Corrections

- Section 1.2.2. "...say, we have do[to] move ..."
- Section 2.3.1 $R(x, h) = o(h^4)$ should be $R_2(x, h) = o(h^4)$

Dos and Don'ts

- Use Sheather-Jones to estimate densities. `bw = "SJ"`
- tripel kolon kan bruges til at få adgang til interne funktioner (som ikke er i namespace) af en pakke.
- use range when both min and max are needed.

Ideas

- Why not use another punishment for integrated error? For instance use

$$\text{IPE}_p(\hat{f}_h) = \int (\hat{f}_h(x) - f_0(x))^p dx = ||\hat{f}_h - f_0||_p^p$$

- Use Hill-like plot to find $||f_0''||_2^2$, as we know that AMISE should behave like $Cn^{-4/5}$, for some C that depends on $||f_0''||_2^2$.

1 Fun

"R doesn't stop you from shooting yourself in the foot, but as long as you don't aim the gun at your toes and pull the trigger, you won't have a problem."

2 S3 class

Density Estimation

Let

$$H(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

then

$$H''(x) = \frac{1}{\sqrt{2\pi}} (x^2 - 1) e^{\frac{-x^2}{2}}$$

We now want to calculate

$$\begin{aligned}
||\tilde{f}''||_2^2 =& \frac{1}{n^2 r^2} \int H'' \left(\frac{x - x_i}{r} \right) H'' \left(\frac{x - x_j}{r} \right) dx \\
=& \frac{1}{n^2 r^2} \int \frac{1}{2\pi} \left(\left(\frac{x - x_i}{r} \right)^2 - 1 \right) e^{\frac{-\left(\frac{x-x_i}{r}\right)^2}{2}} \left(\left(\frac{x - x_j}{r} \right)^2 - 1 \right) e^{\frac{-\left(\frac{x-x_j}{r}\right)^2}{2}} dx \\
=& \frac{1}{n^2 r^2} \int \frac{1}{2\pi} \left(\frac{(x - x_i)^2}{r^2} - 1 \right) \left(\frac{(x - x_j)^2}{r^2} - 1 \right) e^{\frac{-\left((x-x_i)^2 + (x-x_j)^2\right)}{2r^2}} dx
\end{aligned}$$

Define

$$w_{ij} := \sqrt{z_i^2 + z_j^2}$$

for

$$z_i = x - x_i$$

and note that

$$\begin{aligned}
\left(z_i^2 - r^2\right)\left(z_j^2 - r^2\right) =& z_i^2 z_j^2 - r^2 w_{ij}^2 + r^4 \\
=& \frac{w_{ij}^4}{2} - r^2 w_{ij}^2 + r^4 - \frac{(z_i^4 + z_j^4)}{2} \\
=& r^4 - r^2 w_{ij}^2 + \frac{w_{ij}^4}{4} - \frac{w_{ij}^2 x_i^2}{2} - \frac{w_{ij}^2 x_j^2}{2} + w_{ij} x_i x_j + \frac{(x_i - x_j)^4}{4}
\end{aligned}$$