Quantitative Spatial Models in Economics: A Simple Commuting Model of Chicago

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This is a live document and subject to change.

Hog Butcher for the World,
Tool Maker, Stacker of Wheat,
Player with Railroads and the Nation's Freight Handler;
Stormy, husky, brawling,
City of the Big Shoulders...

Come and show me another city with lifted head singing
so proud to be alive and coarse and strong and cunning.

— Carl Sandburg

Abstract

This repository demonstrates the basics of conducting economics research with quantitative spatial models (QSMs). I derive and calibrate a simple quantitative spatial model of Chicago and conduct two counterfactual exercises. I then repeat this process for a richer model and compare the results. This exercise is intended for pedagogical purposes. To this end, the models below are simplified versions of QSMs commonly used in economics research. I welcome any comments and questions.

1. Introduction

The models presented below might strike you as restrictive and unrealistic. I have opted for an especially simple pair of models to demonstrate the basic mechanics of quantitative spatial models. This document will hopefully make the richer models of the literature more accessible. All quantitative spatial models (indeed, all economic models) necessarily abstract from certain features of reality. Results must always be interpreted in light of a researcher's modeling choices and the appropriateness of these choices for the research question at hand.

Ahlfeldt et al. (2015) and Monte, Redding, and Rossi-Hansberg (2018) are popular examples of such models and provide a good introduction to the literature. Redding and Rossi-Hansberg (2017) review the literature and common components of economic geography models. Models of economic geography often draw from the seminal work of Eaton and Kortum (2002) in international trade.

2. Model

I begin with a simple model of commuting to demonstrate the basic mechanics of a common form of QSM. I then extend this model to include (some) other relevant features of a spatial economy.

2.1. A Simple Model (Model A)

Chicago is comprised of discrete neighborhoods $i, n, k, l \in \mathcal{L}$. Each location i has a fixed mass R_i of residents.

2.1.1. Workers

Each agent inelastically supplies one unit of labor. An agent ω residing in location i and working in location n receives indirect[1] utility $\mathcal{U}_{ni}$, where

$$\mathcal{U}_{in\omega} = \left(\frac{w_n}{\kappa_{in}}\right) b_{in\omega}. \tag{1}$$

w_n is the wage paid in location n. κ_{in} is a commuting cost of the iceberg form in the units of utility. $b_{in\omega}$ is an idiosyncratic preference shock with a Fréchet distribution. The cumulative distribution function of $b_{in\omega}$ is given by $F_{in}(b_{in\omega}) = \exp\left(-b_{in\omega}^{-\theta}\right)$. θ governs the dispersion of this preference shock.

A worker ω in location i chooses the workplace that maximizes their indirect utility:

$$n_{i\omega}^* \overset{\text{def}}{=} \arg\max_{n \in \mathcal{L}} \mathcal{U}_{in\omega}. \tag{2}$$

Since workers differ only in their draws of $\{b_{in\omega}\}_{i,n \in \mathcal{L}}$ of preference shocks, we can drop the ω subscript in what follows. The Fréchet-distributed preference shock implies

$$\pi_{in \mid i} \overset{\text{def}}{=} \mathbb{P}\{n_i^* = n\} = \varphi_{in} \Phi_i^{-1},$$
$$\text{where} \quad \varphi_{in} \overset{\text{def}}{=} \left(\frac{w_n}{\kappa_{in}}\right)^\theta \tag{3}$$
$$\text{and} \ \Phi_i \overset{\text{def}}{=} \sum_{k \in \mathcal{L}} \varphi_{ik}.$$

Train (2003), especially the first three chapters, is an excellent resource on discrete choice models. If you are unfamiliar with the result in Equation 3, I recommend starting there. You can access the book at https://eml.berkeley.edu/books/choice2.html.[2]

2.1.2. Firms

A unit mass of perfectly competitive firms in each neighborhood produce a freely traded final good priced at 1 with the technology

$$Y_n = \tilde{A}_n L_n^\beta K_n^{1-\beta}, \tag{4}$$

where $\tilde{A}_n$ is a productivity parameter, L_n is labor input, and K_n is capital input. We assume that each neighborhood has a fixed stock of immobile capital $\overline{K}_n$, so it suffices to consider the production function

$$Y_n = A_n L_n^\beta, \tag{5}$$

where $A_n = \tilde{A}_n \overline{K}_n^{1-\beta}$. These perfectly competitive firms will pay workers their marginal product; wage and labor demand in neighborhood n are given by

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[1] I omit the subproblem of utility maximization given location choice for parsimony. Model B will explicitly discuss this subproblem, which nests the subproblem of utility maximization in this model. We can set $\alpha = 1$ to see the equivalence.

[2] I welcome your feedback. Would it be useful to include the derivation of Equation 3 here?

$$w_n = \beta A_n L_n^{\beta-1}$$
$$\implies L_n = \left(\frac{\beta A_n}{w_n}\right)^{\frac{1}{1-\beta}}. \tag{6}$$

We assume that capital owners spend all their rental income in the city so that goods market clearing still holds. This model shuts down a potentially important margin of adjustment: changing capital stocks. We could also consider firms that compete in the same land markets as residents (we introduce a residential land market in model B). Again, results must always be interpreted in light of such modeling choices and their appropriateness for the research question.

2.1.3. Commuting Equilibrium

For the commuting market to clear, labor demand in location n must equal labor supply to location n across all residential locations i:

$$L_n = \sum_{i\in\mathcal{L}} \pi_{in\,|\,i} R_i. \tag{7}$$

We can substitute Equation 3 and Equation 6 into this expression to obtain an equilibrium characterization:

$$\underbrace{\left(\frac{\beta A_n}{w_n}\right)^{\frac{1}{1-\beta}}}_{\text{Labor Demand}} = \underbrace{\sum_{i\in\mathcal{L}} \varphi_{in}\Phi_i^{-1} R_i}_{\text{Labor Supply}}. \tag{8}$$

We can now define the competitive equilibrium. Given the model parameters $\{\beta,\theta\}$, the vector of residential populations $\boldsymbol{R}$, and the exogenous location characteristics $\{\overline{\boldsymbol{K}}, \boldsymbol{A}, \boldsymbol{\kappa}\}$, the equilibrium is a vector of quantities $\boldsymbol{L}$ and a vector of prices $\boldsymbol{w}$ that satisfy Equation 3, Equation 7, and Equation 8.

> This section does not discuss the existence and uniqueness of the equilibrium. I will
> add sections on these topics in the future. As a note of caution, we cannot compare
> welfare between the two models, given the different utility functions.

