Class36-38

November 16, 2020

0.1 Inference for simple linear regression

0.1.1 A hypothesis test for β

We can find optimal point estimates for β_0, β_1 by minimizing the least squares function, but point estimates do not give us information about how β_0, β_1 with different samples from our population. Every sample of data would give us a different "optimal" point estimate for β_0, β_1. It is natural to ask whether or not β_1 will be statistically different than zero—whether or not a relationship between random variables X and Y is probable.

A natural hypothesis to test β_1 is

$$H_{\text{null}} : \beta_1 = 0 \tag{1}$$
$$H_{\text{Alte.}} : \beta_1 \neq 0 \tag{2}$$
$$\tag{3}$$

If we can collect enough data to disprove that $\beta_1 = 0$ then there may be a relationship between X and Y. In addition to a hypothesis, we need a signfiicance level α and most important: a test statistic.

Test statistic A (probably expected by now) test statistic for β_1 is

$$t = \frac{\beta_1 - \beta_{1\,\text{Null}}}{se(\beta_1)} \tag{4}$$

From above, our null value for β_1 ($\beta_{1\,\text{Null}}$) is zero.

$$t = \frac{\beta_1 - 0}{se(\beta_1)} = \frac{\beta_1}{se(\beta_1)} \tag{5}$$

If we can find an expression for the standard error of β_1 then we can compute our test statistic and compare our test stat to a distribution when we assume the null hypothesis, when we assume that β_1 is zero.

The standard error (you'll derive in this week's homework) for β_1 is

$$se(\beta_1) = \frac{\sigma^2}{\sum_i (x_i - \bar{x})^2} \tag{6}$$

where σ^2 is the variance from our linear regression.

pvalue If we can show that our estimate of β_1 is normally distributed then we know (from an earlier class) our test will have a student's t distribution. It turns out (you'll show in your homework) that we assumed $y_1, y_2, \cdots, y_n$ come from a normal distribution and any linear combination of random variables following a normal distribution also has a normal distribution. We will see (in your homework) that the estimate for β_1 is a linear combination of Ys which are normally distributed. So we can assume then β_1 follows a normal ditribution and our test statistic has a student's t distribution.

The two-sided pvalue from our hypothesis test is computed as

$$\text{pvalue} = p(T_{null} > t_{\text{observed}}) + p(T_{null} < -t_{\text{observed}}) \tag{7}$$

Small pvalues indicate the null hypothesis is unlikely and that β_1 is probably not zero.

0.1.2 Example dataset

The data is a classic set of 442 diabetes patients. The dataset contains 10 variables related to diabetes and a continuous measure of diseaese progression.

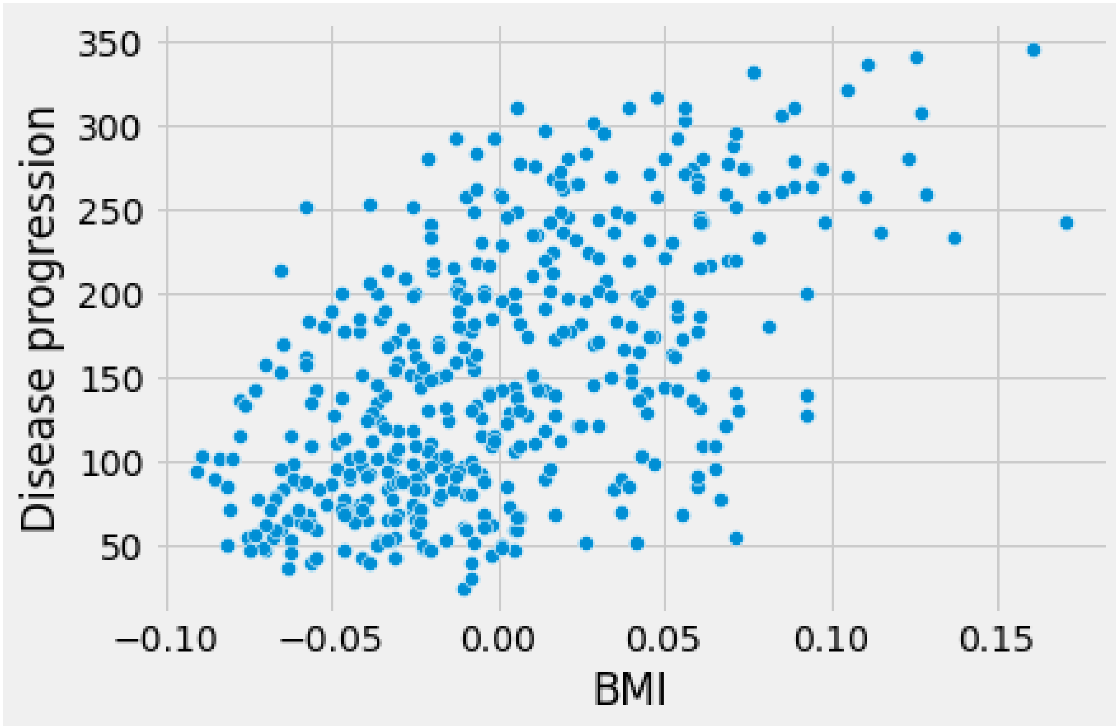
We will plot one of the covariates—BMI— against this measure of disease prgoression and fit a simple linear regression.

