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ORIGINAL ARTICLE

The FRANEC stellar evolutionary code

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Abstract We summarize the main physical assumptions and numerical procedures adopted by the FRANEC code to compute stellar models in all the evolutionary phases at hydrostatic and thermal equilibrium. An application to the Standard Solar Model is also briefly presented.

Keywords Stars: evolution · Stars: interiors · Sun: evolution

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1 Introduction

FRANEC (Frascati Raphson Newton Evolutionary Code) is an evolutionary stellar code developed by a group of italian researchers in Frascati about 30 years ago and then modified and updated during the years. An exhaustive description can be found in Chieffi and Straniero (1989), whereas the last updates of the version of the code adopted by the Pisa-Naples group, mostly related to changes in the input physics, are reported by Ciacio et al. (1997), Cassisi et al. (1998), Cariulo et al. (2004), and, for the application to white dwarf (WD) structures, Prada Moroni and Straniero (2002). FRANEC has been used to compute models in all the evolutionary phases, from the pre-main sequence (PMS) to the cooling sequence of WDs, covering a mass range from ≈ 0.1 to $\approx 25 M_{\odot}$ for several metallicities and Helium abundances. Our code assumes spherical symmetry for the star and negligible effects of rotation and magnetic fields in the stellar interior. Some of the computed models and the related isochrones are available electronically.[1] In Fig. 1 we present selected pre-main sequence (upper panel) and MS and post-MS (lower panel) models.

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2 Integration of the stellar structure

To integrate the four equations of stellar structure and evolution the model is divided into three integration zones: the atmosphere, the sub-atmosphere and the interior, adopting as the independent variable of integration the optical depth, the pressure and the mass, respectively.
The variations of the physical quantities in a time step cannot exceed for each point of the stellar structure a prefixed value. For example for the models computed for the CoRoT ESTA Task 1 we adopted the percentage variations reported in Table 1. The first four columns of this table report the percentage variations in radius, luminosity, pressure and temperature, whereas the fifth column refers to hydrogen abundance and the last one to the nuclear burning luminosity variation, that assumes the same value for the three most important reactions, namely pp, CNO and 3α.

[1] See the URL http://astro.df.unipi.it/SAA/PELZ0.html and http://www.mporzio.astro.it/~marco/GIPSY/homegipsy.html.