2.1.4. Welfare

We can compute expected utility by considering indirect utility. Expected utility for an agent residing in location i will be given by the expected utility in their most attractive workplace:

$$U_i \stackrel{\text{def}}{=} \mathbb{E}\left[\mathcal{U}_{in_{i\omega}^*\omega}\right] = \sum_n \pi_{in\,|\,i}\mathbb{E}[\mathcal{U}_{in\omega}\mid n_{i\omega}^* = n], \tag{9}$$

where we have used the law of iterated expectations. In order to compute this expectation, we need the distribution of $\mathcal{U}_{in\omega}$ conditional on $n_{i\omega}^* = n$. We derive this distribution below:

$$\mathbb{P}\{\mathcal{U}_{in\omega} < u \mid n_{i\omega}^* = n\} = \frac{\mathbb{P}\{\mathcal{U}_{in\omega} < u \wedge n_{i\omega}^* = n\}}{\mathbb{P}\{n_{i\omega}^* = n\}}$$

$$= \frac{1}{\pi_{in \mid i}} \mathbb{P}\left\{b_{in\omega} < u\left(\frac{\kappa_{in}}{w_n}\right) \wedge n_{i\omega}^* = n\right\}$$

$$= \frac{1}{\pi_{in \mid i}} \int_0^{u\left(\frac{\kappa_{in}}{w_n}\right)} \prod_{k \neq n} F_{ik}\left(\frac{w_{in}}{\kappa_{in}}\frac{\kappa_{ik}}{w_{ik}}b\right) dF_{in}(b)$$

$$= \frac{1}{\pi_{in \mid i}} \int_0^{u\left(\frac{\kappa_{in}}{w_n}\right)} \prod_k F_{ik}\left(\frac{w_{in}}{\kappa_{in}}\frac{\kappa_{ik}}{w_{ik}}b\right) \theta b^{-\theta-1} db$$

$$= \frac{1}{\pi_{in \mid i}} \int_0^{u\left(\frac{\kappa_{in}}{w_n}\right)} \exp\left(-\sum_k \left(\frac{w_{in}}{\kappa_{in}}\frac{\kappa_{ik}}{w_{ik}}b\right)^{-\theta}\right) \theta b^{-\theta-1} db \qquad (10)$$

$$= \frac{1}{\pi_{in \mid i}} \int_0^{u\left(\frac{\kappa_{in}}{w_n}\right)} \exp\left(-\pi_{in \mid i}^{-1} b^{-\theta}\right) \theta b^{-\theta-1} db$$

$$= \frac{1}{\pi_{in \mid i}} \int_0^{u\left(\frac{\kappa_{in}}{w_n}\right)} \exp\left(-\pi_{in \mid i}^{-1} b^{-\theta}\right) \theta b^{-\theta-1} db$$

$$= \frac{1}{\pi_{in \mid i}} \left[\pi_{in \mid i} \exp\left(-\pi_{in \mid i}^{-1} b^{-\theta}\right) \Big|_0^{u\left(\frac{\kappa_{in}}{w_n}\right)}\right]$$

$$= \exp\left(-\Phi_i u^{-\theta}\right),$$

which is a Fréchet distribution with shape θ and scale parameter $\Phi_i^{\frac{1}{\theta}}$; this result is part of the Fréchet magic! We can then use the expression for the mean of a Fréchet distribution to compute the expected utility of an agent residing in location i:

$$U_i = \sum_n \pi_{in \mid i} \mathbb{E}[\mathcal{U}_{in\omega} \mid n_{i\omega}^* = n]$$

$$= \sum_n \pi_{in \mid i} \left[\Gamma\left(1 - \frac{1}{\theta}\right) \Phi_i^{\frac{1}{\theta}}\right] \qquad (11)$$

$$= \Gamma\left(1 - \frac{1}{\theta}\right) \Phi_i^{\frac{1}{\theta}}.$$

These derivations are used widely across the literature and usually relegated to an appendix or omitted entirely. Researchers will appeal to the standard results from discrete choice, as the steps are usually similar or identical across models. I would encourage you to derive these results yourself once or twice before appealing to the standard results.[3]

2.1.5. Counterfactual Equilibria

I will denote the vector-collection of a variable x_i over all locations with boldface: $\{x_i\}_{i \in \mathcal{L}} \overset{\text{def}}{=} \boldsymbol{x}$. We consider a baseline equilibrium $\{\boldsymbol{w}^0, \boldsymbol{\pi}^0\}$ for parameters $\{\boldsymbol{A}^0, \boldsymbol{\kappa}^0, \boldsymbol{R}^0\}$ and a counterfactual equilibrium $\{\boldsymbol{w}', \boldsymbol{\pi}'\}$ for parameters $\{\boldsymbol{A}', \boldsymbol{\kappa}', \boldsymbol{R}'\}$. We denote proportional changes with hats, e.g.,

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[3]Are any of the steps above unclear? Please let me know and I can add more exposition.

$$\hat{w}_n = \frac{w_n'}{w_n^0} \implies w_n^0 \hat{w}_n = w_n'. \tag{12}$$

This representation leads us to "exact hat algebra," a popular method to model and summarize counterfactual equilibria. We start by expressing the market clearing condition for the counterfactual equilibrium and then substitute in Equation 6:

$$L_n^0 \hat{L}_n = \left(\sum_{i \in \mathcal{L}} \left(\pi_{in \,|\, i}^0 R_i^0 \right) \left(\hat{\pi}_{in \,|\, i} \hat{R}_i \right) \right)$$

$$\implies \left(\frac{\hat{A}_n}{\hat{w}_n} \right)^{\frac{1}{1-\beta}} = \frac{\sum_{i \in \mathcal{L}} \left(\pi_{in \,|\, i}^0 R_i^0 \right) \left(\hat{\pi}_{in \,|\, i} \hat{R}_i \right)}{L_n^0}. \tag{13}$$

We can use Equation 3 to write

$$\hat{\pi}_{in \,|\, i} = \hat{\varphi}_{in} \hat{\Phi}_i^{-1},$$

$$\text{where} \quad \hat{\varphi}_{in} \stackrel{\text{def}}{=} \left(\frac{\hat{w}_n}{\hat{\kappa}_{in}} \right)^{\theta} \tag{14}$$

$$\text{and} \quad \hat{\Phi}_i \stackrel{\text{def}}{=} \sum_{k \in \mathcal{L}} \pi_{ik \,|\, i}^0 \hat{\varphi}_{ik}$$

The substantive piece of this expression is $\hat{\Phi}_i$. We derive it below:

$$\hat{\Phi}_i = \frac{\sum_{k \in \mathcal{L}} {\color{red}\varphi_{ik}^0} \hat{\varphi}_{ik}}{{\color{red}\sum_{l \in \mathcal{L}} \varphi_{il}^0}} = \sum_{k \in \mathcal{L}} \pi_{ik \,|\, i}^0 \hat{\varphi}_{ik}, \tag{15}$$

where we have used Equation 3 to substitute in for $\pi_{ik \,|\, i}^0$ (see the portions colored red). From Equation 14, we can then express the change in welfare

$$\hat{U}_i = \hat{\Phi}_i^{\frac{1}{\theta}}. \tag{16}$$

We now combine Equation 13 and Equation 14 to obtain

$$\left(\frac{\hat{A}_n}{\hat{w}_n} \right)^{\frac{1}{1-\beta}} = \left[\sum_{i \in \mathcal{L}} \frac{\pi_{in \,|\, i}^0 R_i^0 \hat{R}_i (\hat{w}_n / \hat{\kappa}_{in})^{\theta}}{\sum_{k \in \mathcal{L}} \pi_{ik \,|\, i}^0 (\hat{w}_k / \hat{\kappa}_{ik})^{\theta}} \right] \frac{1}{L_n^0}. \tag{17}$$

What does this characterization of a counterfactual equilibria buy us? If we express a counterfactual as a set of proportional changes to the parameter values $\left\{ \hat{A}, \hat{\kappa}, \hat{R} \right\}$, then we only need data on initial conditional commuting probabilities π^0, workplace population L^0, and residential population R^0 to solve for the proportional changes in wages $\hat{w}$ (using Equation 17) and conditional commuting probabilities $\hat{\pi}$ (using Equation 14).

