

# Intro to ML

## Naïve Bayes, kNN

**Vladislav Goncharenko**

ML Teamlead, DZEN



MSU, spring 2024

# Team

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# Vladislav Goncharenko

- Author of machine learning courses and Masters program at MIPT
- ML researcher (MIPT)
- Team lead of video ranking team at Dzen (yandex.ru)
- Ex-team lead of perception team at self-driving trucks
- Open source fan



# Outline

1. ML and AI overview
2. Thesaurus and notation
3. Maximum Likelihood Estimation
4. Some Machine Learning problems
  - a. Classification
  - b. Regression
  - c. Dimensionality reduction
5. Naïve Bayes classifier
6. k Nearest Neighbours (kNN)

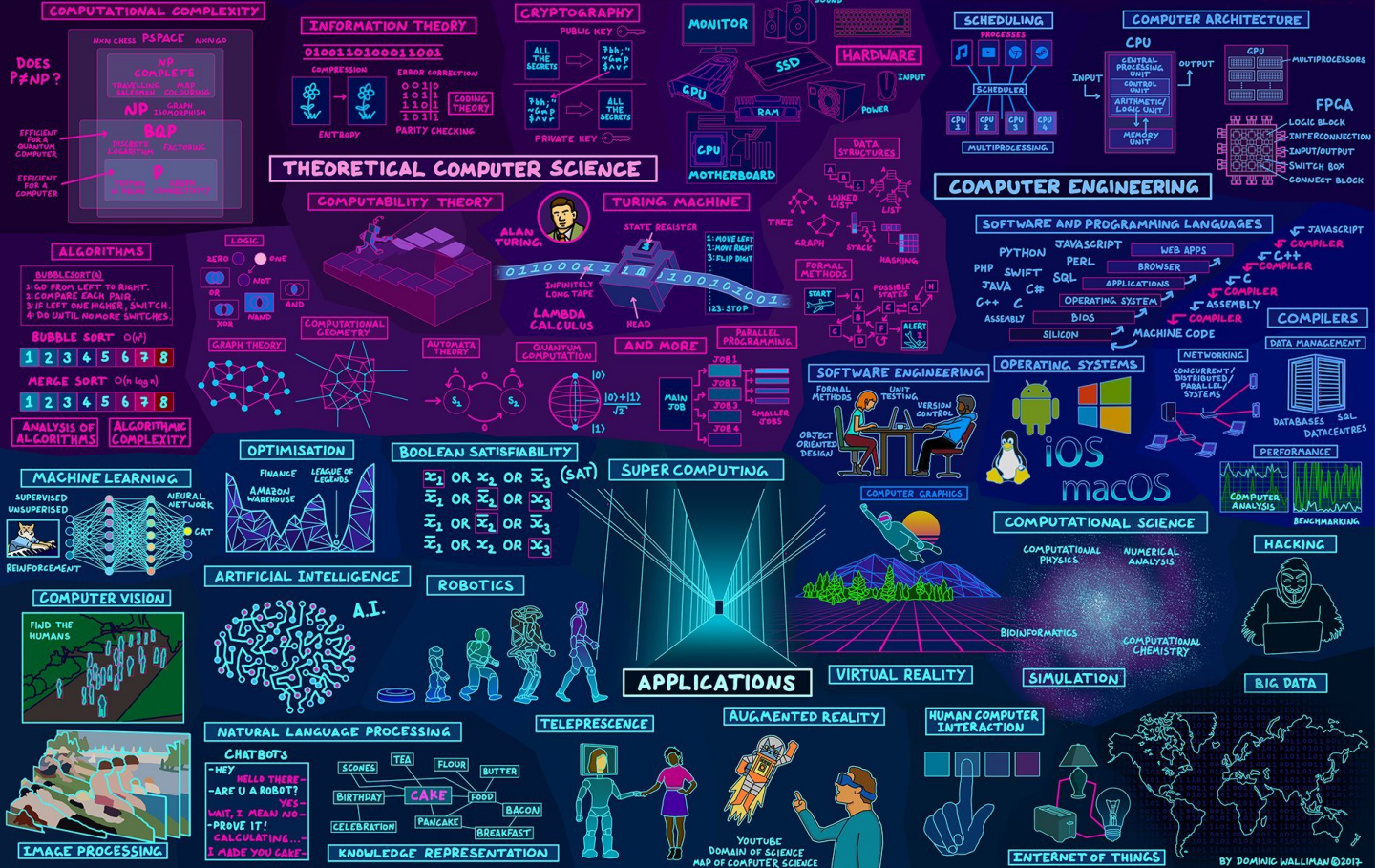
# ML and AI overview

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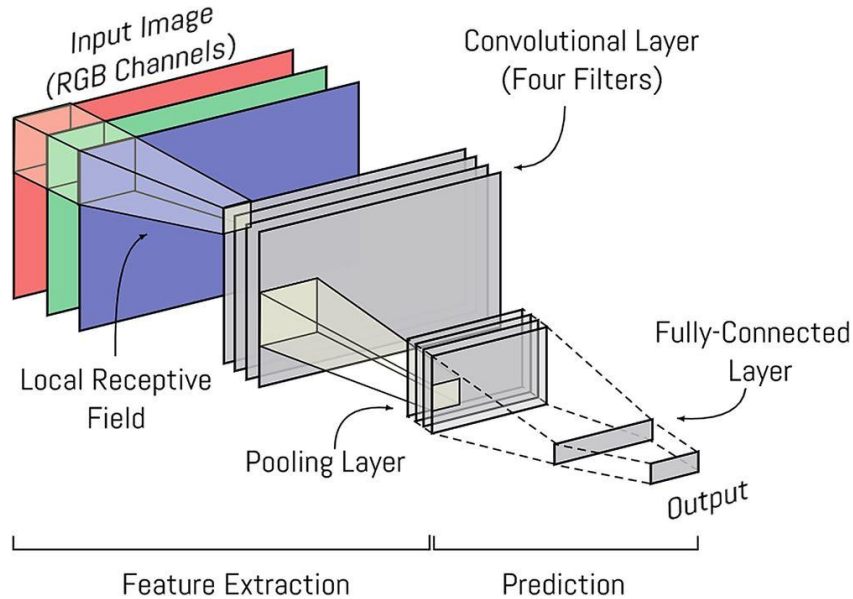
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01

