The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Rev. Dr. Paul A. McNally, S.J., Director of the Georgetown College Observatory, Washington, D.C., U.S.A. (proposed by H. H. Turner); and

Cyril Young, c/o Sir Howard Grubb, Parsons & Co., Heaton Works, Newcastle-on-Tyne (proposed by Sir Charles Parsons).

Sixty-nine presents were announced as having been received since the last meeting, including, amongst others :—

Catania, Reale Osservatorio : Catalogo Astrofotografico, vol. 7, parte 2 (fasc. 50); vol. 8, parte 2 (fasc. 58).

The President announced that the gold medal of the Society had been awarded to Professor Ejnar Hertzsprung, for his determination of the distance of the Lesser Magellanic Cloud and other pioneering work in stellar astronomy.

Professor Hertzsprung has been appointed George Darwin Lecturer for 1929.

On the Rotation of the Stars. By G. Shajn and O. Struve.

1. Captain W. de W. Abney (1) was, we believe, the first to express (in 1877) the idea that the axial rotation of the stars could be determined from measurements of the widths of spectral lines. This opinion met with severe criticism on the part of H. C. Vogel (2), who pointed out that the great width of the hydrogen lines in certain stars could not possibly be due to rotation, since other lines in the same spectra usually appear narrow. It is now known, however, that in many stars all lines are broad, much broader in fact than the limiting width imposed by the resolving power of the spectrograph.

Professor Frank Schlesinger (3) was the first to actually observe the rotation of stars. This he did by measuring the limb-effect in the eclipsing variables δ Libræ and λ Tauri. The same problem was later discussed by G. Forbes (4).

In 1922 J. Hellerich (5) published an important paper on the rotational effects as observed during the time of eclipse in a number of Algol-type variables.

A complete investigation of the rotational effect in β Lyræ and in

Algol was carried out by R. A. Rossiter (6) and Dean B. McLaughlin (7). More recently J. S. Plaskett (8) has shown that the effect is also present in 21 Cassiopeiæ, and McLaughlin (9) has rediscussed the eclipsing variable λ Tauri on the basis of Ann Arbor spectrograms.

All the investigations on eclipsing binaries mentioned above prove that the absorption lines are actually widened by rotation. Adams and Joy (10) have successfully predicted in a number of cases that spectroscopic binaries with very wide and diffuse lines have very short periods. Since it is probable that the rotational periods in spectroscopic binaries are equal to their orbital periods, it is clear that very short periods should be associated with wide and diffuse lines. A similar effect was also noticed by Miss Antonia C. Maury (11) in the spectroscopic binaries μ^1 Scorpii and V Puppis.

The contour of spectral lines in a rotating or pulsating star was investigated by H. Shapley and S. B. Nicholson (12) in connection with Cepheid variables.

The important theoretical discussion by J. A. Carroll (13) on the form of an absorption line in the spectrum of a rotating or expanding star unfortunately arrived only after most of our own work had been completed and prepared for publication. Both papers show that the intensities of spectral lines form a sensitive criterion for the detection of rotation. Additional sides of the problem in question are considered in this paper; we have particularly emphasized certain practical applications.

J. H. Jeans (14) has recently shown that the flow of radiation, carrying with it momentum from one part of the star to another, seriously affects the internal motions of the stars, and in the first place their axial rotations. This factor of radiative viscosity, so called because of its analogy with the ordinary viscosity of gases, leads to the interesting result that the stars do not rotate as solid bodies, but that each spherical shell has its own angular velocity roughly proportional to r^{-2}. Only the innermost portions of the stars deviate markedly from this law. The period of rotation of the outer layer is approximately nine times longer than that of the inner layers. One would, therefore, expect that the effect of axial rotation upon the spectrum would be slight. This, however, may not be true for close double stars. Immediately after fission (if double stars really originate through fission) the periods of axial rotation and of orbital revolution must be identical.

There is also observational evidence in support of this equality. Thus in the β Lyræ stars the light-variation is continuous, indicating that the components are nearly in contact. A difference between the two periods would result in marked changes in the light curve from one revolution to the next, this being caused by the elliptical shape of the components. No such changes have been observed.

One of the stars in which Adams and Joy (15) suspect rotational widening of the lines is the well-known variable W Ursæ Majoris. Being a dwarf of spectral type F8p, the radii of its components cannot be very large. Adams and Joy found that the greatest semi-axis of each component is 540,000 km. or 0·78 in terms of the Sun's radius. The total mass of this system is very nearly that of the Sun, and the period is one of the shortest yet discovered, $0^{d}.334$. The lines are described as "so widened and weakened that measures for velocity and estimates of line-intensity can be made only with the greatest difficulty." It is suggested that "the unusual character of the spectral lines is due mainly to the rotational effect in each star, which may cause a difference of velocity in the line of sight of as much as 240 km. between the two limbs of the star." In this connection an interesting attempt was made by J. Schilt (16) to consider W Ursæ Majoris as a rotating star just preceding fission.

It can be shown that if the rotational velocity of W Ursæ Majoris produces appreciable widening of the lines and decrease in their depth (or central intensity), a similar effect should exist in binaries of early spectral type and of somewhat longer period (17).

Let ω be the angular velocity of the star, let the radius of the star, in km.'s, be designated as r (we neglect here any possible deformations due to tidal effects), and let the inclination of the orbit be i. The actual rotational velocity will then be

$$\mathrm{V}_{\mathrm{rot.}} = r\frac{d\theta}{dt} = r\omega = \frac{r}{\mathrm{P}}\,.\,\mathrm{const.},$$

and the observed component is

$$\mathrm{V}_0 = \frac{r}{\mathrm{P}}\,.\,\sin i\,.\,\mathrm{const.} \qquad . \quad . \quad . \qquad (1)$$

We now make the assumption that in close spectroscopic binaries the periods of rotation and of revolution are actually identical, so that the value of P in our formula is given by the observations.