Inspired by this representation, we define

$$\mathcal{Z}_n(\tilde{w}) \stackrel{\text{def}}{=} \left(\frac{\hat{A}_n}{\tilde{w}_n} \right)^{\frac{1}{1-\beta}} - \left[\sum_{i \in \mathcal{L}} \frac{\pi_{in \,|\, i}^0 R_i^0 \hat{R}_i (\tilde{w}_n / \hat{\kappa}_{in})^{\theta}}{\sum_{k \in \mathcal{L}} \pi_{ik \,|\, i}^0 (\tilde{w}_k / \hat{\kappa}_{ik})^{\theta}} \right] \frac{1}{L_n^0}. \tag{18}$$

We can use this vector-valued function $\mathcal{Z}(\tilde{\boldsymbol{w}})$ to compute the proportional changes in wages (and other equilibrium objects) in counterfactual equilibria. I provide pseudocode for this procedure below. I implement the algorithm in `analysis/qsm_implementation.ipynb`.

Model A Algorithm:
1. s = 0
2. ε = tolerance + 1
3. $\tilde{\boldsymbol{w}}^0 = \vec{1}$
4. **while** $\varepsilon >$ tolerance **do**
5. $\tilde{\boldsymbol{w}}^{s+1} = \tilde{\boldsymbol{w}}^s + \kappa_w \mathcal{Z}(\tilde{\boldsymbol{w}}^s)$
6. $\varepsilon = \max\{|\mathcal{Z}(\tilde{\boldsymbol{w}}^s)|\}$
7.   s = s + 1
8. **end while**
9. **return** $\tilde{\boldsymbol{w}}^s$

It would be appropriate to discuss zero flows here. I will do so in the future.

2.2. A Richer Model (Model B)

We now consider a model with a housing market and residential choice. The mass of agents is denoted $\overline{R}$. We no longer fix an agent's residential location.

2.2.1. Setup

Utility for an agent ω residing in location i and working in location n is given by

$$U_{in\omega} = \left(\frac{c_{in\omega}}{\alpha}\right)^{\alpha} \left(\frac{h_{in\omega}}{1-\alpha}\right)^{1-\alpha} \frac{b_{in\omega}}{\kappa_{in}} \tag{19}$$

where $c_{in\omega}$ is final good consumption, $h_{in\omega}$ is housing consumption, and $F_{in}(b_{in\omega}) = \exp\left(-B_{in} b_{in\omega}^{-\theta}\right)$. We've added a parameter B_{in} that governs average utility for agents that live in location i and work in location n. The Cobb-Douglas form of Equation 19 implies that agents spend a constant fraction α of their income on the final good and $(1-\alpha)$ on housing. The price of the final good is again 1, and we denote the price of housing in location i by q_i. Accordingly, indirect utility for an agent ω residing in location i and working in location n with wage w_n is given by

$$\mathcal{U}_{in\omega} = \left(\frac{\alpha w_n}{\alpha}\right)^{\alpha} \left(\frac{\alpha w_n}{q_i(1-\alpha)}\right)^{1-\alpha} \frac{b_{in\omega}}{\kappa_{in}} = \left(\frac{w_n q_i^{\alpha-1}}{\kappa_{in}}\right) b_{in\omega}. \tag{20}$$

A worker ω now chooses both a residence and workplace:

$$\{i, n\}_{\omega}^* \stackrel{\text{def}}{=} \arg\max_{i,n\in\mathcal{L}} \mathcal{U}_{in\omega}. \tag{21}$$

Similar to before, the Fréchet-distributed preference shock implies the following expression for the *unconditional* residential and commuting probability

$$\pi_{in} \stackrel{\text{def}}{=} \mathbb{P}\{\{i,n\}^* = \{i,n\}\} = \varphi_{in}\Phi^{-1},$$

$$\text{where} \quad \varphi_{in} \stackrel{\text{def}}{=} B_{in}\left(\frac{w_n q_i^{\alpha-1}}{\kappa_{in}}\right)^{\theta} \tag{22}$$

$$\text{and} \quad \Phi \stackrel{\text{def}}{=} \sum_{k \in \mathcal{L}} \sum_{l \in \mathcal{L}} \varphi_{kl}.$$

In what follows, it will be useful to define the mass of residents in each location i

$$R_i \stackrel{\text{def}}{=} \sum_{n \in \mathcal{L}} \pi_{in}\overline{R}, \tag{23}$$

following the notation from Model A.

2.2.2. Housing Market

Each location i has a fixed stock of land available for rent H_i. Landlords face no costs and spend all of their rental income on the final good to ensure goods market clearing. Let $\overline{\nu}_i$ denote the average income of residents in location i. We can than express *aggregate* income for resident in location i

$$\overline{\nu}_i R_i = \sum_{n \in \mathcal{L}} \pi_{in} w_n \overline{R}. \tag{24}$$

Land market clearing implies that housing expenditure (given by utility maximization) must equal landlord income in neighborhood i:

$$\underbrace{(1-\alpha)\overline{\nu}_i R_i}_{\text{Housing Expenditure}} = \underbrace{H_i q_i}_{\text{Landlord Income}} \tag{25}$$

2.2.3. Firms

We maintain the same set of assumptions on the firm side as in Model A. This yields the wage equation and labor demand

$$w_n = \beta A_n L_n^{\beta-1}$$

$$\implies L_n = \left(\frac{\beta A_n}{w_n}\right)^{\frac{1}{1-\beta}}. \tag{26}$$

2.2.4. Commuting Equilibrium

We now use the unconditional commuting probability in Equation 22 to define the commuting market clearing condition:

$$L_n = \sum_{i \in \mathcal{L}} \pi_{in}\overline{R}. \tag{27}$$

We can now define the competitive equilibrium. Given the model parameters $\{\alpha, \beta, \theta\}$, the residential population $\overline{R}$, and the exogenous location characteristics $\{\boldsymbol{H}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\kappa}\}$, the equilibrium is a vector of quantities $\{\boldsymbol{L}, \boldsymbol{R}\}$ and a vector of prices $\{\boldsymbol{w}, \boldsymbol{q}\}$ that satisfy Equation 22, Equation 24, Equation 25, Equation 26, and Equation 27.

2.2.5. Welfare

Given free residential mobility, utility is now equalized across space:

$$U \stackrel{\text{def}}{=} \mathbb{E}\left[\mathcal{U}_{\{in\}^*_\omega \omega}\right] = \Gamma\left(1 - \frac{1}{\theta}\right)\Phi^{\frac{1}{\theta}}.$$ (28)

The derivation follows the same steps as in Model A.

