

# Chapter 5

## Stellar Energy Production

Stars have three primary sources of energy:

1. Heat left over from earlier processes,
2. Gravitational energy,
3. Thermonuclear energy.

We shall see that

- Gravitational energy plays an important role in various stages of star birth and star death.
- Objects like white dwarfs shine because of heat left over from earlier thermonuclear and gravitational energy generation.
- However, the virial theorem indicates that the gravity and left-over heat of formation can supply the energy of the Sun only on a  $10^7$  year timescale.

Thus, thermonuclear reactions are the only viable long-term stellar energy source.

## 5.1 Nuclear Energy Sources

The luminosity of the Sun is

$$L_{\odot} \simeq 3.8 \times 10^{33} \text{ erg s}^{-1}$$

and that of the most luminous stars is about  $10^6 L_{\odot}$ . From the Einstein relation  $m = E/c^2$ , the rate of mass conversion to energy that is required to sustain the Sun's luminosity is

$$\Delta m = \frac{1}{c^2} \Delta E \rightarrow \frac{\Delta m}{\Delta t} = \frac{1}{c^2} \frac{\Delta E}{\Delta t} = \frac{L_{\odot}}{c^2} = 4.2 \times 10^{12} \text{ g s}^{-1},$$

and the most luminous stars require conversion rates a million times larger. Let us now discuss how nuclear reactions in stars can account for mass-to-energy conversion on this scale.

### 5.1.1 The Curve of Binding Energy

The binding energy for a nucleus of atomic number  $Z$  and neutron number  $N$  is

$$B(Z, N) \equiv \left[ \underbrace{Zm_p + Nm_n}_{\text{free nucleons}} - \underbrace{m(Z, N)}_{\text{bound system}} \right] c^2,$$

where

- $m(Z, N)$  is the mass of the nucleus
- $m_p$  is the mass of a proton
- $m_n$  is the mass of a neutron.

The binding energy may be interpreted either as

- the energy released in assembling a nucleus from its constituent nucleons, or
- the energy required to break a nucleus apart into its constituents.

The more relevant quantity is often the *binding energy per nucleon*,  $B(Z, N)/A$ , where  $A = Z + N$  is the atomic mass number.

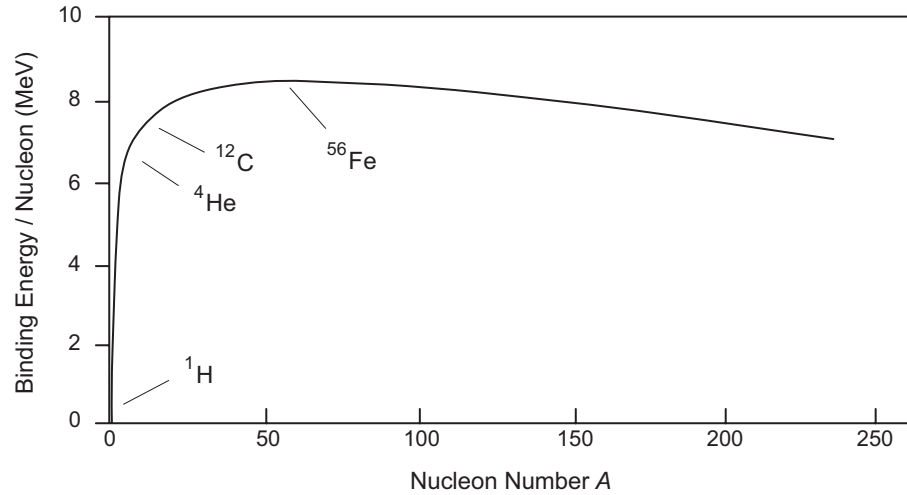


Figure 5.1: The curve of binding energy. Only the average behavior is shown; local fluctuations have been suppressed, as has the isotopic dependence on  $(Z, N)$  for a given  $A$ .

The average behavior of binding energy per nucleon as a function of the atomic mass number  $A$  is shown in Fig. 5.1. The general behavior of the curve of binding energy may be understood from simple nuclear physics considerations, which are elaborated in the typeset chapter but we won't go over here.

### 5.1.2 Masses and Mass Excesses

It is convenient to define the *mass excess*,  $\Delta(A, Z)$ , through

$$\Delta(A, Z) \equiv (m(A, Z) - A) m_{\text{H}} c^2,$$

- $m(A, Z)$  is measured in atomic mass units (amu),
- $A = Z + N$  is the atomic mass number, and
- the atomic mass unit  $m_{\text{H}}$  (which is defined to be  $\frac{1}{12}$  the mass of a  $^{12}\text{C}$  atom) is given by

$$m_{\text{H}} = \frac{1}{N_{\text{A}}} = 1.660420 \times 10^{-24} \text{ g} = 931.478 \text{ MeV}/c^2,$$

with  $N_{\text{A}} = 6.02 \times 10^{23} \text{ mole}^{-1}$  Avogadro's number.

The mass excess is useful because

- The number of nucleons is constant in low-energy nuclear reactions (baryon number conservation).
- Therefore, the atomic mass numbers cancel on both sides of any equation.
- Thus, sums and differences of masses (*large numbers*) may equivalently be replaced by the corresponding sums and differences of mass excesses (*small numbers*).

**Example:** Using the definition of the mass excess, the binding energy equation

$$B(Z, N) \equiv [Zm_p + Nm_n - m(Z, N)]c^2,$$

may be rewritten as

$$\begin{aligned} B(Z, N) &= [Zm_p + Nm_n - m(Z, N)]c^2 \\ &= [Zm_p + (A - Z)m_n - m(Z, N)]c^2 \\ &= [Z\Delta_p + (A - Z)\Delta_n - \Delta(A, Z)]c^2 \\ &= [Z\Delta_p + (A - Z)\Delta_n - \Delta(A, Z)] \times 931.478 \text{ MeV}, \end{aligned}$$

where we have abbreviated the mass excess of the neutron and proton by  $\Delta(1, 0) \equiv \Delta_n$  and  $\Delta(1, 1) \equiv \Delta_p$ , respectively.

In many mass tables, the mass excesses rather than the masses themselves are tabulated.

**Example:** Let us calculate the binding energy of  ${}^4\text{He}$ . The relevant mass excesses are

$$\Delta_p = 7.289 \text{ MeV} \quad \Delta_n = 8.071 \text{ MeV} \quad \Delta(4, 2) = 2.425 \text{ MeV}$$

and the binding energy of  ${}^4\text{He}$  is then

$$\begin{aligned} B(Z, N) &= [Z\Delta_p + (A - Z)\Delta_n - \Delta(A, Z)]c^2 \\ &= 2 \times 7.289 + 2 \times 8.071 - 2.425 = 28.3 \text{ MeV}. \end{aligned}$$

Thus more than 28 MeV of energy is required to separate  ${}^4\text{He}$  into free neutrons and protons.

### 5.1.3 $Q$ -Values

The  $Q$ -value for a reaction is the *total mass of the reactants minus the total mass of the products*, which is equivalent to the corresponding difference in mass excesses,

$$\begin{aligned} Q &= \text{Mass of reactants} - \text{Mass of products} \\ &= \text{Mass excess of reactants} - \text{Mass excess of products.} \end{aligned}$$

It is common to specify the  $Q$ -value in energy units rather than mass units.



**Example:** For the nuclear reaction



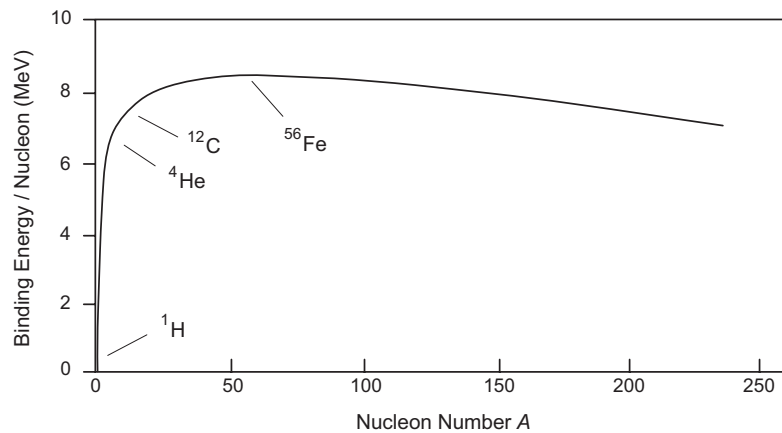
the tabulated mass excesses are

$$\begin{aligned}\Delta({}^2\text{H}) &= 13.136 \text{ MeV} & \Delta({}^{12}\text{C}) &= 0 \text{ MeV} \\ \Delta({}^1\text{H}) &= 7.289 \text{ MeV} & \Delta({}^{13}\text{C}) &= 3.1246 \text{ MeV}.\end{aligned}$$

The  $Q$ -value for this reaction is then

$$Q = \Delta({}^2\text{H}) + \Delta({}^{12}\text{C}) - \Delta({}^1\text{H}) - \Delta({}^{13}\text{C}) = +2.72 \text{ MeV}.$$

- The positive  $Q$  indicates that this is an *exothermic reaction*: 2.72 MeV is liberated from binding energy in the reaction, appearing as kinetic energy or internal excitation of the products.
- Conversely, a negative value of  $Q$  would indicate an *endothermic reaction*: additional energy must be supplied to make the reaction viable.



### 5.1.4 Efficiency of Hydrogen Fusion

Examination of the curve of binding energy suggests two potential nuclear sources of energy:

- Fission of heavier elements into lighter elements.
- Fusion of lighter elements into heavier ones.

Since stars are composed mostly of hydrogen and helium,

- It is clear that their primary energy source must be fusion of lighter elements to heavier ones.
- Coulomb repulsion between charged nuclei will inhibit the possibility of fusion, so hydrogen ( $Z = 1$ ) will be easier to fuse than helium ( $Z = 2$ ).
- In particular, we shall see shortly that main sequence stars are powered by thermonuclear processes that convert four  $^1\text{H}$  into  $^4\text{He}$ .

The total rest mass energy in one gram of material is

$$E = mc^2 = 9 \times 10^{20} \text{ erg},$$

Energy released in fusing 1 gm  $^1\text{H}$  into  $^4\text{He}$  (Exercise 5.1):

$$\Delta E(\text{fusion H} \rightarrow ^4\text{He}) = 6.3 \times 10^{18} \text{ erg g}^{-1}.$$

Therefore, less than 1% of the initial rest mass is converted into energy in the stellar fusion of hydrogen into helium:

$$\frac{\Delta E(\text{fusion H} \rightarrow ^4\text{He})}{\text{total rest-mass energy}} = \frac{6.3 \times 10^{18} \text{ erg g}^{-1}}{9 \times 10^{20} \text{ erg g}^{-1}} \simeq 0.007.$$

- Hydrogen fusion is actually a rather *inefficient* source of energy.
- Furthermore, we shall see shortly that the fusion rates in the cores of lower-mass main sequence stars such as the Sun are actually quite small.

The solar luminosity is produced by a power source equivalent to *several 100 W lightbulbs per  $m^3$*  of the solar core (Exercise 5.18).

- Fusion powers stars not because of its intrinsic efficiency, but rather because of the enormous mass of stars, which implies that they have *large reservoirs of hydrogen*.

## 5.2 Thermonuclear Hydrogen Burning

The primary energy source of stars on the main sequence is fusion of hydrogen into helium. Two general sets of reactions can accomplish this:

1. The *proton–proton chain (PP chain)*
2. The *CNO cycle*

Generally we find that

- The proton–proton chain produces most of the present energy of the Sun and generally is dominant in stars of a solar mass or less.
- The CNO cycle quickly surpasses the proton–proton chain in energy production as soon as the mass exceeds about a solar mass.
- The reason for this rapid switchover is that the PP chain and the CNO cycle have strong and very different dependence on temperature.

### 5.2.1 The Proton–Proton Chain

The most important reactions of the PP chain are summarized in the following figures:

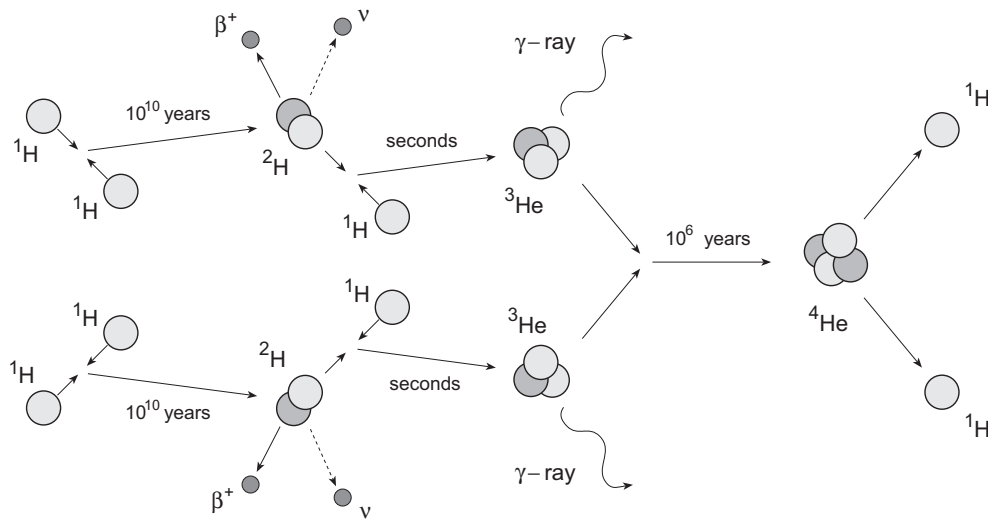


Figure 5.2: Main branch PP-I of the proton–proton chain. The inverses of the times indicate the relative rates for each step under solar conditions and will be discussed in more detail later.

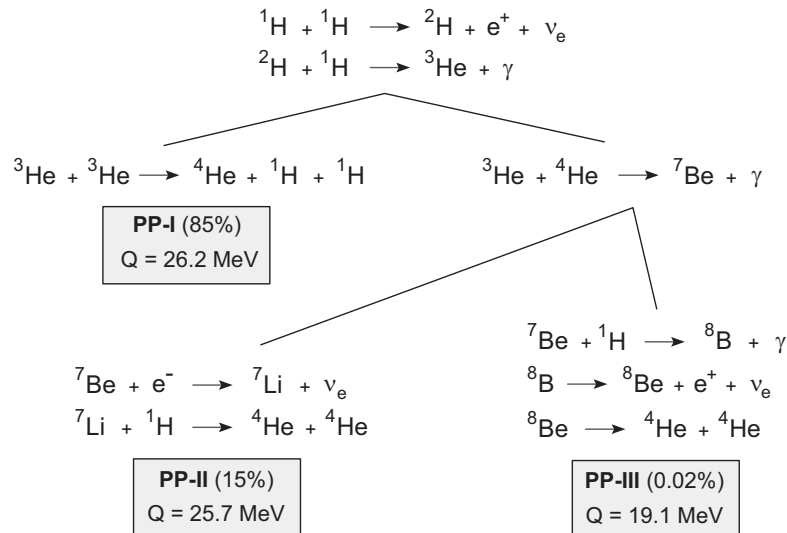


Figure 5.3: The three branches of the PP chain. The percentage contribution to solar energy production and the effective  $Q$ -value are shown for each branch.

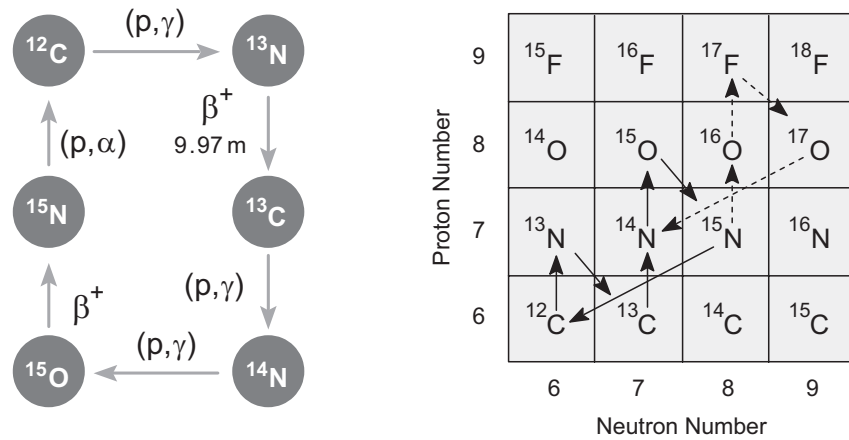


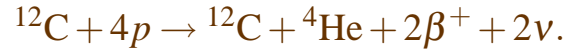
Figure 5.4: The CNO cycle. The main part of the cycle is illustrated schematically on the left side. On the right side the main part of the cycle is illustrated with solid arrows and a side branch is illustrated with dashed arrows. The notation  $(p, i)$  means a proton capture followed by emission of  $i$ ; for example  $^{12}\text{C}(p, \gamma)^{13}\text{N}$ . The notation  $\beta^+$  indicates beta decay by positron emission; for example,  $^{13}\text{N} \rightarrow ^{13}\text{C} + e^+ + \nu_e$ .

### 5.2.2 The CNO Cycle

The name of the carbon–nitrogen–oxygen or CNO cycle derives from the role played by carbon, nitrogen, and oxygen in the corresponding sequence of reactions. The CNO cycle is summarized in Fig. 5.4.