Consider the specific case of a spectroscopic binary of type B0 having an absolute magnitude of -3.0 for the brighter component and an orbital period of $1^{d}.5$. The approximate diameter of the brighter component is, according to Russell's tables (18), about eight times that of the Sun, or ten times that of the diameter of each component in W Ursæ Majoris. Consequently the term $\frac{r}{\mathrm{P}}$ is about 2.2 times larger in our hypothetical star than in W Ursæ Majoris. Provided that sin i is in both cases the same, the spectral lines of our hypothetical star (19) should be much more affected by the rotation than those of W Ursæ Majoris, and this difference may be even larger if we consider that, according to Stebbins (20), "it seems plausible that there is progressive darkening at the limb in the spectral types ranging from almost nothing in the B stars to extreme effects in classes K and M."

The value of the term sin i is not known unless the star is an eclipsing binary, and it is not possible, in general, to compute the individual values of V_0 for each star. The inclination depends upon the orbital elements in the following manner :—

$$\sin i = \mathrm{const.}\ \mathrm{K}\,.\,\mathrm{P}^{\frac{1}{3}}(1 - e^2)^{\frac{1}{2}}\frac{(m_1 + m_2)^{\frac{2}{3}}}{m_2} \qquad . \quad . \qquad (2)$$

If we consider stars of only one spectral class, we may put

$$m_1 + m_2 = \text{const.},$$
$$m_2/m_1 = \text{const.},$$

and consequently

$$\sin i = \text{const. K} \,.\, \text{P}^{\frac{1}{3}} \,.\, (1 - e^2)^{\frac{1}{2}} \quad . \quad . \quad . \quad (3)$$

Finally we may assume for spectroscopic binaries, where e is nearly always small,

$$(1 - e^2)^{\frac{1}{2}} = \text{const.}$$

Then

$$\sin i = \text{const. K} \,.\, \text{P}^{\frac{1}{3}}.$$

Substituting this in equation (1) we have

$$\text{V}_0 = \text{const.} \frac{r \,.\, \text{K}}{\text{P}^{\frac{2}{3}}} \quad . \quad . \quad . \quad . \quad (4)$$

We see that in a spectroscopic binary the rotational velocity is directly proportional to K and inversely proportional to $\text{P}^{\frac{2}{3}}$. Accordingly we may expect that the effects produced by rotation will be particularly strong in binaries with short periods and large amplitudes of their velocity curves. In section 3 we shall show that this is actually the case.

2. *Influence of Rotation upon Line-contour.*—There are a number of factors that influence the width of a spectral line and the distribution of intensities in it (21): (1) Disturbing effects of neighbouring atoms which interfere with the perfect periodicity of the absorbing atom so that its quantum states are not entirely sharp; (2) Doppler effect due to the temperature motion of the particles; (3) Doppler effect due to ascending and descending currents of matter; (4) Doppler effect due to rotation; (5) Compton scattering by free electrons having different velocities; (6) Rayleigh scattering; etc.

The distribution of intensity in a spectral line, F(λ), is of importance in our problem. The theory of the contour of spectral lines has been developed chiefly through the researches of Stewart (22) and of Unsöld (23), based upon the remarkable pioneering work of Schuster (24). The influence of the spectrograph itself must also be considered. Wadsworth and others have shown that this factor produces a distribution which may be represented by an exponential function. We attempted to use the following expression for the distribution of intensity in a spectral line as produced by the combination of all factors:—

$$\text{I} = \text{F}(\lambda) = \text{I}_0 e^{-\kappa^2(\lambda - \lambda_0)^2} \quad . \quad . \quad . \quad (5)$$

I_0 and λ_0 are the intensity and wave-length respectively, corresponding to the middle of the line.

Spectrograms made with one-prism and three-prism spectrographs attached to the 40-inch Simeis reflector and analysed by means of a self-registering Koch microphotometer show that the lines occasionally deviate considerably from this expression, and that the form of the intensity distribution is not the same for all lines. This may also be derived from the observational data relating to the contours of lines, of von Klüber, Unsöld, and Miss Payne (25).

Rotation produces a symmetrical widening of the lines, if the figure of the star is symmetrical with respect to the axis of rotation. Let us suppose that the star is spherical in shape and that its axis of rotation is perpendicular to the line of sight. The rectangular co-ordinates on the apparent disc of the star are connected with the spherical co-ordinates in the following way :—

$$\left.\begin{aligned} x &= \cos b \,.\, \sin l \\ y &= \sin b \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (6)$$

The radial component of the rotational velocity is

$$v = \cos b \sin l \quad . \quad . \quad . \quad . \qquad (7)$$

so that all points on the disc having identical values of x have also identical components of rotational velocity.

Consider a star without any darkening at the limb, and let the original distribution of intensity be given by (5). For a rotating star we integrate (5) over the whole disc. For an arbitrary point the middle of the line is given by

$$\lambda_0' = \lambda_0 + \lambda_0 \frac{v}{c},$$

where

$$v = v_0 \cos b \sin l \quad . \quad . \quad . \quad . \qquad (8)$$

Since v is numerically (in terms of its maximum value) equivalent to the distance x on the stellar disc from the axis of rotation, we can write for the distribution of intensity in the line of the integrated spectrum,

$$\mathrm{I}_1' = \mathrm{I}_0' \int_{-1}^{+1} \int_0^y e^{-\kappa^2 \left[\lambda - \lambda_0 \left(1 \pm \frac{v}{c}\right)\right]^2} dv dy \quad . \quad . \qquad (9)$$

Since

$$y = \sqrt{1 - v^2}$$

we may write in place of (9),

$$\mathrm{I}_1' = \mathrm{I}_0' \int_{-1}^{+1} \sqrt{1 - v^2} e^{-\kappa^2 \left[\lambda - \lambda_0 \left(1 \pm \frac{v}{c}\right)\right]^2} dv \quad . \quad . \qquad (10)$$

Equation (10) can be integrated mechanically.

For our numerical computations we substituted in (10), in place of the expression given in (5), the distribution $\mathrm{F}(\lambda)$ actually recorded by a Koch microphotometer in lines of the Moon's spectrum. While this intensity curve may not hold true for all lines in stellar spectra, it is probably more reliable than equation (5). Since we are concerned at present only with the general aspect of rotational effects, we can safely neglect the small differences in $\mathrm{F}(\lambda)$ for individual lines.

