2017–2018 Alexander-Graham-Bell **NSERC³** Master's award
Summer 2016 Undergraduate Research Awards, **NSERC** and **FRQNT**, Yvan Saint-Aubin
Summer 2015 Undergraduate Research Award, **NSERC**, Matilde Lalín

Academic work experience

- 2021 **Teaching Assistant** at Ghent University (in Dutch)
Mathematics for economics IA
Helped animating practical sessions for 60-80 students, invigilated exams.
- 2016–2018 **Teaching Assistant** at Université de Montréal (in French)
Discrete Mathematics, Analysis 1 and Fundamental Mathematics
Animation of exercises for 20-60 students, office hours, invigilated and corrected exams.

Graduate students advising and evaluation

Current master students

- 2022–2023 Stijn Dezeure. **Mentor** with S. Lazendic (UGent under Prof. A. Pizurica)
Subject: *Quantum computing with applications to image processing*
- 2021–2023 Régis Schulze. **Mentor** with S. Lazendic and A. Guzmán Adán (UGent under Prof. A. Pizurica)
Thesis title: *Classical and quantum approaches to belief propagation algorithms*

Past master students

- 2021–2022 Pieter-Jan Vandaele. **Mentored** with S. Lazendic and A. Guzmán Adán (UGent under Prof. A. Pizurica)
Thesis title: *Classical coding approaches to quantum applications*
- Spring 2021 Bert Christiaens. **Jury member** (UGent under Prof. A. Pizurica)
Thesis title: *Network explainability via content based image retrieval*

Publications and preprints

List available at <https://alexisl-r.github.io/publications/>.

See appended list of publications.

¹Fonds de Recherche du Québec – Nature et Technologies (Québec Research Funds on Natural Sciences)

²Fonds Wetenschappelijk Onderzoek – Vlaanderen (The Research Foundations – Flanders)

³National Science and Engineering Research Council of Canada

Talks and Workshop

Complete list available at <https://alexisl-r.github.io/talks/>.

Invited talks:

- October 2022 **Dartmouth combinatorics seminar**, Dartmouth College, Hanover.
Sandwich cellularity and constructing idempotent in quotients of the affine Temperley-Lieb algebras
- October 2022 **Integrable systems, exactly solvable models and algebras**, CRM Montréal.
Representations and Wenzl-Jones elements of quotients of the affine Temperley-Lieb algebra
- May 2021 **EOS meeting**, online *Finite-dimensional representations of the 3D dihedral Dunkl-Dirac symmetry algebra*
- November 2018 **Seminar: representation theory of algebras**, Université de Sherbrooke
Idempotence, cellularité et algèbres diagrammatiques
- October 2018 **Mathematical physics seminar**, Université de Montréal
Roots of unity and the representation theory of boundary seam algebras

Workshop:

- June 2018 **Canadian Mathematical Society Summer Meeting**, Fredericton
Atelier d'enseignement actif (with Marie-Andrée B. Langlois)

Contributed seminar and conference talks:

- July 2022 **Group34**, Strasbourg
Monogenic representations of the algebra of symmetries of the generalised Dirac operator
- May 2022 **Non-commutative algebras, representation theory and special functions**, CRM, Montréal
On the representation theory of a symmetry algebra associated to a generalised Dirac operator
- February 2022 **Clifford research seminar**, Universiteit Gent, Online
Bases for Dunkl monogenics by generalised symmetries
- June 2021 **Lie Theory and its applications to physics XIV**, Online
Finite-dimensional Representations of the 3D dihedral Dunkl-Dirac symmetry algebra
- April 2021 **Clifford research seminar**, Universiteit Gent, Online
Generalizing the Deligne category, Khovanov's and Sazdanovic's approach
- December 2020 **CMS Winter meeting**, Canada, online
The symmetry algebra of the Dunkl-Dirac operator: the dihedral cases
- March 2020 **Clifford research seminar**, Universiteit Gent
The symmetry algebra of the 3D dihedral Dunkl-Dirac equation.
- November 2019 **SPAS**, Västerås
Cellular structure of seam algebras and avenues for deformations
- June 2019 **Clifford research seminar**, Universiteit Gent
Representation theory and cellular structure of seam algebras
- September 2018 **XXXth Meeting on representation theory of algebras**, Sherbrooke
Cellular structure of boundary seam Temperley-Lieb algebras
- June 2018 **Canadian Mathematical Society Summer Meeting**, Fredericton
Bratteli and the morphisms of boundary seam algebras