**CNO Catalysis**

The main part of the CNO cycle is termed the *CN cycle*. Summing net reactants and products around the CN cycle,



(The  $\gamma$ -rays have been neglected since they do not correspond to a conserved quantity.)

- Therefore, the  $^{12}\text{C}$  serves as a *catalyst* for the conversion of four protons to  $^4\text{He}$ .
- It is required for the sequence to take place, but it is not consumed in the process because a  $^{12}\text{C}$  is returned in the last step of the cycle.

We have written the CNO sequence as if the (p,  $\gamma$ ) reaction on  $^{12}\text{C}$  were the first step, but it is a closed cycle and we may consider any step to be the initial one.

- This implies that any of the carbon, nitrogen, or oxygen isotopes appearing in the cycle may be viewed as a catalyst that serves to convert protons into helium.
- The closed nature of the cycle also implies that
  1. Any mixture of these isotopes will play the same catalytic role.
  2. If any one of the CNO isotopes is present initially a mixture of the others will inevitably be produced by the cycle of reactions.



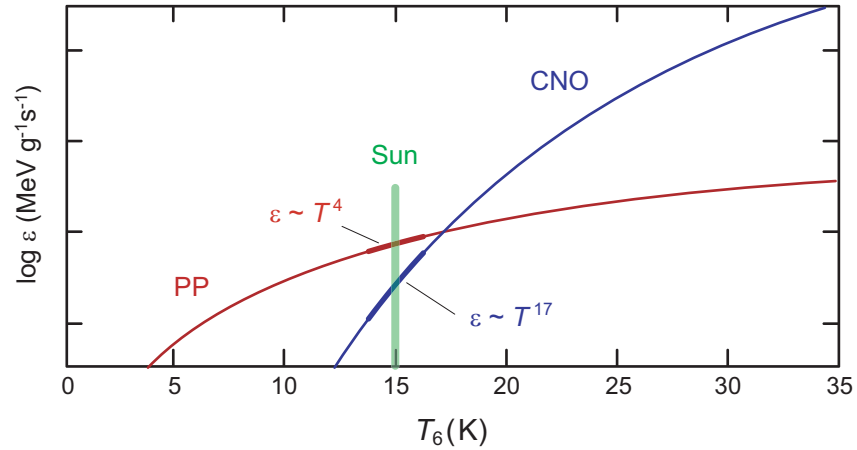


Figure 5.5: Rate of energy release in the PP chain and in the CNO cycle.  $T_6$  denotes the temperature in units of  $10^6$  K.

The rates of energy release from hydrogen burning for the PP chain and CNO cycle are illustrated in Fig. 5.5.

- Note that the CNO cycle has a much stronger temperature dependence than the PP chain.
- This temperature dependence implies that the star's mass on the main sequence is generally the most important factor governing the competition of PP and CNO energy production, because that strongly influences the core temperature.

We will see how to calculate these rates shortly.

The PP cycle can occur in any star containing H, but the CNO cycle requires the presence of C, N, or O as catalysts.

- Therefore, the CNO cycle should be relatively more important in Pop I stars.
- However, this is generally of secondary importance for the competition between the PP chain and CNO cycle in producing stellar energy.
- This issue is also of importance in understanding the very first generation of stars that formed (Pop III).
  - No CNO isotopes produced in the big bang.
  - Thus the first generation of stars must have operated by the PP chain until some of those stars could produce carbon by triple- $\alpha$  (see below).
- These considerations are important for understanding the early Universe because relatively massive stars formed surprisingly early.
  1. CNO is more efficient at producing energy in massive stars than PP because of its very strong temperature dependence.
  2. Thus the pace of early structure evolution likely depended on when the earliest stars produced and distributed sufficient CNO isotopes to allow a succeeding generation of stars to switch to the more efficient CNO cycle.

### 5.3 Cross Sections and Reaction Rates

To proceed with a more quantitative analysis of energy production in stars, we need to introduce the rudiments of nuclear reaction theory as applied in stellar environments. Consider the representative nuclear reaction



where  $Z^*$  denotes an excited compound nucleus as an intermediate state [we will often write this equation in the shorthand notation  $X(\alpha, \beta)Y$ ].

- A compound nucleus is an excited composite formed in the initial collision that quickly decays into the final products of the reaction.
- In the above reaction the left side ( $\alpha + X$ ) is called the *entrance channel* and the right side ( $Y + \beta$ ) is called the *exit channel* for the reaction.
- It is common to classify nuclear reactions according to the number of (nuclear) species in the entrance channel; thus this is a **2-body reaction**.
- We shall often use 2-body reactions to illustrate but
  - 1-body reactions like  $A \rightarrow B + C$  and
  - 3-body reactions like  $A + B + C \rightarrow D$
 also play roles in stellar energy production.

Let us first consider a laboratory setting where the reaction is initiated by a beam of projectiles  $\alpha$  directed onto a target containing nuclei  $X$ .

- The *cross section*  $\sigma_{\alpha\beta}(v)$  (a function of the velocity  $v$ ) is

$$\sigma_{\alpha\beta}(v) \equiv \frac{\rho_{\alpha\beta}}{F(v)} = \left( \frac{\text{reactions per unit time per target nucleus}}{\text{incident flux of projectiles}} \right),$$

and has units of area (a commonly-used unit of cross section is the *barn* ( $b$ ), which is defined to be a *cross section of*  $10^{-24} \text{ cm}^2$ ).

- The incident particle flux  $F(v)$  is given by

$$F(v) = n_{\alpha}v,$$

where  $n_{\alpha}$  is the number density of projectiles  $\alpha$  in the beam and  $v$  is their relative velocity.

- The number of reactions per unit time (reaction rate) per target nucleus  $\rho_{\alpha\beta}$  is

$$\rho_{\alpha\beta} = F(v)\sigma_{\alpha\beta} = n_{\alpha}v\sigma_{\alpha\beta},$$

and the *total reaction rate per unit volume*  $r_{\alpha\beta}(v)$  results from multiplying  $\rho_{\alpha\beta}$  by the number density  $n_X$  of target nuclei  $X$ :

$$r_{\alpha\beta}(v) = \rho_{\alpha\beta}n_X = n_{\alpha}n_Xv\sigma_{\alpha\beta}(v)(1 + \delta_{\alpha X})^{-1},$$

and has units of  $\text{cm}^{-3} \text{ s}^{-1}$  in the CGS system.

- The factor involving the Kroenecker  $\delta$  ( $\delta_{ab}$  is one if  $a = b$  and zero is  $a \neq b$ ) is introduced to prevent overcounting when the colliding particles are identical.
  1. The product  $n_\alpha n_X v \sigma_{\alpha\beta}(v)$  is the rate per unit volume for the 2-body reaction
  2.  $n_\alpha n_X$  is the number of unique particle pairs  $(\alpha, X)$  contained in the unit volume.
  3. But for the collision of identical particles ( $\alpha = X$ ), the number of independent particle pairs  $(\alpha, \alpha)$  is not  $N_\alpha^2$  but  $\frac{1}{2}N_\alpha^2$ .
  4. Therefore, for identical particles the rate expression must be multiplied by a factor of  $1/(1 + \delta_{\alpha X}) = \frac{1}{2}$  to avoid double counting.
  5. More generally, one finds that for  $N$  identical particles a factor of  $1/N!$  is required to prevent double counting.
- Normally we will work in the *center of mass coordinate system*, so velocities, energies, momenta, and cross sections will be center of mass quantities, unless otherwise noted:

$$E = E_{\text{CM}} = \left( \frac{m_X}{m_\alpha + m_X} \right) E_{\text{lab}} \quad v = \sqrt{2E/\mu} \quad \mu \equiv \frac{m_\alpha m_X}{m_\alpha + m_X},$$

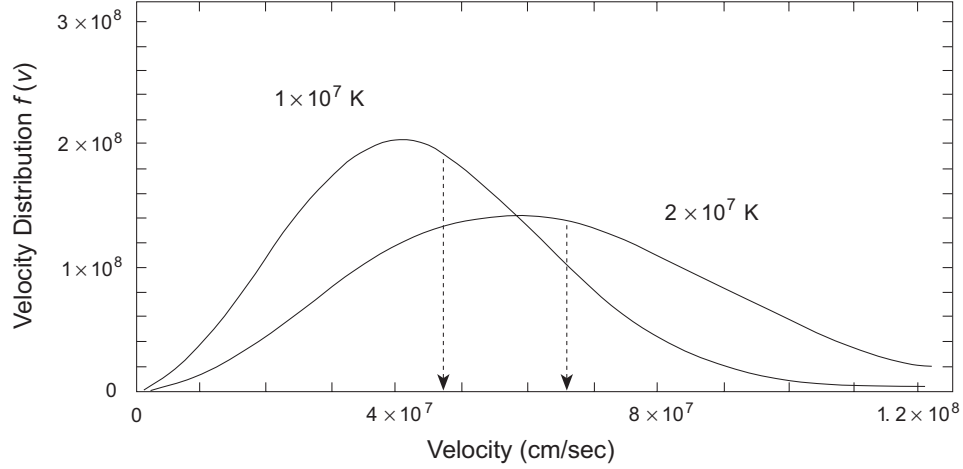


Figure 5.6: Maxwellian velocity distribution for two temperatures; the dashed arrows indicate the mean velocities for each distribution.

#### 5.4 Thermally-Averaged Reaction Rates

The preceding equations assume a beam of monoenergetic particles.

- In a stellar environment we instead have a gas in approximate thermal equilibrium.
- If we assume that the gas can be described classically, at equilibrium it has a *Maxwell–Boltzmann distribution*  $\psi(E)$  of energies

$$\psi(E) = \frac{2}{\pi^{1/2}} \frac{E^{1/2}}{(kT)^{3/2}} \exp(-E/kT).$$

This distribution is illustrated for two different temperatures in Fig. 5.6 as a function of  $v = \sqrt{2E/\mu}$ .

- A thermally-averaged cross section  $\langle \sigma v \rangle_{\alpha\beta}$  may be defined by averaging the velocity-dependent cross section over the velocities in the gas,

$$\begin{aligned}\langle \sigma v \rangle_{\alpha\beta} &\equiv \int_0^\infty \psi(E) \sigma_{\alpha\beta}(E) v dE, \\ &= \sqrt{\frac{8}{\pi\mu}} (kT)^{-3/2} \int_0^\infty \sigma_{\alpha\beta}(E) e^{-E/kT} E dE,\end{aligned}$$

where the second form follows from substituting  $v = \sqrt{2E/\mu}$ .

- The units of  $\langle \sigma v \rangle_{\alpha\beta}$  are  $\text{cm}^3 \text{s}^{-1} = \text{cm}^2 \cdot (\text{cm/s})$  or cross section times velocity. We may then introduce a thermally-averaged reaction rate:

$$\begin{aligned}r_{\alpha\beta} &= n_\alpha n_X \int_0^\infty \psi(E) \sigma_{\alpha\beta}(E) v dE = n_\alpha n_X \langle \sigma v \rangle_{\alpha\beta} \\ &= \rho^2 N_A^2 \frac{X_\alpha X_X}{A_\alpha A_X} \langle \sigma v \rangle_{\alpha\beta} = \rho^2 N_A^2 Y_\alpha Y_X \langle \sigma v \rangle_{\alpha\beta},\end{aligned}$$

where in the interest of compact notation

1. We drop explicit display of the  $\delta$ -function factor necessary when the colliding particles are identical, and
2. In the second line we introduce the mass fractions  $X_i$  and the abundances (molar fractions)  $Y_i$  defined in Ch. 3.

- The units of  $r_{\alpha\beta}$  are  $\text{cm}^{-3} \text{s}^{-1}$  (rate per unit volume),
- The clear physical interpretation of  $r_{\alpha\beta} = n_\alpha n_X \langle \sigma v \rangle_{\alpha\beta}$  is that the total rate for this 2-body reaction is the (thermally averaged) rate for a single  $\alpha$  to react with a single  $X$  to produce  $Y + \beta$ , multiplied by the number of  $\alpha$ s per unit volume and the number of  $X$ s per unit volume.

## 5.5 Parameterization of Cross Sections

To proceed, we require the cross section evaluated in the center of mass system in order to calculate the thermally-averaged rates. This cross section may be parameterized in the general form

$$\sigma_{\alpha\beta}(E) = \pi g \lambda^2 \frac{\Gamma_{\alpha} \Gamma_{\beta}}{\Gamma^2} f(E),$$

- The *energy widths*  $\Gamma_i \equiv \hbar/\tau_i$  are expressed in terms of the corresponding mean life  $\tau_i$  for decay of the compound system through channel  $i$ ,
- the entrance channel is denoted by  $\alpha$ ,
- the exit channel by  $\beta$ ,
- the total width is  $\Gamma = \sum_i \Gamma_i$ , where the sum is over all open channels  $i$ ,
- the probability to decay to channel  $i$  is  $P_i = \Gamma_i/\Gamma$ ,
- the reduced deBroglie wavelength is defined through  $\lambda^2 = \hbar^2/2\mu E$ ,
- The statistical factor  $g$  contains information on the spins of projectile, target, and compound nucleus (typically of order 1)
- The detailed reaction information resides in  $f(E)$ .



The parameters  $\Gamma$  appearing in

$$\sigma_{\alpha\beta}(E) = \pi g \lambda^2 \frac{\Gamma_\alpha \Gamma_\beta}{\Gamma^2} f(E),$$

have the units of  $\hbar$  divided by time, which is energy.

- They are called *energy widths* because states with short lifetimes for decay (large decay rates) correspond to spectral peaks (resonances) broad in energy, by a  $\Delta E \cdot \Delta t \simeq \hbar$  uncertainty principle argument.
- Conversely, states with long decay lifetimes (small decay rates) correspond to narrow resonances. The limiting case is a state that is completely stable, which then corresponds to a vanishing decay rate and has a sharply defined energy.
- The factor  $f(E)$  is generally either
  1. *Resonant*, in which case it is strongly peaked in energy because of a narrow (quasibound) state in the compound nucleus, or
  2. *Nonresonant*, because the reaction energy is far from a resonance, or there are no resonances (quasibound states) in the channel of interest.
- The total rates will typically be a sum of contributions for resonant and nonresonant pieces.

## 5.6 Resonant Cross Sections

In the simplest case of an isolated resonance,  $f(E)$  can be expressed in the *Breit–Wigner form*

$$f(E)_{\text{res}} = \frac{\Gamma^2}{(E - E_r)^2 + (\Gamma/2)^2},$$

where the resonance energy  $E_r$  is related to a corresponding excitation energy  $E^*$  for a quasibound state in the compound nucleus through

$$E_r = (m_Z - m_\alpha - m_X)c^2 + E^*.$$

The corresponding Breit–Wigner cross section is

$$\begin{aligned}\sigma_{\alpha\beta} &= \pi g \lambda^2 \frac{\Gamma_\alpha \Gamma_\beta}{\Gamma^2} f(E) \\ &= \pi g \lambda^2 \frac{\Gamma_\alpha \Gamma_\beta}{(E - E_r)^2 + (\Gamma/2)^2},\end{aligned}$$

and will generally exhibit a strong peak near  $E_r$ .

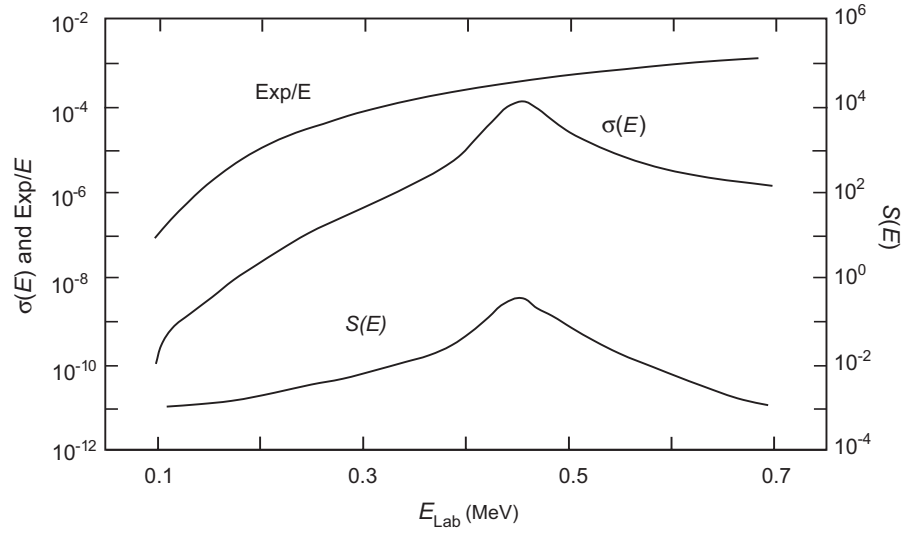


Figure 5.7: Cross section  $\sigma(E)$  in barns for the reaction  $^{12}\text{C}(p, \gamma)^{13}\text{N}$ . The  $S$ -factor  $S(E)$  is defined later. The curve labeled  $\text{Exp}/E$  is the factor  $\exp(-2\pi\eta)/E$ .

As an example, we consider the reaction  $^{12}\text{C}(p, \gamma)^{13}\text{N}$  illustrated in Fig. 5.7. This reaction has a resonance corresponding to a state in  $^{13}\text{N}$  at an excitation energy of 2.37 MeV that is strongly excited at a laboratory proton energy of 0.46 MeV.

