Stuff to fix

- Jeg kommer med flere påstande om industrien som jeg ikke er sikker på har hold i virkeligheden. fx at forsikringsselskaber ikke har bekymret sig for om deres udlodningsstrategier er fair på policeniveau.

Introduction

With-profit insurance contracts are to this day one of the most popular life insurance contracts. They arose as a natural way to distribute the systematic surplus that develops due to the prudent assumptions on which the contract is made. In recent years, sensible questions accompanied by a lot of attention, have been aimed at the surplus, to name a few; is it distributed fairly? what is the risk of negative surplus? how should it be invested? One might look for answers in the existing literature e.g. Møller and Steffensen (2007), Norberg (1999), Steffensen (2000) or Steffensen (2001), where partial differential equations are used to describe the prospective second order reserve and its interplay with surplus. While these PDE's provide a conceptually powerful tool, they are limited to simple market dynamics, and they do not provide new realistic models for long term financial markets. Norberg (1999) considers the development of the surplus in a Markov chain environment, allowing for great model flexibility but even still, an important element is completely neglected. In the existing literature, the human element is left out of the equation. Insurance companies are governed by humans, and the decisions they make, have an influence on their portfolio - in particular concerning surplus and dividends. In a with-profit insurance contract many quantities are fixed at initialisation of the policy, but the rate at which dividends are accrued is not. The insurance company has a certain amount of freedom when it comes to the distribution of surplus, and the

one of their free variable

The human element is encapsulated in the so-called Management Actions.

Future management actions are mathematically intangible

In order to ensure a

In this paper we derive a retrospective differential equation for the savings account, which aids in determining how much each individual policy has contributed to the collective surplus. Apart from providing a tool for fair distribution of surplus, the method provides insight on the financial risks carried by the insurance companies.

The notion of bonus has been studied before; see chapter 6 of Møller and Steffensen (2007), Norberg (1999), Steffensen (2000) and Steffensen (2001). As stated by CEIPOS[1] w

> *The actuarial and statistical methods used to calculate the best estimate should take account of the effect on these future cash-flows of potential future actions by the management of (re)insurance undertakings based upon current and credible information.*

It seems as though previous contributions to the subject have focused on deriving closed form solutions for the reserve and surplus. The closed form solutions come at a price - a distribution of the financial market is a necessity. In essence, these models need to know how the market is expected to develop, in order to provide information about how the prospective reserves are expected to develop. Appealing as these analytical solutions are, However, the real-world complexity of S can rarely be captured by a simple distribution with analytically known moments, and therefore simulations are used instead

Our method is fundamentally different

The contribution of this paper, is to provide a tool that can be used for prognostication based on simulation of arbitrary complexity. In a sense, we embrace the complexity of the financial market and its intricate influence on life insurance... It is infeasible to do nested monte-carlo simulations, so we average over the biometric risks only. In principle providing the true estimate

The savings account, henceforth referred to as savings, differs from the reserves that classically are used to asses insurance liabilities, for in particular one reason: it depends on the entire history of the policy. This poses a problem, as the Markov property that usually saves us from the hassle of dealing with the past, no longer can be directly applied. Furthermore, the prospective reserves are a tool to find the liabilities of today, while the savings of today already are known.

With the methods from this paper, any [but as insurance companies have been satisfied with fairness on portfolio level, there has been little effort invested in the surplus generation on the individual policy]. Much of the theory concerning surplus has not taken into account that dividends usually are spent on increasing the policies benefits. This increase in benefits has an impact on the dynamics of the savings, and several of the [(Norberg, 1999) concerned with the

[1]https://eiopa.europa.eu/CEIOPS-Archive/Documents/Advices/CEIOPS-L2-Final-Advice-TP-Assumptions-future-management-actions.pdf

size of the surplus belonging to each policy, we are interested in the case where surplus is used to buy more insurance.]. Bonus emerges due to the difference between the first and second order basis. The first order basis

"Life can only be understood backwards; but it must be lived forwards"

The contribution of this article is to embrace the fact that we, as humans, make decisions based on our past. Even though I twist the wise words of Søren Kierkegaard, they

Setup

We consider the classic multi-state life insurance setup, comprised of a state process Z denoting the state of the policy in a finite state space $\mathcal{J} = \{0, 1, ..., J\}$.

- Definition of X and Y
- Deterministic second order basis, and discussion regarding simulation.
- remark on continuity of $b^i(t)$
- model limitations.

The dynamics of X are assumed to be affine

$$\begin{aligned} dX(s) =& X(s)g_1(s, Z(s))ds + g_2(s, Z(s))ds \\ &+ \sum_{h \neq Z(s-)} \left(X(s-)h_1(s, Z(s-), h) + h_2(s, Z(s-), h)\right) dN^h(s). \end{aligned}$$

In practice, the surplus account is shared among policyholders. The increased benefits are also guaranteed, which wi do not consider.

Due to the fact that there are risk premiums on the bonus benefits, we get a system of two quantities that influence each other - the savings and the surplus. In the case of cash dividends, one simply has to find an expression for the expected surplus, as the reserves of the policies are unaffected by cash dividends.

