Introduction

The Solvency II Directive states that Something about FMA's and which quantity to develop in order to create the balance sheet for all periods.

State-Wise Probability Weighted Reserve

Define

$$\tilde{X}^{j}(t) := \mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=i\}}]$$

and note that

$$E_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}] = E_{Z(0)}[X(t)|Z(t)=j]p_{Z(0),j}(0,t), \tag{1}$$

by the definition of conditional expectation. We can think of \tilde{X}^j as the probability weighted state-wise reserves. The relation between \tilde{X}^j and $\mathrm{E}[X(t)]$ is

$$\begin{split} \mathbf{E}_{Z(0)}[X(t)] = & \mathbf{E}_{Z(0)}[\mathbf{E}_{Z(0)}[X(t)|Z(t)]] \\ = & \mathbf{E}_{Z(0)} \left[\sum_{j \in \mathcal{J}} \mathbb{1}_{\{Z(t)=j\}} \mathbf{E}_{Z(0)}[X(t)|Z(t) = j] \right] \\ = & \mathbf{E}_{Z(0)} \left[\sum_{j \in \mathcal{J}} \mathbb{1}_{\{Z(t)=j\}} \frac{\mathbf{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]}{p_{0j}(0,t)} \right] \\ = & \sum_{j \in \mathcal{J}} p_{0j}(0,t) \frac{\tilde{X}^{j}}{p_{0j}(0,t)} \\ = & \sum_{j \in \mathcal{J}} \tilde{X}^{j}. \end{split}$$

Life Insurance Setup

The setup differs from the classical \dots The dynamics of X are assumed to be affine

$$dX(s) = X(s)g_1(s, Z(s))ds + g_2(s, Z(s))ds + \sum_{h \neq Z(s-)} (X(s-)h_1(s, Z(s-), h) + h_2(s, Z(s-), h)) dN^h(s).$$

Life-Death Model

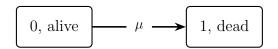
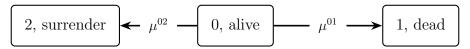
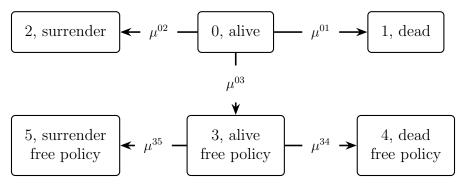


Figure 1: Life-Death model

Life-Death-Surrender



Life-Death-Surrender With Free Policy



Thoughts

- Same build-up as ?. Hierarchical examples \Rightarrow general transient.
- Refer to ?
 - Introduction and motivation stochastic reserve, Monte Carlo method. A little comment on the fact that the problem is still hard to solve.
 - Life-death (simple analytic solution).
 - Life-death free policy (how to deal with extra states).
 - General model without duration.
 - Life-death-surrender free policy, including discussion of free policy factor.
 - Lost all trick works.
 - General model with duration dependence.

- Inclusion of surplus. Use independence when dividend is assigned on discrete points in time.
- Deterministic intensities.
- Market dependent intensities allowed when directly dependent on the market.

References

Kristian Buchardt, Thomas Møller, and Kristian Schmidt. Cash flows and policyholder behaviour in the semi-markov life insurance setup. *Scandinavian Actuarial Journal*, 2015(8), 2015. ISSN 03461238. URL http://search.proquest.com/docview/1718994569/.

Ragnar Norberg. Reserves in life and pension insurance. Scand. Actuar. J. 1, pages 3–24, 1991.