If the Maxwell–Boltzmann distribution  $\psi(E)$  and the widths  $\Gamma_i$  vary slowly over a resonance, we may assume

$$\psi(E) \rightarrow \psi(E_r) \quad \Gamma_\alpha \rightarrow \Gamma_\alpha(E_r) \quad \Gamma_\beta \rightarrow \Gamma_\beta(E_r),$$

and we obtain the resonant velocity-averaged cross section

$$\begin{aligned} \langle \sigma v \rangle_{\alpha\beta} = & \frac{\pi \hbar^2 g}{2\mu} \sqrt{\frac{8}{\pi\mu}} (kT)^{-3/2} e^{-E_r/kT} \\ & \times \Gamma_\alpha(E_r) \Gamma_\beta(E_r) \int_0^\infty \frac{1}{(E - E_r)^2 + (\Gamma/2)^2} dE. \end{aligned}$$

If the resonance is broad or  $E_r$  is small, the preceding assumptions may be invalid and it may be necessary to integrate over the resonance energy numerically using the data.

The integrand peaks near  $E_r$ ; extending the lower limit of the integral to minus infinity and assuming the widths to be constant gives

$$\langle \sigma v \rangle_{\alpha\beta} = 2.56 \times 10^{-13} \frac{(\omega\gamma)_r}{(\mu T_9)^{3/2}} \exp(-11.605 E_r/T_9) \text{ cm}^3 \text{ s}^{-1},$$

where  $E_r$  is in MeV,  $T_9$  indicates temperature in units of  $10^9$  K, and

$$(\omega\gamma)_r \equiv g \frac{\Gamma_\alpha \Gamma_\beta}{\Gamma}$$

has units of MeV and is tabulated for reactions of interest.

## 5.7 Nonresonant Cross Sections

Most (but not all) reactions relevant for stellar energy production are, not surprisingly, exothermic

$$Q \equiv (\text{mass of reactants}) - (\text{mass of products}) \\ = m_{\alpha}c^2 + m_{\chi}c^2 - m_{\beta}c^2 - m_{\gamma}c^2 > 0.$$

- Typical  $Q$ -values for the reactions of interest are  $\sim 1$  MeV, with this energy going into kinetic energy of particles in the exit channel and any internal excitation of the products.
- This additional energy leads to a marked asymmetry in the entrance and exit channels for charged particle reactions of interest in stellar energy production.
- In the entrance channel, the thermal energies available are set by the temperatures through

$$kT = 8.6174 \times 10^{-8} T \text{ keV},$$

with the temperature expressed in kelvin.

- Hydrogen burning in main sequence stars typically occurs in a temperature range  $10^7 \text{ K} \lesssim T \lesssim 10^9 \text{ K}$ , implying kinetic energies in the plasma of  $1 \text{ keV} \lesssim kT \lesssim 100 \text{ keV}$ .
- Thus, for average  $Q \sim 1 \text{ MeV}$  noted above, we often have

$$E(\text{entrance}) \ll E(\text{exit})$$

for reactions of interest.

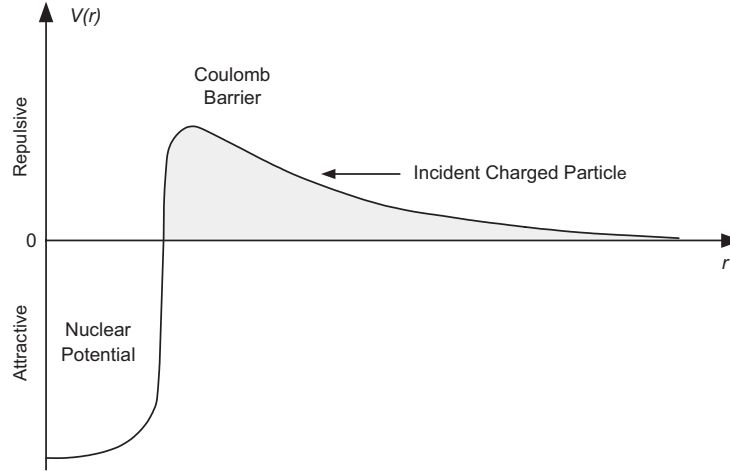


Figure 5.8: The Coulomb barrier for charged-particle reactions.

### 5.7.1 Coulomb Barriers

Because of the low average energies in the entrance channel, charged-particle reactions are strongly influenced by the Coulomb barrier

$$E_{\text{CB}} = 1.44 \frac{Z_{\alpha} Z_{\text{x}}}{R(\text{fm})} \text{ MeV},$$

where  $Z_i$  is the atomic number of particle  $i$ , the barrier separation distance  $R$  is

$$R \simeq 1.3(A_{\alpha}^{1/3} + A_{\text{x}}^{1/3}) \text{ fm},$$

and  $A_i$  is the atomic mass number (in amu) of particle  $i$ . As is common in nuclear physics, energies have been expressed in MeV and distances in fermis (fm).

**Example:** Consider a proton scattering from a  $^{28}\text{Si}$  nucleus. From

$$E_{\text{CB}} = 1.44 \frac{Z_{\alpha} Z_x}{R(\text{fm})} \text{ MeV} \quad R \simeq 1.3(A_{\alpha}^{1/3} + A_x^{1/3}) \text{ fm}$$

we obtain

$$E_{\text{CB}} = 1.44 \frac{(1)(14)}{1.3(1^{1/3} + 28^{1/3})} \text{ MeV} = 3.8 \text{ MeV}.$$

Table 5.1: Coulomb barriers for  $p + X$ 

X	$Z_\alpha Z_\beta$	$R$ (fm)	$E_{CB}$ (MeV)
$^1_1\text{H}$	1	2.6	0.55
$^{12}_6\text{C}$	6	4.3	2.0
$^{28}_{14}\text{Si}$	14	5.2	3.8
$^{56}_{26}\text{Fe}$	26	6.3	6.0

Some typical Coulomb barriers for proton reactions  $p + X$  are shown in Table 5.1, where we note that

- Entrance channel energies for hydrogen fusion in stars (which lie in the range  $10^{-3}$  to  $10^{-1}$  MeV) are typically orders of magnitude lower than the Coulomb barrier.
- As we shall see, this implies a *dramatic temperature dependence* for hydrogen fusion reactions.
- On the other hand, exit channel energies (approximately 1 MeV in typical cases) are comparable to the barrier energies for fusion of protons with lighter ions.



### 5.7.2 Barrier Penetration Factors

Classically, characteristic energies in a stellar plasma are *too small to surmount the Coulomb barrier* for typical charged-particle reactions.

- However, it is possible for *quantum-mechanical tunneling* to take place for energies that lie below the height of the barrier, albeit with small probability.
- Assuming  $E_{\text{CB}} \gg E$ , the barrier penetration probability for a collision with zero relative orbital angular momentum (termed *s-waves in scattering theory*) is

$$P(E) \propto e^{-2\pi\eta},$$

where the dimensionless Sommerfeld parameter  $\eta$  is defined by

$$\eta = \frac{Z_\alpha Z_X e^2}{\hbar v} = \sqrt{\frac{\mu}{2}} \frac{Z_\alpha Z_X e^2}{\hbar E^{1/2}}.$$

- For representative values of the parameters,  $P(E) \approx \exp(-12)$ . Thus, charged particle reactions crucial to power generation in stars are *highly improbable events*.
- Since the reaction probability for barrier penetration will be dominated by the probability to penetrate the barrier, we take as an *entrance channel width for nonresonant reactions*

$$\Gamma_\alpha \simeq e^{-2\pi\eta},$$

which clearly has a strong energy dependence.

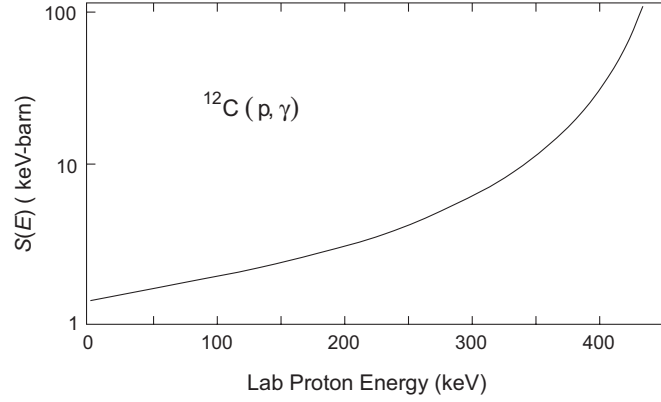


Figure 5.9: S-factor versus laboratory proton energy for a typical capture reaction. This is the portion of the  $S$ -factor for the region below the resonance in Fig. 5.7.

### 5.7.3 Astrophysical S-Factors

Exit channel energies for charged-particle reactions are comparable to the barrier energies; thus we assume that  $\Gamma_\beta$  is a weakly varying function of  $E$ , as is  $\Gamma = \Gamma_\alpha + \Gamma_\beta$ , and we express the nonresonant cross section as

$$\sigma_{\alpha\beta}(E) = \pi g \lambda^2 \frac{\Gamma_\alpha \Gamma_\beta}{\Gamma^2} f(E) \equiv \frac{S(E)}{E} e^{-2\pi\eta},$$

where  $\lambda^2 \propto 1/E$  and  $\Gamma_\alpha \propto e^{-2\pi\eta}$  have been used.

- The factor  $S(E) \equiv \sigma_{\alpha\beta}(E) E e^{2\pi\eta}$ , is termed the *astrophysical S-factor*.
- It is assumed to vary slowly with  $E$  and contains all energy dependence not introduced explicitly through the factors  $\lambda^2 \sim 1/E$  or  $\exp(-2\pi\eta)$ .

The S-factor for a  $(p, \gamma)$  reaction is illustrated in Fig. 5.9.

- Because of the low energies corresponding to  $kT$  that characterize stellar plasmas, reaction cross sections are often needed at *lower energies than can feasibly be measured in a laboratory*.
- Then, experimental measurements at higher energy must be *extrapolated to the lower energy of interest*.
- This is usually done by assuming no resonance at the lower energy and plotting

$$S(E) \equiv \sigma_{\alpha\beta}(E)Ee^{2\pi\eta},$$

which has smooth behavior and therefore is more easily extrapolated than the full cross section.

Then from

$$\langle \sigma v \rangle_{\alpha\beta} = \sqrt{\frac{8}{\pi\mu}} (kT)^{-3/2} \int_0^\infty \sigma_{\alpha\beta}(E) e^{-E/kT} E dE,$$

$$\sigma_{\alpha\beta}(E) = \pi g \lambda^2 \frac{\Gamma_\alpha \Gamma_\beta}{\Gamma^2} f(E) \equiv \frac{S(E)}{E} e^{-2\pi\eta},$$

the thermally-averaged nonresonant cross section is

$$\begin{aligned} \langle \sigma v \rangle_{\alpha\beta} &= \sqrt{\frac{8}{\pi\mu}} (kT)^{-3/2} \int_0^\infty S(E) e^{-E/kT - 2\pi\eta} dE \\ &= \sqrt{\frac{8}{\pi\mu}} (kT)^{-3/2} \int_0^\infty S(E) e^{-E/kT} e^{-bE^{-1/2}} dE \end{aligned}$$

where  $b \equiv 2\pi(\mu/2)^{1/2} Z_\alpha Z_x e^2 / \hbar$  and  $S(E)$  is in  $\text{erg cm}^2$ .

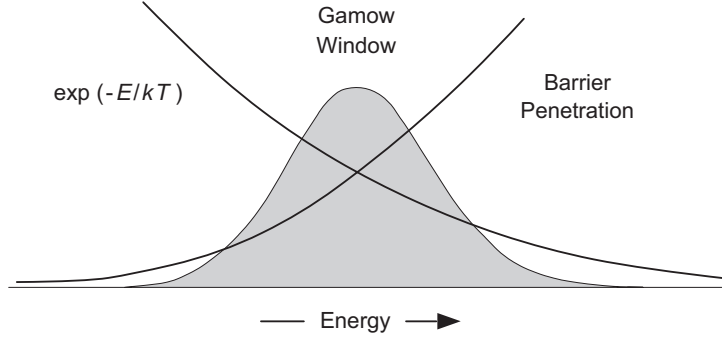


Figure 5.10: The Gamow window.

### 5.7.4 The Gamow Window

The energy dependence of

$$\langle \sigma v \rangle_{\alpha\beta} = \sqrt{(8/(\pi\mu))(kT)^{-3/2}} \int_0^\infty S(E) e^{-E/kT} e^{-bE^{-1/2}} dE$$

resides primarily in the factor

$$F_G \equiv e^{-E/kT} e^{-bE^{-1/2}},$$

which is termed the *Gamow window*.

- The first factor  $\exp(-E/kT)$ , arising from the *Maxwell–Boltzmann velocity distribution*, decreases rapidly with energy.
- The second factor  $\exp(-bE^{-1/2})$ , arising from the *barrier penetration factor*, increases rapidly with energy.
- The product is strongly *localized in energy* (Fig. 5.10).

Only if the energies fall within the relevant Gamow window will charged particle reactions in stellar environments have significant probability to occur.

The maximum of the Gamow peak is found at (see Exercise 5.4)

$$E_0 = 1.22(Z_\alpha^2 Z_X^2 \mu T_6^2)^{1/3} \text{ keV}.$$

- For many reactions of interest this corresponds to only *tens of keV*.
- This implies that laboratory cross sections often must be *extrapolated to these low energies* to calculate astrophysical processes because it is very difficult to do reliable experiments at such low energies.

Useful approximate expressions for the width of the Gamow peak and for the nonresonant cross section can be obtained by

- assuming the Gamow peak to be a Gaussian having the same peak position and curvature at the peak as the realistic Gamow peak (see Exercise 5.5).

In this approximation the width of the Gamow peak is

$$\Delta = \frac{4}{3^{1/2}}(E_0 kT)^{1/2} = 0.75(Z_\alpha^2 Z_X^2 \mu T_6^5)^{1/6} \text{ keV}$$

and the cross section is

$$\langle \sigma v \rangle_{\alpha\beta} \simeq \frac{0.72 \times 10^{-18} S(E_0) a^2}{\mu Z_\alpha Z_X T_6^{2/3}} \exp(-a T_6^{-1/3}),$$

in units of  $\text{cm}^3 \text{s}^{-1}$ , where  $a = 42.49(Z_\alpha^2 Z_X^2 \mu)^{1/3}$  and  $S(E_0)$  is evaluated at the energy of the Gamow peak in units of keV barns.

From the gaussian approximation we find that for the interaction of two protons at a temperature of  $T_6 = 20$  (that is,  $T = 20 \times 10^6$  K),

$$kT = 1.7 \text{ keV} \quad E_0 = 7.2 \text{ keV} \quad \Delta = 8.2 \text{ keV},$$

and from the earlier table the corresponding Coulomb barrier is about 500 keV.

## 5.8 Libraries of Cross Sections

The Gamow window is not Gaussian.

- By using expansions to characterize the deviation from Gaussian behavior of the realistic curve, correction terms may be derived that give a more accurate representation of the thermally-averaged cross section.
- One parameterization that incorporates such correction terms and is often used in reaction rate compilations is

$$\langle \sigma v \rangle = a(f_0 + f_1 T^{1/3} + f_2 T^{2/3} + f_3 T + f_4 T^{4/3} + f_5 T^{5/3}) \frac{e^{-bT^{-1/3}}}{T^{2/3}}$$

where  $a$ ,  $b$ , and  $f_n$  parameterize the cross section.

The Caughlan and Fowler compilation used in Exercises for this chapter is parameterized in this manner.

## 5.9 Total Rate of Energy Production

The total reaction rate per unit volume  $r_{\alpha\beta}$  is given by

$$r_{\alpha\beta} = \rho^2 N_A^2 \frac{X_\alpha X_\beta}{A_\alpha A_\beta} \langle \sigma v \rangle_{\alpha\beta} = \rho^2 N_A^2 Y_\alpha Y_\beta \langle \sigma v \rangle_{\alpha\beta},$$

The corresponding total *rate of energy production per unit mass* is then given by the product of the rate and the  $Q$ -value, divided by the density:

$$\epsilon_{\alpha\beta} = \frac{r_{\alpha\beta} Q}{\rho},$$

which has CGS units of  $\text{erg g}^{-1} \text{s}^{-1}$ . The  $Q$ -value entering this expression is defined by

$$Q \equiv (\text{mass of reactants}) - (\text{mass of products})$$

but with the proviso that *if a reaction produces a neutrino that removes energy from the star without appreciable interaction in the core, this neutrino energy should be subtracted from the total  $Q$ -value.*



## 5.10 Temperature and Density Exponents

It is often useful to parameterize the energy production rate of a star in the power-law form

$$\varepsilon = \varepsilon_0 \rho^\lambda T^\nu,$$

and characterize the behavior of the energy production in terms of the *temperature exponent*  $\nu$  and the *density exponent*  $\lambda$ .

- Although the energy production is not universally of this form, this approximation with constant exponents is usually valid for a limited range of temperatures and densities.
- Since the energy production mechanisms for stars are often operative only in a very narrow range of temperatures and densities, this can provide a useful parameterization for the regions of physical interest.

From the preceding expression, we may define temperature and density exponents for an arbitrary energy production function  $\varepsilon(\rho, T)$  through

$$\lambda = \left( \frac{\partial \ln \varepsilon}{\partial \ln \rho} \right)_T \quad \nu = \left( \frac{\partial \ln \varepsilon}{\partial \ln T} \right)_\rho.$$

Table 5.2: Density and temperature exponents

Stellar process	Density ( $\lambda$ )	Temperature ( $\nu$ )
PP chain	1	$\sim 4$
CNO cycle	1	$\sim 16$
Triple- $\alpha$	2	$\sim 40$

Temperature and density exponents are displayed in Table 5.2 for the PP chain and CNO cycle, and for the triple- $\alpha$  process that burns helium to carbon in red giant stars.