[27]:
```
import seaborn as sns
from statsmodels.regression.linear_model import OLS
import statsmodels.api as sm

import sklearn
from sklearn.datasets import load_diabetes

X, y = load_diabetes(return_X_y=True)
bmi = X[:,2]

bmi=sm.add_constant(bmi)
plt.style.use("fivethirtyeight")

fig,ax = plt.subplots()
sns.scatterplot(bmi[:,1],y, ax=ax)

ax.set_xlabel("BMI")
ax.set_ylabel("Disease progression")
```

```python
results = OLS(y,bmi).fit()
results.summary()
```

```
/usr/local/lib/python3.9/site-packages/seaborn/_decorators.py:36: FutureWarning:
Pass the following variables as keyword args: x, y. From version 0.12, the only
valid positional argument will be `data`, and passing other arguments without an
explicit keyword will result in an error or misinterpretation.
  warnings.warn(
```

[27]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.344
Model:                            OLS   Adj. R-squared:                  0.342
Method:                 Least Squares   F-statistic:                     230.7
Date:                Mon, 16 Nov 2020   Prob (F-statistic):           3.47e-42
Time:                        10:31:41   Log-Likelihood:                -2454.0
No. Observations:                 442   AIC:                             4912.
Df Residuals:                     440   BIC:                             4920.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const        152.1335      2.974     51.162      0.000     146.289     157.978
x1           949.4353     62.515     15.187      0.000     826.570    1072.301
==============================================================================
Omnibus:                       11.674   Durbin-Watson:                   1.848
Prob(Omnibus):                  0.003   Jarque-Bera (JB):                7.310
Skew:                           0.156   Prob(JB):                       0.0259
Kurtosis:                       2.453   Cond. No.                         21.0
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```



0.2 R^2

The coefficient of determination (R^2) describes one minus the variance in the errors made by a regression model divided by the variance in errors if we used the mean as a predictor. If we call the variance in errors made by a regression model SSE and variance in errors made by using the simple mean as SST then

$$R^2 = 1 - \frac{SSE}{SST}$$

Values of R^2 close to 1 mean we make smaller errors whe nusing our regression model and values of R^2 clsoe to zero says our errors using a regression model are the same as if we used the simple mean.

How do we find the variance of the errors made when we choose a regression model? Well that is

$$SSE = \sum_i (y_i - \hat{y}_i)^2$$

where $\hat{y}_i$ is our prediction of the true value y_i form our regression. We can do the same for $\bar{y}$.

$$SST = \sum_i (y_i - \bar{y})^2$$

The acronym **SSE** stands for "Sum Squares Error" and the acronym **SST** stands for "Sum Squares Total". The expression for R^2 is then the relative reduction in variance from our regression model.