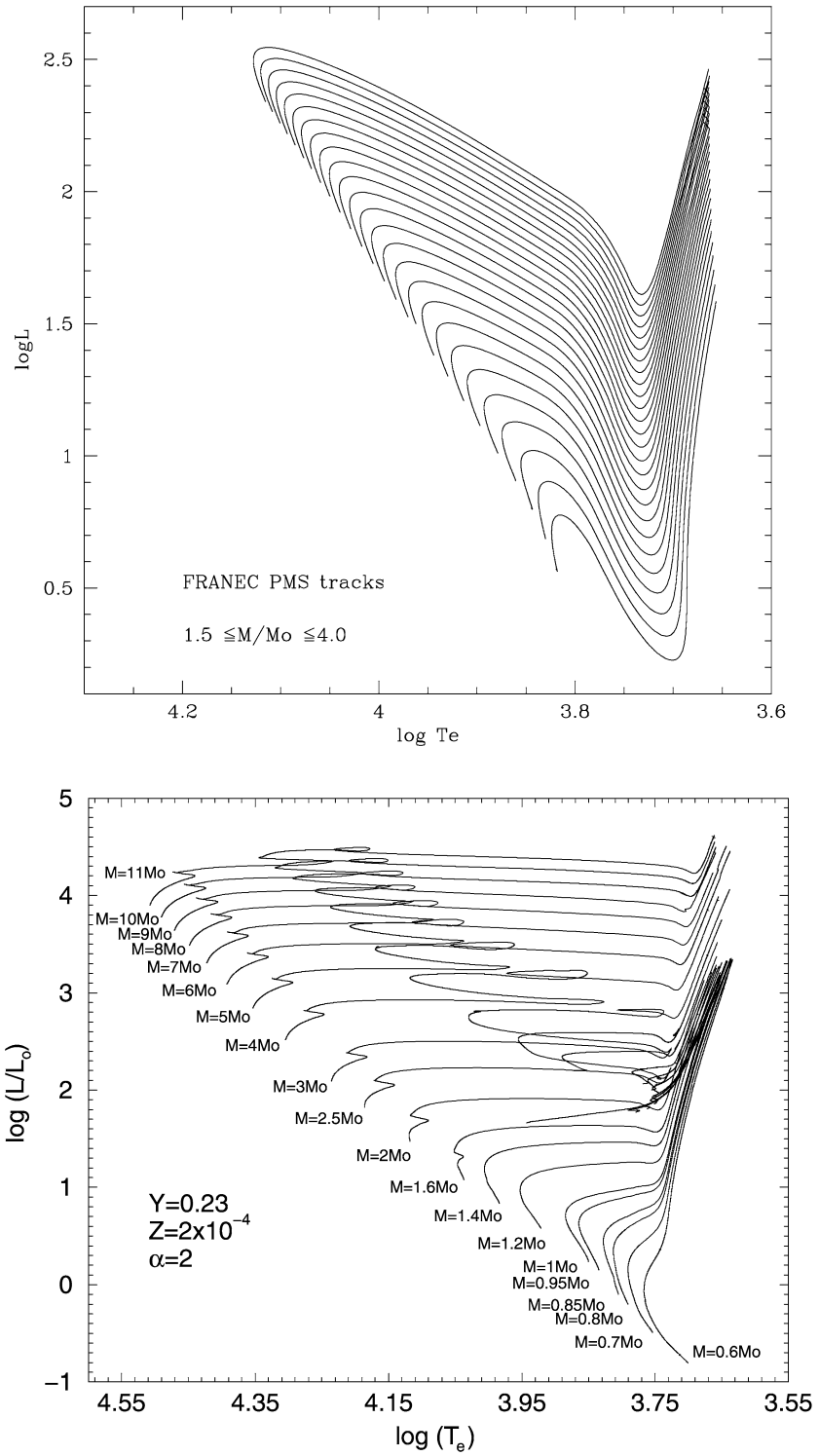

Fig. 1 *Upper panel*: Pre-main sequence evolutionary tracks for the labeled stellar masses and solar chemical composition ($Z = 0.02$, $Y = 0.27$). *Lower panel*: evolutionary tracks for the labeled stellar masses and chemical composition from the main sequence (MS) to the first thermal pulse during the asymptotic giant branch (AGB) phase or (for the most massive models) to the onset of central C ignition

The computed evolution can start either from the homogeneous zero age main sequence (ZAMS) structures or from the chemically homogeneous quasi-static pre-main sequence models with central temperature values lower than $T_c \approx 10^6$ K. The equations for the chemical evolution of the stellar matter are solved by means of a classical Raphson-Newton method. In each mesh the chemical abundances are evolved in time, taking into account microscopic diffusion, if required. The convective zones are assumed to be chemically homogeneous. The chemical abundance in the subatmospheric and atmospheric regions is taken as the one of the most external mesh of the stellar interior. The reference values adopted in the code are shown in Table 2.

Table 1 Maximum allowed variation in a time step for the physical quantities of each mesh, for CoRoT ESTA Task 1 models. The term $\delta L_n/L_n$ refers to the percentage variation in the luminosity associated to nuclear burning reactions, and it is the same for pp, CNO and 3α

$\delta R/R$	$\delta L/L$	$\delta P/P$	$\delta T/T$	$\delta X/X$	$\delta L_n/L_n$
0.002	0.3	0.1	0.3	0.1	0.1

Table 2 Reference values for the stellar code

$L_\odot$ [erg/s]	$M_\odot$ [g]	$R_\odot$ [cm]	G [dyne cm^2 g^{-2}]
$3.846 \cdot 10^{33}$	$1.989 \cdot 10^{33}$	$6.9599 \cdot 10^{10}$	$6.668 \cdot 10^{-8}$

Table 3 Maximum allowed variation of the physical quantities from one mesh point to the next one, for CoRoT ESTA Task 1 models.