The influence of the darkening at the limb, as will be seen below, is small. In one of the numerical examples we have used a darkening coefficient equal to 0·75 (see fig. 1 ; the intensity curve for Y Cygni, initial line-width 2 Å., drawn between the two curves for the rotating and non-rotating star, represents the solution with darkening at the limb).

The resulting curves will also be influenced by the character of the lines, from the point of view of their energy level. The light coming from the edges of the disc traverses thinner layers of the star's atmosphere and the line originates in higher and cooler regions. Accordingly certain lines must appear strengthened near the edges, while other lines are weakened and perhaps even reversed. It is also clear that the wings of certain lines may disappear near the edges. Obviously

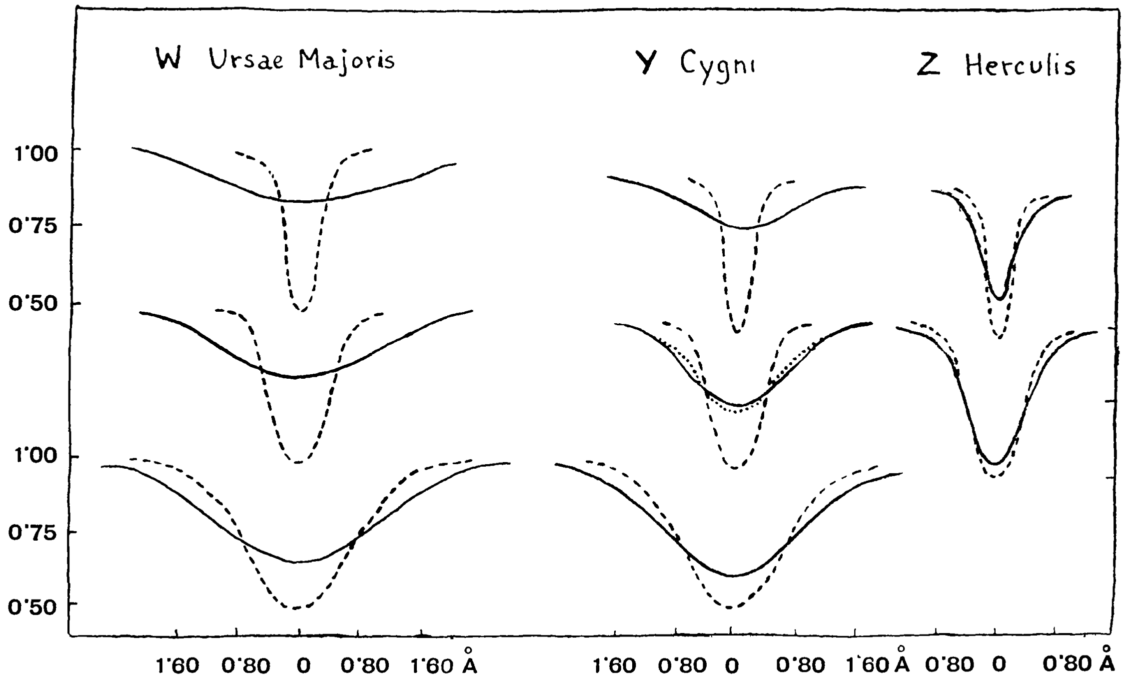

Fig. 1.—Contour of spectral lines, for initial widths of 1, 2, and 4 Å. The initial intensity of the middle of the lines was assumed to be 0·50.

- - - - - Non-rotating stars. —— Effect of rotation. . . . . . Solution with darkening.

the low-temperature lines will be the most sensitive ones with respect to rotation.

To illustrate the order of magnitude to be expected for the rotational effect we have made computations for a number of spectroscopic binaries under the assumption that the periods of rotation and of revolution are equal. The effect will, of course, depend upon the initial width and upon the central intensity of the line. We have carried through the computations for lines of 1, 2, and 4 Å. width. The greater values are encountered more frequently in the earlier spectral types, and it is in these that fast rotations are especially probable.

The results are shown in fig. 1 and in Table I. We assume that the initial intensity of the line is $0^{m}{\cdot}75$, this representing the difference between the continuous spectrum and the centre of the line.

TABLE I.

Central Intensity of Lines for Rotating Stars.
(The initial intensity was assumed to be $0^m.75$.)

Star.	Width. 1 Å.	2 Å.	4 Å.
	m	m	m
W Ursæ Majoris	0·12	0·24	0·43
Y Cygni	·17	·35	·52
Z Herculis	·52	·69	
u Herculis	. .	·23	

The above results apply to the larger components of the binaries under consideration. Photographically the phenomenon will be more complicated, but the essential features will remain the same. There is not only a very pronounced widening but also a large decrease in the depth of the line. This latter should constitute a very sensitive criterion of stellar rotation (26). With our present-day instruments the effect should be noticeable for rotational velocities exceeding 30 km./sec. It is of interest to note that wide and narrow lines react in varying degrees to the effect of rotation.

It is readily seen that the effect of rotation depends also upon the initial intensity of the line. Deeper lines will be more affected than shallow ones. We have computed the contours of two lines in the spectrum of Y Cygni of the same initial width, 2 Å., but of different initial central intensities, 0·50 and 0·20, in units of the intensity of the neighbouring portion of the continuous spectrum. The results are shown in fig. 2. The intensity-ratio of the two lines in question is not the same in a non-rotating and in a rotating star :

	Intensity-ratio.
For a non-rotating star	2·50
For a rotating star	1·29

Owing to rotation the unequal initial depths become more nearly uniform. This criterion is perhaps less applicable to spectroscopic binaries since the superposition of the continuous spectrum of the secondary star leads to a similar effect. However, in single stars, or in binaries where there is no interference on the part of the fainter component, this difficulty does not exist.

