Escuela de Ciencias Físicas y Matemáticas
Tarea 4
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Problema 1. *Resolver*

$$x^2y'' + x(x + \frac{1}{2})y' + xy = 0$$

Comencemos a ver los puntos singulares, para ver si toca Frobenius.

$$y'' + (1 + \frac{1}{2x})y' + \frac{1}{x}y = 0$$

$$\text{Donde los puntos singulares serán:} \frac{1}{2x}, \frac{1}{x} | x = 0 \in A'$$

$$y = \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty} (n + r)a_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n + r) * (n + r - 1)a_n x^{n+r-2}$$

$$\sum_{n=0}^{\infty} (n + r) * (n + r - 1)a_n x^{n+r} + \sum_{n=0}^{\infty} (n + r)a_n x^{n+r+1} + \frac{1}{2}\sum_{n=0}^{\infty} (n + r)a_n x^{n+r} + \sum_{n=0}^{\infty} a_n x^{n+r+1} = 0$$

$$m = n + 1, n = m - 1 | m = n$$

$$\sum_{m=0}^{\infty} (m + r) * (m + r - 1)a_m x^{m+r} + \sum_{m=1}^{\infty} (m - 1 + r)a_{m-1} x^{m+r} + \frac{1}{2}\sum_{m=0}^{\infty} (m + r)a_m x^{m+r} + \sum_{m=1}^{\infty} a_{m-1} x^{m+r} = 0$$

$$m = 1$$

$$\sum_{m=1}^{\infty} (m + r) * (m + r - 1)a_m x^{m+r} + \sum_{m=1}^{\infty} (m - 1 + r)a_{m-1} x^{m+r} + \frac{1}{2}\sum_{m=1}^{\infty} (m + r)a_m x^{m+r} + \sum_{m=1}^{\infty} a_{m-1} x^{m+r}$$

$$+(r) * (r - 1)a_0 x^r + \frac{1}{2}(r)a_0 x^r = 0$$

$$\sum_{m=1}^{\infty} [[(m + r) * (m + r - 1) + \frac{1}{2}(m + r)]a_m + [(m - 1 + r) + 1]a_{m-1}]x^{m+r} + (r) * (r - 1)a_0 x^r + \frac{1}{2}(r)a_0 x^r = 0$$

por independencia lineal

$$r^2 - \frac{1}{2}(r) = 0 \Longrightarrow r(r - \frac{1}{2}) = 0$$

$$\begin{cases} r_1 = 0 \\ r_2 = \frac{1}{2} \end{cases} \tag{1}$$

$$a_m = -\frac{[(m-1+r)+1]a_{m-1}}{[(m+r)*(m+r-1)+\frac{1}{2}(m+r)]}$$

$$r_1 = 0$$

$$a_m = -\frac{[m]a_{m-1}}{[(m)*(m-1)+\frac{1}{2}(m)]}$$

$$m >= 1$$

$$\begin{cases} a_1 = -2a_0 \\ a_2 = \frac{4a_0}{3} \\ a_3 = -\frac{8a_0}{15} \end{cases} \tag{2}$$

$$r_2 = \frac{1}{2}$$

$$a_m = -\frac{[(m+\frac{1}{2})]a_{m-1}}{[(m+\frac{1}{2})*(m-\frac{1}{2})+\frac{1}{2}(m+\frac{1}{2})]}$$

$$m >= 1$$

$$\begin{cases} a_1 = -a_0 \\ a_2 = \frac{a_0}{2} \\ a_3 = -\frac{a_0}{6} \end{cases} \tag{3}$$

Así siendo la solución

$$y = A(1 - 2x + \frac{4x^2}{3} - \frac{8x^3}{15} + \cdots) + \sqrt{x}B(1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots)\blacksquare$$

Problema 2. *Resolver*

$$2x^2y'' + 3xy' - (2x-1)y = 0$$

sucesivas.

Comencemos a ver los puntos singulares, para ver si toca Frobenius.

$$2x^2y'' + 3xy' - (2x-1)y = 0$$

Donde los puntos singulares serán:

$$\frac{2}{3x}, \frac{1}{x} - \frac{1}{2x^2}|x = 0 \in A'$$

$$y = \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty} (n+r)a_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty} (n+r) * (n+r-1)a_n x^{n+r-2}$$

$$2\sum_{n=0}^{\infty} (n+r) * (n+r-1)a_n x^{n+r} + 3\sum_{n=0}^{\infty} (n+r)a_n x^{n+r} + 2\sum_{n=0}^{\infty} a_n x^{n+r+1} - \sum_{n=0}^{\infty} a_n x^{n+r} = 0$$