- Notice in Table 5.2 the exquisite temperature dependence exhibited by these reactions.
- This enormous sensitivity of energy production to temperature is a central feature of stellar structure and stellar evolution.

## 5.11 Reaction Selection Rules

Sometimes it is possible to make statements about the astrophysical significance of various nuclear reactions based on selection rules and conservation laws, without having to calculate any detailed rates.

- For example, angular momentum is conserved in all reactions and the angular momentum  $\mathbf{J}$  of a compound nucleus state populated in a two-body reaction must satisfy

$$\mathbf{j}_1 + \mathbf{j}_2 + \mathbf{l} = \mathbf{J}.$$

where  $\mathbf{j}_1$  and  $\mathbf{j}_2$  are the angular momenta associated with the colliding particles and  $\mathbf{l}$  is the angular momentum of relative orbital motion in the entrance channel.

- Likewise, isotopic spin (an abstract approximate symmetry) is conserved to a high degree in strong interactions and the isotopic spins in a two-body reaction must approximately satisfy

$$\mathbf{t}_1 + \mathbf{t}_2 = \mathbf{T},$$

where  $\mathbf{t}_1$  and  $\mathbf{t}_2$  are the isotopic spins associated with the colliding particles and  $\mathbf{T}$  is the isotopic spin of the state that is populated in the reaction.

- Parity (symmetry of the wavefunction under space reflection) is maximally broken in the weak interactions, but is essentially conserved in the strong and electromagnetic reactions. Moreover, in a nuclear reaction that does not involve the weak force, the parities must satisfy

$$(-1)^l \pi(j_1) \pi(j_2) = \pi(J).$$

where  $\pi = \pm$  denotes the parity and  $j_i$  the angular momentum of the states.

Compound nucleus states with angular momentum, isospin, and parity quantum numbers that do not satisfy these selection rules will generally not be populated significantly in reactions.

For nuclei with even numbers of protons and neutrons (*even–even nuclei*) the ground states always have angular momentum and parity  $J^\pi = 0^+$ .

- Therefore, if the colliding particles in the entrance channel are even–even nuclei in their ground states, the angular momentum and the parity of the state excited in the compound nucleus are *both* determined completely by the orbital angular momentum of the entrance channel:

$$J = l \quad \pi(J) = (-1)^l.$$

- Resonance states satisfying this condition are said to have *natural parity*.

Consider the reaction  $\alpha + {}^{16}\text{O} \rightarrow {}^{20}\text{Ne}^*$  (where the \* on the Ne indicates that it is in an excited state).

- Under normal astrophysical conditions the  $\alpha$ -particle and  ${}^{16}\text{O}$  will be in their ground states and thus will each have  $J^\pi = 0^+$ .
- Therefore, parity conservation requires that any state excited in  ${}^{20}\text{Ne}$  by this reaction have parity

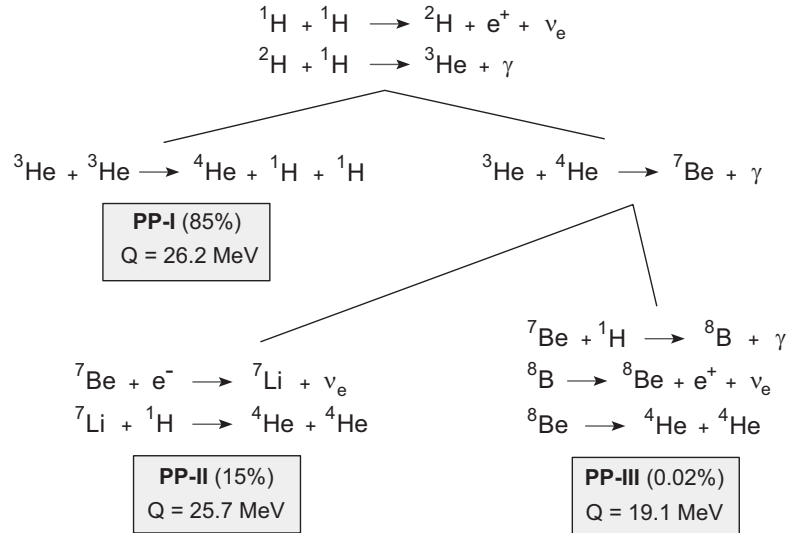
$$\pi({}^{20}\text{Ne}) = (-1)^l = (-1)^J.$$

- We conclude that states in  ${}^{20}\text{Ne}$  having  $J^\pi = 0^+, 1^-, 2^+, 3^-, \dots$  may be populated (because they are natural parity) but that population of states having (say)  $J^\pi = 2^-$  or  $3^+$  is forbidden by parity conservation.
- In the  ${}^{20}\text{Ne}$  spectrum there is a state at 4.97 MeV of excitation relative to the ground state having  $J^\pi = 2^-$ .
- This state cannot be excited in the capture reaction  $\alpha + {}^{16}\text{O} \rightarrow {}^{20}\text{Ne} + \gamma$  because to do so would violate parity conservation (it is not a natural parity state).

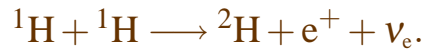
As we shall see in the later discussion of helium burning, if this seemingly obscure state had the opposite parity, we would not exist (!).

## 5.12 Reactions of the Proton–Proton Chain

The slowest reaction in the PP chain,



and therefore the one that governs the overall rate at which the chain produces power, is the initial step



This reaction is very slow because it proceeds by the weak interaction rather than the strong interaction (the appearance of the neutrino on the right side is the sure sign that this is a weak interaction). There is a reason why weak interactions are called weak: *their reaction rates are typically many orders of magnitude smaller than corresponding strong interaction rates.*

The reaction  ${}^1\text{H} + {}^1\text{H} \longrightarrow {}^2\text{H} + \text{e}^+ + \nu_{\text{e}}$  is nonresonant.

- The reaction rate is found to be

$$r_{\text{pp}} = \frac{1}{2} n_{\text{p}}^2 \langle \sigma v \rangle_{\text{pp}} = \frac{1.15 \times 10^9}{T_9^{2/3}} X^2 \rho^2 \exp(-3.38/T_9^{1/3}) \text{ cm}^{-3} \text{ s}^{-1},$$

where  $X$  is the hydrogen mass fraction.

- As shown in Exercise 5.8, the temperature exponent is

$$\nu_{\text{pp}} = 11.3/T_6^{1/3} - 2/3,$$

implying that  $\nu_{\text{pp}} \simeq 4$  for  $T_6 = 15$ .

- The rate of change for proton number because of this reaction is given by the usual radioactive decay law,

$$\frac{dn}{dt} = -\frac{1}{\tau} n,$$

where  $\tau$  is the mean life.

- Thus, the *mean life for a proton* with respect to being converted in the PP chain is

$$\tau_{\text{p}} = -\frac{n_{\text{p}}}{dn_{\text{p}}/dt} = \frac{n_{\text{p}}}{2r_{\text{pp}}},$$

where a factor of two appears in the denominator because two protons are destroyed in each reaction.

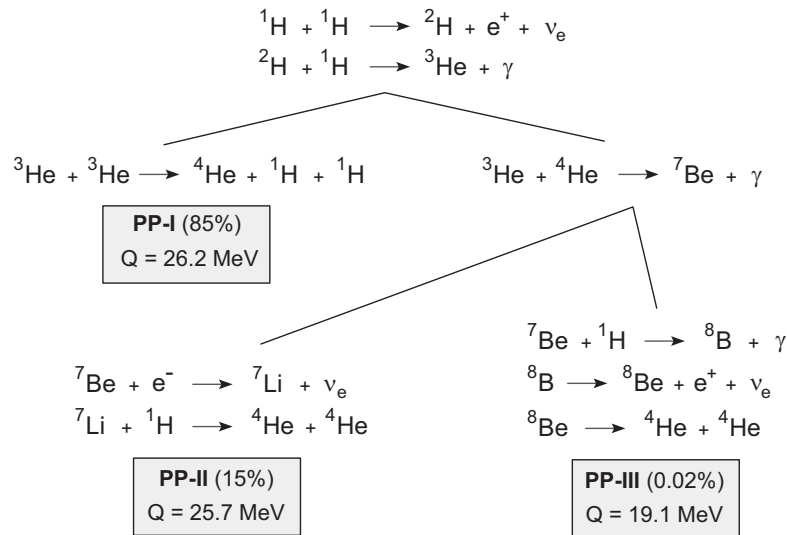
For typical solar conditions we may take a temperature of  $T_6 = 15$ , a central density of  $\rho = 100 \text{ g cm}^{-3}$ , and a hydrogen mass fraction in the center of  $X = 0.5$ .

- This yields an estimate of  $\tau_p \simeq 6 \times 10^9 \text{ years}$ .
- This is remarkably long and sets the scale for the main sequence life of the Sun.

The slowness of the initial step in the PP-chain is ultimately related to the diproton ( ${}^2\text{He}$ ) not being a bound system.

- If the diproton were bound, the first step of the PP-chain could be a strong interaction and the lifetime would be much shorter.
- Instead, the first step must wait for a highly improbable event: a weak decay of a proton from a broad  $p$ - $p$  resonance having a very short lifetime.
- In contrast, the mean life for the deuterium produced in the first step of the PP chain and consumed in the next step ( $p + d \rightarrow {}^3\text{He} + \gamma$ ) is about one minute under the conditions prevailing in the solar core.
- The final fusion of two helium-3 isotopes to form helium-4 is much slower ( $\tau \sim 10^6 \text{ years}$ ), but is orders of magnitude faster than the first step.
- Thus, the initial step of the PP chain governs the rate of the reaction and in turn sets the main sequence lifetime for stars running on the PP chain.





The relative importance of PP-I versus PP-II and PP-III depends on the competition between the reactions



- Over the range of temperatures where PP is expected to be important, the first reaction generally is faster than the second by about four orders of magnitude, ensuring the dominance of PP-I over PP-II and PP-III.
- The branching between PP-II and PP-III depends on the competition between electron capture and proton capture on  ${}^7\text{Be}$ .
- At the temperature of the Sun, electron capture dominates and PP-II is much stronger than PP-III.
- At somewhat higher temperatures, PP-III will begin to make much larger relative contributions (however, at higher temperatures the CNO process will quickly become more important than the PP chains in stellar energy production).

Table 5.3: Effective  $Q$ -values

Process	$Q_{\text{eff}}$ (MeV)	% Solar energy
PP-I	26.2	83.7
PP-II	25.7	14.7
PP-III	19.1	0.02
CNO	23.8	1.6

The effective  $Q$ -values for the PP chain depends on which subchain is followed, since the energy carried off by neutrinos is different in the 3 cases. The effective  $Q$ -values are listed in Table 5.3.

- The average energy released per PP chain fusion in the Sun is

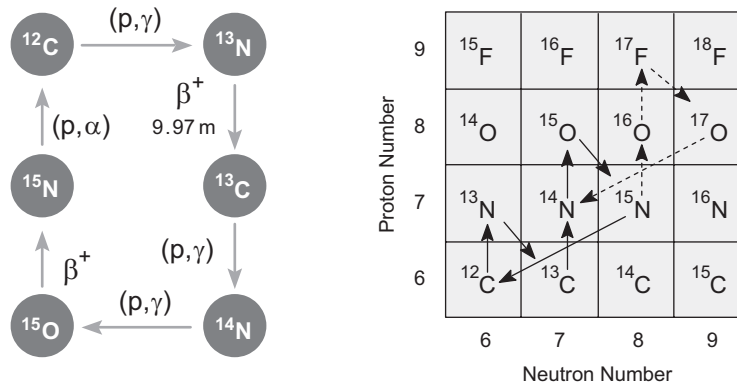
$$\overline{\Delta E}_{\text{pp}} = 0.85 \left( \frac{26.2}{2} \right) + (0.15)(25.7) = 15 \text{ MeV},$$

where PP-III has been ignored and the factor of 2 in the denominator of the first term results from the first two steps of PP-I needing to run twice to provide the two  $^3\text{He}$  isotopes required for the last step.

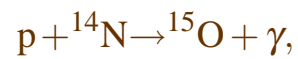
- Although PP-III has negligible influence on solar energy production, the preceding discussion indicates that it produces much higher energy neutrinos than PP-I or PP-II.
- The PP-III chain is highly temperature dependent because it is initiated by proton capture on a  $Z = 4$  nucleus (Coulomb barrier).
- Therefore, detection of the high-energy neutrinos emitted from the PP-III chain can provide a very sensitive probe of the central temperature of the Sun. We shall return to this issue when we discuss the solar neutrino anomaly.

### 5.13 Reactions of the CNO Cycle

Because of Coulomb barriers and  $S$ -factors, the slowest reaction in the CNO cycle



is typically



which has  $S = 3.3$  keV barns in the energy range of interest.

- The corresponding mean life for  $^{14}\text{N}$  against this reaction in the core of the Sun is approximately  $\tau_{^{14}\text{N}} = 5 \times 10^8$  years.
- The concentration of  $^{14}\text{N}$  at the core of the Sun in the Standard Solar Model is  $2.6 \times 10^{22} \text{ cm}^{-3}$  (corresponding to a molar abundance  $Y$  of 0.6%).
- The hydrogen concentration is  $3 \times 10^{25} \text{ cm}^{-3}$  and previously the mean life for consumption of a proton by PP chain fusion was estimated to be  $6 \times 10^9$  years.
- These numbers imply that the ratio of PP chain to CNO cycle reactions in the core of the Sun is approximately

$$\left( \frac{\text{rate for PP}}{\text{rate for } ^{14}\text{N} + \text{p}} \right) = \left( \frac{\tau_{^{14}\text{N}}}{\tau_{\text{pp}}} \right) \left( \frac{3 \times 10^{25}}{2.6 \times 10^{22}} \right) \simeq 100,$$

- We conclude that for conditions prevailing in the Sun the PP chain dominates the CNO cycle.

Detailed calculations within the Standard Solar Model indicate that the Sun is producing 98.4% of its energy from the PP chain and only 1.6% from the CNO cycle.

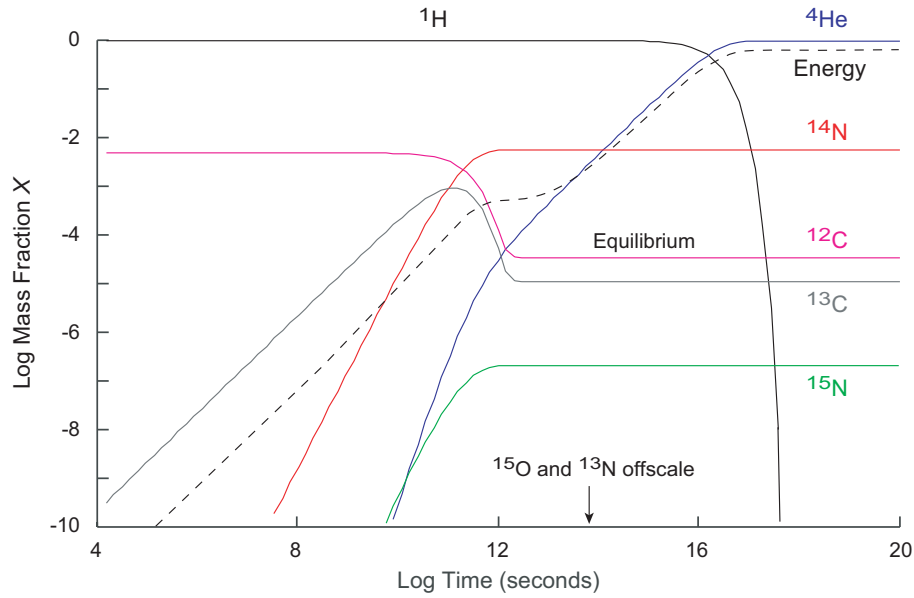
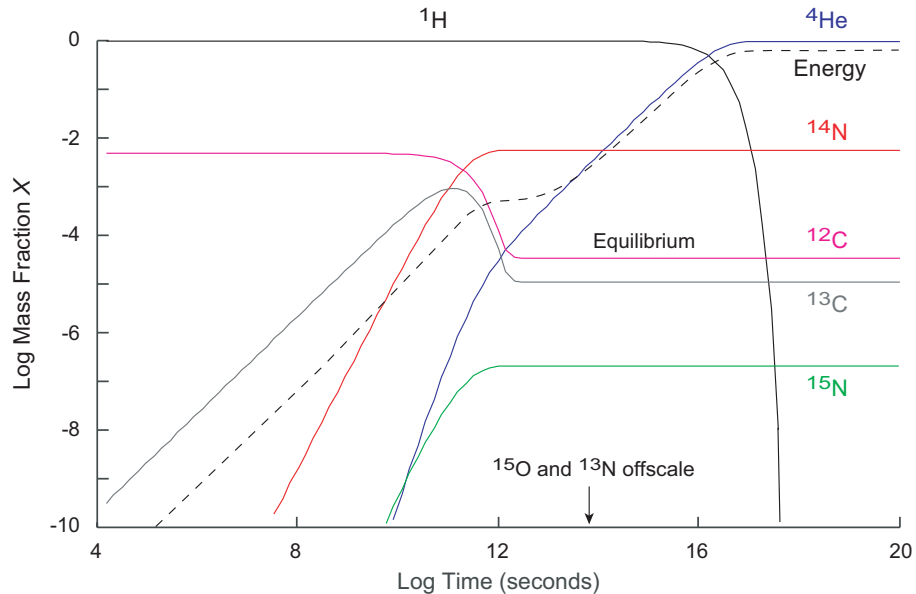


Figure 5.11: The CNO cycle run to completion starting with only hydrogen and a small amount of  $^{12}\text{C}$ . Isotopic mass fractions are shown as solid lines (the mass fractions for  $^{13}\text{N}$  and  $^{15}\text{O}$  are of order  $10^{-14}$  or smaller and are offscale on this plot). The dashed line is the integrated energy release on an arbitrary log scale. The temperature and density were assumed constant at  $T_6 = 20$  and  $\rho = 100 \text{ g cm}^{-3}$ , respectively.