# MAP OF COMPUTER SCIENCE



# Computer Vision



Basics:

- Classical CV (filters, border detectors)
- Convolutional Neural Networks



# Computer Vision

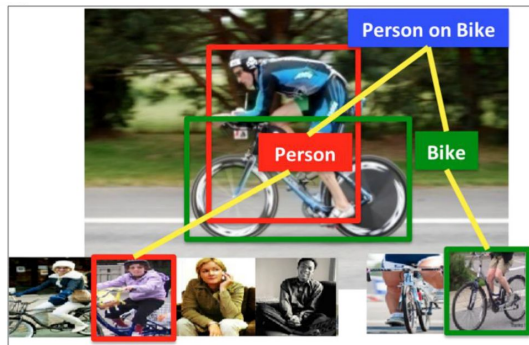


Some achievements:

- Object detection
- Semantic segmentation
- Generative models



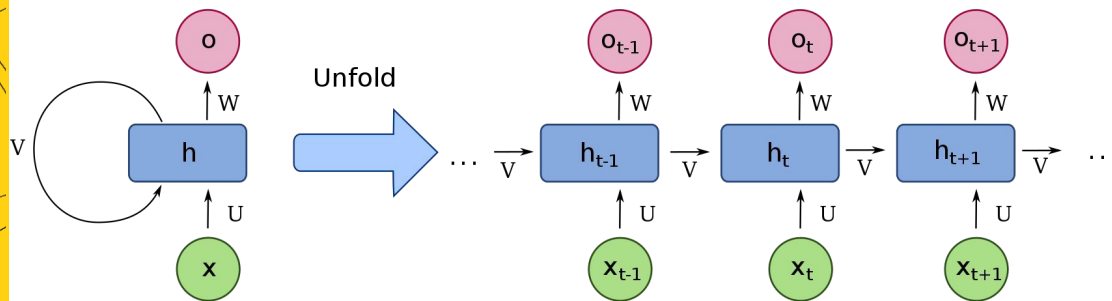
person  
hammer  
flower pot  
power drill







# Natural Language Processing



Basics:

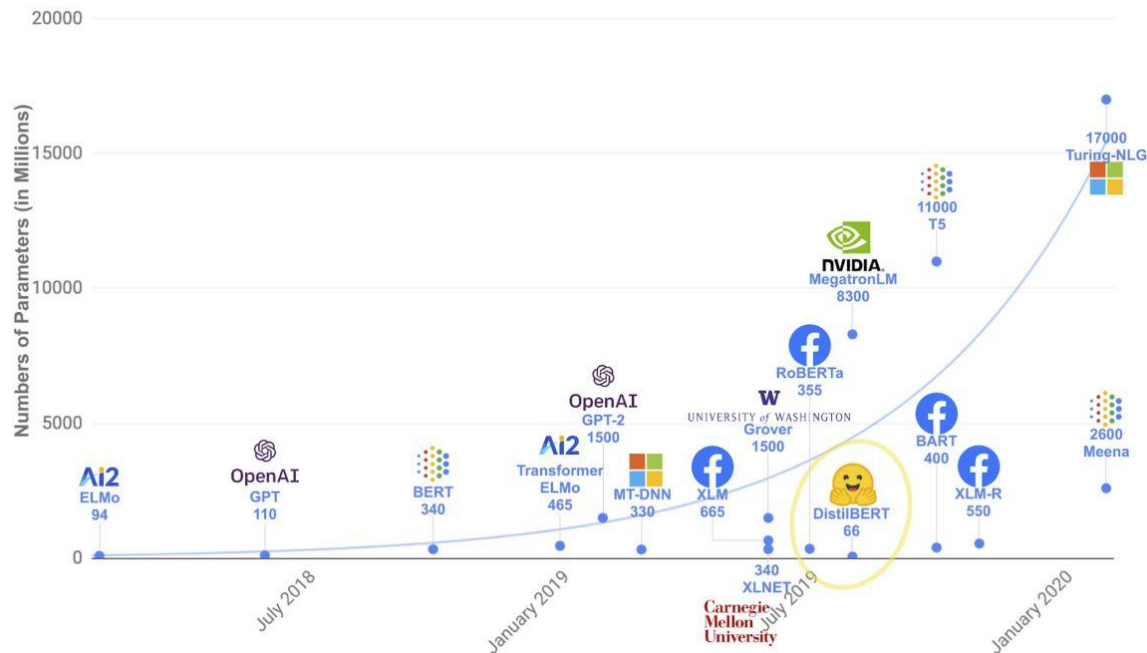
- Language models
- Recurrent Neural Networks
- Attention module

