Escuela de Ciencias Físicas y Matemáticas
Tarea 5
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Problema 1.

$$J_{-k}(x) = (-1)^k J_k(x)$$

$$J_{-k}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)\Gamma(n+1-k)}(\frac{x}{2})^{2n-k}$$

$$(-1)^k J_k(x) = (-1)^k \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)\Gamma(n+1+k)}(\frac{x}{2})^{2n+k}$$

$$(-1)^k J_k(x) = \sum_{n=0}^{\infty} \frac{(-1)^{k+n}}{\Gamma(n+1)\Gamma(n+1+k)}(\frac{x}{2})^{2n+k}$$

$$n+k = u | n = u-k$$

$$\sum_{u=k}^{\infty} \frac{(-1)^u}{\Gamma(u-k+1)\Gamma(u+1)}(\frac{x}{2})^{2(u-k)+k}$$

$$\sum_{u=k}^{\infty} \frac{(-1)^u}{\Gamma(u-k+1)\Gamma(u+1)}(\frac{x}{2})^{2u-k} = J_{-k}(x)$$

con u = k, con n =0, asi sabiendo que empezara desde cero, para cualquier k, mayor o igual que cero

$$(-1)^k J_k(x) = J_{-k}(x) | \forall k \in I \blacksquare$$

Problema 2.

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^k J_k(x)] = x^k J_{k-1}(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^k J_k(x)] = x^k J_{k-1}(x)$$

$$J_k(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}(\frac{x}{2})^{2n+k}$$

$$x^k J_k(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}\left(\frac{x^{2n+2k}}{2^{2n+k}}\right)$$

$$\frac{d}{dx}\left[x^k J_k(x)\right] = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}\left(2(n+k)\frac{x^{2n+2k-1}}{2^{2n+k}}\right)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k-1)!}\left(\frac{x^{2n+2k-1}}{2^{2n+k-1}}\right)$$

$$= x^k \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k-1)!}\left(\frac{x^{2n+k-1}}{2^{2n+k-1}}\right)$$

$$= x^k \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k-1)!}\left(\frac{x}{2}\right)^{2n+k-1}$$

$$= x^k J_{k-1}(x) \blacksquare$$

Problema 3.

$$J_k'(x) + kx^{-1}J_k(x) = J_{k-1}(x)$$

$$J_k(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}(\frac{x}{2})^{2n+k}$$

$$J_k'(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}(\frac{2n+k}{2})(\frac{x}{2})^{2n+k-1}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}(2n+k)(\frac{x^{2n+k-1}}{2^{2n+k}})$$

$$kx^{-1}J_k(x) = \sum_{n=0}^{\infty} \frac{(-1)^n * k}{(n)!(n+k)!}(\frac{x^{2n+k-1}}{2^{2n+k}})$$

$$J_k'(x) + kx^{-1}J_k(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}[2n+k+k](\frac{x^{2n+k-1}}{2^{2n+k}})$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}[2n+2k](\frac{x^{2n+k-1}}{2^{2n+k}})$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k)!}[n+k](\frac{x^{2n+k-1}}{2^{2n+k-1}})$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+k-1)!}[n+k](\frac{x}{2})^{2n+k-1}$$

$$= J_{k-1}(x)\blacksquare$$

Problema 4.

$$\frac{\mathrm{d}}{\mathrm{d}x}[J_0(x)] = -J_1(x)$$

donde

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n)!}(\frac{x}{2})^{2n}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!^2}(\frac{x}{2})^{2n}$$

Así

$$\frac{\mathrm{d}}{\mathrm{d}x}[J_0(x)] = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!^2}(2n\frac{x^{2n-1}}{2^{2n}})$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!^2}(2n\frac{x^{2n-1}}{2^{2n}})$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n-1)!(n)!}(\frac{x}{2})^{2n-1}$$

$$= J_{-1}(x) \text{ esto por el Ejercicio 1}$$

Entonces se obtiene

$$J_{-1}(x) = (-1)J_1(x)$$

$$= -J_1(x)\blacksquare$$

Problema 5.

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$$

$$J_{\frac{1}{2}}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+\frac{1}{2})!}(\frac{x}{2})^{2n+\frac{1}{2}}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+\frac{1}{2})!} (\frac{x}{2})^{2n} \sqrt{\frac{x}{2}}$$

$$= \sqrt{\frac{x}{2}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n)!(n+\frac{1}{2})!} (\frac{x}{2})^{2n}$$

$$= \sqrt{\frac{x}{2}} [\frac{1}{(\frac{1}{2}!)} - \frac{x^2}{4 * \frac{3}{2}!} + ...]$$

$$= \sqrt{\frac{x}{2}} [\frac{2}{\sqrt{\pi}} - \frac{x^2}{4} \frac{4}{3\sqrt{\pi}} + \frac{x^4}{16 * 2!} \frac{8}{15\sqrt{\pi}} - ...]$$

$$= \sqrt{\frac{2}{\pi}} \sqrt{x} [1 - \frac{x^2}{6} + \frac{x^4}{120} - ...]$$

$$= \sqrt{\frac{2}{\pi x}} [x - \frac{x^3}{3!} + \frac{x^5}{5!} - ...(-1)^n \frac{x^{2n+1}}{(2n+1)!} + ...]$$

$$= \sqrt{\frac{2}{\pi x}} \sin x \blacksquare$$