Poster:

- May 2022 **PhD Day meeting of the BMS**, Liège
Weavings weights: double dihedral deformation (with Marcelo De Martino and Roy Oste)

Outreach

More available at <https://alexisl-r.github.io/popularization/>.

Popularization work for a broad audience

See appended list of publications

Popularization talks

- September 2022 **Club mathématique**, Université de Montréal
La domination, une histoire d'échecs
- November 2021 **PRIME problem-solving avond**, UGent
Koninginnen en (bijzondere) borden
- September 2020 **Club mathématique**, Université de Montréal
Des pentagones aux heptagones, une infinité de différences
- November 2018 **Club mathématique**, Université de Montréal
Huit dames pour un échiquier
- July 2018 **Camp mathématique de l'AMQ**, Dawson College, Montréal
Le carrérouel du géomètre
- November 2017 **Club mathématique**, Université de Montréal
Excursion typographique: la matrice des fontes

Outreach activities

- 2021– **Chess and mathematics**. I collected a set of chess-inspired mathematical problems and created activities around them, both for general audience and university mathematics students.
- 2016-2018 **Institut des sciences mathématiques**. I created an activity on tatami tiling in 2016 for the *MathFest* and co-created the activity *Le carrérouel du géomètre* for the festival *Eureka!* 2018 at the Science center of Montréal.
- 2018-2019 **L'Axiomatique**. *Corrector-in-chief and writer* for a mathematics student journal at Université de Montréal.
- 2015-2017 **JÉMUM**. *Co-created and co-edited* a student mathematical journal to showcase summer research by undergraduate students.

Leadership and implication

Conference organiser

- 2022-23 **Kleine Seminar mini-course**. We will organise a mini-course in April 2023 on p -Kazhdan-Lusztig theory given by Maud De Visscher and Chris Bowman as a doctoral course for Ghent University
- 2019 **SAMARI**. One-day conference on possibilities after graduation in mathematics. *Co-initiator and co-organizer*
- 2016-2018 **Seminars in Undergraduate Mathematics in Montréal**. Annual provincial undergraduate weekend conference. *President for 2016-2017 and member of the organizing committee in 2018*
- 2018 **SAPHARI**. One-day conference on possibilities after graduation in physics. *Organizer of the mathematics session*
- 2017 **Canadian Undergraduate Mathematics Conference** Annual national undergraduate one-week conference. *President for the Montréal 2017 edition*

Session and seminar organizer

- 2019-present **Kleine seminar**. Organizing a postgraduate mathematical seminar at UGent on various algebraic subjects related to representation theory, https://alexisl-r.github.io/kleine_seminar/
- 2017-2018 **AARMS-STUDC student poster session**. Co-organizer for 2018 CMS Summer Meeting, University of New-Brunswick, Fredericton and 2017 CMS Winter Meeting, University of Waterloo, Waterloo.
- 2017-2018 **Séminaire étudiant en mathématiques**. Co-organizer for the graduate students mathematics seminar at Université de Montréal
- 2015-2017 **Club mathématique**. Co-organizer for a weekly seminar of talks aimed at undergraduate students at Université de Montréal

Community service

- 2020-present **Reviewer** for zbMATH Open. 13 articles reviewed and one book in progress [zbMATH:langlois-remillard.alexis](#)
- 2016-2019 **Student committee** of the Canadian Mathematical Society. *Committee member*: involved in the French translation of general activities and in the edition of French articles in the annual publication *Notes from the Margin*.
- 2017-2019 **AECSMS**: Graduate mathematics and statistic students association in Université de Montréal. *President* (2018-2019) and *Treasurer* (2017-2018).
- 2014-2019 Students representative on the **mathematics and statistic departmental assembly** at Université de Montréal: 2014-2016 for undergraduate and 2018-2019 for graduate students.