In actual practice the effect of rotation can be studied in two different ways. Consider a single spectral line. Its width and central intensity are functions of many factors, rotation being one of them. In a given number of stars there will be various intensities and line-widths, and it is not *a priori* possible to distinguish the influence of rotation from that of the other factors. We may, however, subdivide our stars into a number of groups, such that the expected rotations differ systematically from one group to the next, while the influences of the other factors remain, on the average, constant. In that case rotation will stand out and can be investigated separately. This method is purely statistical, and we have followed it in section 3.

Since stellar spectra contain usually many lines, wide and narrow, intense and weak, it should, theoretically, be possible to investigate the effect of rotation for each star individually. Suppose we know, from the solar spectrum, the initial intensities and widths of certain lines. If the spectrum of a star of the solar type shows the narrow lines appreciably too weak, while the wide lines remain about the same, we may conclude that the weakening is due to rotation (provided, of course, that the particular line is not sensitive to changes in absolute magnitude).

An entirely different method can be used in eclipsing variables. We have mentioned before that the effect of rotation has actually been measured from the small variations in radial velocity at the epochs just

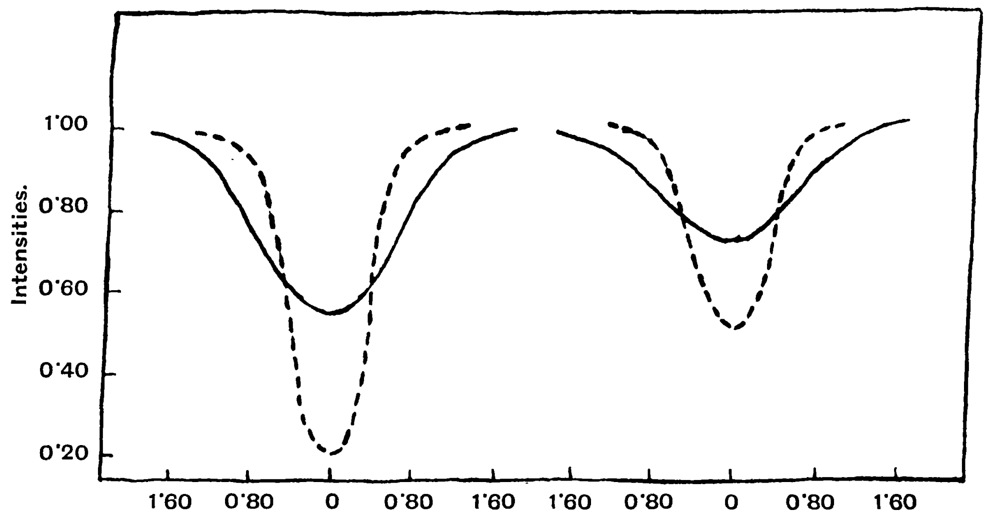

FIG. 2.—Line-contours for Y Cygni. The initial intensities are 0·20 and 0·50.
- - - - - Non-rotating star. —— Rotating star.

preceding and following the middle of eclipse. The line-contours observed during the same epochs will also be subject to rotational changes. The intensity distribution will be asymmetric and the degree of asymmetry will change with the phase. It should also be noted that during eclipse the central depths of the lines are less in a rapidly rotating star than in one having slow rotation. We have computed the contours of a line of the primary component of Y Cygni, for various phases (see fig. 3). We have assumed that (1) the star is spherical, (2) there is no darkening at the limb, (3) the spectrum of the principal star is not affected by that of the fainter. Our last assumption is invalid in the case of Y Cygni, so that, strictly speaking, our computations do not refer to that star. There are, however, many other eclipsing variables that sufficiently closely resemble this hypothetical object. The solution was obtained by a mechanical quadrature of equation (10) with the substitution of our empirical function F(λ), as explained above. The computations refer to a line of initial intensity $0^{m}75$. Table II. shows the change in the central depth of the line as a function of phase.

TABLE II.

Central Intensity of a Line of Initial Width 1 Å. *and Intensity* 0m·75 *during Eclipse for Y Cygni.*

Phase.	Intensity.
d	m
0·129	0·33
·110	·29
·089	·26
·070	·24
·052	·22
·018	·18

(The phases are counted from the middle of the eclipse.)

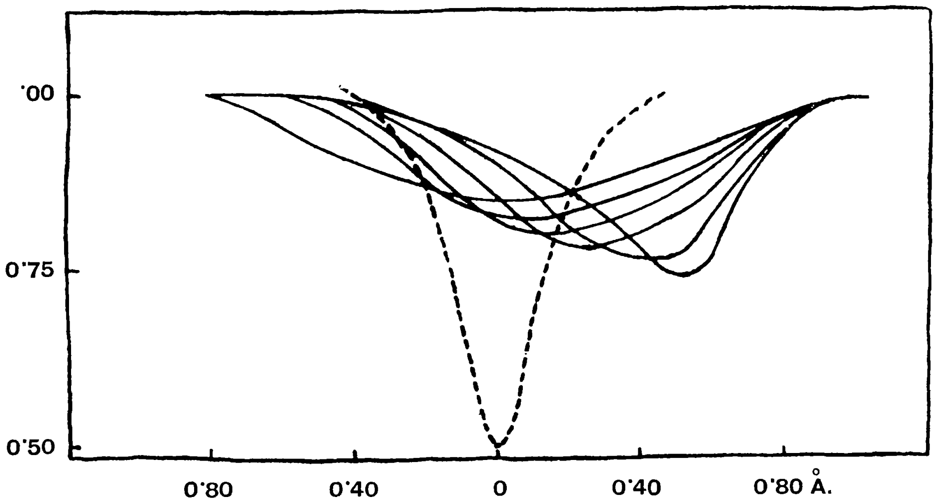

FIG. 3.—Line-contours at different phases during eclipse, for Y Cygni.
- - - - - Non-rotating star.

Investigations, made during an eclipse, of the line-contours in eclipsing variables would doubtless give useful results for the further study of their rotation.

3. *Observational Results for Spectroscopic Binaries.*—We have shown theoretically in section (1) that the rotational effects, if they are at all measurable, should predominate in binaries with short periods and large velocity amplitudes. We shall now show that this is actually the case, and that indeed the effects are so pronounced that they can easily be recognized in a statistical discussion.

