Introduction

With-profit insurance contracts are to this day one of the most popular life insurance contracts. They arose as a natural way to distribute the systematic surplus that develops due to the prudent assumptions on which the contract is made, and in the recent years a lot of attention has been aimed at how this distribution of surplus should be conducted fairly. In this regard, fairness means that the value of contributions to the surplus correspond to the value of the bonus received. If one is content with fairness on the portfolio level, the simple requirement that the surplus should be zero when all contracts are terminated, is sufficient. However, if we wish to distribute fairly on the policy level, more information about the development of each individual policy is needed.

The Solvency II Directive states that .... Something about FMA's and which quantity to develop in order to create the balance sheet for all periods.

State-Wise Probability Weighted Reserve

Define

$$\tilde{X}^j(t) := \mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]$$

and note that

$$\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}] = \mathrm{E}_{Z(0)}[X(t)|Z(t)=j]p_{Z(0),j}(0,t), \tag{1}$$

by the definition of conditional expectation. We can think of $\tilde{X}^j$ as the probability weighted state-wise reserves. The relation between $\tilde{X}^j$ and $\mathrm{E}[X(t)]$ is

$$\begin{aligned}
\mathrm{E}_{Z(0)}[X(t)] =& \mathrm{E}_{Z(0)}[\mathrm{E}_{Z(0)}[X(t)|Z(t)]] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\mathrm{E}_{Z(0)}[X(t)|Z(t)=j]\right] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\frac{\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]}{p_{0j}(0,t)}\right] \\
=& \sum_{j\in\mathcal{J}} p_{0j}(0,t)\frac{\tilde{X}^j}{p_{0j}(0,t)} \\
=& \sum_{j\in\mathcal{J}} \tilde{X}^j.
\end{aligned}$$

Life Insurance Setup

The setup differs from the classical ... The dynamics of X are assumed to be affine

$$\begin{aligned} dX(s) = & X(s)g_1(s, Z(s))ds + g_2(s, Z(s))ds \\ & + \sum_{h \neq Z(s-)} \left(X(s-)h_1(s, Z(s-), h) + h_2(s, Z(s-), h) \right) dN^h(s). \end{aligned}$$

Life-Death Model

Dynamics given by

$$dX(s) = \mathbb{1}_{\{Z(s-)=0\}} \left(X(s)g_1(s) + g_2(s) \right) ds - X(s-)dN^1(s).$$

We have not specified how the pay-out of the reserve upon death is distributed between the insured and the insurer - only that the reserve is nullified.

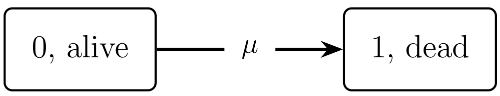
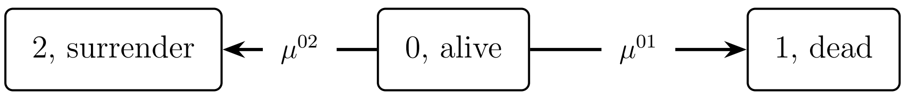
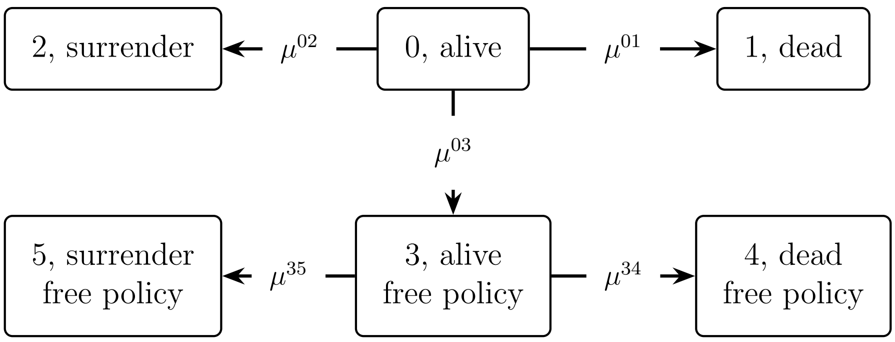


Figure 1: Life-Death model

Life-Death-Surrender



Life-Death-Surrender With Free Policy



Thoughts

- With-profit insurance! Expected reserve including accumulation of dividends.
- Same build-up as Buchardt et al. (2015). Hierarchical examples $\Rightarrow$ general transient.
- Refer to Norberg (1991)
  - Introduction and motivation - stochastic reserve, Monte Carlo method. A little comment on the fact that the problem is still hard to solve.
  - Life-death (simple analytic solution).
  - Life-death free policy (how to deal with extra states).
  - General model without duration.
  - Life-death-surrender free policy, including discussion of free policy factor.
  - Lost all trick works.
  - General model with duration dependence.
  - Inclusion of surplus. Use independence when dividend is assigned on discrete points in time.
- Deterministic intensities.
- Market dependent intensities - allowed when directly dependent on the market, making them deterministic.
- We are only concerned with the reserve.
- Maybe we should use a different wording? **Savings**/stash/backlog/accumulation/hoard/reservoir instead of reserve, to distinguish between the Danish words for "reserve" and "depot"

References

Kristian Buchardt, Thomas Møller, and Kristian Schmidt. Cash flows and policyholder behaviour in the semi-markov life insurance setup. *Scandinavian Actuarial Journal*, 2015(8), 2015. ISSN 03461238. URL `http://search.proquest.com/docview/1718994569/`.

Ragnar Norberg. Reserves in life and pension insurance. *Scand. Actuar. J. 1*, pages 3–24, 1991.