Languages

- Français :** Native language
English: Fluent
Nederlands: Proficient (B2 CEFRL 2021-05)
Deutsch: Basic (Level around A2 CEFRL)

Other Interests and Activities

- Chess** I am a Canadian and Québécois expert and play with the KGSRL chess club.
- Lifeguard** I taught swimming lessons and was a lifeguard from 2011 to 2016 and a lifeguard instructor from 2013 to 2016. I received a Commonwealth service citation for my volunteer work in 2015 and 2014.
- Literature** I greatly enjoy reading, writing and discussing literary works.

Publications

Published and accepted articles

- A3) De Bie, Hendrik; Langlois-Rémillard, Alexis; Oste, Roy, and Van der Jeugt, Joris (2022) Generalised symmetries and bases for Dunkl monogenics. 18 p. To appear in Rocky Mountains Journal of Mathematics [arXiv:2203.01204](https://arxiv.org/abs/2203.01204) (accepted). Submitted: 10/03/2022
- A2) De Bie, Hendrik; Langlois-Rémillard, Alexis; Oste, Roy, and Van der Jeugt, Joris (2022) Finite-dimensional representations of the symmetry algebra of the dihedral Dunkl-Dirac operator, J Algebra 591: 170-216, [doi:10.1016/j.jalgebra.2021.09.025](https://doi.org/10.1016/j.jalgebra.2021.09.025) and [arXiv:2010.03381](https://arxiv.org/abs/2010.03381)
- A1) Langlois-Rémillard, Alexis, and Saint-Aubin, Yvan (2020) The representation theory of seam algebras, SciPost Phys. 8, 019, 34p. [doi:10.21468/SciPostPhys.8.2.019](https://doi.org/10.21468/SciPostPhys.8.2.019)

Published and accepted proceedings contributions

- P3) Langlois-Rémillard, Alexis (2021+) The dihedral Dunkl-Dirac symmetry algebra with negative Clifford signature. To appear in the proceedings of Lie Theory and Its Applications in Physics XIV, PROMS vol 396, 2021. 7p. [arXiv:2209.06599](https://arxiv.org/abs/2209.06599) (in print). Submitted: 22/12/2021
- P2) Langlois-Rémillard, Alexis (2020+) Deforming algebras with anti-involution via twisted associativity. To appear in the proceeding of the International conference on stochastic processes and algebraic structures, volume II: algebraic structures and applications (Västerås, Sweden, October 2019), ed. Sergei Silvestrov. 21 p. [arXiv:2106.01855](https://arxiv.org/abs/2106.01855) (accepted). Submitted: 29/03/2020.
- P1) Langlois-Rémillard, Alexis, and Oste, Roy (2020) An Exceptional Symmetry Algebra for the 3D Dirac-Dunkl Operator. In Dobrev V. (ed) Lie Theory and Its Applications in Physics. LT Varna 2019. Springer Proceedings in Mathematics & Statistics, vol 335, pp 399-405. Springer, Singapore. [doi:10.1007/978-981-15-7775-8_30](https://doi.org/10.1007/978-981-15-7775-8_30) and [arXiv:2009.13904](https://arxiv.org/abs/2009.13904)

Outreach articles

- O3) Langlois-Rémillard, Alexis, and Senécal, Charles (2022) Des dames sur d'étranges échiquiers. Accromath, 17.2, pp. 2-7. Available online at <https://accromath.uqam.ca/2022/09/des-dames-sur-detranges-echiquiers/>
- O2) Langlois-Rémillard, Alexis (2022). Huit dames et un échiquier. Accromath, 17.1, pp. 8-13. Available online at <https://accromath.uqam.ca/2022/02/huit-dames-et-un-echiquier/>
- O1) Boutet, Véronique; Godin, Jonathan, and Langlois-Rémillard, Alexis (2017). Excursion typographique : La matrice des fontes, Accromath, 12.2, pp. 26-29. Available online at <https://accromath.uqam.ca/2017/09/la-matrice-des-fontes/>