1 Active state model

We consider a simple model where the expected future savings are described by an easily derived differential equation. The model consists of n inactive states where there are no payments, and one active state with a non-zero savings-dependent payment process b. On transition to one of the inactive states, the savings are nullified. We need not specify what happens to the savings on a transition - they may be paid out to the customer or the insurance company, or any combination of the two - the only important requirement is that the savings are zero in all inactive states. As there are no payments in the inactive states, $\chi^{0j}(t,x) = 0$ for all inactive states j, implying that $h_x(t,0,j,x,y) = -x$. While the policy is in the active state, it contributes and receives dividend from a surplus account, Y. The survival model with and without surrender options are special cases of this model. Denote by 0 the active state. The dynamics of X are

$$dX(s) = \mathbb{1}_{\{Z(s-)=0\}} g(s,0,X(s))ds - \sum_{h=1}^{n} X(s-)dN^h(s).$$

Note that there are no risk premiums, as there is no risk for the insurance company company related to the transitions of the policy. Therefore, the only risk carried by the insurance company, relates to the interest of the savings account.

Remark: If the benefits are identical after age 65, the states 0,1,3 and 4 can be lumped, as well as 2,5 and 6, thus creating a survival model. If the dynamics in two states are identical, they can be viewed as one. Life annuity at age 65.

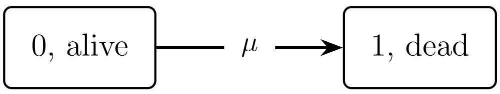


Figure 1: Life-Death model

2 Active state model

n-state model

State-Wise Probability Weighted Reserve

Define

$$\tilde{X}^j(t) := \mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]$$

and note that

$$\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}] = \mathrm{E}_{Z(0)}[X(t)|Z(t)=j]p_{Z(0),j}(0,t), \tag{1}$$

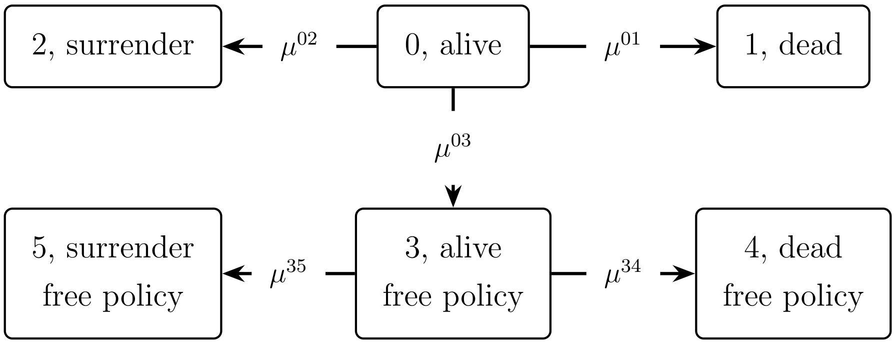
by the definition of conditional expectation. We can think of $\tilde{X}^j$ as the probability weighted state-wise reserves. The relation between $\tilde{X}^j$ and $\mathrm{E}[X(t)]$ is

$$\begin{aligned}
\mathrm{E}_{Z(0)}[X(t)] =& \mathrm{E}_{Z(0)}[\mathrm{E}_{Z(0)}[X(t)|Z(t)]] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\mathrm{E}_{Z(0)}[X(t)|Z(t)=j]\right] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\frac{\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]}{p_{0j}(0,t)}\right] \\
=& \sum_{j\in\mathcal{J}} p_{0j}(0,t)\frac{\tilde{X}^j}{p_{0j}(0,t)} \\
=& \sum_{j\in\mathcal{J}} \tilde{X}^j.
\end{aligned}$$

Life-Death-Surrender

$$\begin{tikzcd}
\boxed{\text{2, surrender}} & \boxed{\text{0, alive}} \arrow[l, "\mu^{02}"'] \arrow[r, "\mu^{01}"] & \boxed{\text{1, dead}}
\end{tikzcd}$$

Life-Death-Surrender With Free Policy



Use of Savings account

Thoughts

- With-profit insurance! Expected reserve including accumulation of dividends.
- Same build-up as Buchardt et al. (2015). Hierarchical examples $\Rightarrow$ general transient.
- Refer to Norberg (1991)
  - Introduction and motivation - stochastic reserve, Monte Carlo method. A little comment on the fact that the problem is still hard to solve.
  - Life-death (simple analytic solution).
  - Life-death free policy (how to deal with extra states).
  - General model without duration.
  - Life-death-surrender free policy, including discussion of free policy factor.
  - Lost all trick works.
  - General model with duration dependence.
  - Inclusion of surplus. Use independence when dividend is assigned on discrete points in time.
- Deterministic intensities.
- Market dependent intensities - allowed when directly dependent on the market, making them deterministic.

- We are only concerned with the reserve.
- Maybe we should use a different wording? **Savings**/stash/backlog/accumulation/hoard/reservoir instead of reserve, to distinguish between the Danish words for "reserve" and "depot"

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