# Natural Language Processing



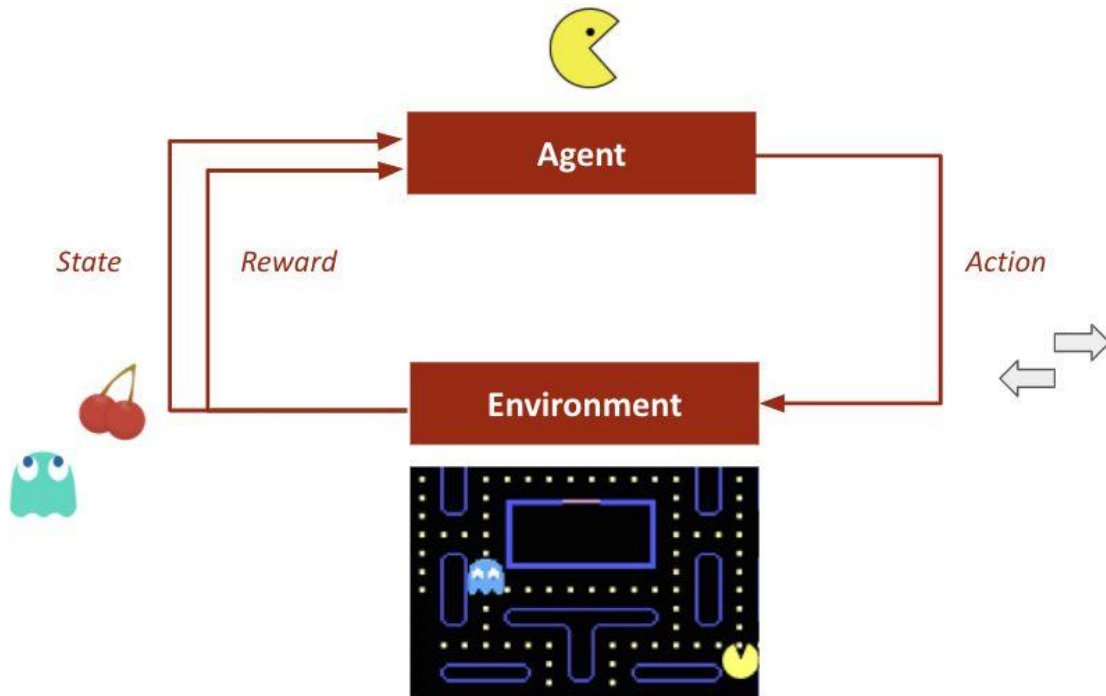
Some achievements:

- Machine translation
- Texts classification
- Texts generation





# Reinforcement Learning



Basics:

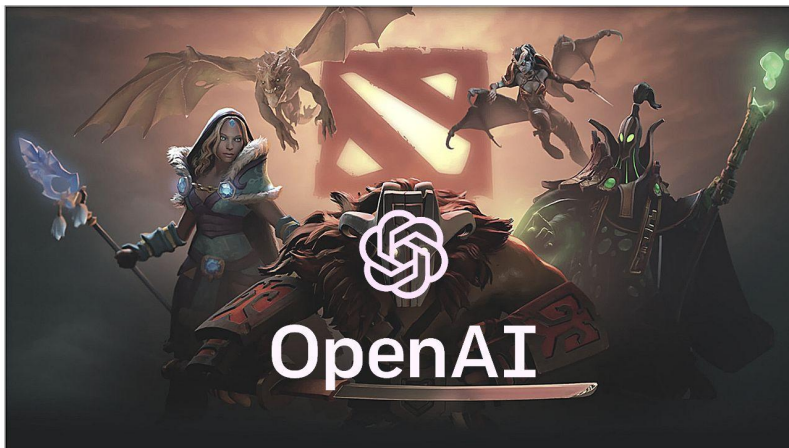
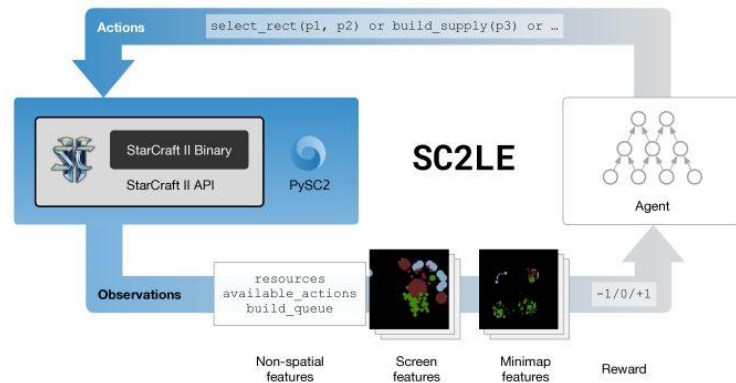
- Q-learning
- DQN
- REINFORCE



# Reinforcement Learning

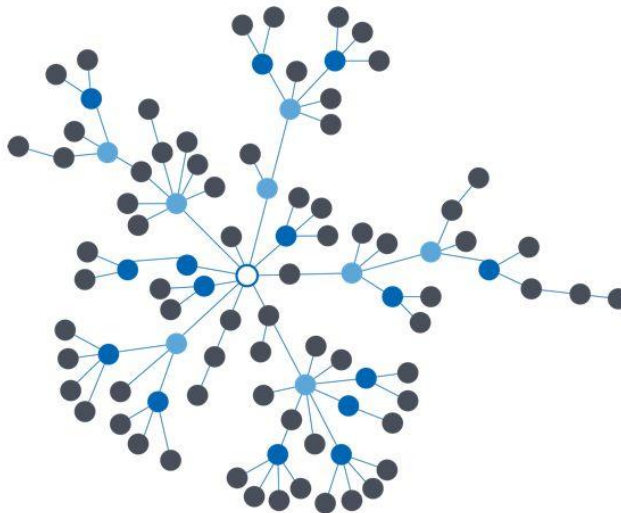
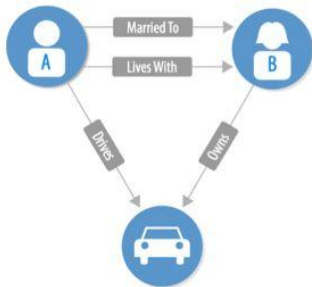
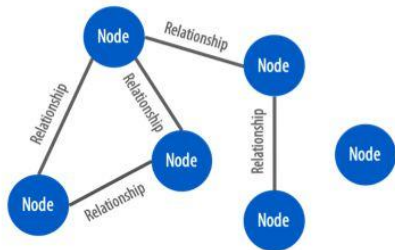
Achievements:

- Alpha Go
- OpenAI Five
- DeepMind Star Craft 2





# Machine Learning on Graphs



Basics:

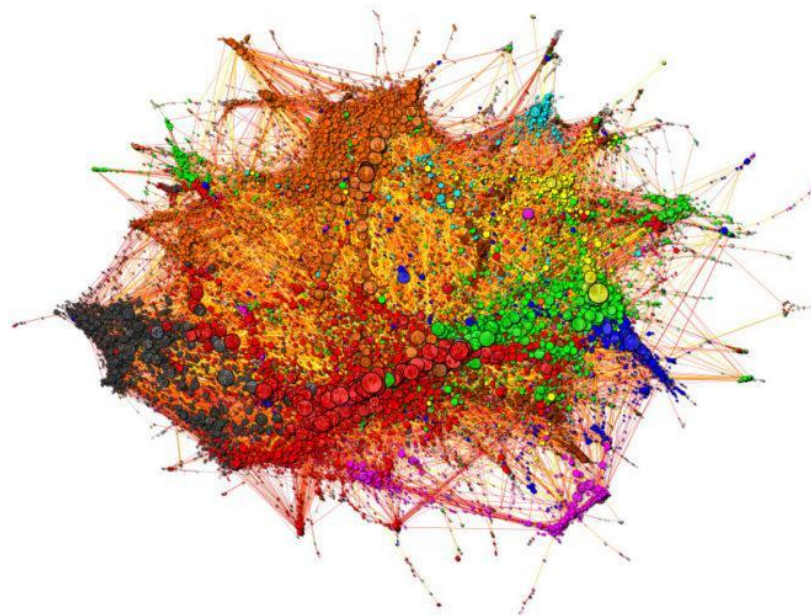
- Random graphs
- Small world model
- Graphs convolutions

# Machine Learning on Graphs



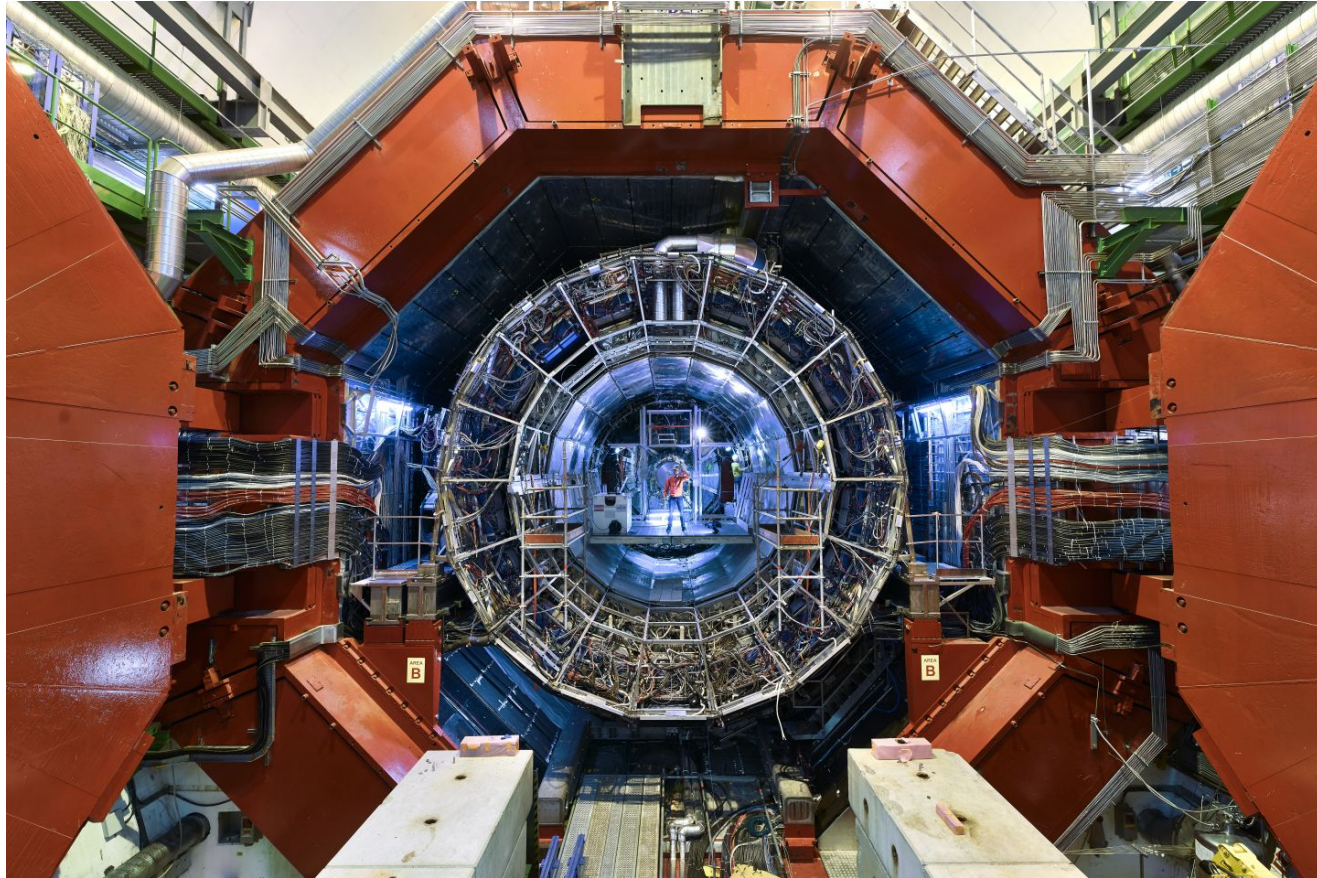
Some achievements:

- Communities detection
- Recommender systems





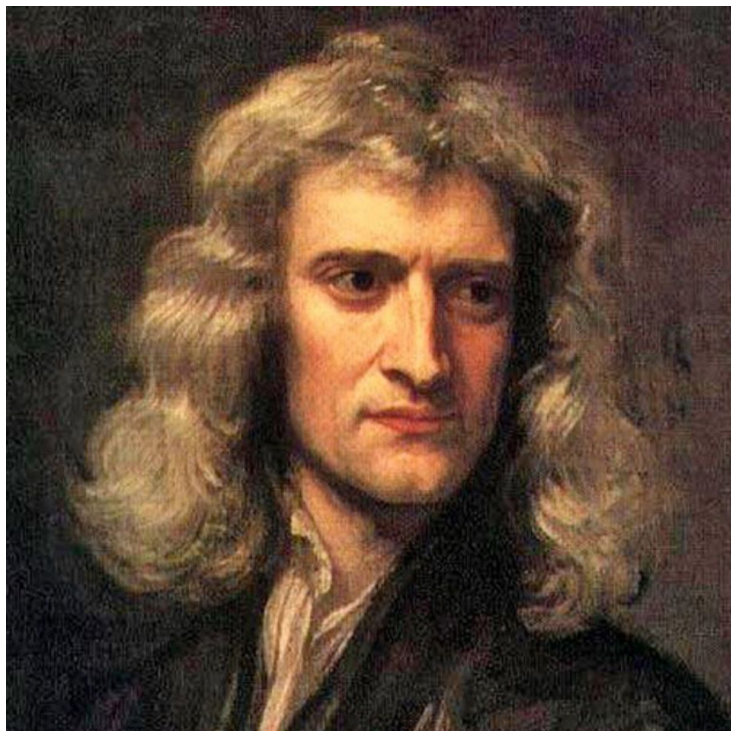
# Machine Learning applications





Data → Knowledge

# Long before the ML

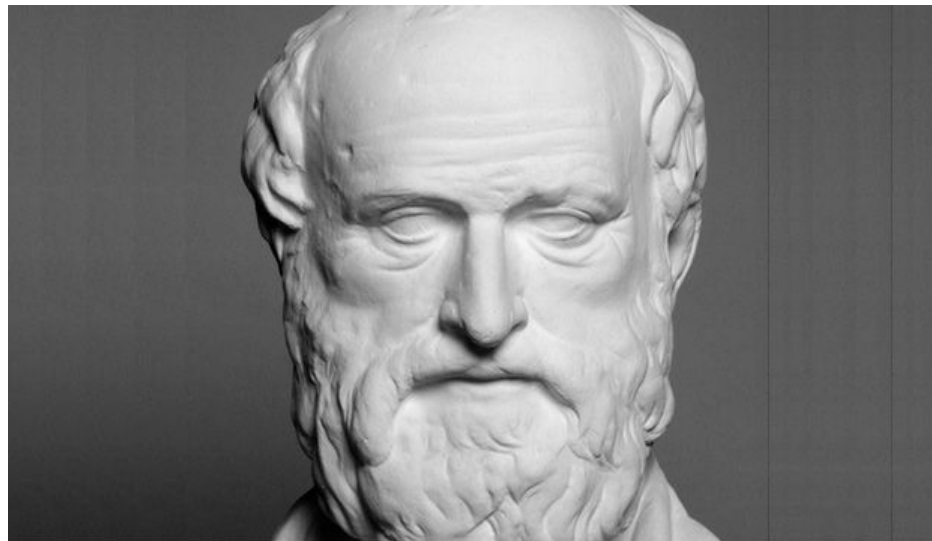


Isaac Newton



Johannes Kepler

# Long before the ML



Eratosthenes

# ML thesaurus

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# ML thesaurus



Denote the **dataset**.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE



# ML thesaurus



**Observation** (or datum, or data point) is one piece of information.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

In many cases the **observations** are supposed to be *i.i.d.*

- ***independent***
- ***identically distributed***

# ML thesaurus



**Feature** (or predictor) represents some special property.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

# ML thesaurus



These all are features

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

# ML thesaurus



These all are features

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

# ML thesaurus



These all are features

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

# ML thesaurus



These all are features

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE



# ML thesaurus



And even the name is a **feature**

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

# ML thesaurus



The ***design matrix or feature matrix*** contains all the observations and their features

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Features can even be multidimensional, we will discuss it later in this course



# Matrix notation: features

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Feature matrix is usually denoted as  $X \in R^{n \times p}$

where  $n$  is number of objects in dataset and  $p$  is number of properties

# ML thesaurus



**Target** represents the information we are interested in.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Target can be either a **number** (real, integer, etc.) – for **regression** problem

# ML thesaurus



**Target** represents the information we are interested in.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Or a **label** – for **classification** problem

# ML thesaurus



**Target** represents the information we are interested in.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

*Mark can be treated as a label too (due to finite number of labels: 1 to 5)*



# ML thesaurus



Further we will work with the numerical target (mark)

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)
John	22	5	4	Brown	English	5
Aahna	17	4	5	Brown	Hindi	4
Emily	25	5	5	Blue	Chinese	5
Michael	27	3	4	Green	French	5
Some student	23	3	3	NA	Esperanto	2

# ML thesaurus



**Target** represents the information we are interested in.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Target (passed)
John	22	5	4	Brown	English	5	TRUE
Aahna	17	4	5	Brown	Hindi	4	TRUE
Emily	25	5	5	Blue	Chinese	5	TRUE
Michael	27	3	4	Green	French	5	TRUE
Some student	23	3	3	NA	Esperanto	2	FALSE

Target can be either a **number** (real, integer, etc.) – for **regression** problem



# Matrix notation: target

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)
John	22	5	4	Brown	English	5
Aahna	17	4	5	Brown	Hindi	4
Emily	25	5	5	Blue	Chinese	5
Michael	27	3	4	Green	French	5
Some student	23	3	3	NA	Esperanto	2

Target matrix is usually denoted as  $Y \in R^n$

where  $n$  is number of objects in dataset

# ML thesaurus



The **prediction** contains values we predicted using some **model**.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	4.5
Aahna	17	4	5	Brown	Hindi	4	4.5
Emily	25	5	5	Blue	Chinese	5	5
Michael	27	3	4	Green	French	5	3.5
Some student	23	3	3	NA	Esperanto	2	3

One could notice that prediction just averages of Statistics and Python marks. So our **model** can be represented as follows:

$$\text{mark}_{ML}^{\hat{}} = \frac{1}{2}\text{mark}_{Statistics} + \frac{1}{2}\text{mark}_{Python}$$