Research Statement

Max Planck Institute for Mathematics, Universität Bonn
 Max Planck Postdoc applications 2023
 23rd October 2022

Alexis Langlois-Rémillard
 PhD Candidate | Ghent University
alexislangloisremillard@gmail.com
<https://alexisl-r.github.io/>

Diagrammatics and representations of algebras related to Howe and Schur–Weyl dualities

General context

The concept of **Howe duality** originates from the influential work of Howe [How89]. In its most classical inception, it relates the representations of a dual pair G, G' that are mutually centralising subgroups of the double cover of a symplectic group. The methods of Howe have been applied over the last 30 years in a vast array of cases [CW12].

The original **Schur–Weyl duality** [Sch01] states that the actions of GL_m and the symmetric group S_n on the tensor product of a fundamental representation $(\mathbb{C}^m)^{\otimes n}$ are each other's centraliser. So we can decompose $(\mathbb{C}^m)^{\otimes n}$ into a direct sum of simple S_n module tensor simple GL_m -modules. It also fits under the theme of Howe duality.

My doctoral research tackled an algebra related to the Howe dual pair $(\text{Pin}(d), \mathfrak{osp}(1|2))$ in a deformed version of Weyl–Clifford algebras, where deformations occurred by mean of a reflection group. Previously it was concentrated on a class of diagrammatic algebras, most notably the Temperley–Lieb algebra [TL71], related to problems inspired by physics and that were in a Schur–Weyl type dual pair with quantum groups.

In fact, many algebras related to these types of dualities admit a graphical calculus. This enables the statement of complex conditions by simple topological rules. Figure 1 below presents a famed example of knot theory and recoupling theory in the Temperley–Lieb algebra [KL94].

$$\begin{aligned}
 P_n &= P_n^2, & \vdots \begin{array}{|c|} \hline n \\ \hline \end{array} \vdots &= \vdots \begin{array}{|c|} \hline n \\ \hline \end{array} \vdots \begin{array}{|c|} \hline n \\ \hline \end{array} \vdots; \\
 e_i P_n &= 0 = P_n e_i, & \begin{array}{|c|} \hline \vdots \\ \hline \end{array} \begin{array}{|c|} \hline n \\ \hline \end{array} \begin{array}{|c|} \hline \vdots \\ \hline \end{array} &= \begin{array}{|c|} \hline \vdots \\ \hline \end{array} \begin{array}{|c|} \hline n \\ \hline \end{array} \begin{array}{|c|} \hline \vdots \\ \hline \end{array} = 0 = \begin{array}{|c|} \hline \vdots \\ \hline \end{array} \begin{array}{|c|} \hline n \\ \hline \end{array} \begin{array}{|c|} \hline \vdots \\ \hline \end{array} = \begin{array}{|c|} \hline \vdots \\ \hline \end{array} \begin{array}{|c|} \hline n \\ \hline \end{array} \begin{array}{|c|} \hline \vdots \\ \hline \end{array}; \\
 P_n &= P_{n-1} - \frac{[n-1]_q}{[n]_q} P_{n-1} e_1 P_{n-1}, & \vdots \begin{array}{|c|} \hline n \\ \hline \end{array} \vdots &= \vdots \begin{array}{|c|} \hline 1 \\ \hline n \\ \hline \end{array} \vdots - \frac{[n-1]_q}{[n]_q} \vdots \begin{array}{|c|} \hline 1 \\ \hline n \\ \hline \end{array} \vdots \begin{array}{|c|} \hline 1 \\ \hline n \\ \hline \end{array} \vdots.
 \end{aligned}$$

Figure 1: On the left, the algebraic properties of the elements P_n [Jon83; Wen87] in the Temperley–Lieb calculus, and their diagrammatic counterparts.