Table III. contains the line-widths for a number of B and A type spectroscopic binaries for which spectrograms were available at the Yerkes Observatory. For the B stars the widths of He λ 4472 and Mg^+ λ 4481 have been measured. The results agree reasonably well, showing that for these two lines the widening is frequently due to a common cause. The hydrogen lines are, of course, unsuitable for such a study, since their width is quite abnormal. In the A stars only Mg^+ λ 4481 has been measured.

The probable error of the measures was not determined. But in a previous series it was found to be equal to ± 0·12 Å. from measures of the same plate. It is believed that the line-widths given in Table III. are consistent within about 0·2 Å.

From the resolving power of the Yerkes spectrograph (97,600 for three prisms) we have computed that the minimum width imposed by the properties of the spectrograph is about 0·68 Å. (for one prism). Only line-widths greater than this value have a real physical meaning.

TABLE III.

Measures of Line-widths in Spectroscopic Binaries of Types B and O.

Boss No.	Name.	Width in Å. λ 4472.	Width in Å. λ 4481.	Qual.	P. (d)	K. (Km./sec.)
123	29 π Andromedæ	1·6	1·2	g	143·67	47·66
159	23 Cassiopeiæ	1·3	0·7	g	33·75	16·32
425	46 ω Cassiopeiæ	1·9	1·0	g	69·92	29·64
641	42 π Arietis	2·0	1·9	g	3·854	24·77
816	19 τ^5 Eridani	1·0	1·0	g	6·2236	107
913	46 ξ Persei	4·1	..	g	6·951	7·87
920	35 λ Tauri	2·6	..	f	3·953	56·18
1107	94 τ Tauri	2·7	1·7	g	1·505	44·34
1139	9 α Camelopard.	2·7	..	g	7·996	9·0
1147	3 π^4 Orionis	1·3	1·3	g	9·519	25·93
1159	8 π^5 Orionis	2·5	2·7	g	3·700	57·88
1213	66 Eridani	..	1·3	g	5·522	97
1250	19 β Orionis	1·0	0·8	g	21·90	3·771
1301	28 η Orionis	2·0	1·7	g	7·990	144·75
1314	30 ψ Orionis	3·3	2·4	g	2·526	144·12
1333	25 χ Aurigæ	1·1	1·3	g	655·16	20·53
1339	34 δ Orionis	3·2	2·4	g	5·7325	100·12
1353	37 φ^1 Orionis	1·4	1·4	g	3066	13·3
1365	43 θ^2 Orionis	3·3	1·8	g	21·029	93·7
1366	44 ι Orionis	2·2	..	g	29·136	109·90
1388	125 Tauri	1·6	1·7	g	27·864	25·5
1399	−1° 1004 Orion.	2·6	1·6	g	27·160	93·04
1525	67 ν Orionis	1·2	0·9	g	131·26	34·09
1872	27 Can. Maj.	2·2	..	g	121	25
1899	29 Can. Maj.	3·8	..	g	4·393	218·44
1901	30 τ Can. Maj.	2·8	..	g	154·80	52·1
2445	76 κ Cancri	1·3	0·8	g	6·393	67·8
3138	31 Corvi	3·1	2·2	g	1·503	118·19

TABLE III.—*continued.*

Boss No.	Name.	Width in Å. λ 4472.	Width in Å. λ 4481.	Qual.	P.	K.
					d	Km./sec.
3281	5 ϰ Draconis	2·1	2·1	g	0·89038	18·9
3476	67 α Virginis	3·0	..	g	4·014	126·1
3988	7 ζ Coronæ	2·9	1·7	g	12·585	134·82
4008	25 A Serpentis	2·7	1·6	g	38·95	50·52
4062	6 π Scorpii	3·8	2·9	g	1·571	138
4388	68 u Herculis	3·6	3·2	g	2·051	99·50
4604	13 μ Sagittarii	1·1	1·2	g	180·2	64·5
4779	112 Herculis	1·9	1·4	f	6·3624	18·0
4794	11 δ¹ Lyræ	2·8	1·8	g	88·112	33·68
4864	18 Y Aquilæ	1·3	1·5	g	1·302	27·59
4934	46 υ Sagittarii	1·4	0·8	g	137·937	48·15
5018	44 σ Aquilæ	2·5	..	g	1·950	163·52
5375	57 Cygni	1·8	1·1	g	2·855	110·4
5532	8 β Cephei	1·5	..	g	0·190	16·9
5565	56° 2617 Cephei	2·0	..	p	1·364	77·01
5764	2 Lacertæ	1·4	1·1	g	2·616	80·3
5913	16 Lacertæ	1·2	1·2	g	12·3106	22·2
6046	1 Hev. Cassiopeiæ	1·6	1·3	g	6·067	59·06

Measures of Line-widths in Spectroscopic Binaries of Type A.

Boss No.	Name.	Width λ 4481.	Qual.	P.	K.
				d	Km./sec.
10	21 α Andromedæ	1·1	f	96·67	30·75
82	+43° 92 Andromedæ	0·8	g	3·956	41·7
145	20 π Cassiop.	1·3	f	1·964	117·32
428	6 β Arietis	1·4	g	107·0	32·6
923	36 τ Eridani	0·8	g	0·85347	35·7
986	b¹ Persei	1·7	g	1·527	41·89
1244	14 Aurigæ	1·1	f	3·789	21·56
1452	31 Camelop.	1·6	g	2·933	76·02
1457	136 Tauri	1·1	g	5·969	48·9
1492	2 Monocerotis	0·9	g	9·355	57·1
1501	61 μ Orionis	0·6	g	4·447	30·8
1515	40 Aurigæ	1·1	f	28·28	51·38
1690	24 γ Geminorum	0·9	g	2175	6·12
1979	br α Geminorum	0·7	g	9·219	13·56
1979	ft α Geminorum	1·0	g	2·928	31·76