# ML thesaurus



The **prediction** contains values we predicted using some **model**.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	4.5
Aahna	17	4	5	Brown	Hindi	4	4.5
Emily	25	5	5	Blue	Chinese	5	5
Michael	27	3	4	Green	French	5	3.5
Some student	23	3	3	NA	Esperanto	2	3

*Different models can provide different predictions:*

$$\text{mark}_{ML} = \frac{1}{2}\text{mark}_{Statistics} + \frac{1}{2}\text{mark}_{Python}$$

# ML thesaurus



The **prediction** contains values we predicted using some **model**.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	1
Aahna	17	4	5	Brown	Hindi	4	5
Emily	25	5	5	Blue	Chinese	5	2
Michael	27	3	4	Green	French	5	4
Some student	23	3	3	NA	Esperanto	2	3

*Different models can provide different predictions:*

$$\text{mark}_{ML}^{\hat{}} = \text{random}(\text{integer from } [1; 5])$$

# ML thesaurus



The **prediction** contains values we predicted using some **model**.

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	1
Aahna	17	4	5	Brown	Hindi	4	5
Emily	25	5	5	Blue	Chinese	5	2
Michael	27	3	4	Green	French	5	4
Some student	23	3	3	NA	Esperanto	2	3

*Different models can provide different predictions.*

*Usually some **hypothesis** lies beneath the model choice.*

# ML thesaurus



**Loss function** measures the error rate of our model.

Square deviation	Target (mark)	Predicted (mark)
16	5	1
1	4	5
9	5	2
1	5	4
1	2	3

- **Mean Squared Error** (where  $\mathbf{y}$  is vector of targets):

$$MSE(\mathbf{y}, \hat{\mathbf{y}}) = \frac{1}{N} \|\mathbf{y} - \hat{\mathbf{y}}\|_2^2 = \frac{1}{N} \sum_i (y_i - \hat{y}_i)^2$$



# ML thesaurus



**Loss function** measures the error rate of our model.

Absolute deviation	Target (mark)	Predicted (mark)
4	5	1
1	4	5
3	5	2
1	5	4
1	2	3

- **Mean Absolute Error** (where  $\mathbf{y}$  is vector of targets):

$$MAE(\mathbf{y}, \hat{\mathbf{y}}) = \frac{1}{N} \|\mathbf{y} - \hat{\mathbf{y}}\|_1 = \frac{1}{N} \sum_i |y_i - \hat{y}_i|$$

# ML thesaurus



To learn something, our **model** needs some degrees of freedom:

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	4.5
Aahna	17	4	5	Brown	Hindi	4	4.5
Emily	25	5	5	Blue	Chinese	5	5
Michael	27	3	4	Green	French	5	3.5
Some student	23	3	3	NA	Esperanto	2	3

$$\text{mark}_{ML}^{\hat{}} = w_1 \cdot \text{mark}_{Statistics} + w_2 \cdot \text{mark}_{Python}$$

# ML thesaurus



To learn something, our **model** needs some degrees of freedom:

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	4.447
Aahna	17	4	5	Brown	Hindi	4	4.734
Emily	25	5	5	Blue	Chinese	5	5.101
Michael	27	3	4	Green	French	5	3.714
Some student	23	3	3	NA	Esperanto	2	3.060

$$\hat{\text{mark}}_{ML} = w_1 \cdot \text{mark}_{Statistics} + w_2 \cdot \text{mark}_{Python}$$

# ML thesaurus



To learn something, our **model** needs some degrees of freedom:

Name	Age	Statistics (mark)	Python (mark)	Eye color	Native language	Target (mark)	Predicted (mark)
John	22	5	4	Brown	English	5	1
Aahna	17	4	5	Brown	Hindi	4	5
Emily	25	5	5	Blue	Chinese	5	2
Michael	27	3	4	Green	French	5	4
Some student	23	3	3	NA	Esperanto	2	3

$$\text{mark}_{ML}^{\hat{}} = \text{random}(\text{integer from } [1; 5])$$

# ML thesaurus



Last term we should learn for now is ***hyperparameter***.

***Hyperparameter*** should be fixed before our model starts to work with the data.

We will discuss it later with kNN as an example.

# ML thesaurus



Recap:

- Dataset
- Observation (datum)
- Feature
- Design matrix
- Target
- Prediction
- Model
- Loss function
- Parameter
- Hyperparameter

# Maximum Likelihood Estimation

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# Parametric and nonparametric models

Nonparametric statistics is a type of statistical analysis that makes minimal assumptions about the underlying distribution of the data being studied. Often these models are infinite-dimensional, rather than finite dimensional, as is parametric statistics.

Nonparametric statistics can be used for descriptive statistics or statistical inference. Nonparametric tests are often used when the assumptions of parametric tests are evidently violated.

[© Common knowledge site](#)





# Likelihood maximization

Consider the most simple case of discrete features and target.

Denote dataset  $X, Y$  generated by distribution with parameter  $\theta$

Likelihood of a parameter is defined as probability of sampling this particular data in case underlying distribution is defined by this parameter.

Maximization of likelihood means we choose the most probable parameters having this particular dataset

$$L(\theta|X, Y) = P(X, Y|\theta) \rightarrow \max_{\theta}$$

Note that likelihood is not probability function of  $\theta$



## i.i.d. property

We can employ i.i.d property of data samples to split probability of the whole dataset into independent problems

$$P(X, Y|\theta) = \prod_i P(x_i, y_i|\theta)$$

Then we apply logarithm function to both parts of equation above

$$\log P(X, Y|\theta) = \sum_i \log P(x_i, y_i|\theta)$$

The latter expression is easier to operate with:  
later we will predict log-probability of each object directly