In Fig. 5.11 we illustrate several remarks made earlier by implementing a calculation of the CNO abundances carried to hydrogen depletion (conversion of all hydrogen to helium) for a star with a constant temperature of  $T_6 = 20$  and constant density of  $100 \text{ g cm}^{-3}$ .

- We have assumed an initial mixture having only two isotopes:  $^1\text{H}$  (0.995 mass fraction) and  $^{12}\text{C}$  (0.005 mass fraction).
- Even though we start with only a trace amount of one CNO isotope ( $^{12}\text{C}$ ), the cycle eventually generates an equilibrium abundance of all CNO isotopes and steadily releases energy by converting all the hydrogen to helium.



- Once the cycle is in equilibrium, the mass fractions of the CNO isotopes remain essentially constant, so they may be viewed as catalyzing the conversion of hydrogen to helium, as discussed earlier.
- Notice also the result (which is a general one) that the CNO cycle run long enough to equilibrate tends to produce  $^{14}\text{N}$  as the dominant CNO isotope, even though there was no initial abundance of this isotope at all in this simulation (to understand why, see Exercise 5.14).
- It is thought that most of the  $^{14}\text{N}$  found in the Universe has been produced by the CNO cycle.

The effective  $Q$ -value for the CNO cycle is 23.8 MeV (Table 5.3). The rate of energy production is

$$\epsilon_{\text{CNO}} = \frac{4.4 \times 10^{25} \rho X Z}{T_9^{2/3}} \exp(-15.228/T_9^{1/3}) \text{ erg g}^{-1} \text{ s}^{-1},$$

and the corresponding temperature exponent is

$$\nu_{\text{CNO}} = 50.8/T_6^{1/3} - 2/3,$$

which gives  $\nu_{\text{CNO}} \simeq 18$  for  $T_6 = 20$ .

This *remarkably strong temperature dependence* implies that, were the Sun only slightly hotter, the CNO cycle instead of the PP chain would be the dominant energy production mechanism.

### 5.14 Timescale for Main Sequence Lifetimes

Since the main sequence is defined by stable burning of hydrogen, the rate of hydrogen fusion determines a *timescale for life on the main sequence*.

- Comparison of stellar evolution simulations with observations suggest that *stars leave the main sequence when  $\sim 10\%$  of their original hydrogen has been burned* to helium.
- Let us define a *timescale  $\tau_{\text{nuc}}$  for main sequence lifetimes* by forming the ratio of the energy released from burning 10% of the hydrogen and the luminosity.
- The energy available from the burning of one gram of hydrogen to helium is  $\sim 6 \times 10^{18}$  ergs. Therefore,

$$\tau_{\text{nuc}} = \frac{E_{\text{H}}/10}{L} = 6 \times 10^{17} \frac{XM}{L} \text{ s},$$

- $E_{\text{H}}$  is the energy available from fusing all the hydrogen in the star,
- $L$  is the present luminosity in  $\text{erg s}^{-1}$ ,
- $X$  is the original hydrogen mass fraction,
- $M$  is the mass of the star in grams.



**Example:** Inserting values characteristic of the Sun, we find for the solar main sequence timescale

$$\tau_{\text{nuc}}^{\odot} = \frac{E_{\text{H}}/10}{L} \sim 2.2 \times 10^{17} \text{ s} \sim 10^{10} \text{ yr.}$$

Expressing this timescale in solar units, we may write for any star

$$\tau_{\text{nuc}} = 10^{10} \left( \frac{M}{M_{\odot}} \right) \left( \frac{L_{\odot}}{L} \right) \text{ yr,}$$

and utilizing the mass–luminosity relation

$$\frac{L}{L_{\odot}} \simeq \left( \frac{M}{M_{\odot}} \right)^{3.5},$$

the main sequence timescale may be expressed for  $M \geq M_{\odot}$  as

$$\tau_{\text{nuc}} \simeq 10^{10} \left( \frac{M}{M_{\odot}} \right)^{-2.5} \text{ yr.}$$

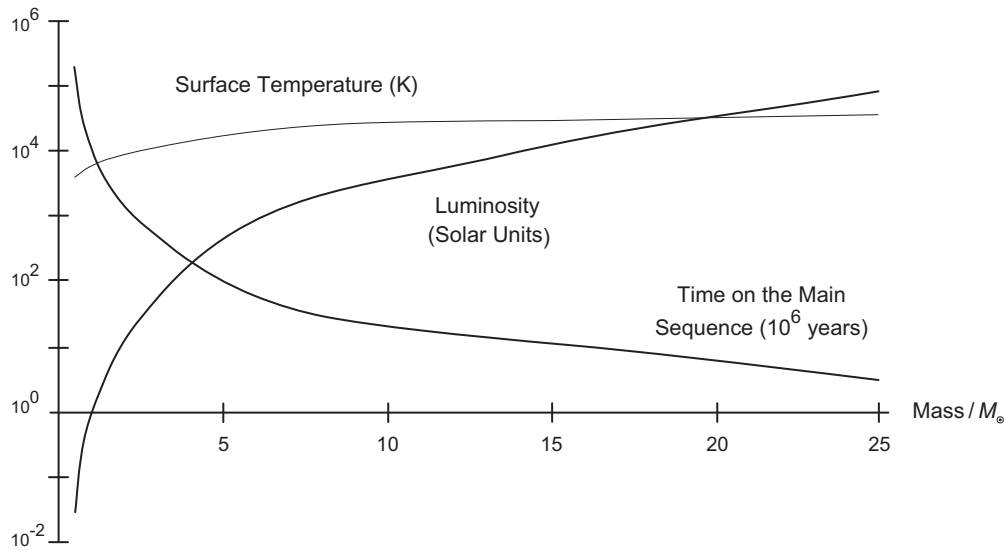


Figure 5.12: Main sequence lifetimes, temperatures, and luminosities.

Some main sequence lifetimes are illustrated in Fig. 5.12

- The Sun has a main sequence lifetime of about *10 billion years*, but
- A  $20 M_{\odot}$  stays on the main sequence for about *5.5 million years* and
- A  $100 M_{\odot}$  star lives on the main sequence for only about *100,000 years*.
- Conversely, for main sequence stars with  $M \ll M_{\odot}$ , we may estimate that the main sequence lifetime greatly *exceeds the present age of the Universe*.

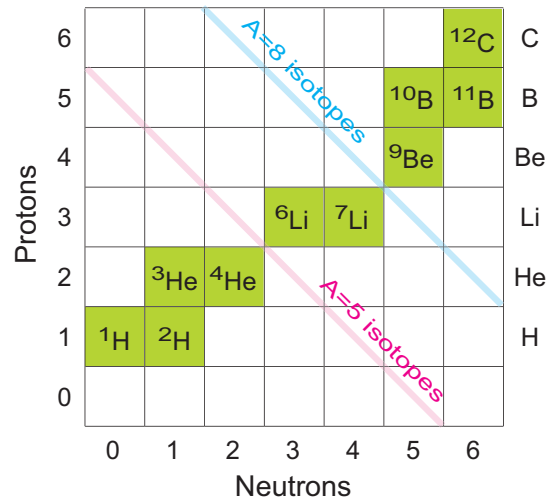
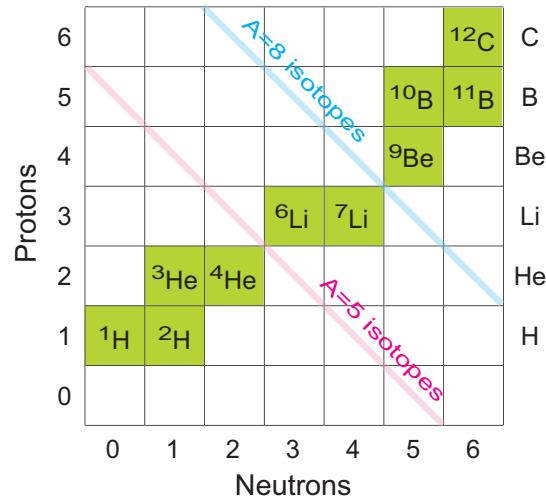


Figure 5.13: The green boxes indicate stable isotopes. There are no stable mass-5 or mass-8 isotopes.

## 5.15 The Triple-Alpha Process

Main sequence stars produce their power by hydrogen fusion. This builds up a thermonuclear ash of helium in the core of the star.

- The star continues to fuse hydrogen to helium in a shell surrounding the central core of helium that is built up.
- This *hydrogen shell burning* adds gradually to the accumulating core of helium and the star remains on the main sequence until about 10% of its initial hydrogen has been consumed.
- Fusion of the helium to heavier elements is difficult because
  - There is a *large Coulomb barrier*,
  - There are *no stable mass-5 and mass-8 isotopes* to serve as intermediaries for the production of heavier elements (Fig. 5.13).



- Thus, helium fusion can occur only at very high temperatures and densities: temperatures in excess of about  $10^8$  K and densities of  $10^2 - 10^5 \text{ g cm}^{-3}$ .
- Such conditions can result when stars exhaust their hydrogen fuel and their cores begin to contract.
- Because there are *no stable mass-8 isotopes*, the resulting fusion of helium must involve a *two-step process* in which
  1. Two helium ions (alpha-particles) combine to form highly unstable  $^8\text{Be}$ ,
 
$$\alpha + \alpha \rightarrow ^8\text{Be}$$
  2. This in turn combines with another helium ion to form carbon.
 
$$\alpha + ^8\text{Be} \rightarrow ^{12}\text{C}$$
- The resulting sequence, which is crucial to the power generated by red giant stars and to the production of most of the carbon and oxygen in the Universe, is called the *triple- $\alpha$  process*.

Our bodies are composed of about 65% oxygen and 18% carbon. *We owe our very existence to the (highly improbable) triple- $\alpha$  process, as we discuss further below!*

The burning of helium to carbon by the triple- $\alpha$  process may be viewed as taking place in *three basic steps*:

1. A small transient population of  ${}^8\text{Be}$  is built up by He + He fusion,



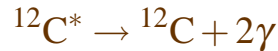
which has  $Q \simeq 92 \text{ keV}$ .

2. A small transient population of  ${}^{12}\text{C}$  in an excited state is built up by the reaction



On general grounds, to produce a finite population of  ${}^{12}\text{C}^*$  *this reaction must be resonant*. Otherwise it would be too slow to compete with the decay of  ${}^8\text{Be}$  to two  $\alpha$ -particles.

3. A small fraction of the  ${}^{12}\text{C}^*$  excited states decay electromagnetically by



to the ground state of carbon-12.

This (highly improbable) sequence of reactions has the net effect of converting three helium ions to  ${}^{12}\text{C}$ , with an energy release  $Q = +7.275 \text{ MeV}$ . Let us consider each of these steps in more detail.

### 5.15.1 Equilibrium Population of Beryllium-8

- The mean life for decay of  ${}^8\text{Be}$  back into two alpha-particles is  $\tau \simeq 10^{-16}$  seconds, which corresponds to a width of  $\Gamma_{8\text{-Be}} = \hbar\tau^{-1} = 6.8 \text{ eV}$ .
- The capture will be too slow to compete with this decay back into alpha-particles unless the corresponding resonance peak overlaps substantially with the Gamow peak.
- Thus, we expect this rate controlling step of the triple- $\alpha$  process to be significant *only when the energy of the Gamow peak is comparable to the  $Q$ -value of 92 keV* for the reaction, and this in turn sets the required conditions for the triple- $\alpha$  process to proceed.
- The maximum of the Gamow peak is given approximately by

$$E_0 = 1.22(Z_\alpha^2 Z_x^2 \mu T_6^2)^{1/3} \text{ keV}.$$

which implies a temperature of  $1.2 \times 10^8 \text{ K}$  for  $E_0 = 92 \text{ keV}$ .

- Therefore, *only for temperatures of order  $10^8 \text{ K}$*  can the initial step of the triple- $\alpha$  reaction produce a sufficient equilibrium concentration of  ${}^8\text{Be}$  to allow the subsequent steps to proceed.

- The preceding simple estimate ignores details such as electron screening that must be considered for a more precise estimate of the temperature for helium burning, but it sets the correct order of magnitude.
- Also note that this temperature estimate raises the question of *why helium was not consumed in big bang nucleosynthesis* by the triple- $\alpha$  mechanism. The answer:
  - The *temperature* was high enough but
  - The *density* was too low.
  - Both high temperatures and high densities, which *awaited the formation of stars*, were required to produce significant amounts of carbon by the triple- $\alpha$  mechanism.



We may estimate the equilibrium concentration of  ${}^8\text{Be}$  under various conditions by application to nuclei (rather than atoms) of a *suitable modification of the Saha equations*. The required changes are

1. Replace the number densities of ions and electrons with the number densities of  $\alpha$ -particles, and the number density of neutral atoms with the number density of  ${}^8\text{Be}$ .
2. Replace the statistical factors  $g$  for atoms with corresponding statistical factors associated with nuclei. This is trivial for the present case: the ground states of both  ${}^8\text{Be}$  and  ${}^4\text{He}$  have angular momentum zero and  $g = 1$  in both cases.
3. The ionization potentials entering the atomic Saha equations are replaced by  $Q$ -values in the nuclear case. In the present example,  $Q = 91.78 \text{ keV}$  for  ${}^8\text{Be} \rightarrow \alpha\alpha$  (“ionization” of  ${}^8\text{Be}$  to two  $\alpha$ -particles).
4. The electron mass in the atomic Saha equations is replaced by the reduced mass  $m_\alpha m_\alpha / 2m_\alpha = \frac{1}{2}m_\alpha$ .

The resulting *nuclear Saha equation* is

$$\frac{n_\alpha^2}{n({}^8\text{Be})} = \left( \frac{\pi k T m_\alpha}{h^2} \right)^{3/2} \exp(-Q/kT).$$

The situation where such equations are applicable is termed *nuclear statistical equilibrium (NSE)*.

**Example:** *Helium flashes* are explosive helium burning events that can occur in red giant stars (see Ch. 9).

Typical helium flash conditions in lower-mass red giant stars correspond to a temperature of  $T_9 \simeq 0.1$  and a density of  $\rho \simeq 10^6 \text{ g cm}^{-3}$ .

For a triple- $\alpha$  powered helium flash in a pure helium core, we obtain from

$$\frac{n_\alpha^2}{n(^8\text{Be})} = \left( \frac{\pi k T m_\alpha}{h^2} \right)^{3/2} \exp(-Q/kT).$$

that

$$\frac{n(^8\text{Be})}{n_\alpha} = 7 \times 10^{-9},$$

corresponding to an equilibrium  $^8\text{Be}$  concentration of  $n(^8\text{Be}) = 10^{21} \text{ cm}^{-3}$  during the flash (see Exercise 5.10).

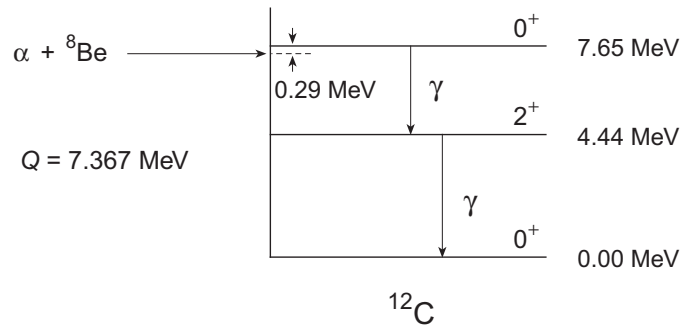
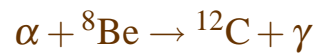


Figure 5.14: Nuclear energy levels in  $^{12}\text{C}$  for the final steps of the triple- $\alpha$  reaction. Levels are labeled by  $J^\pi$  and energy relative to the ground state. The  $0^+$  state at 7.65 MeV is the Hoyle resonance.

### 5.15.2 Formation of the Excited State in Carbon-12

The next step of the triple- $\alpha$  process,



has  $Q = 7.367 \text{ MeV}$  and proceeds through an angular momentum  $J = 0$  resonance in  $^{12}\text{C}$  at an excitation energy relative to the  $^{12}\text{C}$  groundstate of 7.654 MeV, as illustrated in Fig. 5.14.

**Hoyle resonance:** The existence of this resonance was *predicted by Hoyle* to explain the energy production in red giant stars.