TABLE III.—*continued.*

Boss No.	Name.	Width λ 4481.	Qual.	P. d	K. Km./sec.
2754	30 H Ursæ Majoris	1·0	g	11·583	34·07
2900	45 ω Ursæ Majoris	1·0	g	15·840	20·64
2987	55 Ursæ Majoris	1·4	g	2·5	38·5
3123	95 o Leonis	1·0	g	6·6	65
3182	+78° 412 Draconis	2·2	g	1·271	63·2
3210	15 η Virginis	0·7	g	71·9	26·8
3323	32 d^2 Virginis	1·3	f	38·3	40·99
3363	77 ε Ursæ Majoris	1·1	g	1515	3·5
3371	12 α^2 Can. Ven.	0·8	g	5·50	21·5
3474	79 ζ Ursæ Majoris	0·9	g	20·536	69·22
3626	11 α Draconis	0·8	g	51·38	46·25
3825	19 δ Libræ	1·7	g	2·327	76·5
3961	5 α Coronæ	2·0	g	17·36	34·93
4328	58 ε Herculis	1·6	g	4·024	70·39
4462	55 ξ Serpentis	0·8	g	2·292	19·35
4752	6 ζ Lyræ br.	1·2	g	4·300	51·24
4788	50 Draconis	1·3	g	4·118	79·12
5099	61 φ Aquilæ	0·8	g	3·320	38·25
5171	65 θ Aquilæ	0·9	f	17·124	46·0
5469	67 σ Cygni	1·1	g	11·043	1·98
5579	77 Cygni	1·4	p	1·729	109·7
5600	49 δ Capricorni	1·5	g	1·023	65·67

The results of the measurements are plotted in figs. 4 and 5. The abscissæ are expressed in log (10 P) and the ordinates represent the respective values of K. The stars occupy an area (27) limited at the bottom by the x-axis, at the left by a line that seems to be roughly straight and vertical, and at the top by a curve of the equation $K = C \cdot P^{-\frac{1}{3}}$. The conditions for finding a strong rotational widening of the lines (large K and small P) are fulfilled in the upper left portion of the occupied area. A glance at the figure shows that it is in these parts of the diagram that only wide lines are observed (28).

It is interesting to note that a few wide-line stars have large P and small K. If widening of spectral lines were due to rotation only this would indicate that the components of long-period spectroscopic binaries may occasionally have a fast rotation. But it is, of course, quite possible that the broadening depends on other factors. We conclude that the location of a spectroscopic binary in the upper left corner of the diagrams is a sufficient condition for observing wide lines in their spectra. However, if a star is located to the right or to the bottom of the diagrams the lines are not necessarily narrow.

The predominance of fast rotational velocities in short-period spectroscopic binaries can be brought out also in a different way. It is a well-known fact that stellar spectra of the same type greatly differ from one another in the appearance of their lines. This is especially true in the earliest spectral types, for which Adams and Joy have introduced the designations " sharp " and " nebulous." The character of spectrum is, as Adams and Joy have shown, correlated with absolute magnitude. It is probable that a large portion of this effect is due to atmospheric conditions within the stars. However, if we consider only

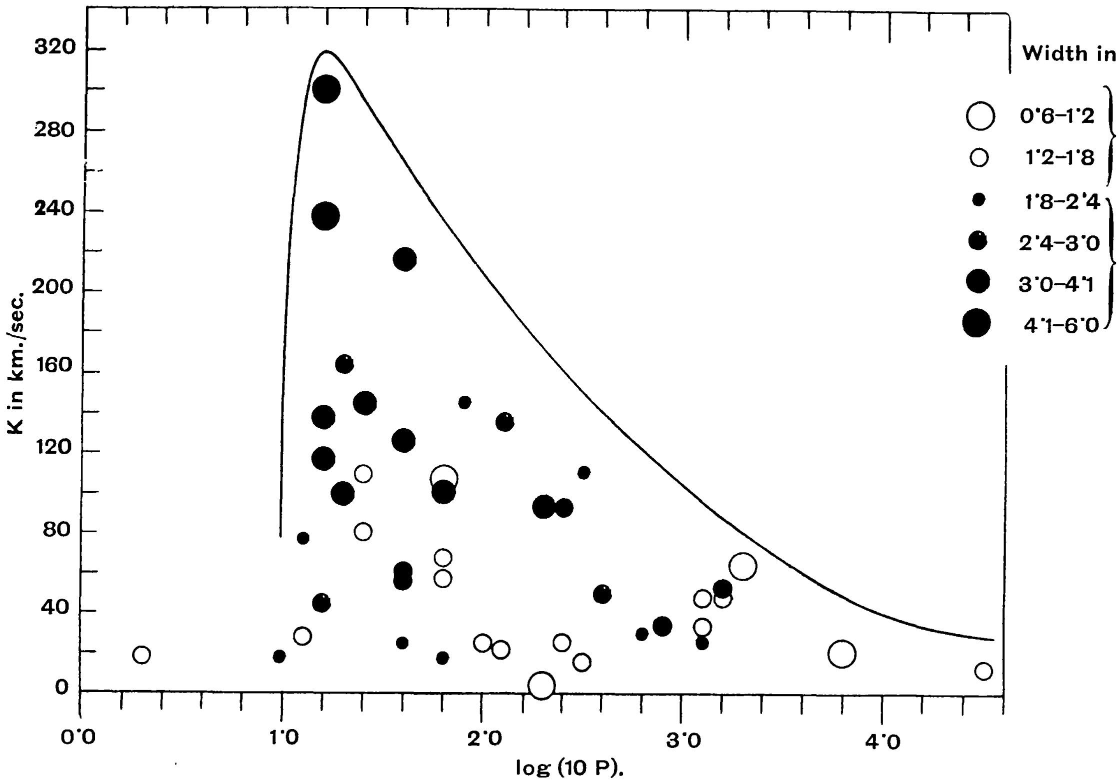

FIG. 4.—Line widths in spectroscopic binaries of types O and B. The two highest stars in the diagram are V Puppis and μ^1 Scorpii, for which Miss Maury finds a width of 5·8 Å and 4·9 Å respectively.

spectroscopic binaries an interesting detail is brought to light. We would naturally expect that rapid rotation would produce " nebulous " lines. In the following table we have used the spectroscopic binaries of Moore's Third Catalogue (29), for which the subdivision " n " or " s " is known. A few additional stars were taken from Beer's recent paper on spectroscopic binaries (30).