# Log-likelihood equivalence



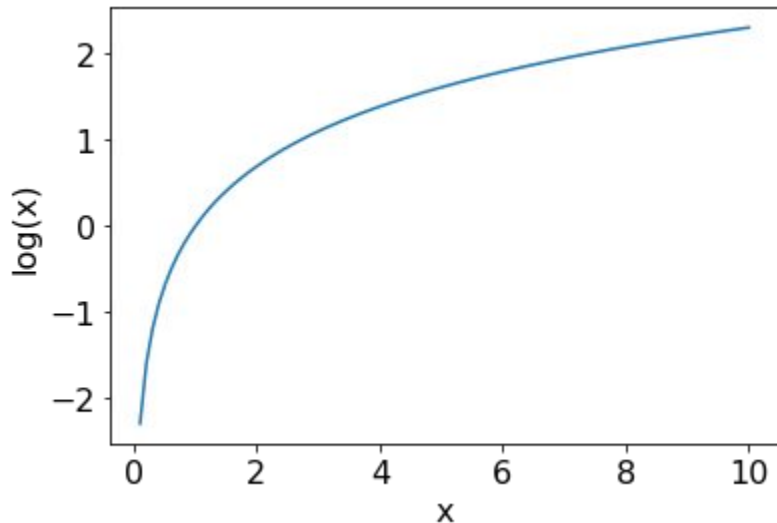
Since logarithm is a convex function on open set, it preserves maximum of expression when applied, so that

$$L(\theta|X, Y) \rightarrow \max_{\theta}$$

and

$$\log L(\theta|X, Y) \rightarrow \max_{\theta}$$

have the same solutions in terms of  $\theta$



# Maximum Likelihood Estimation



$$\hat{\theta} = \arg \max_{\theta} L(\theta|X, Y)$$

is called maximum likelihood estimation of model parameters.

In optimization theory functions are usually minimized, so the same problem could be reformulated using **Negative Log-Likelihood (NLL)** loss

$$\hat{\theta} = \arg \min_{\theta} - \sum_i \log P(x_i, y_i|\theta)$$

# Machine Learning problems overview

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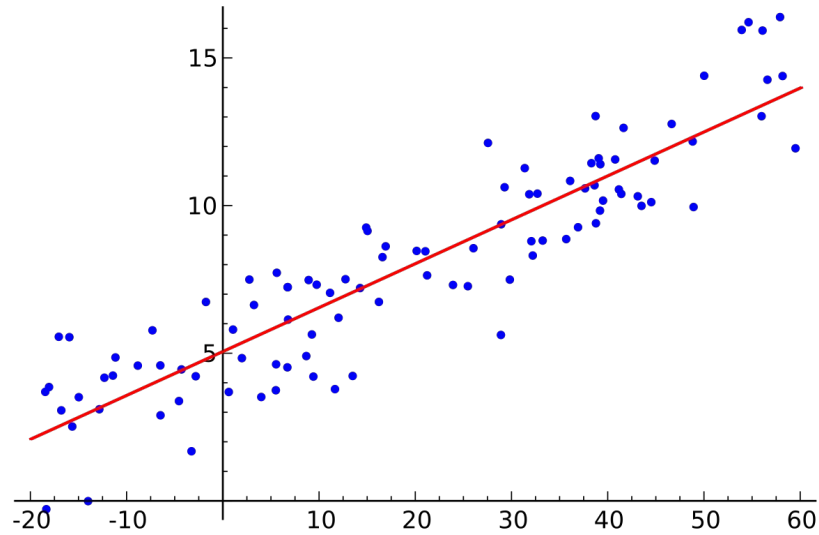
# Supervised learning problem statement

Let's denote:

- Training set  $\mathcal{L} = \{\mathbf{x}_i, y_i\}_{i=1}^n$ , where
  - $(\mathbf{x} \in \mathbb{R}^p, y \in \mathbb{R})$  for regression
  - $\mathbf{x}_i \in \mathbb{R}^p, y_i \in \{+1, -1\}$  for binary classification
- Model  $f(\mathbf{x})$  predicts some value for every object
- Loss function  $Q(\mathbf{x}, y, f)$  that should be minimized