2.2.6. Counterfactual Equilibria

We proceed as in model A and derive the exact hat system.

$$\hat{\varphi}_{in} = \hat{B}_{in}\left(\frac{\hat{w}_n \hat{q}_i^{\alpha-1}}{\hat{\kappa}_{in}}\right)^\theta$$

$$\hat{\pi}_{in} = \frac{\hat{\varphi}_{in}}{\sum_{k\in\mathcal{L}}\sum_{l\in\mathcal{L}} \pi^0_{kl}\hat{\varphi}_{kl}}$$

$$\hat{R}_i = \frac{\overline{R}^0 \widehat{\overline{R}}}{R^0_i} \sum_{n\in\mathcal{L}} \pi^0_{in}\hat{\pi}_{in}$$

$$\left(\frac{\hat{A}_n}{\hat{w}_n}\right)^{\frac{1}{1-\beta}} = \left(\frac{\overline{R}^0 \widehat{\overline{R}}}{L^0_n}\right) \sum_{i\in\mathcal{L}} \pi^0_{in}\hat{\pi}_{in}$$ (29)

$$\hat{\overline{\nu}}_i \hat{R}_i = \widehat{\overline{R}}\left(\frac{\sum_{n\in\mathcal{L}} \pi^0_{in} w^0_n \hat{\pi}_{in}\hat{w}_n}{\sum_{k\in\mathcal{L}} \pi^0_{ik} w^0_k}\right)$$

$$\hat{q}_i = \frac{\hat{\overline{\nu}}_i \hat{R}_i}{\widehat{H}_i}$$

$$\hat{U} = \hat{\Phi}^{\frac{1}{\theta}}.$$

We combine the expressions from above and define

$$\mathcal{Z}_n(\tilde{w}, \tilde{q}) \stackrel{\text{def}}{=} \left(\frac{\hat{A}_n}{\hat{w}_n}\right)^{\frac{1}{1-\beta}} - \left(\frac{\overline{R}^0 \widehat{\overline{R}}}{L^0_n}\right) \sum_{i\in\mathcal{L}} \frac{\pi^0_{in}\hat{B}_{in}(\hat{w}_n \hat{q}_i^{\alpha-1}/\hat{\kappa}_{in})^\theta}{\sum_{k\in\mathcal{L}}\sum_{l\in\mathcal{L}} \pi^0_{kl}\hat{B}_{kl}(\hat{w}_l \hat{q}_k^{\alpha-1}/\hat{\kappa}_{kl})^\theta}$$

$$\mathcal{Q}_i(\tilde{w}, \tilde{q}) \stackrel{\text{def}}{=} \left(\frac{\widehat{\overline{R}}}{\widehat{H}_i}\right)\left(\frac{\sum_{n\in\mathcal{L}} \pi^0_{in} w^0_n \hat{\pi}_{in}\hat{w}_n}{\sum_{k\in\mathcal{L}} \pi^0_{ik} w^0_k}\right).$$ (30)

> **Model B Algorithm:**
> 1. $s = 0$
> 2. $\varepsilon = \text{tolerance} + 1$
> 3. $\tilde{w}^0 = \tilde{q}^0 = \vec{1}$
> 4. **while** $\varepsilon > \text{tolerance}$ **do**
> 5. $\quad \tilde{q}^{s+1} = (1 - \kappa_q)\tilde{q}^s + \kappa_q \mathcal{Q}(\tilde{w}^s, \tilde{q}^s)$
> 6. $\quad \tilde{w}^{s+1} = \tilde{w}^s + \kappa_w \mathcal{Z}(\tilde{w}^s, \tilde{q}^s)$

7. $\varepsilon = \max\{|\mathcal{Z}(\tilde{w}^s, \tilde{q}^s)|\}$

8. $s = s + 1$

9. **end while**

10. **return** $\tilde{w}^s, \tilde{q}^s$
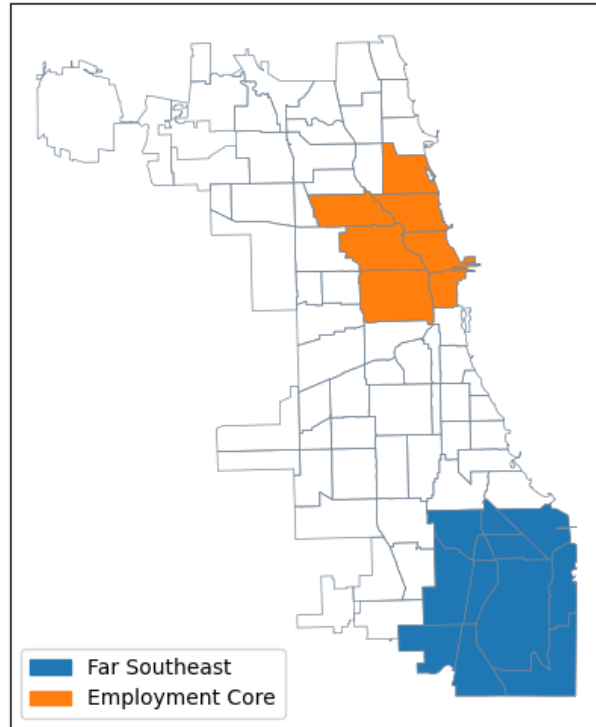
3. Data and Calibration

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I will discuss the data and parameter choices used to generate the results below. The
code to generate the data is available upon request and will be incorporated into a
future version of this repository.
```

4. Counterfactual Exercises