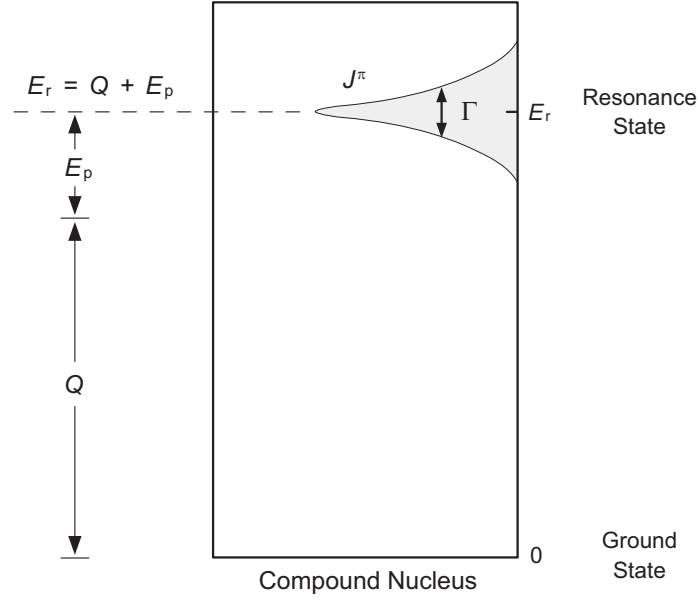


Figure 5.15: Relationship of  $Q$ -value, resonance energy  $E_r$ , and center of mass energy  $E_p$  when an isolated resonance is maximally excited in a reaction.

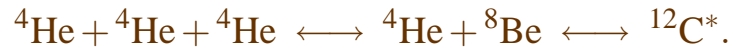
- As illustrated in Fig. 5.15, the population of the Hoyle state is optimized if the center of mass energy plus the  $Q$ -value is equal to the resonance energy relative to the ground state of  $^{12}\text{C}$ .
- Once this excited state is formed, the dominant reaction is a rapid decay back to  $\alpha + {}^8\text{Be}$ , but a small fraction of the time the ground state of  $^{12}\text{C}$  may instead be formed by two  $\gamma$ -ray decays, as also illustrated in Fig. 5.14.
- If nuclear statistical equilibrium is assumed, the concentration of  $^{12}\text{C}^*$  excited states is given by

$$\frac{n(^{12}\text{C}^*)}{n_\alpha^3} = 3^{3/2} \left( \frac{h^2}{2\pi m_\alpha kT} \right)^3 \exp[(3m_\alpha - m_{12}^*)c^2/kT],$$

where  $m_{12}^*$  is the mass of  $^{12}\text{C}$  in the excited state (see Exercise 5.9).

### 5.15.3 Formation of the Ground State in Carbon-12

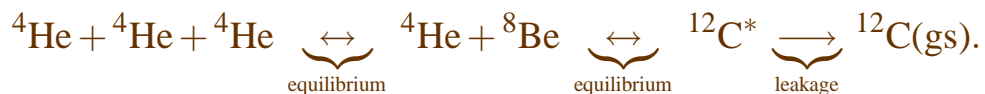
- The preceding considerations determine a dynamical equilibrium



- This produces an equilibrium population of  ${}^{12}\text{C}^*$ , almost all of which decays back to  ${}^4\text{He} + {}^8\text{Be}$ .
- However, the excited state of  ${}^{12}\text{C}$  can decay electromagnetically to its ground state with a mean life of

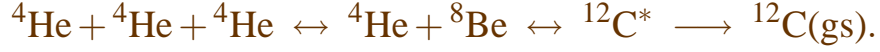
$$\tau \left( {}^{12}\text{C}^* \rightarrow {}^{12}\text{C}(\text{gs}) \right) = 1.8 \times 10^{-16} \text{ s},$$

- This implies that *one in about every 2500 excited carbon nuclei* that are produced decay to the stable ground state.
- Because this decay probability is so small, it does not influence the equilibrium appreciably and we may represent the entire triple- $\alpha$  process schematically as



where left-right arrows indicate approximate nuclear statistical equilibrium and the one-way arrow indicates a leakage from the equilibrium that is a small perturbation and therefore does not disturb it significantly.

- Thus in the approximate equilibrium



The production rate for  ${}^{12}\text{C}$  in its ground state is then the product of the equilibrium  ${}^{12}\text{C}^*$  population and the decay rate to the ground state,

$$\begin{aligned} \frac{dn({}^{12}\text{C}^*)}{dt} &= \underbrace{n({}^{12}\text{C}^*)}_{\text{Number density } {}^{12}\text{C}^*} \times \underbrace{\frac{1}{\tau({}^{12}\text{C}^* \rightarrow {}^{12}\text{C}(\text{gs}))}}_{\text{Electromagnetic decay rate}} \\ &= \frac{n_\alpha^3}{\tau({}^{12}\text{C}^* \rightarrow {}^{12}\text{C}(\text{gs}))} 3^{3/2} \left( \frac{h^2}{2\pi m_\alpha kT} \right)^3 \\ &\quad \times \exp[-(m_{12}^* - 3m_\alpha)c^2/kT], \end{aligned}$$

where we have used

$$\frac{n({}^{12}\text{C}^*)}{n_\alpha^3} = 3^{3/2} \left( \frac{h^2}{2\pi m_\alpha kT} \right)^3 \exp[(3m_\alpha - m_{12}^*)c^2/kT],$$

We see that the rate of carbon production depends primarily on two things:

1. An activation energy given by  $(m_{12}^* - 3m_\alpha)c^2 = 0.3795 \text{ MeV}$  that must be borrowed to create the  ${}^{12}\text{C}^*$  intermediate state.
2. The mean life for the decay  ${}^{12}\text{C}^* \rightarrow {}^{12}\text{C}(\text{gs})$ , which is

$$\tau({}^{12}\text{C}^* \rightarrow {}^{12}\text{C}(\text{gs})) = 1.8 \times 10^{-16} \text{ s}.$$

Table 5.4: Parameters for the triple- $\alpha$  reaction

$T$ (K)	$kT$ (keV)	$Q/kT$	$\exp(Q/kT)$
$5 \times 10^7$	4.309	-88.08	$5.6 \times 10^{-39}$
$1 \times 10^8$	8.617	-44.04	$7.5 \times 10^{-20}$
$2 \times 10^8$	17.234	-22.02	$2.7 \times 10^{-10}$

The strong temperature dependence for the triple- $\alpha$  reaction results primarily from the exponential Boltzmann factor in

$$\begin{aligned}
 \frac{dn(^{12}\text{C}^*)}{dt} &= \frac{n(^{12}\text{C}^*)}{\tau(^{12}\text{C}^* \rightarrow ^{12}\text{C}(\text{gs}))} \\
 &= \frac{n_\alpha^3}{\tau(^{12}\text{C}^* \rightarrow ^{12}\text{C}(\text{gs}))} 3^{3/2} \left( \frac{h^2}{2\pi m_\alpha kT} \right)^3 \\
 &\quad \times \exp[-(m_{12}^* - 3m_\alpha)c^2/kT],
 \end{aligned}$$

because at helium burning temperatures the average thermal energy  $kT$  is typically much less than the activation energy of  $Q = -379.5$  keV. This is illustrated in Table 5.4, where we see that doubling the temperature in the vicinity of  $10^8$  K changes the Boltzmann factor by tens of orders of magnitude.

### 5.15.4 Energy Production in the Triple- $\alpha$ Reaction

The total energy released in the triple- $\alpha$  reaction is  $Q = 7.275 \text{ MeV}$  and the energy production rate is given by

$$\epsilon_{3\alpha} = \frac{5.1 \times 10^8 \rho^2 Y^3}{T_9^3} \exp(-4.4027/T_9) \text{ erg g}^{-1} \text{ s}^{-1},$$

where  $Y$  is the helium abundance. From

$$\lambda = \left( \frac{\partial \ln \epsilon}{\partial \ln \rho} \right)_T \quad \nu = \left( \frac{\partial \ln \epsilon}{\partial \ln T} \right)_\rho.$$

- This implies density and temperature exponents

$$\lambda_{3\alpha} = 2 \quad \nu_{3\alpha} = 4.4/T_9 - 3,$$

- The quadratic dependence on the density occurs because the reaction is *effectively 3-body*, since three alpha-particles must fuse to form the carbon; the dependence on temperature is very strong, as we have already seen in connection with Table 5.4.
- For  $T_8 = 1$ , we obtain a temperature exponent  $\nu_{3\alpha} \simeq 40$ , implying that *a helium core is a very explosive fuel*.<sup>a</sup>

---

<sup>a</sup>An explosion is runaway burning (whether ordinary chemical burning or thermonuclear burning). If a thermonuclear fuel has a large temperature exponent, in the corresponding temperature range the rate of burning can increase enormously if the temperature increases only a little. This greatly increases the probability that burning becomes explosive once it is initiated.



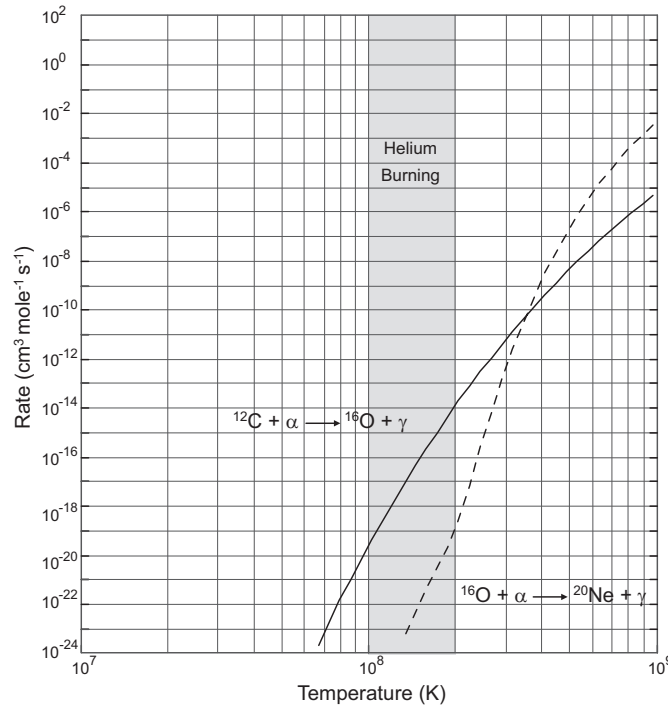


Figure 5.16: Radiative alpha capture rates for  $^{12}\text{C}$  and  $^{16}\text{O}$ .

## 5.16 Burning of Carbon to Oxygen and Neon

- Once carbon has been formed by the triple- $\alpha$  sequence, oxygen can be produced by the radiative capture reaction



- There are *no resonances near the Gamow window* for this reaction, so the rate is *slow* (and experimentally uncertain). The currently accepted rate is shown in the figure above.
- The uncertainty in this rate has *significant astrophysical consequences* because it determines the ratio of carbon to oxygen to neon production in stars.

- In addition to the impact on abundance of these elements in the Universe, the carbon–oxygen ratio in stellar cores can have large influence on various aspects of late stellar evolution.

For example, the composition of white dwarfs and of the cores of massive stars late in their lives depend critically on this rate, so it can have a large impact on how stars die and what is left behind when they do.

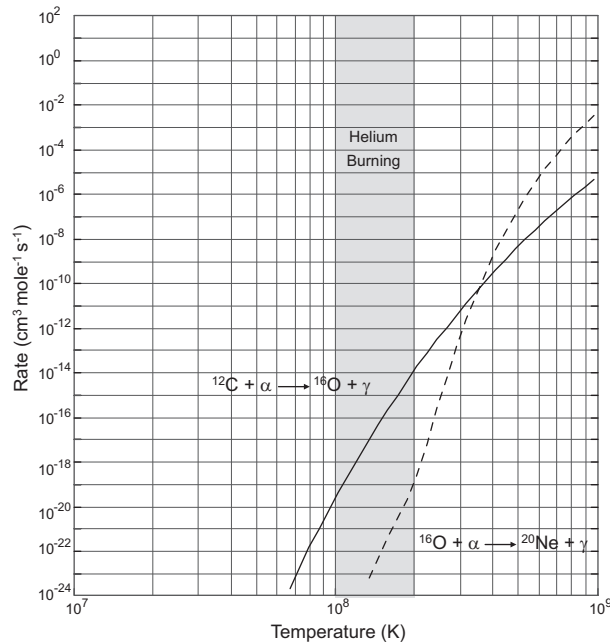


Figure 5.17: Radiative alpha capture rates for  $^{12}\text{C}$  and  $^{16}\text{O}$ .

- Once oxygen is produced by  $^4\text{He} + ^{12}\text{C} \rightarrow ^{16}\text{O} + \gamma$ , neon can be formed by  $^4\text{He} + ^{16}\text{O} \rightarrow ^{20}\text{Ne} + \gamma$ . The rate is plotted in Fig. 5.17.
- It is *slow* under helium burning conditions because *it is nonresonant* and has a *larger Coulomb barrier* than  $\alpha$  capture on  $^{12}\text{C}$ .
- This implies that *little neon is produced* during helium burning.

The primary residue of helium burning is a *carbon–oxygen core*.

- The carbon is produced by the triple- $\alpha$  sequence and the oxygen by radiative capture on the carbon, with the ratio of carbon to oxygen depending strongly on the uncertain rate for the radiative capture reaction.

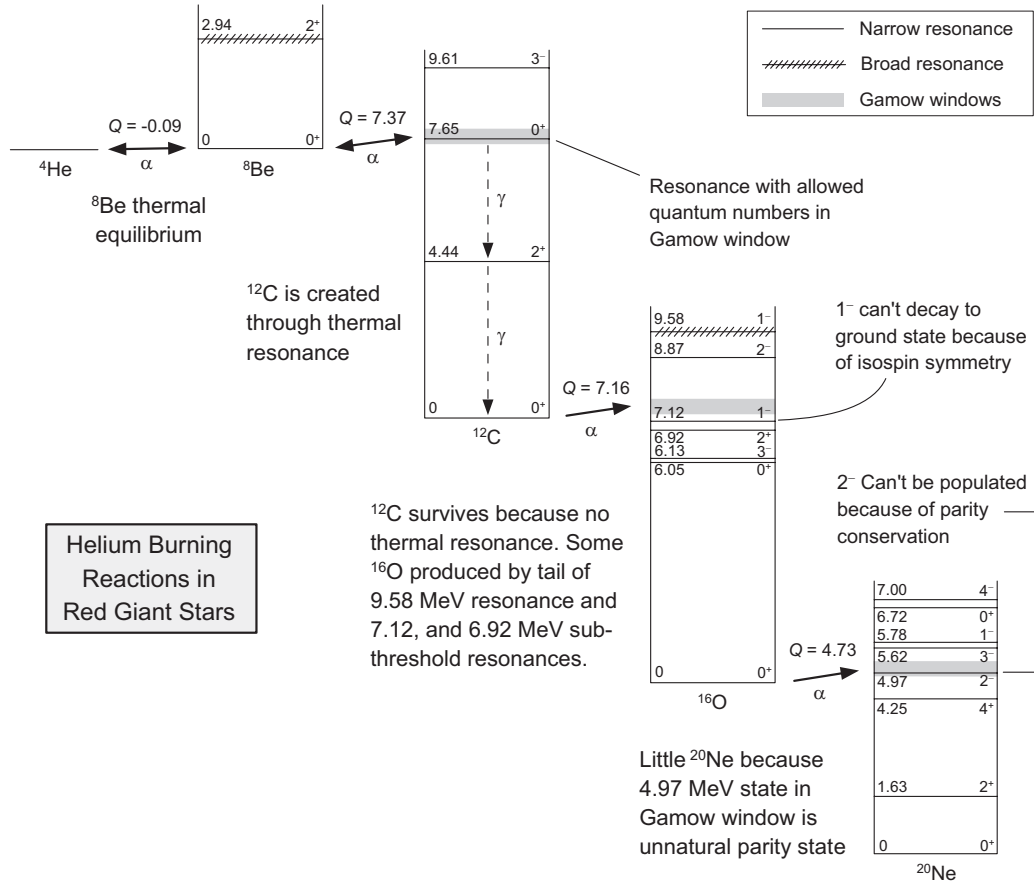
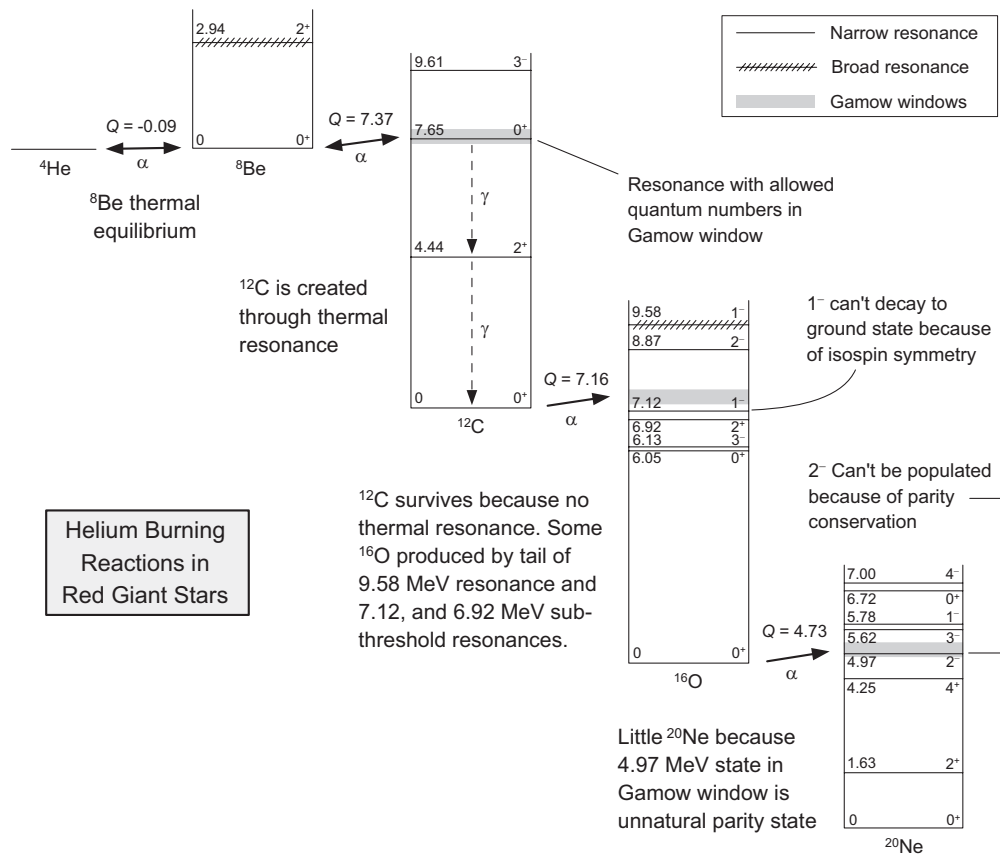


