6.4

(i)

$$\dfrac{\dfrac{}{\{\,\} \vdash f x = 1 : \forall \alpha . \alpha \to \ int}\,(p1) \quad \dfrac{\dfrac{}{\{\,\}[f : (\mathrm{B} \to \mathrm{int}) \to \mathrm{int}] \vdash f : (\mathrm{B} \to int) \to int}\,(p1) \quad \dfrac{}{\{\,\}[f : \mathrm{B} \to \mathrm{int}] \vdash f : \mathrm{B} \to int}\,(p1)}{\{\,\}[f \,:\, \forall \alpha . \alpha \to int] \vdash f\ f\ : int}\,(p9)}{\{\,\} \vdash let\ f\ x = 1\ in\ f\ f\ end : int}\,(p8)$$

f has to be polymorphic otherwise f f would force an impossible infinite type.

(ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\{f : int \to int, x : int\} \vdash x : int}\,(p1) \quad \dfrac{}{\{f : int \to int, x : int\} \vdash 10 : int}\,(p1)}{\{f : int \to int, x : int\} \vdash \ x \ < \ 10 : \ bool} \quad \dfrac{}{\{f : int \to int, x : int\} \vdash \ 42 : \ int}\,(p1) \quad \dfrac{\dfrac{}{\{f : int \to int, x : int\} \vdash \ f \ : \ int \to int}\,(p1) \quad \dfrac{\dfrac{}{\{f : int \to int, x : int\} \vdash x : int}\,(p1) \quad \dfrac{}{\{f : int \to int, x : int\} \vdash 1 : int}\,(p1)}{\{f : int \to int, x : int\} \vdash \ (x \ + \ 1) : int}\,(p4)}{\{f : int \to int, x : int\} \vdash \ f\ (x \ + \ 1) : int}\,(p8)}{\{\,\}[x : int, f : int \to int] \vdash \ if\ x < 10\ then\ 42\ else\ f(x+1) : int}\,(p7) \quad \dfrac{\dfrac{}{\{f : int \to int\} \vdash \ f \ : \ int \to int}\,(p1) \quad \dfrac{}{\{f : int \to int\} \vdash \ 20 \ : \ int}\,(p1)}{\{\,\}[f : int \to int] \vdash \ f\ 20 : \ int}\,(p9)}{\{\,\} \vdash \ let\ rec\ f\ x \ = \ if\ x < 10\ then\ 42\ else\ f(x+1) in\ f\ 20 : \ int}\,(p8)$$

f is recursive (it calls itself in its own definition). In a recursive let (or let rec), the type of f is assumed during the inference of its own body So f cannot have a polymorphic type like 'a → int; it must have a fixed (monomorphic) type int → int.