```
I plan to summarize equilibrium changes for a larger set of endogenous variables and a
discussion of the differences between the two models.
```

I compare the equilibrium impact of two parameter shocks: a 5% increase in productivity in the Far Southeast Side and a 5% reduction in commuting costs from the Far Southeast to Chicago's employment core (the Loop, Near North Side, and Near West Side). For interpretation, it is important to note that the $\hat{w}$ reports the changes in wages *paid* to agents working in a given location.

Figure 1: Impacted Neighborhoods

4.1. Wages

Figure 2: $\hat{w}$
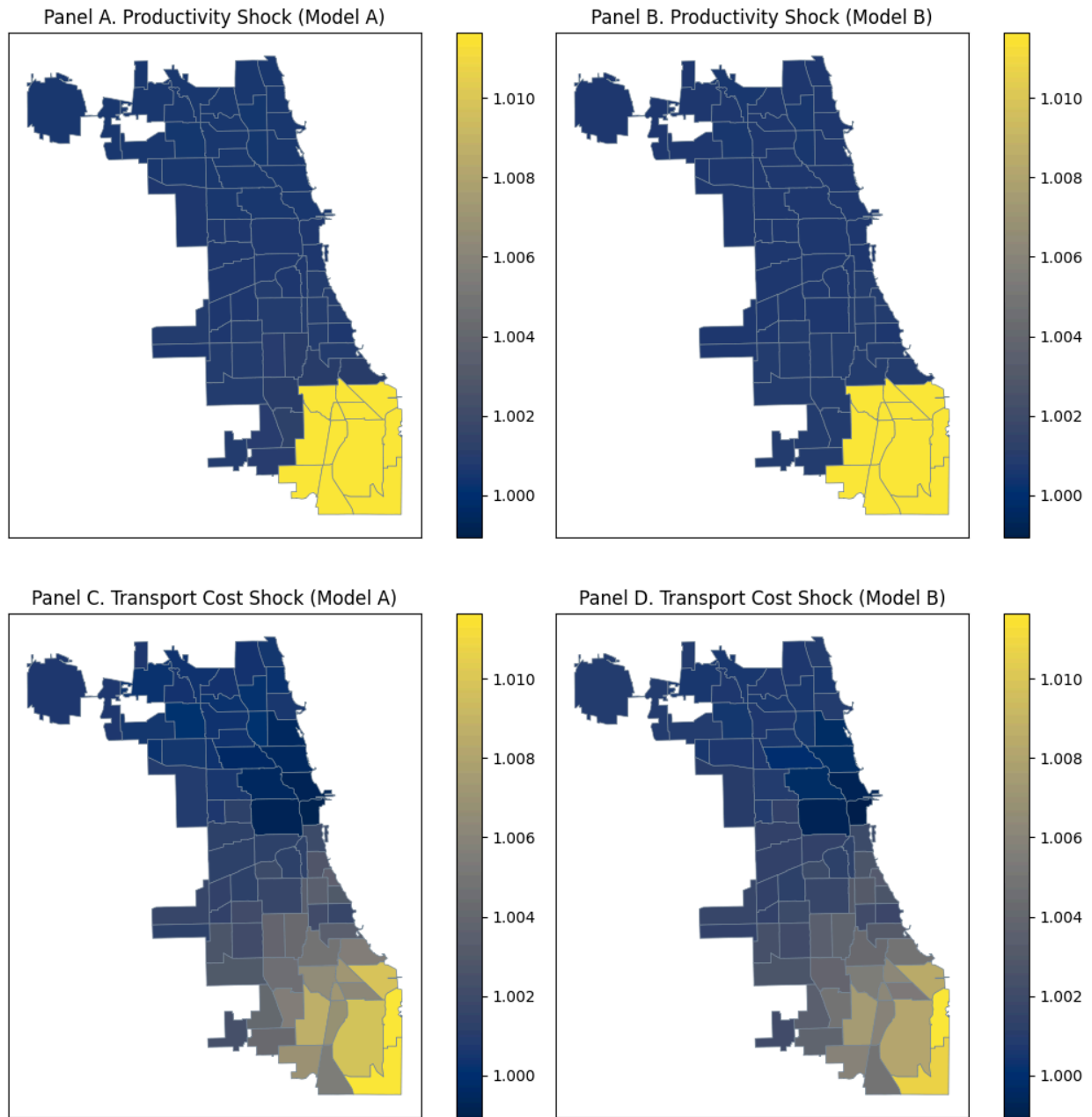
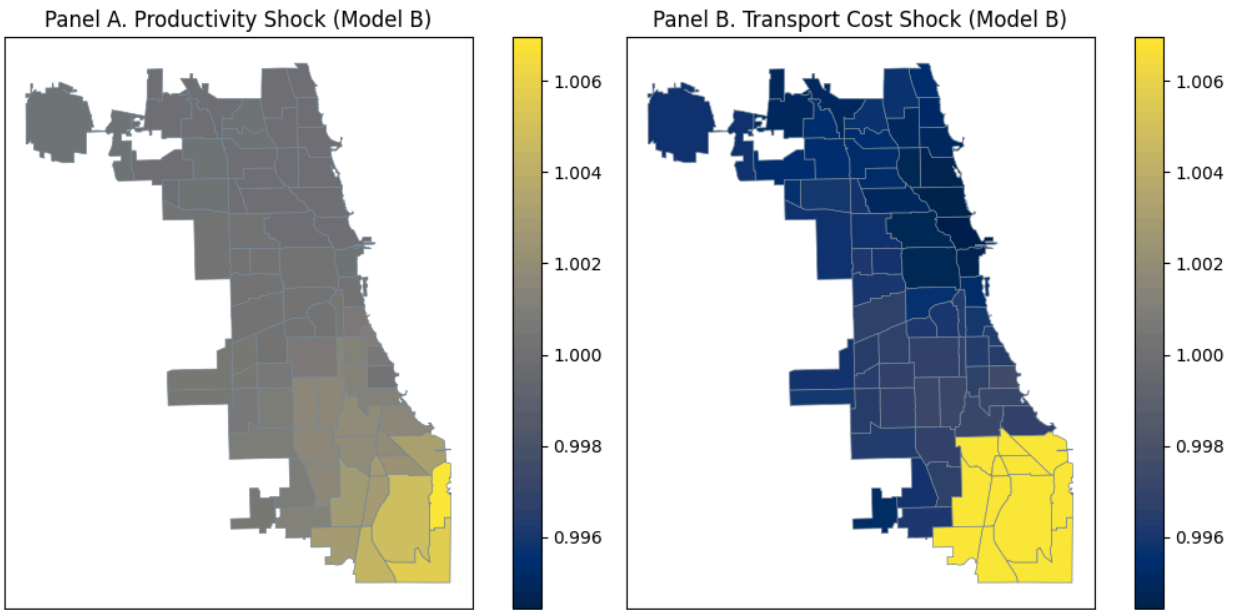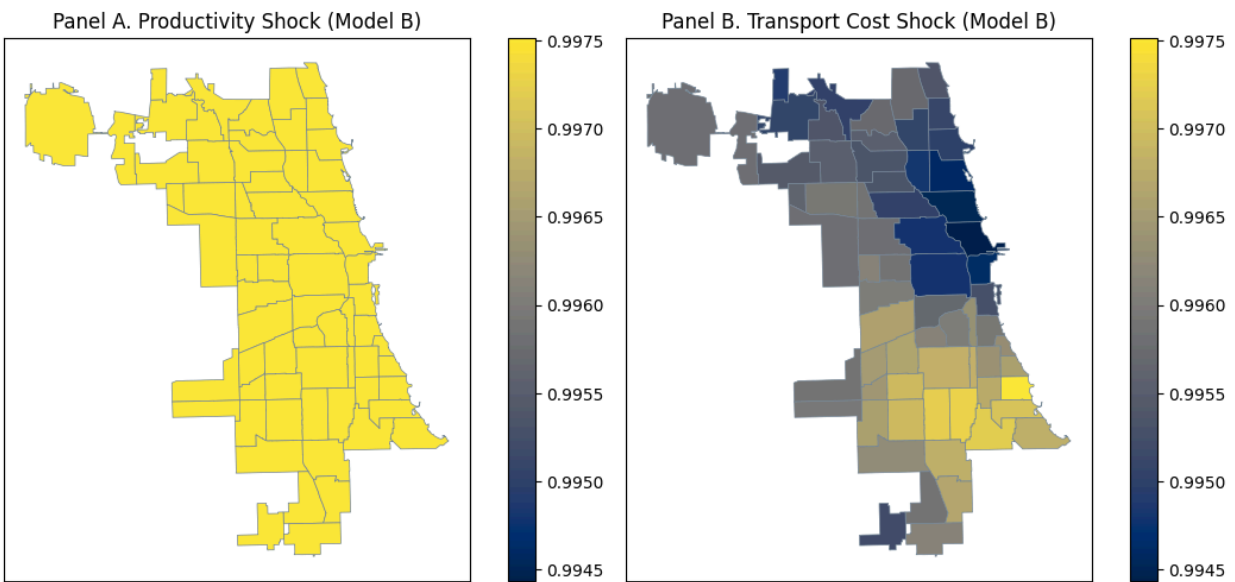Figure 3: $\hat{w}$

4.2. Rents

Figure 4: $\hat{q}$

Panel A. Productivity Shock (Model B)

