Stuff to fix

- Jeg kommer med flere påstande om industrien som jeg ikke er sikker på har hold i virkeligheden. fx at forsikringsselskaber ikke har bekymret sig for om deres udlodningsstrategier er fair på policeniveau.

Introduction

With-profit insurance contracts are to this day one of the most popular life insurance contracts. They arose as a natural way to distribute the systematic surplus that develops due to the prudent assumptions on which the contract is made, and in the recent years a lot of attention has been aimed at how this distribution of surplus should be conducted fairly. In this regard, fairness means that the value of contributions to the surplus correspond to the value of the bonus received. If one is content with fairness on the portfolio level, it is sufficient to require that the collective surplus should be zero when all contracts are terminated. However, if we wish to distribute fairly on the policy level, more information about the development of each individual policy is needed. In this paper we derive a retrospective differential equation for the savings account, which aids in determining how much each individual policy has contributed to the collective surplus. Apart from providing a tool for fair distribution of surplus, the method provides insight on the financial risks carried by the insurance companies.

The savings account, henceforth referred to as savings, differs from the reserves that classically are used to asses insurance liabilities, for in particular one reason: it depends on the entire history of the policy. This poses a problem, as the Markov property that usually saves us from the hassle of dealing with the past, no longer can be directly applied. Furthermore, the prospective reserves are a tool to find the liabilities of today, while the savings of today already are known. The notion of bonus has been studied before; see chapter 6 of Møller and Steffensen (2007), Norberg (1999), Steffensen (2000) and Steffensen (2001). Partial differential equations describing the prospective second order reserve have been derived by (hvem? Mogens? I hvert fald i kapitel 6 i Liv2), and while they provide a powerful tool, they are limited to simple market dynamics, and as stated in chapter 6 of Møller and Steffensen (2007)

> *[This chapter] serves to present the concept of surplus-linked life insurance payments and demonstrate the partial differential equation methodology for calculation*

of reserves based on this concept. It does not *provide new realistic models for long term financial markets and stochastic intensities.*

The contribution of this paper, is to provide a tool [but as insurance companies have been satisfied with fairness on portfolio level, there has been little effort invested in the surplus generation on the individual policy]. Much of the theory concerning surplus has not taken into account that dividends usually are spent on increasing the policies benefits. This increase in benefits has an impact on the dynamics of the savings, and several of the [(Norberg, 1999) concerned with the size of the surplus belonging to each policy, we are interested in the case where surplus is used to buy more insurance.]. Bonus emerges due to the difference between the first and second order basis. The first order basis

Setup

We consider the classic multi-state life insurance setup, comprised of a state process Z denoting the state of the policy in a finite state space $\mathcal{J} = \{0, 1, ..., J\}$.

- Definition of X and Y
- Deterministic second order basis, and discussion regarding simulation.
- remark on continuity of $b^i(t)$
- model limitations.

The dynamics of X are assumed to be affine

$$\begin{aligned} dX(s) =& X(s)g_1(s, Z(s))ds + g_2(s, Z(s))ds \\ &+ \sum_{h \neq Z(s-)} \left(X(s-)h_1(s, Z(s-), h) + h_2(s, Z(s-), h)\right) dN^h(s). \end{aligned}$$

In practice, the surplus account is shared among policyholders. The increased benefits are also guaranteed, which wi do not consider.

Due to the fact that there are risk premiums on the bonus benefits, we get a system of two quantities that influence each other - the savings and the surplus. In the case of cash dividends, one simply has to find an expression for the expected surplus, as the reserves of the policies are unaffected by cash dividends.

1 Active state model

We consider a simple model where the expected future savings are described by an easily derived differential equation. The model consists of n inactive states where there are no payments, and one active state with a non-zero savings-dependent payment process b. On transition to one of the inactive states, the savings are nullified. We need not specify what happens to the savings on a transition - they may be paid out to the customer or the insurance company, or any combination of the two - the only important requirement is that the savings are zero in all inactive states. As there are no payments in the inactive states, $\chi^{0j}(t,x) = 0$ for all inactive states j, implying that $h_x(t,0,j,x,y) = -x$. While the policy is in the active state, it contributes and receives dividend from a surplus account, Y. The survival model with and without surrender options are special cases of this model. Denote by 0 the active state. The dynamics of X are

$$dX(s) = \mathbb{1}_{\{Z(s-)=0\}} g(s,0,X(s))ds - \sum_{h=1}^{n} X(s-)dN^h(s).$$

Note that there are no risk premiums, as there is no risk for the insurance company company related to the transitions of the policy. Therefore, the only risk carried by the insurance company, relates to the interest of the savings account.

