Stuff to fix

- Jeg kommer med flere påstande om industrien som jeg ikke er sikker på har hold i virkeligheden. fx at forsikringsselskaber ikke har bekymret sig for om deres udlodningsstrategier er fair på policeniveau.

Introduction

With-profit insurance contracts are to this day one of the most popular life insurance contracts. They arose as a natural way to distribute the systematic surplus that develops due to the prudent assumptions on which the contract is made, and in the recent years a lot of attention has been aimed at how this distribution of surplus should be conducted fairly. In this regard, fairness means that the value of contributions to the surplus correspond to the value of the bonus received. If one is content with fairness on the portfolio level, it is sufficient to require that the collective surplus should be zero when all contracts are terminated. However, if we wish to distribute fairly on the policy level, more information about the development of each individual policy is needed. In this paper we derive a retrospective differential equation for the savings account, which aids in determining how much each individual policy has contributed to the collective surplus. Apart from providing a tool for fair distribution of surplus, the method provides insight on the financial risks carried by the insurance companies.
The savings account, henceforth referred to as savings, differs from the reserves that classically are used to asses insurance liabilities, for in particular one reason: it depends on the entire history of the policy. This poses a serious problem, as the Markov property that usually saves us from the hassle of dealing with the past, no longer can be directly applied. Furthermore, the prospective reserves are used to find the liabilities of today, while the savings of today already are known. the notion of bonus as been studied See chapter 6 of Møller and Steffensen (2007)... [but as insurance companies have been satisfied with fairness on portfolio level, there has been little effort invested in the surplus generation on the individual policy]. [(**?**) concerned with the size of the surplus belonging to each policy, we are interested in the case where surplus is used to buy more insurance.]

State-Wise Probability Weighted Reserve

Define

$$\tilde{X}^j(t) := \mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]$$

and note that

$$\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}] = \mathrm{E}_{Z(0)}[X(t)|Z(t)=j]p_{Z(0),j}(0,t), \tag{1}$$

by the definition of conditional expectation. We can think of $\tilde{X}^j$ as the probability weighted state-wise reserves. The relation between $\tilde{X}^j$ and $\mathrm{E}[X(t)]$ is

$$\begin{aligned}
\mathrm{E}_{Z(0)}[X(t)] =& \mathrm{E}_{Z(0)}[\mathrm{E}_{Z(0)}[X(t)|Z(t)]] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\mathrm{E}_{Z(0)}[X(t)|Z(t)=j]\right] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\frac{\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]}{p_{0j}(0,t)}\right] \\
=& \sum_{j\in\mathcal{J}} p_{0j}(0,t)\frac{\tilde{X}^j}{p_{0j}(0,t)} \\
=& \sum_{j\in\mathcal{J}} \tilde{X}^j.
\end{aligned}$$

Life Insurance Setup

The setup differs from the classical ... The dynamics of X are assumed to be affine

$$\begin{aligned}
dX(s) =& X(s)g_1(s,Z(s))ds + g_2(s,Z(s))ds \\
&+ \sum_{h\neq Z(s-)} \left(X(s-)h_1(s,Z(s-),h) + h_2(s,Z(s-),h)\right) dN^h(s).
\end{aligned}$$

Life-Death Model

Dynamics given by

$$dX(s) = \mathbb{1}_{\{Z(s-)=0\}}\left(X(s)g_1(s) + g_2(s)\right)ds - X(s-)dN^1(s).$$

We have not specified how the pay-out of the reserve upon death is distributed between the insured and the insurer - only that the reserve is nullified.

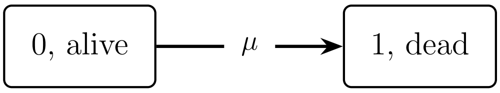
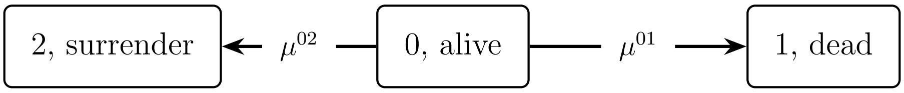
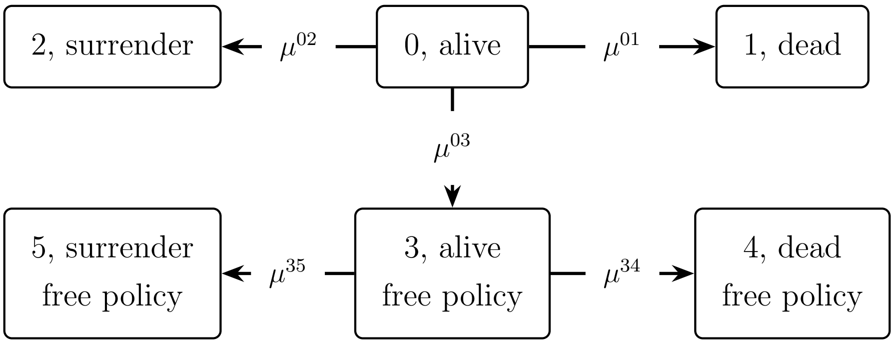


Figure 1: Life-Death model

Life-Death-Surrender



Life-Death-Surrender With Free Policy



Thoughts

- With-profit insurance! Expected reserve including accumulation of dividends.
- Same build-up as Buchardt et al. (2015). Hierarchical examples $\Rightarrow$ general transient.
- Refer to Norberg (1991)
  - Introduction and motivation - stochastic reserve, Monte Carlo method. A little comment on the fact that the problem is still hard to solve.
  - Life-death (simple analytic solution).
  - Life-death free policy (how to deal with extra states).

- General model without duration.
- Life-death-surrender free policy, including discussion of free policy factor.
- Lost all trick works.
- General model with duration dependence.
- Inclusion of surplus. Use independence when dividend is assigned on discrete points in time.

- Deterministic intensities.
- Market dependent intensities - allowed when directly dependent on the market, making them deterministic.
- We are only concerned with the reserve.
- Maybe we should use a different wording? **Savings**/stash/backlog/accumulation/hoard/reservoir instead of reserve, to distinguish between the Danish words for "reserve" and "depot"

References

Kristian Buchardt, Thomas Møller, and Kristian Schmidt. Cash flows and policyholder behaviour in the semi-markov life insurance setup. *Scandinavian Actuarial Journal*, 2015(8), 2015. ISSN 03461238. URL `http://search.proquest.com/docview/1718994569/`.

Thomas Møller and Mogens Steffensen. *Market-valuation methods in life and pension insurance*. International Series on Actuarial Science. Cambridge University Press, Cambridge, 2007. ISBN 0521868777.

Ragnar Norberg. Reserves in life and pension insurance. *Scand. Actuar. J. 1*, pages 3–24, 1991.