Panel B. Transport Cost Shock (Model B)

Figure 5: $\hat{q}$

Panel A. Productivity Shock (Model B)

Panel B. Transport Cost Shock (Model B)
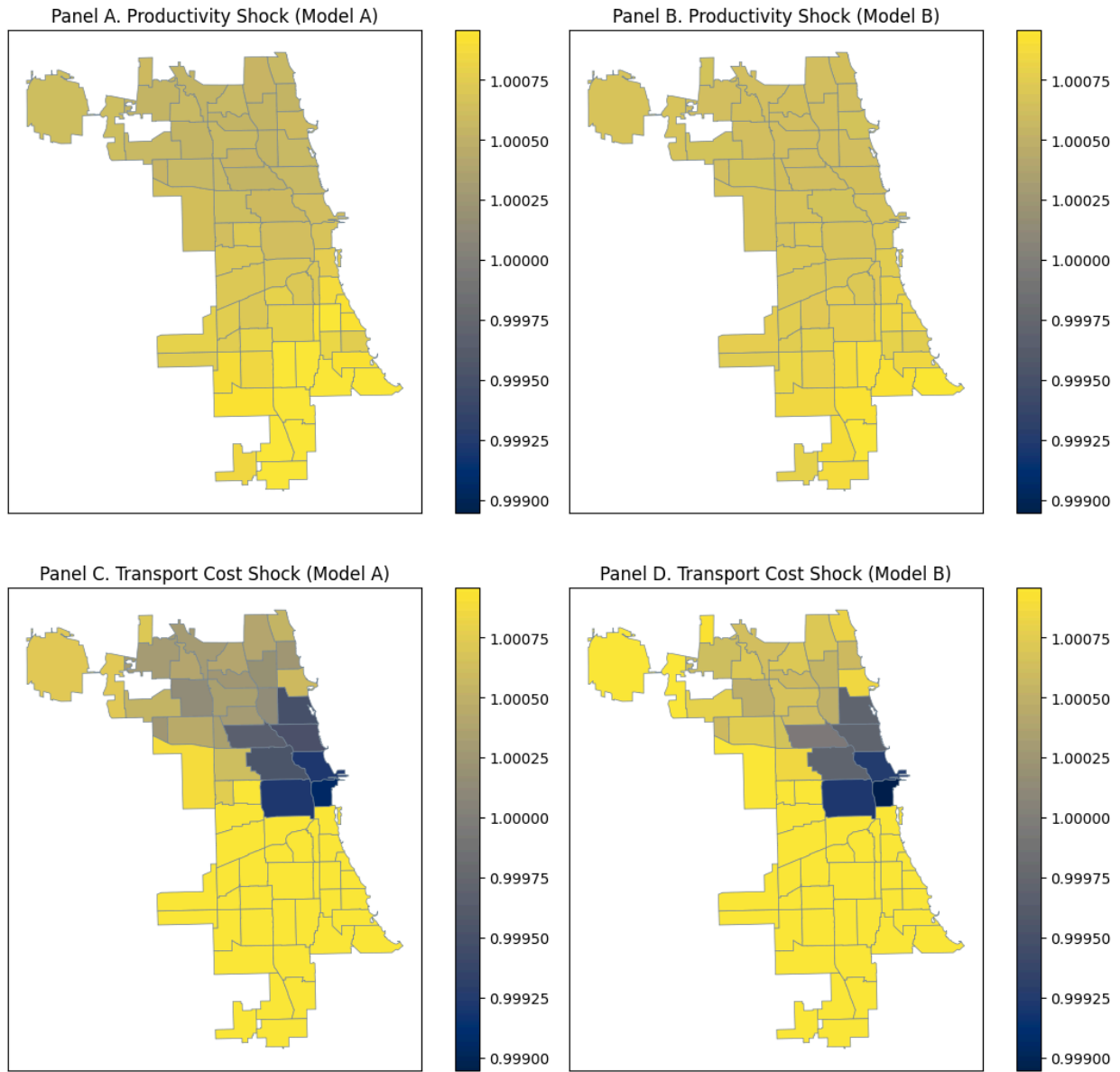
4.3. Labor Supply

Figure 6: $\hat{L}$
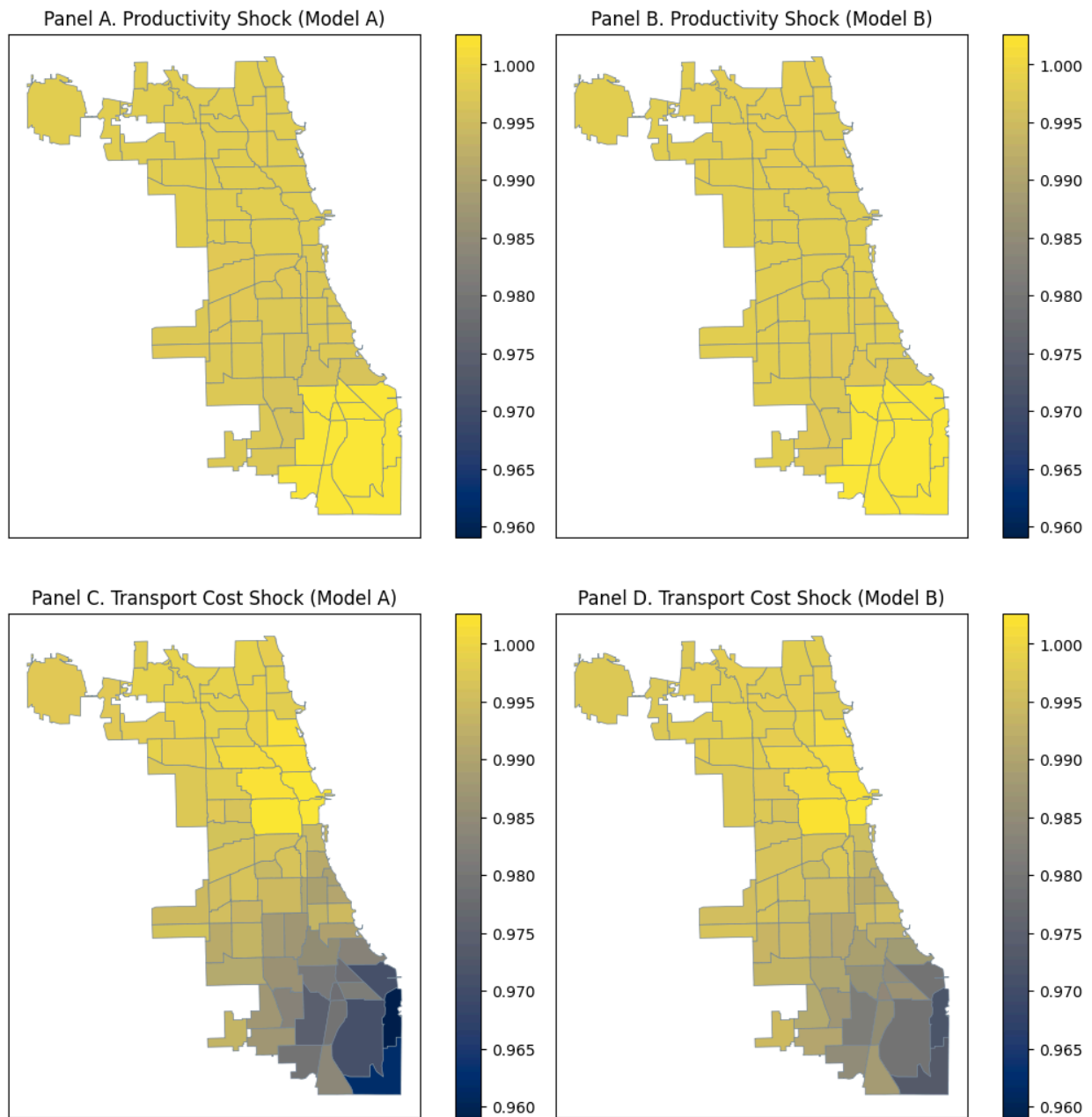
Figure 7: $\hat{L}$
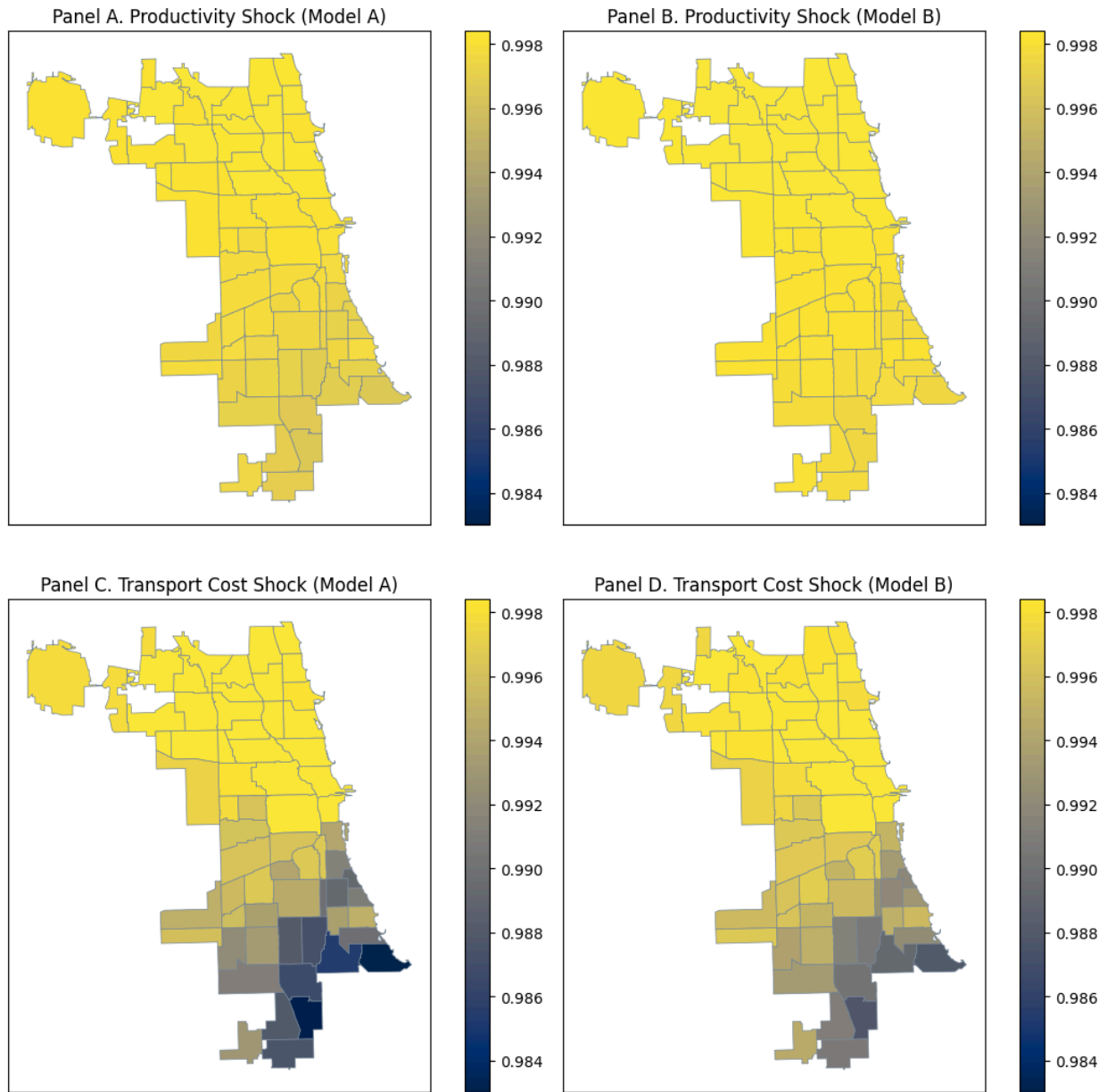
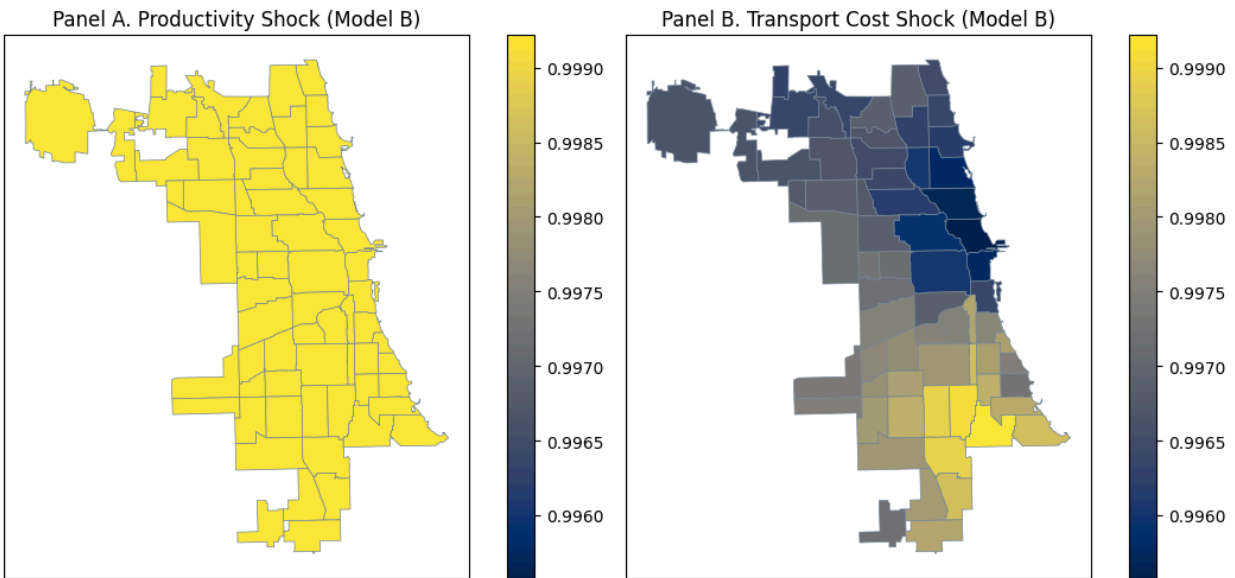## 4.4. Residential Population

Figure 8: $\widehat{R}$