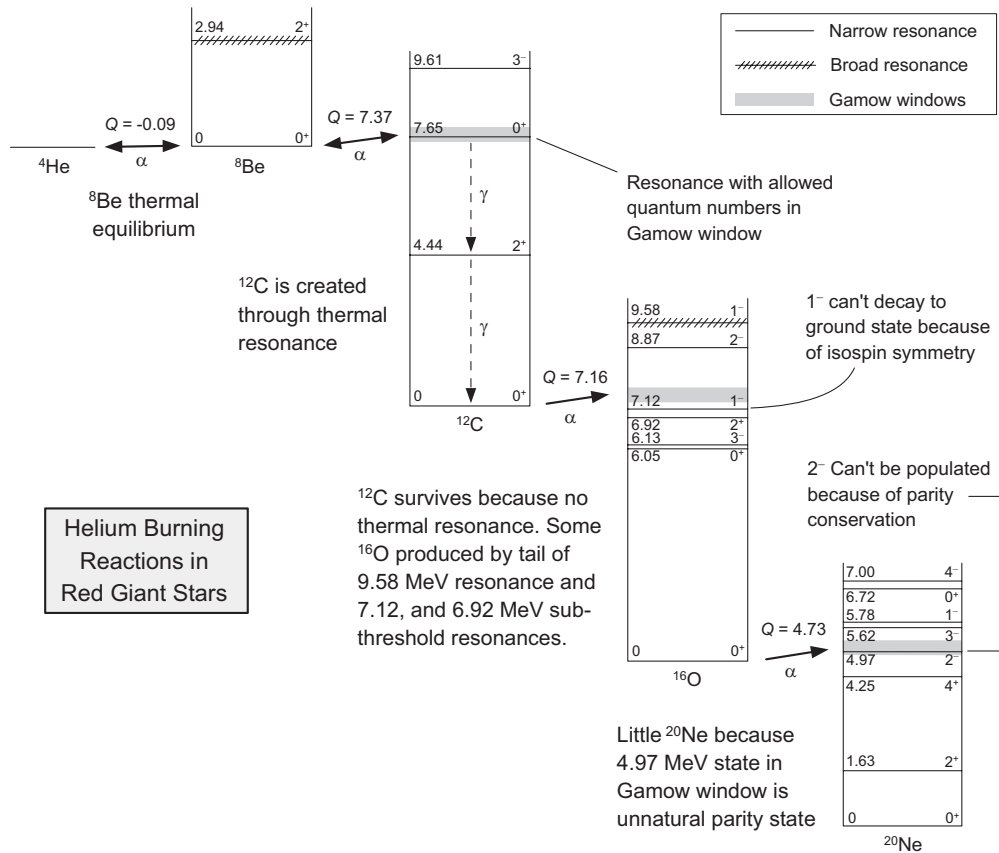
Figure 5.18: Summary of helium burning.

## 5.17 The Outcome of Helium Burning

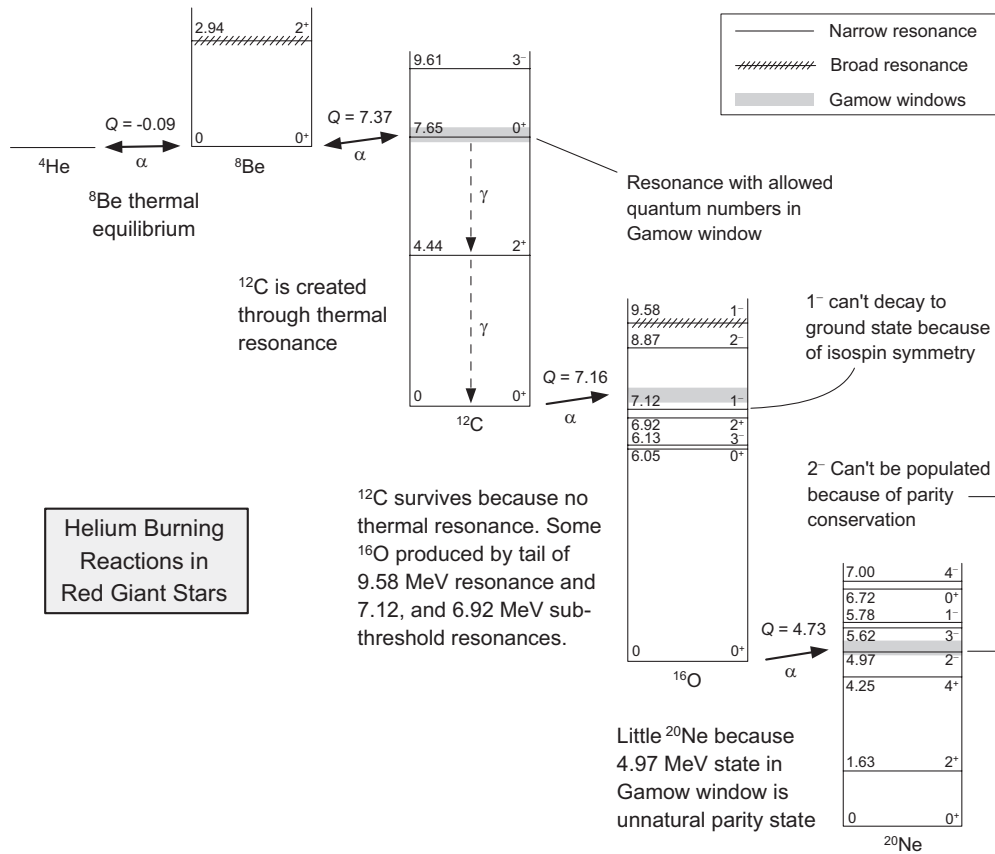
The outcome of helium burning is summarized in Fig. 5.18. This outcome is a remarkable example of how fundamentally different our Universe would be if just a few seemingly boring details of nuclear physics were slightly different.



- The proportion of carbon to oxygen in the Universe is determined by the competition between the carbon-producing triple- $\alpha$  reaction and the carbon-depleting, oxygen-producing radiative capture reaction  ${}^4\text{He} + {}^{12}\text{C} \rightarrow {}^{16}\text{O} + \gamma$ .
- Further, that much carbon or oxygen exists at all is crucially dependent on the slowness of the neon-producing reaction  ${}^4\text{He} + {}^{16}\text{O} \rightarrow {}^{20}\text{Ne} + \gamma$ .



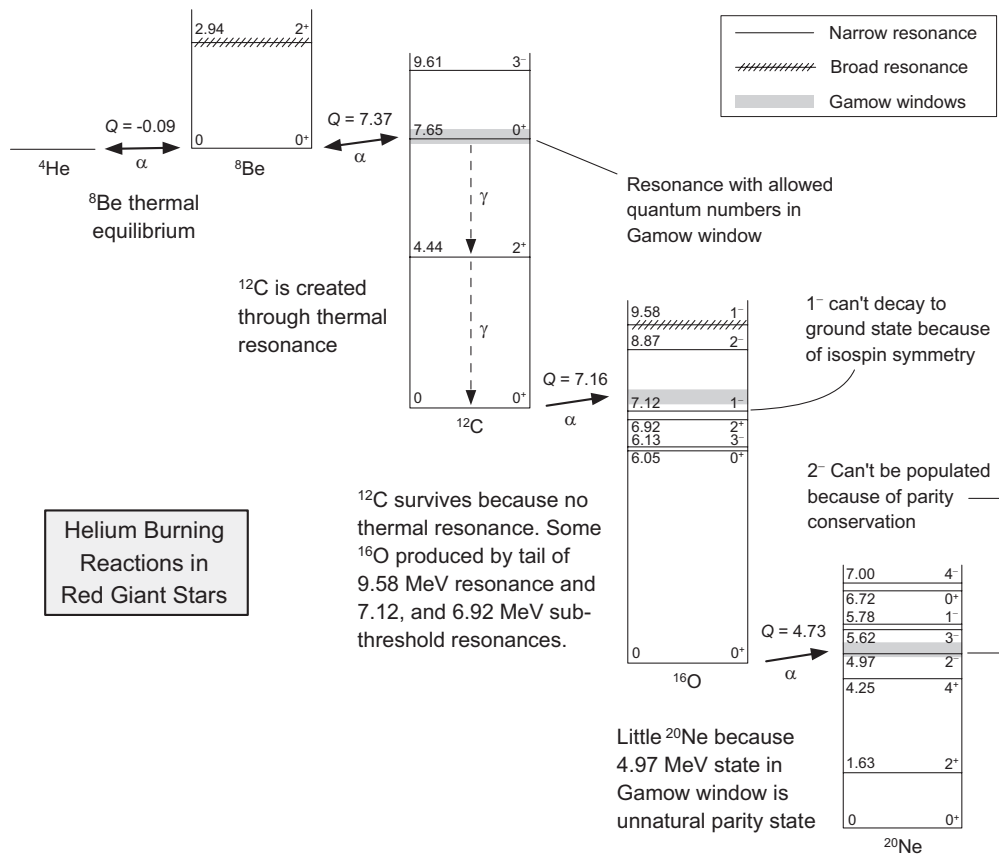
- If, contrary to fact, a resonance existed near the fusion window for the reaction  $^4\text{He} + ^{12}\text{C} \rightarrow ^{16}\text{O} + \gamma$ , the corresponding rate would be large and almost all carbon produced by triple- $\alpha$  would be converted rapidly to oxygen, leaving little carbon in the Universe.



- A similar fate would follow if the 0<sup>+</sup> excited state in  $^{12}\text{C}$  at 7.65 MeV were a little higher in energy, since this would greatly slow the triple- $\alpha$  rate by virtue of the Boltzmann factor in

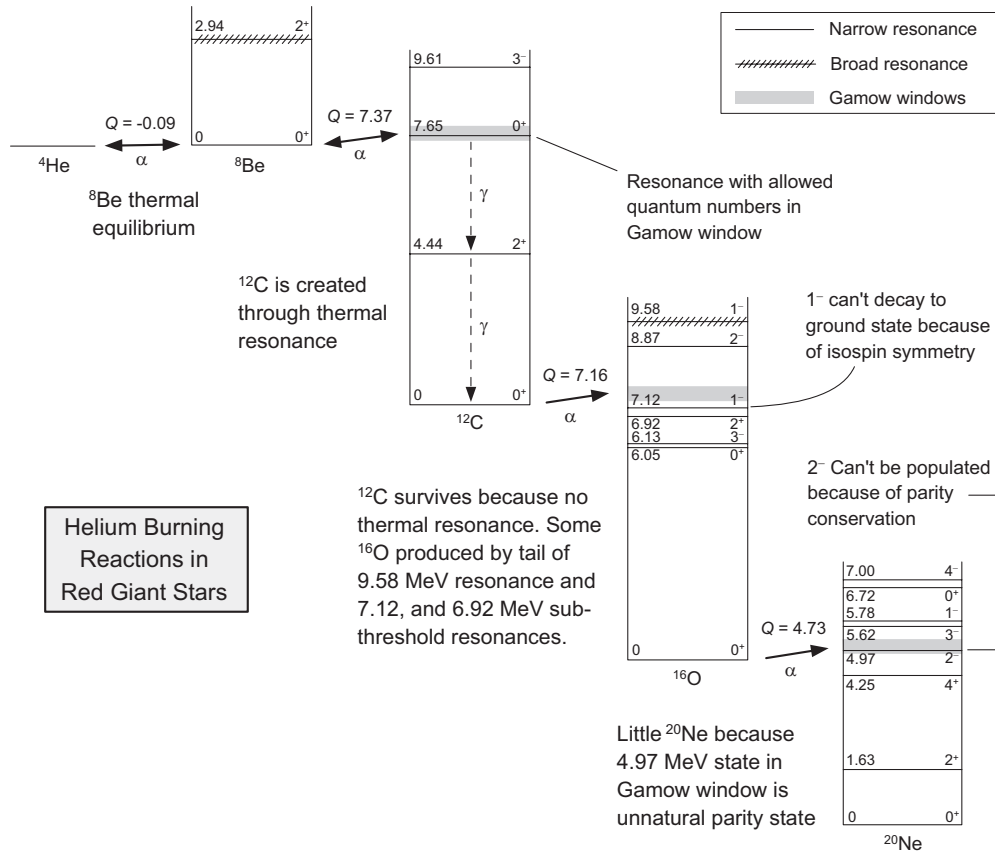
$$\frac{dn(^{12}\text{C}^*)}{dt} = \frac{n_\alpha^3}{\tau(^{12}\text{C}^* \rightarrow ^{12}\text{C}(\text{gs}))} 3^{3/2} \left( \frac{h^2}{2\pi m_\alpha kT} \right)^3 \times \exp[-(m_{12}^* - 3m_\alpha)c^2/kT],$$

and any carbon that was produced would be converted rapidly to oxygen through  $^4\text{He} + ^{12}\text{C} \rightarrow ^{16}\text{O} + \gamma$ .

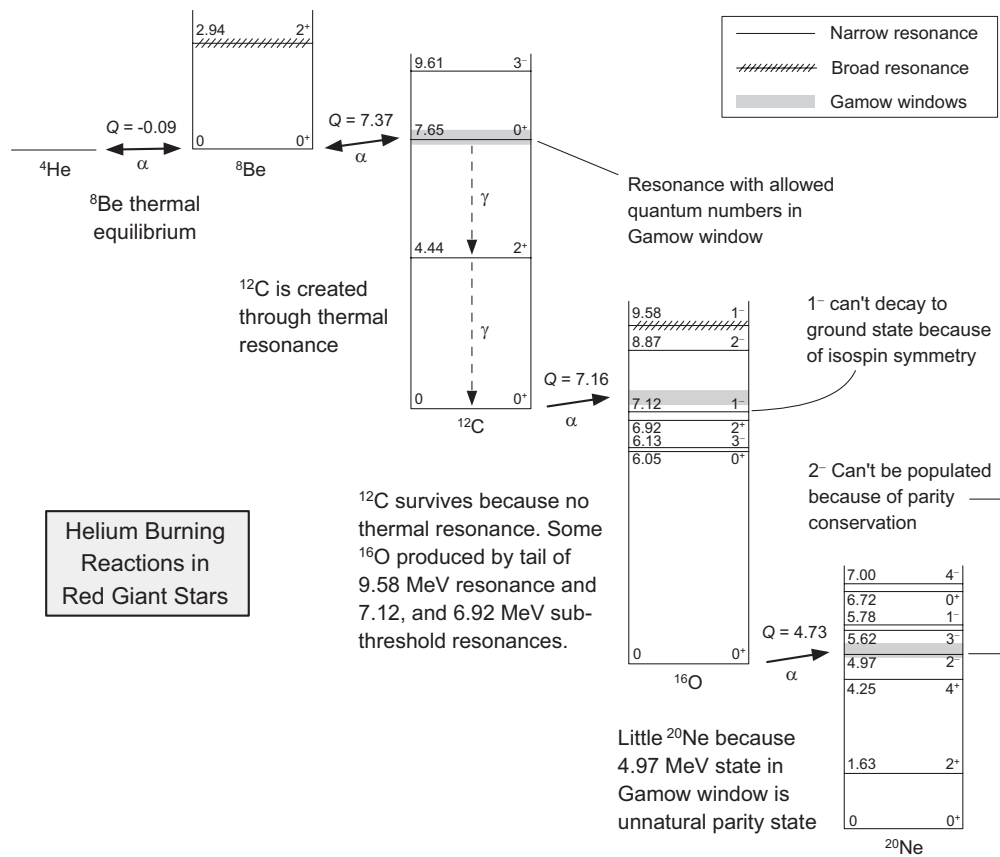


- Conversely, if the resonance at 7.65 MeV in  $^{12}\text{C}$  did not exist, the triple- $\alpha$  reaction would not work at all in red giant stars and there would be little of either carbon or oxygen in the Universe.





- Finally, if the neon-producing reaction  ${}^4\text{He} + {}^{16}\text{O} \rightarrow {}^{20}\text{Ne} + \gamma$  were (contrary to fact) resonant—which it would be if the parity of a single excited state in neon were positive instead of negative,
  - Most of the carbon and oxygen produced by helium burning would be transformed by this reaction to neon.
  - Neon is a noble gas and therefore essentially chemically inert, in contrast to the rich chemistry of carbon that makes biology as we know it possible.