Statement of research problems

Main research aim

Construct representations and new diagrammatic calculus of algebras related to Howe and Schur–Weyl dualities.

Many algebras can be studied via these methods; I will focus first on three concrete problems to initiate the research. The overarching goal would be to then express links between the project, let it be by using similar methods, or directly by finding functorial relations between the families.

First concrete projects

1. Study the representation theory of the total angular momentum algebra for specific groups and define a diagrammatic calculus for general ones.
2. Study the untangled affine Temperley–Lieb algebras in characteristic 0 and p and define appropriate Jones–Wenzl elements.
3. Define an infinite symmetric webs calculus to study LKB representations.

Proposed research plan

Total angular momentum algebra The first problem is a continuation of my PhD thesis. The total angular momentum algebra studied here can be defined abstractly by generators and relations [DOV18a] or as the supercentraliser of an $\mathfrak{osp}(1|2)$ realisation inside the tensor product of a rational Cherednik algebra [EG02] and a Clifford algebra. This means that the algebra depends on a reflection group W and a weight function κ invariant on W -orbits. This instances is the algebra coming from the Howe dual pair $(\mathrm{Pin}(d), \mathfrak{osp}(1|2))$ [ØSS09] present in the product, but other Howe dualities have also been studied [CD20; Ciu+20]. Relatively little was known over the representation theory of this algebra, only the groups $W = \mathbb{Z}_2^N$ [DGV16] and $W = S_3$ [DOV18b] had been studied. In my doctoral thesis, I presented the representation theory of $W = D_{2m} \times \mathbb{Z}_2$ [De +22a] and $W = D_{2m} \times D_{2n}$, and I gave a realisation as polynomial solutions to the Dunkl–Dirac equation for any group W [De +22b].

At the moment, an ongoing collaboration with Marcelo De Martino and Roy Oste aims to extend the two first results to a stack of dihedral groups, and to consider the representation theory at “exotic” values of κ . Our preliminary computations hint that the general case will divide into 4-dimensional “slices” and, for odd dimension, with an extra 3-dimensional “slice”; precisely the two cases we already studied, leaving only the question of how to coordinate the slices. Furthermore, in most previous works, we avoided values of κ that do not permit unitarity. In the low-dimensional cases, the values we avoided did not result in interesting behaviour, but we expect that having many values simultaneously conflicting could allow for remarkable types of representation, as is the case for representations of rational Cherednik algebras.

Once this first project is done, I propose to investigate the following directions, focussing on general W , and therefore much more difficult.

1. We know from [De +22b] that generalised symmetries can be used to create a basis for a realisation of an important representation: the polynomial null-solutions of a Dirac operators in which the derivatives are changed to Dunkl derivatives [Dun89]. This is only one representation, but we know the monogenic polynomials are one of, if not the, most important representation of the total angular momentum algebra, often encoding the behaviour of the representation outside exotic values of κ . An interesting avenue seems thus to study an extended algebra instead: a deformation of the conformal algebra defined in [CD15] and use a reduction to the total angular momentum algebra to obtain concrete information on the admissible representations.
2. Create diagrammatics for this algebra by combining Webster’s diagrammatics for rational Cherednik algebras [Web17] with a modification of Brundan’s, Comes’s and Kujawa’s diagrammatics for Brauer–Clifford supercategory [BCK19]. A hint that these algebras encode interesting diagrammatics was already pointed out in [FH15] where crossing relations that could be represented via Temperley–Lieb algebras elements were found.

The untangled affine Temperley–Lieb algebras The affine Temperley–Lieb algebra is an algebra of very high relevance for physicists and algebraist. It is an infinite-dimensional algebra that

appears in conformal field theory and is linked to Virasoro algebras. If the normal Temperley–Lieb algebra is naturally understood via Schur–Weyl duality, the duality breaks for the affine Temperley–Lieb algebra.