Panel A. Productivity Shock (Model B)

Panel B. Transport Cost Shock (Model B)

Figure 9: $\widehat{R}$

Panel A. Productivity Shock (Model B)

Panel B. Transport Cost Shock (Model B)
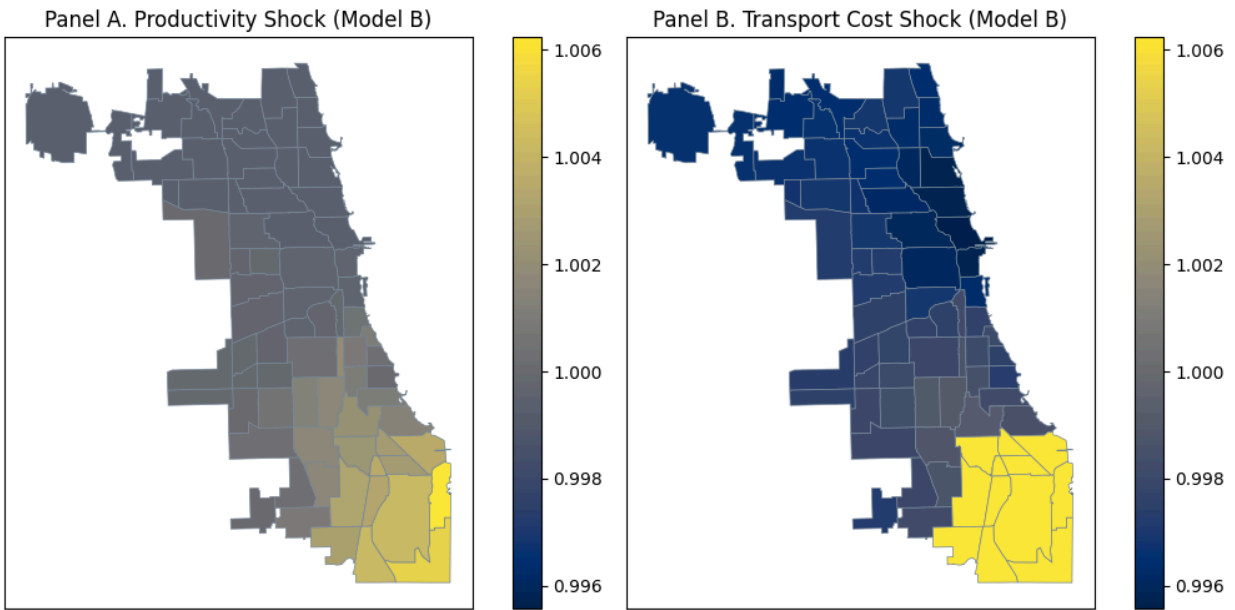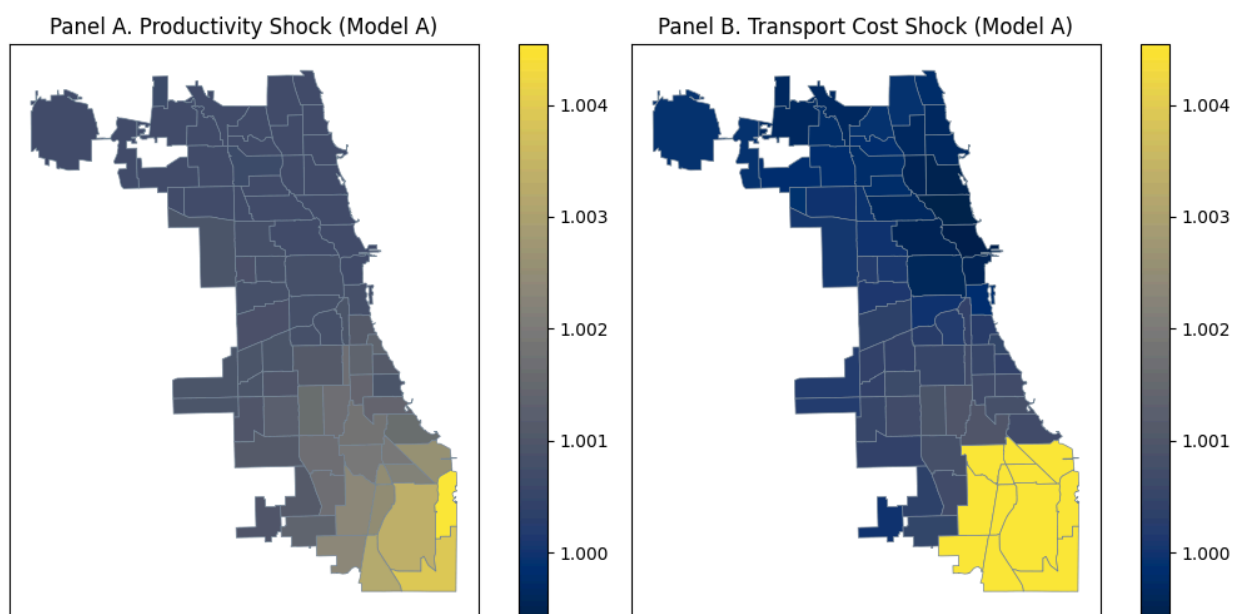
4.5. Welfare

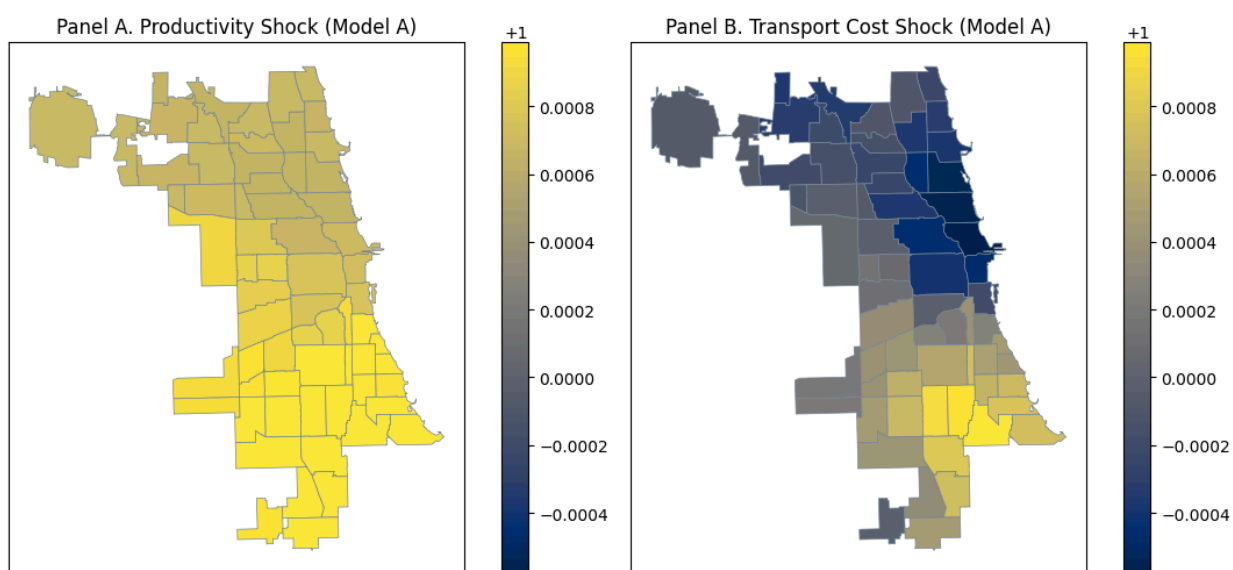Figure 10: $\widehat{U}$

Figure 11: $\widehat{U}$

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