In agreement with our expectation, short periods show a distinct tendency to be associated with " nebulous " lines. It might be noted that for spectroscopic binaries of type B8–F2 the relative number of " s " is much greater than for single stars. But this may be due to selection, since it is easier to measure sharp-lined stars than those having diffuse lines.

TABLE IV.

Spectroscopic Binaries arranged according to Period and Character of Spectrum.

Spectrum.	Period.	" n."	" s."
B0–B6	Longer than 3^d	20	10
,,	Shorter ,, ,,	8	0
B8–F2	Longer than 3^d	9	28
,,	Shorter ,, ,,	8	4
B0–F2	Longer than 3^d	29	38
,,	Shorter ,, ,,	16	4

The existence of fast rotation in spectroscopic binaries of short period seems thus to be established beyond any doubt. The line-

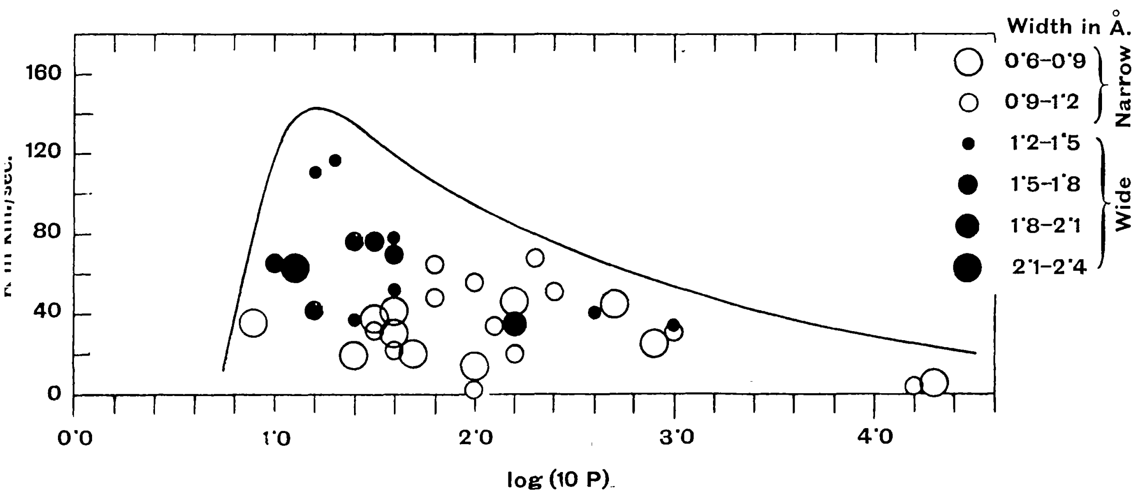

FIG. 5.—Line widths in spectroscopic binaries of type A.

widths in Table III. are strongly correlated with central intensity, so that the effect of widening is associated with one of decrease in the residual intensity of the lines, as is demanded by the theory of section 2. We hope in future to apply some of the differential methods of section 2 to individual stars, in order to get more quantitative results regarding the actual speeds of rotation.

4. *The Rotational Effect in the Spectrum of Jupiter.*—To find what might actually be expected in the case of a rotating star, spectrograms of Jupiter were obtained in 1927 by means of a single-prism spectrograph attached to the 40-inch Simeis reflector. The rotation of Jupiter causes in the reflected sunlight a Doppler shift of about 25 km./sec. at the equatorial limb. The experiment consisted in comparing two spectrograms of Jupiter. The one was taken with the slit perpendicular to the equator and with the image of the planet accurately bisected by the slit. In this case there was, of course, no effect of rotational widening nor any decrease in line-depth. The next plate was taken with approximately the same exposure time, the image of the planet

being oscillated uniformly across the slit by means of an electric motor for very slow motion, thus giving an integrated spectrum for the whole planet. This is probably a fair approximation to the spectrum of a rotating star, in spite of the notoriously strong darkening of Jupiter at the limb. The exposures, usually made near the meridian, were about 25 minutes long. The linear dispersion in the region measured was about 12·5 Å. per mm. The total number of plates was ten. The spectrograms were run through a Koch microphotometer with gear ratio 48·5 to 1.

The widths of a number of lines were measured on the microphotometer tracings (see Tables V. and VI.). The results are affected by blending with other lines so that the precision is not great. However, since we are here concerned only with differential effects, these uncertainties, as well as instrumental or other errors, are of no consequence.

There is a slight widening of the spectral lines in the integrated spectrum, but its amount is of the same order of magnitude as the mean error, and is therefore not certain. Evidently an equatorial velocity of rotation of 25 km./sec. produces an amount of widening that is at the very limit of what the measurements can give. The rotation should probably be appreciably faster, perhaps of the order of 50 to 100 km./sec., to produce effects upon the line-widths that are definitely measurable.

The measures of line-intensities give very much better results. Since the exposure times of the spectrograms were such that the photographic densities of the continuous spectrum are the same, we have measured the depths of the lines in mm.'s and summarised in Table VII. the differences for each line.

The results show that the lines of the integrated spectrum are appreciably shallower than those which are not affected by rotation, but this difference is not very large and is noticeable only after a careful study of the material.

We may thus conclude that the effect produced by a rotational velocity of the order of 25 km./sec. is about at the limit of what the measurements can reveal. Since in actual practice we shall never be able to compare with each other two spectra that are as much alike as two plates of Jupiter, the lower limit should be taken rather above 25 km./sec. However, there is every reason to believe that rotational velocities exceeding 50 km./sec. should be measurable without much difficulty. That such velocities are not infrequent, is shown by our statistical discussion of spectroscopic binaries. Whether or not single stars also frequently rotate with such high velocities remains to be seen.

TABLE V.

The Widths of Spectral Lines.

