

GHANA COMMUNICATION TECHNOLOGY UNIVERSITY



FACULTY OF COMPUTING AND INFORMATION SYSTEMS

Bachelor's in Computer Science (LEVEL 100)

END-OF-SEMESTER EXAMINATIONS, APRIL/MAY 2021

MATH 121 / 103: DISCRETE MATHEMATICS

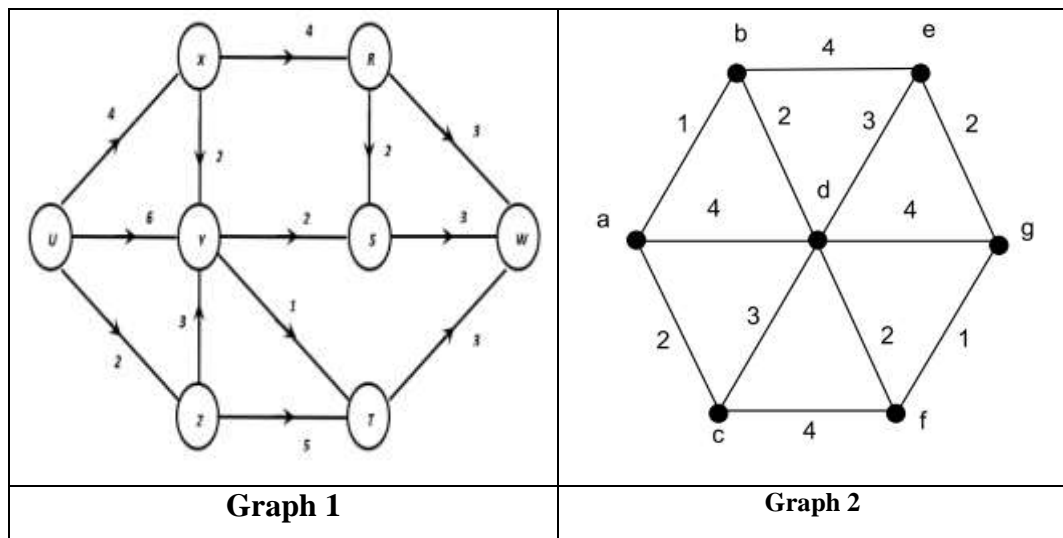
Duration: 24 Hours

***[THIS PAPER IS MADE UP OF 2 SECTIONS
SECTION A IS COMPULSORY WORTH 40 MARKS
SECTION B CHOOSE ANY 2 WORTH 20 MARKS]***

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SECTION A (ANSWER ALL QUESTIONS) – [40 MARKS]

1. a) Consider the following directed and undirected weighted graphs below:



- (i) Determine the weighted matrix, adjacency list for both graphs
Use Dijkstra's Algorithm to find the shortest distance from **[2 marks]**
- (ii) Graph 1: vertex ***U*** and **[5 marks]**
- (iii) Graph 2: vertex ***a*** **[5 marks]**
- to all the other vertices. Determine if there exist any two shortest paths from ***U*** to ***W*** and from ***a*** to ***g*** in each graph?
- b) Suppose cells in a computer memory indexed from 0 to 10. From the

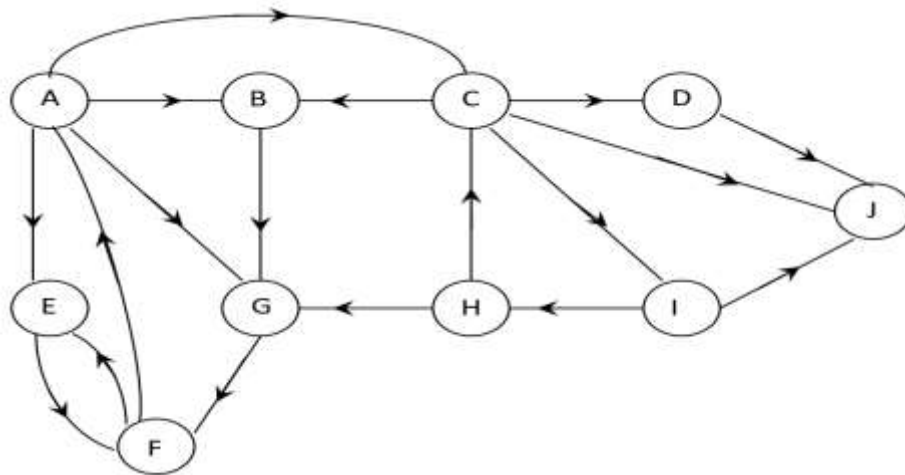
0	1	2	3	4	5	6	7	8	9	10

Hash function below, show how the data would be inserted in the order given in initially empty cells. Use Collision Resolution policy if required:

$$H(n) = n \bmod 11$$

Cells indexed 0 to 10: Data: 55, 15, 285, 743, 375, 22, 10, 800 **[8 marks]**

2. a) Using the graph below:
- (i) Draw the adjacency list **[2 marks]**
- (ii) Perform a breadth first search (BFS) and a depth first search (DFS) using ***A*** as your source. **[4 marks]**
- (iii) Derive the BFS tree diagram **[4 marks]**



- b) Given two primes $p = 11$, $q = 3$ and $e = 17$. Determine the following:
- The totient function (Φ) ? [2 marks]
 - Show that the $\text{GCD}(\Phi, e) = 1$ and determine d ? [6 marks]
 - State the Public and Private key? [2 marks]
 - Starting with letter "A" as "0", encrypt the word **GO**? [8 marks]
 - State and explain one property of the decryption key d ? [2 marks]

SECTION B (CHOOSE ANY 2) – [20 MARKS]

3. a) Using the Handshaking Principle, determine the number of edges of a graph with fourteen (14) vertices and each with degree six (6)? [5 marks]
- b) Prove using method of induction for all $n \geq 1$

$$P(n) : 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + n(n+1)(n+2) = \frac{1}{4} n(n+1)(n+2)(n+3) ? \quad [5 \text{ marks}]$$

4. a) Use the Laws of Algebra of sets and Logic to prove that:
- $P \rightarrow Q$ is logically equivalent to $(\neg P \vee Q)$? [1 mark]
 - Find the negations of $P \rightarrow Q$? [3 marks]
 - Using Truth Table that: $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$? [2 marks]
- b) Determine the duality of the Boolean equations
- $(a \cdot 1) + (0 + \bar{a}) = 0$ [1 mark]
 - $a \cdot (\bar{a} + b) = a \cdot b$ [1 mark]
 - $a + (\bar{a} \cdot b) = a + b$ [1 mark]

(iv) $(a+1) \cdot (a+0) = a$ [1 mark]

5. a) Determine the *converse*, *inverse* and *contrapositive* of the following statements by indicating the conditional statement.

(i) If we are in binary system then $1 + 1 = 10$ [1 mark]

(ii) Good food are not cheap [1 mark]

(iii) If $9x + 36 = 9$ then $x \neq 17$. [1 mark]

(iv) If $\cos(x) = 1$ then $x = 0$. [1 mark]

b) If $f(x) = 8x$, $g(x) = x^4$ and $h(x) = 4x + 3$ then find $(f \circ g) \circ h$ and $f \circ (g \circ h)$. Show that $(f \circ g) \circ h = f \circ (g \circ h)$? [2 marks]

c) Let $f : \mathbb{R} \rightarrow (-1, 1)$ be defined by $f(x) = \frac{x}{x-1}$, $x \in \mathbb{R}$. Find the inverse of the above function if it exists, where \mathbb{R} is the set of real numbers. [4 marks]

6. Given the Boolean function:

a) $F(P, Q, R) = \overline{P}\overline{Q}\overline{R} + \overline{P}Q\overline{R} + \overline{P}QR + P\overline{Q}\overline{R} + P\overline{Q}R + PQR$

i) Represent the function in a Karnaugh Map and minimize? [2 marks]

ii) Draw the minimized digital circuit? [2 marks]

b) A relation $R = \{(x, y) : 4x = 6y\}$ is defined on the set $A = \{1 \leq x \leq 12\}$. Find the reflexive, symmetric and transitive closure of this relation? [4 marks]

c) A customer at an ATM realizes that he has forgotten the last two digits of his pin code. He however remembers that the two digits were greater than five. He does have 3 entry tries before the ATM confiscates his card. How many possible combinations of the 2 digits can he try? [2 marks]