Since the influential work of Graham and Lehrer [GL98], its representation theory has been a central object of interest, mainly via the study of its monoidal category. It is presented via periodic planar diagrams, or diagrams on the cylinder. Another pair of generators is also added: the twist Ω and its inverse Ω^{-1} . Its presentation by generators and relations is given below, with $i, j \in \{0, \dots, n-1\}$ being periodic and id , the identity:

$$\begin{aligned} e_i^2 &= \beta e_i, & e_i e_{i\pm 1} e_i &= e_i, \\ e_i e_j &= e_j e_i, \quad |i-j| > 1, & \Omega e_j \Omega^{-1} &= e_{j-1}, \\ \Omega^2 e_1 &= e_{n-1} \dots e_2 e_1, & \Omega \Omega^{-1} &= \Omega^{-1} \Omega = \text{id}. \end{aligned}$$

Diagrammatically, it is given by:

$$e_1 = \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \vdots \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array}, \quad \Omega = \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \vdots \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array}, \quad \Omega^{-1} = \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \vdots \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} \quad \text{and} \quad \text{id} = \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \vdots \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array}.$$

Recently, Martin and Spencer proved a modular version of the famed Jones–Wenzl projectors [MS22], extending on the work of Burrull, Libedinsky and Sentinelli [BLS19]. It enabled Spencer to generalise our work [LS20] and the work of Flores and Peltola [FP18] on the boundary seam algebra [MRR15] to the modular case [Spe21].

The goal of this project would be to approach the affine Temperley–Lieb algebra via the following quotient making it finite-dimensional:

$$\Omega^N = \gamma \text{id}; \quad \text{diagrammatically:} \quad \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \vdots \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array} = \gamma \begin{array}{|c|} \hline \text{---} \\ \hline \text{---} \\ \hline \vdots \\ \hline \text{---} \\ \hline \text{---} \\ \hline \end{array}.$$

Diagrammatically, this amounts to unwinding full turns of the strands on the cylinder. It was motivated by a question of Tubbenhauer coming from their recent work with Khovanov and Sitaraman [KST22] where they used representation theory of specific algebras to make cryptographic protocols.

At the moment, we have defined the algebra, the untangled affine Temperley–Lieb algebra $uATL_n(\beta, \gamma)$, and proved it is sandwich cellular [TV22], a generalisation of cellularity [GL96] extending upon the notion of affine cellularity of König and Xi [KX12]. This gives ways to a study of its representation theory via its cell modules.

The algebra $uATL_n(\beta, \gamma)$ has n one-dimensional modules. In normal Temperley–Lieb algebras, the Jones–Wenzl projector is the idempotent linked to the only one-dimensional module. In the untangled affine version, we are able to compute the n Jones–Wenzl-like projectors $Q_{n,r}$ for characteristic 0 and we have linear recurrence formulas that uniquely determine their coefficients. The next step of the project, which should be completed before the start of the research stay, is to give closed forms for the coefficients and use the projector to study the representation theory à la [GL98] in characteristic 0 for roots of unity. This is part of an ongoing collaboration with Alexi Morin-Duchesne and Robert Spencer.

During my stay, I propose to investigate the following directions extending this project.

1. Define the projectors we find in the modular case, doing work similar to [MS22; BLS19]. In our case, the technical difficulties will be greater as the algebra has two parameters we need to tune. The tour de force of Martin and Spencer will need to be reproduced with

care.

