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Diagrammatics and representations of algebras related to Howe and Schur–Weyl dualities

General context

My main research field is concerned with representation theory, a subfield of algebra. Algebra is concerned about the abstract structures that govern mathematics. For an algebraist, the symmetries of a molecule and the shuffling of a deck of cards are two examples of the same structure: group theory. The force of this abstraction is to enable simultaneous advancements in many seemingly unrelated fields. A result in group theory has implication for a magic tricks, or in the study of the formation of crystals.

Representation theory aims to study a structure by considering its action on known objects. Most often, it reduces a question in abstract algebra to one of *linear* algebra. When the representation is faithful, what happens inside it represent what is happening at the abstract level.

Not all algebras have the same ease of approach. I focus on algebras admitting a type of graphical calculus. This enables the statement of complex conditions by simple graphical rules. Figure 1 presents an example.

$$\mathsf{JW}_n = \mathsf{JW}_n^2,$$

$$e_i\mathsf{JW}_n = 0 = \mathsf{JW}_n e_i,$$

$$\mathsf{JW}_n = \mathsf{JW}_{n-1} - \frac{[n-1]_q}{[n]_q}\mathsf{JW}_{n-1}e_1\mathsf{JW}_{n-1},$$

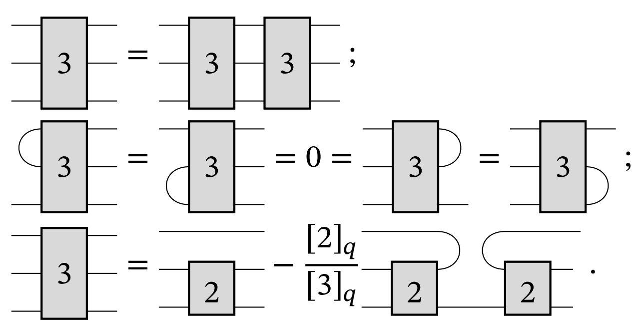

Figure 1: Algebraic properties of elements JW_n in the Temperley–Lieb algebras and their diagrammatic counterparts for $n = 3$.

Main research aims

Construct faithful representations and new diagrammatic calculus for algebras related to Howe and Schur–Weyl dualities. To begin with:

1. Study the representation theory of the total angular momentum algebra.
2. Study modular versions of the affine Temperley–Lieb algebras via relevant quotients.

The two dualities in question are both examples of a double centraliser property [CW12]: an object is studied on which a pair of algebras act. The duality expresses that they are each other commutant. This interaction offers a fruitful way to study the structure and often allows interesting combinatorics.

Proposed research plan

Total angular momentum algebra The first problem continues my PhD. The algebra studied can be defined as the supercentraliser of an $\mathfrak{osp}(1|2)$ realisation inside the tensor product of a rational Cherednik algebra [EG02] and a Clifford algebra, in one of its Howe dualities from the pair $(\mathrm{Pin}(d), \mathfrak{osp}(1|2))$. This means that the algebra depends on a reflection group W and a weight function $\varkappa$ invariant on W-orbits. Only the groups $W = \mathbb{Z}_2^N$ [DGV16] and $W = S_3$ [DOV18] had been studied before. As part of my PhD, I investigated $W = D_{2m}\times\mathbb{Z}_2$ [De +22] and $W = D_{2m}\times D_{2n}$.

As a first step, an ongoing collaboration with Marcelo De Martino and Roy Oste aims to extend to a stack of dihedral groups. Then I will investigate general group W. I aim to create diagrammatics for this algebra by combining Webster's diagrammatics for rational Cherednik algebras [Web17] by smashing in the calculus rules for Brauer–Clifford supercategory [BCK19]. A hint that these algebras encode interesting diagrammatics was already present in [FH15] where crossing relations representable via Temperley–Lieb algebras were found.

Quotients of affine Temperley–Lieb algebras The affine Temperley–Lieb algebra is an infinite-dimensional algebra that appears in conformal field theory and is linked to Virasoro algebras. Since the influential work of Graham and Lehrer [JG98], its representation theory has been a central object of interest, mainly via the study of its monoidal category.

The goal of this project would be to approach the affine Temperley–Lieb algebra via a quotient making it finite-dimensional. It was motivated by a question of Tubbenhauer coming from their recent work with Khovanov and Sitaraman [KST22] where they used the representation theory of diagrammatic algebras to make cryptographic protocols.

At the moment, we have defined the algebras and proved it is sandwich cellular [TV22], a generalisation of cellularity [JG96]. This gives ways to a study of its representation theory via its cell modules. Furthermore, we are able to compute the Jones-Wenzl projector for characteristic 0, so we could study a deformation akin to [LS20], with the modular case taken care of by the work of Spencer and Martin [MS22]. The first step will be to present the construction before diving in the modular case. This is part of an ongoing collaboration with Alexi Morin-Duchesne and Robert Spencer.

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