$\delta R/R$	$\delta L/L$	$\delta P/P$	$\delta T/T$	$\delta M/M_{tot}$
0.07	0.03	0.15	0.05	0.1

2.1 The stellar interior

In the interior the equations are solved with the Henyey method, taking as boundary conditions in the external part the physical values calculated at the base of the sub-atmosphere. To reach the convergence of a stellar model the equations are required to be verified at least at the 10^{-5} level in each mesh.

The mass distribution of the mesh points is fixed by the requirement that the physical quantities R, L, P, T, M should not vary by more than some pre-fixed amount from one mesh point to the next one. The requirements for the models computed in the context of CoRoT ESTA Task 1 are reported in Table 3. With these requirements the typical number of meshes in the stellar interior is ≈800–900.

2.2 The atmosphere and sub-atmosphere

In the atmosphere the independent variable is the optical depth, ranging from about 0 to 2/3 or to the onset of convection. We usually adopt the solar scaled relation between temperature and optical depth by Krishna Swamy (1966); however for the CoRoT ESTA Task 1–Task 3 comparisons the Eddington $T(\tau)$ relation has been used. The radius is fixed to the photospheric radius as obtained from the Stephan-Boltzmann law. In the sub-atmosphere the independent variable is the pressure, from the end of the atmosphere to a defined value in mass. The integration method is a fourth order Runge-Kutta technique with a variable number of meshes, typically ranging from 300 to 400. In the atmosphere and subatmosphere the variation of the chemical composition due to nuclear reactions is required to be negligible. The code checks that the nuclear reaction efficiency is lower than a pre-fixed value; if this occurrence is not verified the code automatically reduces the extension of the subatmospheric region to the level where the nuclear efficiency is the required one.

2.3 The treatment of convection

The extension of the convective zones in the stellar structure is evaluated by means of the classical Schwarzschild criterion and the time scale of convective mixing is always assumed to be much shorter than the nuclear burning time scale. A first order Taylor expansion is used to follow the burnings in the convective regions. During the central He burning phase two additional phenomena, namely the induced core overshooting and the semiconvection, are taken into account.

Mechanical overshooting of the stellar core during the central H and He burning phases can be either included or neglected. When it is considered the extension of the affected region is calculated from the boundary of the Schwarzschild convective region through the parameter:

$$\alpha_{ov} = \Delta M_{ov}/H_{pm}$$

where $H_{pm} = -dM/d\ln P$.

In the external layers the required superadiabaticity is evaluated by following the Cox and Giuli (1968) derivation of the Bohm-Vitense mixing lenght formalism.

2.4 Atomic diffusion

The code has the possibility to include diffusion of He and heavier elements in the interior of the star. In particular for ^{12}C, ^{14}N, ^{16}O, ^{6}Li, ^{7}Li,^{9}B, ^{11}B, ^{56}Fe diffusion is explicitly calculated, whereas the other heavy elements are assumed to diffuse as ^{56}Fe. Electron diffusion is also included. The diffusion coefficients by Thoul et al. (1994) are adopted. These are obtained as the solutions of the Burgers equations for a multi-components fluid, including the gravity (pressure), temperature and composition terms. The rate of change of the mass fractions of the element "m", due to diffusion, is written in dimensionless form as:

$$\frac{\partial X_m}{\partial t} = -\frac{1}{\rho r^2}\frac{\partial [r^2 X_m T^{\frac{5}{2}} \xi_m(r)]}{\partial r} \tag{1}$$