$$m = n+1, n = m-1|m = n$$

$$2\sum_{m=0}^{\infty} (m+r) * (m+r-1)a_m x^{m+r} + 3\sum_{m=0}^{\infty} (m+r)a_m x^{m+r} + 2\sum_{m=1}^{\infty} a_{m-1} x^{m+r} - \sum_{m=0}^{\infty} a_m x^{m+r} = 0$$

por ser Linealmente independiente, queda como:

$$2r^2 + r - 1 = 0$$

$$\begin{cases} r_1 = -1 \\ r_2 = \frac{1}{2} \end{cases} \tag{4}$$

$$\sum_{m=1}^{\infty} [(2(m+r) * (m+r-1) + 3(m+r) - 1)a_m + 2a_{m-1}]x^{m+r} = 0$$

$$a_m = \frac{-2a_{m-1}}{[((m+r) * (2m+2r+1) - 1)]}$$

$$r_1 = -1$$

$$a_m = \frac{-2a_{m-1}}{[((m-1) * (2m-1) - 1)]}$$

$$n >= 1$$

$$\begin{cases} a_2 = 2a_0 \\ a_2 = -2a_0 \\ a_3 = \frac{4a_0}{9} \end{cases} \tag{5}$$

$$r_1 = \frac{1}{2}$$

$$a_m = \frac{-2a_{m-1}}{[((m+\frac{1}{2})*(2m+2)-1)]}$$

$$n >= 1$$

$$\begin{cases} a_1 = -2\frac{a_0}{5} \\ a_2 = 2\frac{a_0}{35} \end{cases} \tag{6}$$

Así siendo la solución

$$y = A * x^{-1}(1 - 2x - 2x^2 + \frac{4x^3}{9} + \cdots) + \sqrt{x}B(1 - \frac{2x^2}{5} + \frac{2x^2}{35} + \cdots)\blacksquare$$

Problema 3. *Resolver*

$$2xy'' + (x+1)y' + 3y = 0$$

$$2xy'' + (x+1)y' + 3y = 0$$

Donde los puntos singulares serán:

$$\frac{1}{2} + \frac{1}{2x}, \frac{3}{2x}|x = 0 \in A'$$

donde $P(x)$ y $Q(x)$, no son analíticas en $x_0 = 0 \in A'$ así que usaremos Frobenius.

$$y = \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$y' = \sum_{n=0}^{\infty}(n+r)a_n x^{n+r-1}$$

$$y'' = \sum_{n=0}^{\infty}(n+r)*(n+r-1)a_n x^{n+r-2}$$

$$2\sum_{n=0}^{\infty}(n+r)*(n+r-1)a_n x^{n+r-1} + \sum_{n=0}^{\infty}(n+r)a_n x^{n+r} + \sum_{n=0}^{\infty}(n+r)a_n x^{n+r-1} + 3\sum_{n=0}^{\infty}a_n x^{n+r} = 0$$

$$m = n-1, n = m+1|m = n$$

$$2\sum_{m=-1}^{\infty}(m+1+r)*(m+r)a_{m+1}x^{m+r}+\sum_{m=0}^{\infty}(m+r)a_mx^{m+r}+\sum_{m=-1}^{\infty}(m+1+r)a_{m+1}x^{m+r}+3\sum_{m=0}^{\infty}a_mx^{m+r}=0$$

$$m=0$$

al ser linealmente independientes las soluciones de series

$$2(r)*(r-1)a_{-1}x^{r-1}+(r)a_{-1}x^{r-1}\longrightarrow r_1=0,r_2=\frac{1}{2}$$

$$[2[(m+1+r)*(m+r)+(m+1+r)]a_{m+1}+[(m+r)+3]a_m]=0$$

$$a_{m+1}=-\frac{[(m+r)+3]a_m}{[2(m+1+r)*(m+r)+(m+1+r)]}$$

$$a_{m+1}=-\frac{[(m+3)]a_m}{[2(m)*(m+1)+(m+1)]}$$

$$\begin{cases} a_2=-3a_1 \\ a_3=2a_1 \\ a_4=-\frac{2a_1}{3} \end{cases} \tag{7}$$

$$a_{m+1}=-\frac{[(m+\frac{7}{2})]a_m}{[2(m+\frac{3}{2})*(m+\frac{1}{2})+(m+\frac{3}{2})]}$$

$$\begin{cases} a_2=-\frac{7a_1}{6} \\ a_3=\frac{21a_0}{40} \end{cases} \tag{8}$$

Asi siendo la solucion

$$y=A(1+3x+2x^2-\frac{2x^3}{3}+\cdots)+x^{\frac{1}{2}}B(1-\frac{7x}{6}+\frac{21x^2}{40}+\cdots)\blacksquare$$