Remark: If the benefits are identical after age 65, the states 0,1,3 and 4 can be lumped, as well as 2,5 and 6, thus creating a survival model. If the dynamics in two states are identical, they can be viewed as one. Life annuity at age 65.

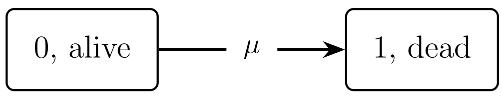


Figure 1: Life-Death model

2 Active state model

n-state model

State-Wise Probability Weighted Reserve

Define

$$\tilde{X}^j(t) := \mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]$$

and note that

$$\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}] = \mathrm{E}_{Z(0)}[X(t)|Z(t)=j]p_{Z(0),j}(0,t), \tag{1}$$

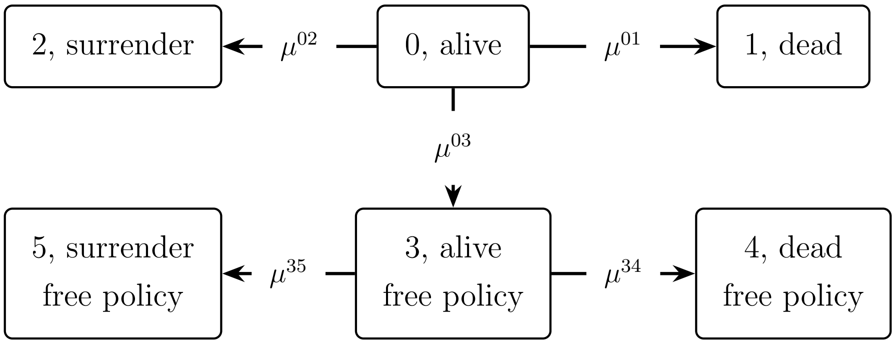
by the definition of conditional expectation. We can think of $\tilde{X}^j$ as the probability weighted state-wise reserves. The relation between $\tilde{X}^j$ and $\mathrm{E}[X(t)]$ is

$$\begin{aligned}
\mathrm{E}_{Z(0)}[X(t)] =& \mathrm{E}_{Z(0)}[\mathrm{E}_{Z(0)}[X(t)|Z(t)]] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\mathrm{E}_{Z(0)}[X(t)|Z(t)=j]\right] \\
=& \mathrm{E}_{Z(0)}\left[\sum_{j\in\mathcal{J}} \mathbb{1}_{\{Z(t)=j\}}\frac{\mathrm{E}_{Z(0)}[X(t)\mathbb{1}_{\{Z(t)=j\}}]}{p_{0j}(0,t)}\right] \\
=& \sum_{j\in\mathcal{J}} p_{0j}(0,t)\frac{\tilde{X}^j}{p_{0j}(0,t)} \\
=& \sum_{j\in\mathcal{J}} \tilde{X}^j.
\end{aligned}$$

Life-Death-Surrender

$$\begin{tikzcd}
\boxed{\text{2, surrender}} & \boxed{\text{0, alive}} \arrow[l, "\mu^{02}"'] \arrow[r, "\mu^{01}"] & \boxed{\text{1, dead}}
\end{tikzcd}$$

Life-Death-Surrender With Free Policy



Use of Savings account

Thoughts

- With-profit insurance! Expected reserve including accumulation of dividends.
- Same build-up as Buchardt et al. (2015). Hierarchical examples $\Rightarrow$ general transient.
- Refer to Norberg (1991)
  - Introduction and motivation - stochastic reserve, Monte Carlo method. A little comment on the fact that the problem is still hard to solve.
  - Life-death (simple analytic solution).
  - Life-death free policy (how to deal with extra states).
  - General model without duration.
  - Life-death-surrender free policy, including discussion of free policy factor.
  - Lost all trick works.
  - General model with duration dependence.
  - Inclusion of surplus. Use independence when dividend is assigned on discrete points in time.
- Deterministic intensities.
- Market dependent intensities - allowed when directly dependent on the market, making them deterministic.

- We are only concerned with the reserve.
- Maybe we should use a different wording? **Savings**/stash/backlog/accumulation/hoard/reservoir instead of reserve, to distinguish between the Danish words for "reserve" and "depot"

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