where $\xi_m(r)$ are the so called "diffusion functions" which (for each element m) are calculated as:

$$\begin{aligned}\xi(m) = {} & a_P(m)\frac{\partial \ln P}{\partial r} + a_T(m)\frac{\partial \ln T}{\partial r} \\ & + \sum_n a_X([-3pt]m,n)\frac{\partial \ln X(n)}{\partial r} \\ & - \sum_n a_X(m,n)\frac{\sum_j \frac{Z(j)X(j)}{A(j)}\frac{\partial \ln X(j)}{\partial r}}{\sum_i \frac{Z(i)X(i)}{A(i)}}\end{aligned} \tag{2}$$

where n runs over chemical elements heavier than helium and the sums over i and j are over all the considered ionic species with atomic weight A and atomic number Z. a_P, a_T, a_X are the gravitational, thermal and chemical diffusion coefficients, respectively. The diffusion velocity is progressively reduced toward the outermost layers to reach a zero value at the stellar surface. Radiative acceleration is not included. The variation of the total metallicity, due to diffusion, is taken into account in the opacity calculation while it is not considered in the equation of state.

3 Physical inputs

In the following we describe the main physical inputs adopted in the FRANEC code, starting with the three fundamental ingredients of a stellar model, namely the energy generation coefficients, the equation of state and the radiative and conductive opacity.

3.1 Energy generation

The adopted reaction rates are taken from the results of the NACRE collaboration (Angulo et al. 1999).

Current version of FRANEC takes into account the burning of light elements (D, ^{7}Li, ^{9}Be, etc.) in the pre-main sequence phase, with the original Deuterium abundance from Geiss and Gloeckler (1998). During H burning the evolution in time toward the equilibrium abundance of ^{3}He, ^{12}C, ^{14}N, ^{16}O is explicitly followed. The original ^{3}He abundance is taken from Geiss and Gloeckler (1998).

The code implements weak (Salpeter 1954), weak-intermediate and intermediate-strong (Graboske et al. 1973; Dewitt et al. 1973) and strong screening (Itoh et al. 1977, 1979). For the CoRoT ESTA Task 1–Task 3 models, as required, only weak screening has been adopted.

For neutrino energy losses the values by Haft et al. (1994) and Itoh et al. (1996) have been adopted.

3.2 Equation of state

The most recent version of the code uses the OPAL 2005 tabulations for the appropriate metallicity value, but for CoRoT ESTA Task 1–Task 3 models the OPAL 2001 EOS has been adopted. The OPAL tables are interpolated linearly in temperature and pressure and with a spline in Hydrogen abundance. For values of the temperature and pressure outside the OPAL 2001 domain, we use the equation of state by Prada Moroni and Straniero (private communications, see also Straniero 1988).

3.3 Opacity

The adopted opacity tables are the latest OPAL ones,[2] with chemical mixtures either from Grevesse and Noels (1993), Grevesse and Sauval (1998) or from Asplund et al. (2005), in the temperature range $\log T \geq 4.2$. In such a way the EOS is consistent with the adopted opacity tables from the same Livermore group. For lower temperatures the molecular opacity tables from Alexander and Ferguson (1994) are adopted for Task 1–Task 3 models and interpolated with a spline function in Z, with a cubic function in temperature and density and with a linear dependence on the hydrogen abundance. The updated version of the code implements also the Ferguson et al. (2005) tables.

A mass-weighted combination of He, C and O is adopted in the H exhausted regions. In case of diffusion the effect of the variation of the global metallicity on the opacity values is taken into account. As for conductive opacity the evaluations by Potekhin (1999) or Itoh et al. (1983) are adopted.