1927	λ 4030·9. Normal.	Integr.	Diff.	λ 4033·2. Normal.	Integr.	Diff.
	mm.	mm.		mm.	mm.	
Oct. 10	0·188	0·190	+0·002	0·140	0·136	−0·004
19	·161	·184	+ ·023	·138	·130	− ·008
24	·159	·173	+ ·014	·126	·140	+ ·014
26	·147	·157	+ ·009	·118	·107	− ·011
28	0·173	0·157	−0·016	0·124	0·132	+0·008
		Mean	+0·006±0·0066 (m. e.)			0·000±0·0048

1927	λ 4034·6. Normal.	Integr.	Diff.	λ 4035·8. Normal.	Integr.	Diff.
	mm.	mm.		mm.	mm.	
Oct. 10	0·099	0·095	−0·004	0·082	0·091	+0·009
19	·118	·103	− ·015	..	..	..
24	·122	·120	− ·002	..	..	..
26	·109	·122	+ ·013	..	..	..
28	0·099	0·089	−0·010	0·122	0·122	0·000
			−0·004±0·0047			+0·004±0·0045

1927	λ 4052·0 Normal.	Integr.	Diff.	λ 4055·0. Normal.	Integr.	Diff.
	mm.	mm.		mm.	mm.	
Oct. 10	0·194	0·184	−0·010	..	..	..
19	·181	·177	− ·004	..	..	..
24	..	..	..	..	..	..
26	·188	·198	+ ·010	0·208	0·212	+0·004
28	0·188	0·179	−0·009	0·165	0·171	+0·006
			−0·003±0·0046			+0·005±0·001

1927	λ 4063·6. Normal.	Integr.	Diff.	λ 4071·9. Normal.	Integr.	Diff.
	mm.	mm.		mm.	mm.	
Oct. 10	0·330	0·318	−0·012	0·293	0·297	+0·004
19	·217	·252	+ ·035	·247	·268	+ ·021
24	·268	·282	+ ·014	·268	·268	·000
26	0·305	0·305	0·000	0·289	0·289	·000
28	..	..	..	0·247	0·250	+0·003
			+0·009±0·010			+0·006±0·0039

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TABLE VI.

Summary of Line-widths.

λ.	Integr.—normal.	Mean Error.	λ.	Integr.—normal.	Mean Error.
4030·9	+0·006	±0·0066	4052·0	− ·003	·0046
4033·2	·000	·0048	4055·0	+ ·005	·0010
4034·6	− ·004	·0047	4063·6	+ ·009	·0100
4035·8	+ ·004	·0045	4071·9	+ ·006	·0039

TABLE VII.

Intensity-differences of Lines.

Normal—Integrated.

λ.	Oct. 10.	Oct. 19.	Oct. 24.	Oct. 26.	Oct. 28.
4030·9	+2·0	+2·2	+0·5	+1·5	+0·8
4033·2	+1·5	−0·3	+1·5	+1·0	+0·5
4034·6	0·0	−0·6	+0·5	0·0	0·0
4035·8	+0·2	+0·4	+0·3	−0·5	..
4037·2	+0·4	+0·5	+0·1	−0·5	0·0
4038·9	+0·7	−0·4	+0·6	+0·8	0·0
4041·6	+0·8	−1·4	+0·2	..	0·0
4044·2	+2·8	+1·5	−0·6	+0·5	+1·0
4045·8	0·0	−0·6	+1·5	0·0	+1·7
4048·8	+0·1	−0·5	−0·3	+2·0	+0·5
4050·8	..	0·0	..	..	..
4052·0	..	−0·8	..	+1·7	+0·3
4055·0	..	..	..	+0·5	+0·4
4057·6	..	..	..	+1·2	..
4063·6	..	+0·9	+2·5	+1·5	+1·0
4067·1	+3·8	+0·7	+0·2	+0·9	+0·7
4071·9	+1·0	+1·0	+0·4	+2·2	+1·1
4074·9	+2·0	+0·5	+0·2	+1·0	+1·0
4076·9	..	+0·5	+2·0	−0·5	+1·3
4077·9	..	+0·7	+1·1	−0·5	..
4078·4	..	−0·2	0·0	−0·2	−0·4
Mean	+1·18±0·33	+0·22±0·19	+0·63±0·22	+0·66±0·21	+0·58±0·14

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(19) The eclipsing variable V Puppis (Sp. B1p; P = $1^{d}\cdot 454$; K = 302 km./sec.) closely resembles our hypothetical B star. According to Hellerich (*loc. cit.*) the rotational velocity for the brighter component of this star is 276 km./sec., and the corresponding line-width should be 7·9 Å. Miss Maury (*Harvard Annals*, **84**, 186, 1920) has measured the widths of several lines in this star and has found a mean of 5·78 Å.—an unusually high value for a B-type star.
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(26) The numerical results cannot be directly compared with those of Carroll, since he uses spectral lines of zero intensity, whereas our lines are taken from observational data, and their depths are very moderate
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(28) In fig. 4 the stars α Camelopardalis and ξ Persei, the orbits of which are based on the detached calcium lines, have been excluded. These stars are peculiar and their velocity curves may not be due to orbital motion. Both have wide lines and small values of K. They would, of course, in no way contradict the above statement, even if they had been included. On the other hand, we have included the two stars V Puppis and μ^1 Scorpii, based on Miss Maury's measurements.
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Shortt Clocks and the Earth's Rotation. By J. Jackson, M.A., D.Sc.

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In *M.N.*, **88**, 465, a description was given of the performance of the clocks Shortt 3 and Shortt 11 from the time of their installation, 1924 November and 1926 May respectively, till the end of 1927. It was there shown that the going of these clocks over long intervals of time far surpassed that of earlier types of clock. The accuracy of the clocks indeed was such that it was found advisable to allow for the non-uniformity of sidereal time resulting from nutation—the principal term of coefficient $1^{s}\cdot 06$ and period 18·6 years indeed varies so slowly that it causes no trouble, but the term of coefficient ${}^{s}\cdot 08$ and six-month period