2. The quotient we use is, somehow, the simplest of a tower of algebras. We can define a family of imbricates untangled Temperley–Lieb algebra $uATL_n^k(\beta, \gamma)$ for which the quotient is changed to $\Omega^{kn} = \gamma \text{id}$. A comment of Théo Pinet suggests considering the limit of the process of successive quotients. Then the inductive limit is conjectured to be $aTL_n(\beta)$ and we could study the representation theory of $aTL_n(\beta)$ by lifting the projective modules in one untangled algebra.
3. Define its fusion rules and see how do they interact with the inductive limit. There are multiple fusion rules proposed for the affine Temperley–Lieb algebra, each with their own physical meanings, advantages and disadvantages. We would follow the recent definition of [IM22] adapted to the quotients.
4. Interestingly, similar work to define a Jones–Wenzl-type projector has been carried on another quotient of the affine Temperley–Lieb algebra by Queffelec and Wedrich [QW18] where they obtained a categorification of the skein algebra on the annulus. To define precisely what our projector categorifies would be of interest.
5. Lastly, I wish to investigate the physical meaning of the algebras $uATL_n(\beta, \gamma)$ and related deformation $Q_{n,r}uTL_{m+n}(\beta, \gamma)Q_{n,r}$ and see if it gives rise to the same type of conformal theory with boundaries as what is explored by Flores and Peltola [FP18; FP20]. On the latter, we could also expect that an inductive process on the sequence of quotients could lead back to spin chains as module on the affine Temperley–Lieb [PS22], but we know the Schur–Weyl duality of Temperley–Lieb breaks down in the affine case, so how and when precisely it breaks in the limit is a key point to understand the affine Temperley–Lieb algebra.

LKB representations and infinite web calculus. In a recent preprint, Lacabanne, Tubbenhauer and Vaz gave a formulation of Verma Howe duality [LTV22] with the pair $U_q(\mathfrak{sl}_2)$ and $U_q(\mathfrak{sl}_n)$. As such, it gives a double centraliser formulation with the action of both quantum enveloping algebras on a tensor product of quantum Verma modules. In it, they found that it realises the Lawrence–Krammer–Bigelow (LKB) representations [JK11].

The last problem stems from a question of Tubbenhauer: *is it possible to find a diagrammatic calculus mimicking symmetric webs to replace $U_q(\mathfrak{sl}_2)$ in the duality?* A motivation to investigate lies in the fact that it is the case in the finite-dimensional case, in the quantum Howe duality outside Verma [RT16]. Symmetric webs offer a diagrammatic calculus for the category of the representations of $U_q(\mathfrak{sl}_n)$ and its presentation by generators and relation was proven in [CKM14]. Furthermore, it is of interest to note that this is somehow an extension of the Temperley–Lieb algebra calculus linked to $U_q(\mathfrak{sl}_2)$.

The goal of this project would be to extend this calculus outside finite-dimensional modules.

References

- [BCK19] J. Brundan, J. Comes, and J. R. Kujawa. “A Basis Theorem for the Degenerate Affine Oriented Brauer–Clifford Supercategory”. In: *Canad. J. Math.* 71.5 (2019), pp. 1061–1101. doi: [10.4153/CJM-2018-030-8](https://doi.org/10.4153/CJM-2018-030-8).
- [BLS19] G. Burrull, N. Libedinsky, and P. Sentinelli. “p-Jones-Wenzl idempotents”. In: *Advances in Mathematics* 352 (2019), pp. 246–264. doi: [10.1016/j.aim.2019.06.005](https://doi.org/10.1016/j.aim.2019.06.005).
- [CD15] K. Coulembier and H. De Bie. “Conformal symmetries of the super Dirac operator”. In: *Rev. Mat. Iberoam.* 31.2 (2015), pp. 373–410. doi: [10.4171/RMI/838](https://doi.org/10.4171/RMI/838).
- [CD20] D. Ciubotaru and M. De Martino. “The Dunkl–Cherednik deformation of a Howe duality”. In: *J. Algebra* 560 (2020), pp. 914–959. doi: [10.1016/j.jalgebra.2020.05.034](https://doi.org/10.1016/j.jalgebra.2020.05.034).
- [Ciu+20] D. Ciubotaru, H. De Bie, Marcelo De M., and R. Oste. “Deformations of unitary Howe dual pairs”. In: *arXiv:2009.05412* (2020). arXiv: 2009.05412.

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