4 The Standard Solar Model

In recent years helioseismology has added important pieces of information on the solar structure, producing severe tests for Standard Solar Model (SSM) calculations. SSMs thus constitute, for each evolutionary code, an important check of the reliability of the physical inputs and of the efficiency of the microscopic mechanisms adopted for the calculations. Bahcall et al. (1995) already showed that, adopting physical inputs very similar to the ones used for the CoRoT ESTA Task 1–Task 3 models, to reach the agreement with helioseismological observables one needs an accurate treatment of helium and heavy element diffusion.

Recent works investigates the effects of the last update of the solar chemical composition (Asplund et al. 2005) on the solar characteristics, pointing out a disagreement between the observed and the predicted sound speed (see e.g. Bahcall et al. 2005; Basu and Antia 2004). However, further investigations are needed and this issue is beyond of the purposes of this contribution.

Table 4 shows selected characteristics of our SSM calculated with the physical inputs of Task 3 models, including He and heavy elements diffusion. Figure 2 shows the comparison between the helioseismological squared isothermal sound speed $U = P/\rho$ from Degl'Innocenti et al. (1997) and the result of our SSM. We notice that the sound speed in the solar interior is nicely reproduced but a small discrepancy at the bottom of the convective zone is present. This problem

[2] These are computed by adopting the facilities at the URL: http://www.phys.llnl.gov/Research/OPAL/

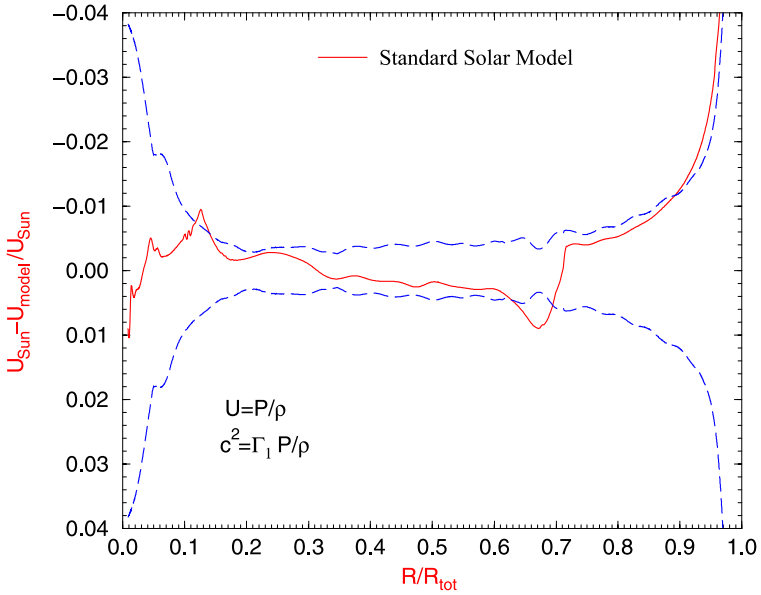

Fig. 2 Relative difference between the helioseismological sound speed and the result of our standard solar model with the physical inputs of CoRoT ESTA Task 1–Task 3 models. The region between the *blue long dashed lines* indicates the allowed region taking into account the uncertainties on the helioseismological results (Degl'Innocenti et al. 1997)

Table 4 Selected characteristics of our SSM with the physical inputs of Task 1, Task 3 models. Original (in), present photospheric (photo), central (c) values and the values at the base of the external convective zone (b) are shown

$t_\odot$ [Gyr]	4.57
$L_\odot$ [erg]	3.846
$R_\odot$ [10^{10} cm]	6.960
$(Z/X)_{photo}$	0.0245
X_{in}	0.712
Y_{in}	0.269
Z_{in}	0.0198
X_{photo}	0.744
Y_{photo}	0.238
Z_{photo}	0.0182
$R_b/R_\odot$	0.716
T_b [10^6 K]	2.16
c_b [10^7 cm s^{-1}]	2.22
T_c [10^7 K]	1.569
ρ_c [100 gr cm^{-}3]	1.517
Y_c	0.63

is well known and discussed in the literature, see e.g. Basu and Antia (1997), Richard et al. (1996).

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