- Thus, our very existence appears to depend on the *parity* of obscure nuclear states in atoms that have nothing whatsoever to do with the chemistry of life!

***The Anthropic Principle and Helium Burning:*** Some would argue, based on the observed diversity of life on Earth and how quickly it arose after formation of the planet, that life in the Universe is inevitable.

- But this point of view assumes the existence of the chemicals on which life (as we know it) is built.
- The preceding discussion suggests that the existence of the building blocks of life depends on arcane facts on the MeV scale (nuclear physics) that have nothing to do with the physics of eV scales (chemistry) that governs life.
- The very possibility of biochemistry may be an accident of physical parameter values in our Universe.
- Such considerations lie at the basis of the (simplest) *anthropic principle*:

The Universe has just the right value of constants and just the detailed physics like the energy and parity of obscure nuclear states required for life because, if it didn't, there would be no life in the Universe and therefore no one to ask the question.

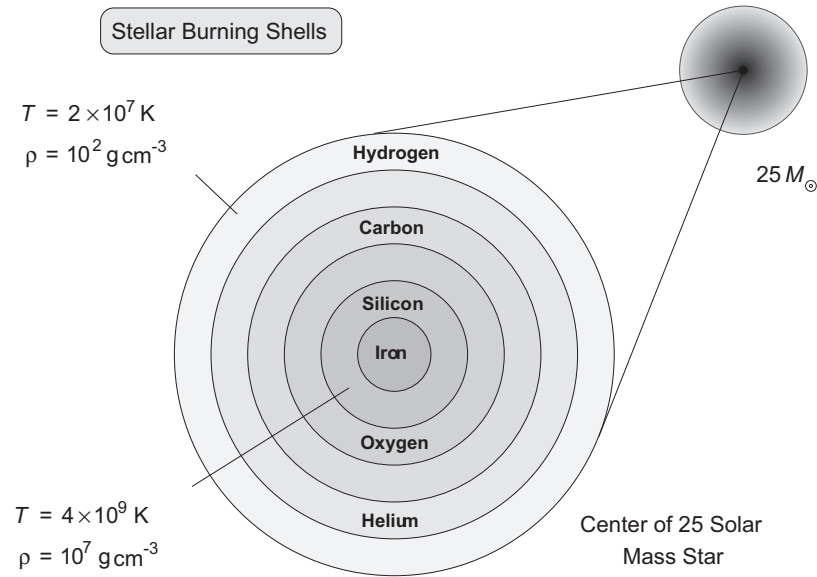
- It is not clear whether this line of thinking, as intriguing as it is, enhances our scientific understanding of the Universe (or other possible universes).

If a star is massive enough, more advanced burnings are possible by virtue of the high temperatures and pressures that result as the core contracts after exhausting its fuel. Typical burning stages in massive stars and their characteristics are illustrated in the following table and figure.

Table 5.5: Burning stages in massive stars (Woosley)

Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_{\odot}$ star
H	He	$4 \times 10^6$ K	$0.1M_{\odot}$	$7 \times 10^6$ years
He	C, O	$1.2 \times 10^8$ K	$0.4M_{\odot}$	$5 \times 10^5$ years
C	Ne, Na, Mg, O	$6 \times 10^8$ K	$4M_{\odot}$	600 years
Ne	O, Mg	$1.2 \times 10^9$ K	$\sim 8M_{\odot}$	1 years
O	Si, S, P	$1.5 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 0.5$ years
Si	Ni-Fe	$2.7 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 1$ day

### 5.18 Advanced Burning Stages



Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_{\odot}$ star
H	He	$4 \times 10^6$ K	$0.1M_{\odot}$	$7 \times 10^6$ years
He	C, O	$1.2 \times 10^8$ K	$0.4M_{\odot}$	$5 \times 10^5$ years
C	Ne, Na, Mg, O	$6 \times 10^8$ K	$4M_{\odot}$	600 years
Ne	O, Mg	$1.2 \times 10^9$ K	$\sim 8M_{\odot}$	1 years
O	Si, S, P	$1.5 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 0.5$ years
Si	Ni-Fe	$2.7 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 1$ day

**Carbon burning:** Carbon burns at a temperature of  $T \sim 5 \times 10^8$  K and a density of  $\rho \sim 3 \times 10^6$  g cm $^{-3}$ , primarily through



As indicated in the table above, such reactions are possible for stars having masses larger than about  $4M_{\odot}$ .

- Burning stages beyond that of carbon require conditions that are probably realized only for stars having  $M \gtrsim 8M_{\odot}$  or so.
- At the required temperatures, a new feature comes into play because *the most energetic photons can disrupt the nuclei produced in preceding burning stages.*

Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_{\odot}$ star
H	He	$4 \times 10^6$ K	$0.1M_{\odot}$	$7 \times 10^6$ years
He	C, O	$1.2 \times 10^8$ K	$0.4M_{\odot}$	$5 \times 10^5$ years
C	Ne, Na, Mg, O	$6 \times 10^8$ K	$4M_{\odot}$	600 years
Ne	O, Mg	$1.2 \times 10^9$ K	$\sim 8M_{\odot}$	1 years
O	Si, S, P	$1.5 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 0.5$ years
Si	Ni-Fe	$2.7 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 1$ day

**Neon burning:** At  $T \sim 10^9$  K, neon can burn by a two-step sequence. First, a neon nucleus is *photodisintegrated by a high-energy photon* (which become more plentiful as the temperature is increased since the average photon energy is  $\sim kT$ )



Then the alpha-particle produced in this step can initiate a radiative capture reaction



This burning sequence produces a core of  ${}^{16}\text{O}$  and  ${}^{24}\text{Mg}$ .

Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_{\odot}$ star
H	He	$4 \times 10^6$ K	$0.1M_{\odot}$	$7 \times 10^6$ years
He	C, O	$1.2 \times 10^8$ K	$0.4M_{\odot}$	$5 \times 10^5$ years
C	Ne, Na, Mg, O	$6 \times 10^8$ K	$4M_{\odot}$	600 years
Ne	O, Mg	$1.2 \times 10^9$ K	$\sim 8M_{\odot}$	1 years
O	Si, S, P	$1.5 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 0.5$ years
Si	Ni–Fe	$2.7 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 1$ day

**Oxygen burning:** At a temperature of  $2 \times 10^9$  K, oxygen can fuse through the reaction



The silicon thus produced can react only at temperatures where the photon spectrum is sufficiently “hard” (has significant high-energy components) that photodissociation reactions begin to play a dominating role.



Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_{\odot}$ star
H	He	$4 \times 10^6$ K	$0.1M_{\odot}$	$7 \times 10^6$ years
He	C, O	$1.2 \times 10^8$ K	$0.4M_{\odot}$	$5 \times 10^5$ years
C	Ne, Na, Mg, O	$6 \times 10^8$ K	$4M_{\odot}$	600 years
Ne	O, Mg	$1.2 \times 10^9$ K	$\sim 8M_{\odot}$	1 years
O	Si, S, P	$1.5 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 0.5$ years
Si	Ni–Fe	$2.7 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 1$ day

**Silicon burning:** At  $T \sim 3 \times 10^9$  K, silicon may be burned to heavier elements.

- At these temperatures the photons are quite energetic and those in the high-energy tail of the Maxwell–Boltzmann distribution can readily photodissociate nuclei.
- A network of photodisintegration and capture reactions in approximate nuclear statistical equilibrium with each other develops and the population in this network evolves preferentially to those isotopes that have the largest binding energies.
- Since from the plot of the binding energy curve the most stable nuclei are in the iron group, silicon burning carried to completion under equilibrium conditions tends to produce iron-group nuclei.

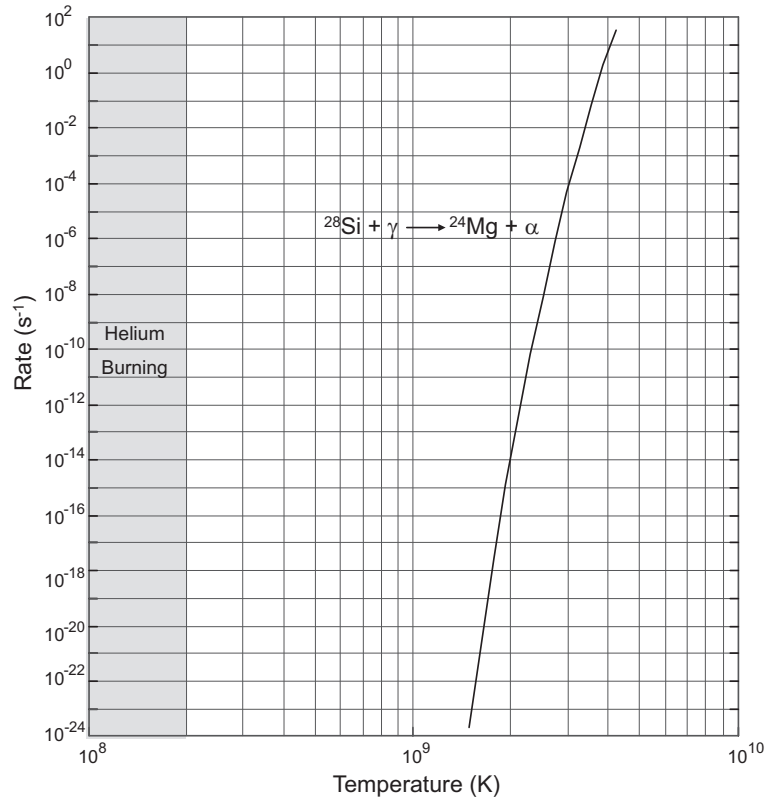


Figure 5.19: The temperature dependence of the rate-controlling step in silicon burning. For reference, the typical range of temperatures corresponding to helium burning is indicated.

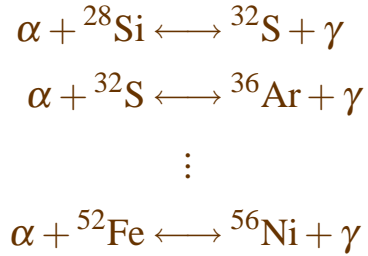
- The initial step in silicon burning is a photodisintegration like



which requires a photon energy of 9.98 MeV or greater.

- From Fig. 5.19 we infer that silicon burning has an extremely strong temperature dependence (near  $T_9 = 3$  the  $T$  exponent is  $\nu \sim 50$ ), and it requires temperatures more than an order of magnitude larger than for helium burning.

- The  $\alpha$  particles thus liberated can now initiate radiative capture reactions on seed isotopes in the gas; a representative sequence is



- The reactions in this series are typically in *equilibrium or quasiequilibrium*, and are much faster than the initial photodisintegration.
- Thus the photodisintegration of silicon is the *rate-controlling step* in silicon burning.

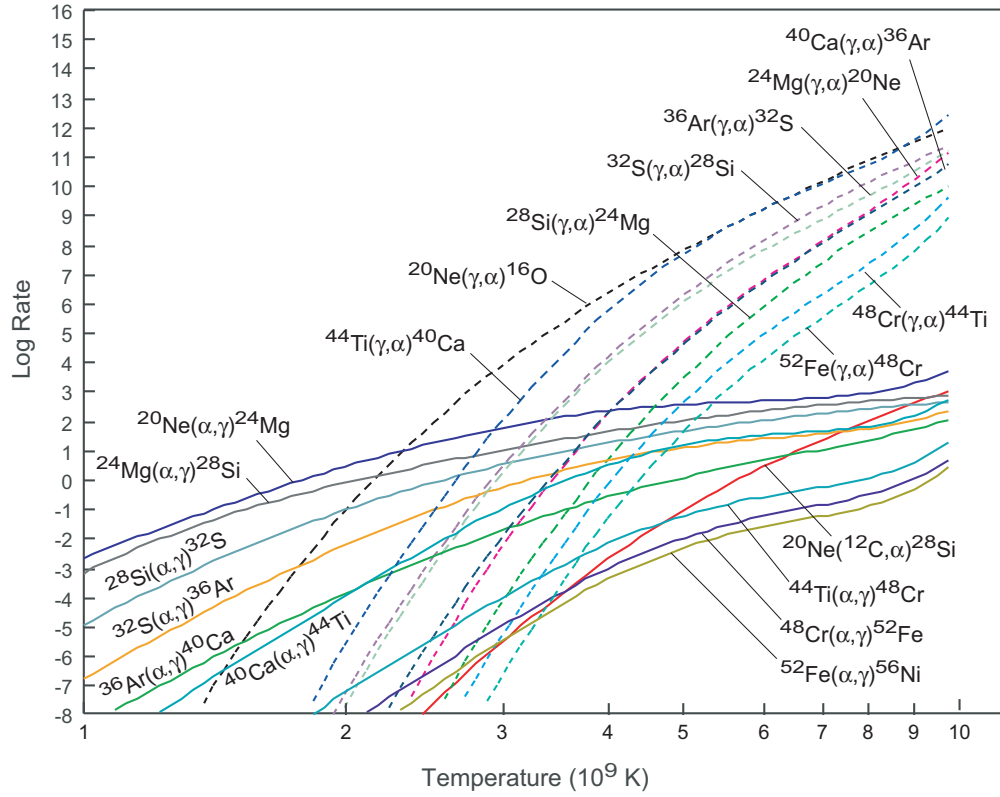


Figure 5.20: Some rates for competing capture reactions  $A(\alpha, \gamma)B$  and photodisintegration reactions  $A(\gamma, \alpha)B$  that are important for silicon burning. The photodisintegration rates are given in units of  $\text{s}^{-1}$ . The  $\alpha$ -capture reaction rates are given in units of  $\text{cm}^3 \text{mole}^{-1} \text{s}^{-1}$ . To convert the  $\alpha$ -capture rate for the reaction of  $\alpha + A$  to units of  $\text{s}^{-1}$ , it must be multiplied by a molar density factor  $\rho Y_\alpha Y_A$ , where  $\rho$  is the mass density,  $Y_\alpha$  is the  $\alpha$ -particle abundance, and  $Y_A$  is the abundance of isotope  $A$ .

- The rates for some of the competing capture and photodisintegration reactions in silicon burning are illustrated in Fig. 5.20.
- Note the steep temperature dependence of the photodisintegration reactions, and that for high enough density and  $\alpha$ -particle abundance many of the photodisintegration rates will become comparable to the rates for their inverse capture reactions somewhere in the temperature range  $10^9$ – $10^{10}$  K.

- Since the iron group nuclei are the most stable in the Universe, silicon burning represents the last stage by which fusion and radiative capture reactions can build heavier elements under equilibrium conditions.
- One might think that we could make still heavier elements by increasing the temperature, so that the required extra energy for fusion presented by higher Coulomb barriers could be provided by the kinetic energy of the gas.
- But this becomes self-defeating in statistical equilibrium because the higher temperatures will also lead to increased photodissociation and iron-group nuclides are still the equilibrium product.
- In subsequent chapters we will address the issue of other mechanisms by which stars can produce the elements heavier than iron that our Universe obviously contains.

Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_{\odot}$ star
H	He	$4 \times 10^6$ K	$0.1M_{\odot}$	$7 \times 10^6$ years
He	C, O	$1.2 \times 10^8$ K	$0.4M_{\odot}$	$5 \times 10^5$ years
C	Ne, Na, Mg, O	$6 \times 10^8$ K	$4M_{\odot}$	600 years
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O	Si, S, P	$1.5 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 0.5$ years
Si	Ni-Fe	$2.7 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 1$ day

### 5.19 Timescales for Advanced Burning

As is apparent from the table above, the timescales for advanced burning are greatly compressed relative to earlier burning stages. These differences are particularly striking for very massive stars, which rush through all stages at breakneck speed. For example, the  $25M_{\odot}$  example used for the above table

- Takes only about 10 million years to advance through its hydrogen and helium burning phases,
- Completes its burning of oxygen in only six months, and
- Transforms its newly-minted silicon into iron group nuclei in a single day.

Nuclear fuel	Nuclear products	Ignition temperature	Minimum main sequence mass	Period in $25M_{\odot}$ star
H	He	$4 \times 10^6$ K	$0.1M_{\odot}$	$7 \times 10^6$ years
He	C, O	$1.2 \times 10^8$ K	$0.4M_{\odot}$	$5 \times 10^5$ years
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Si	Ni-Fe	$2.7 \times 10^9$ K	$\sim 8M_{\odot}$	$\sim 1$ day

These timescales are set by

- The amount of fuel available,
- The energy per reaction derived from burning the fuel, and
- The rate of energy loss from the star, which ultimately governs the burning rate.
- The last factor is particularly important because energy losses are large when the reaction must run at high temperature.

Each factor separately shortens the timescale for advanced burning; taken together they make the timescales for the most advanced burning almost instantaneous on the scale set by the hydrogen burning.

**An Analogy:** To get a perspective on how short the advanced burning timescale is, imagine the lifetime of the  $25M_{\odot}$  star to be compressed into a single year.

- Then the hydrogen fuel would be gone by about December 7 of that year,
- The helium would burn over the next 24 days,
- The carbon would burn in the 42 minutes before midnight,
- The neon and oxygen would burn in the last several seconds before midnight, and
- The silicon would be converted to iron in the last 1/100 second of the year (with a quite impressive New Year's Eve fireworks display in the offing—see the later discussion of core-collapse supernovae).