- Regression problem



Estimated  
(or predicted)  
Y value for  
observation  $i$

Estimate of  
the regression  
intercept

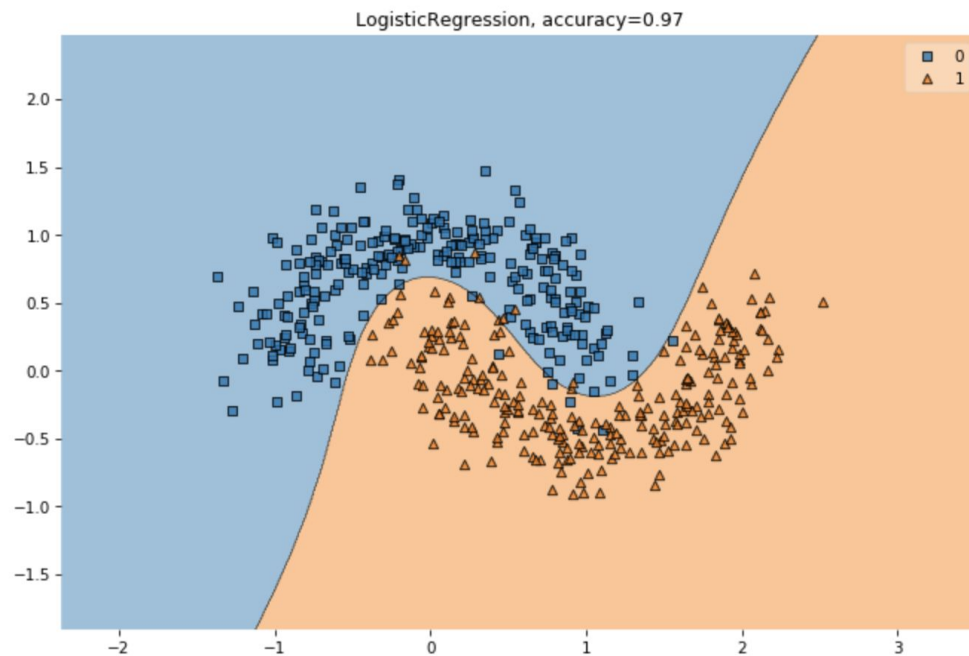
Estimate of the  
regression slope

Value of X for  
observation  $i$

$$\hat{Y}_i = b_0 + b_1 X_i$$



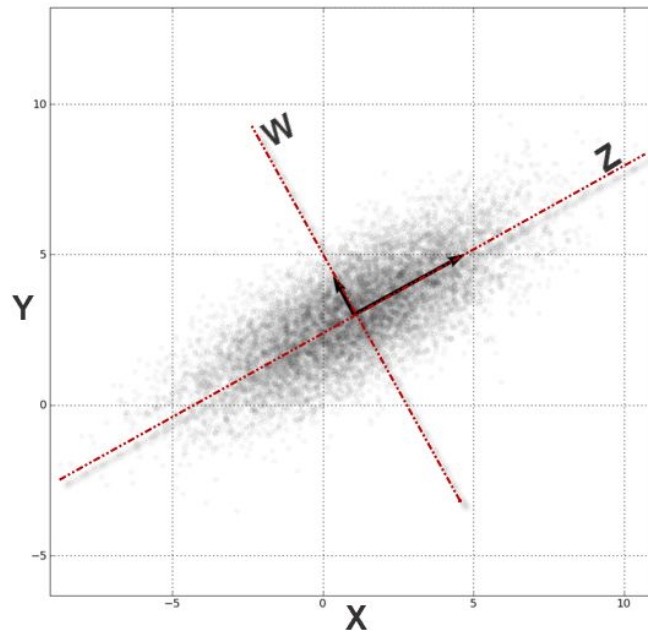
- Regression problem
- Classification problem







- Regression problem
- Classification problem
- Dimensionality reduction



# Naïve Bayes classifier

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# Naïve Bayes classifier

Let's denote:

- Training set  $\mathcal{L} = \{\mathbf{x}_i, y_i\}_{i=1}^n$ , where
  - $\mathbf{x}_i \in \mathbb{R}^p$ ,  $y_i \in \{C_1, \dots, C_k\}$  for k-class classification

# Bayes' theorem



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

or, in our case

$$P(y_i = C_k | \mathbf{x}_i) = \frac{P(\mathbf{x}_i | y_i = C_k)P(y_i = C_k)}{P(\mathbf{x}_i)}$$



# Naïve Bayes classifier

Let's denote:

- Training set  $\mathcal{L} = \{\mathbf{x}_i, y_i\}_{i=1}^n$ , where
  - $\mathbf{x}_i \in \mathbb{R}^p$ ,  $y_i \in \{C_1, \dots, C_K\}$  for K-class classification

$$P(y_i = C_k | \mathbf{x}_i) = \frac{P(\mathbf{x}_i | y_i = C_k) P(y_i = C_k)}{P(\mathbf{x}_i)}$$

Naïve assumption: features are ***independent***

# Naïve Bayes classifier



$$P(y_i = C_k | \mathbf{x}_i) = \frac{P(\mathbf{x}_i | y_i = C_k) P(y_i = C_k)}{P(\mathbf{x}_i)}$$

Naïve assumption: features are ***independent***:

$$P(\mathbf{x}_i | y_i = C_k) = \prod_{l=1}^p P(x_i^l | y_i = C_k)$$



# Naïve Bayes classifier

$$P(y_i = C_k | \mathbf{x}_i) = \frac{P(\mathbf{x}_i | y_i = C_k) P(y_i = C_k)}{\cancel{P(\mathbf{x}_i)}}$$

Optimal class label:

$$C^* = \arg \max_k P(y_i = C_k | \mathbf{x}_i)$$

To find maximum we even do not need the denominator

But we need it to get probabilities

# k Nearest Neighbors

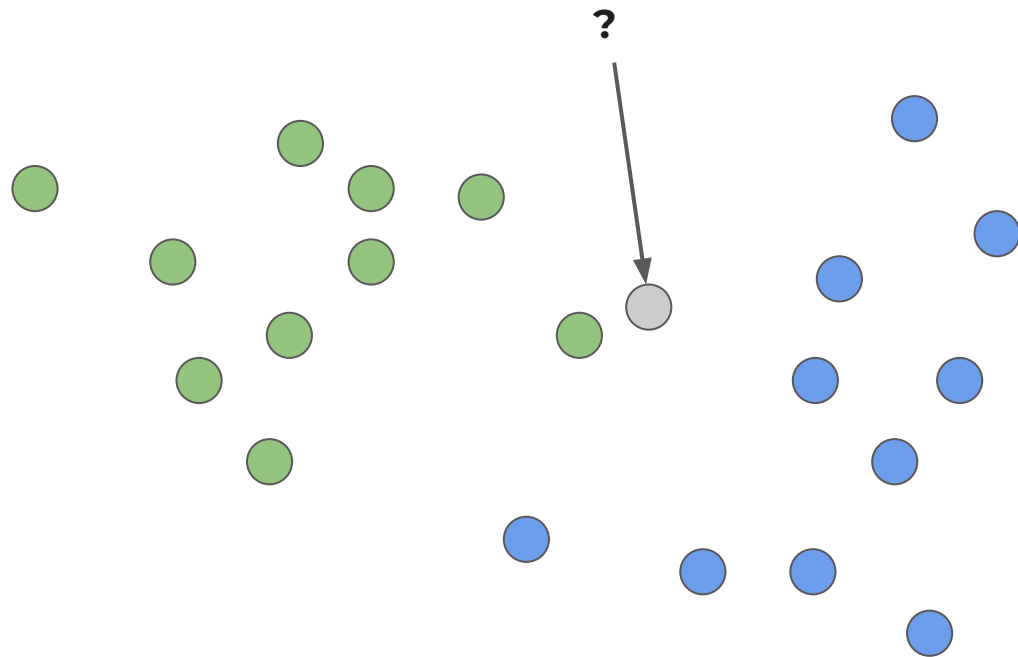
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# Intuition



# kNN model



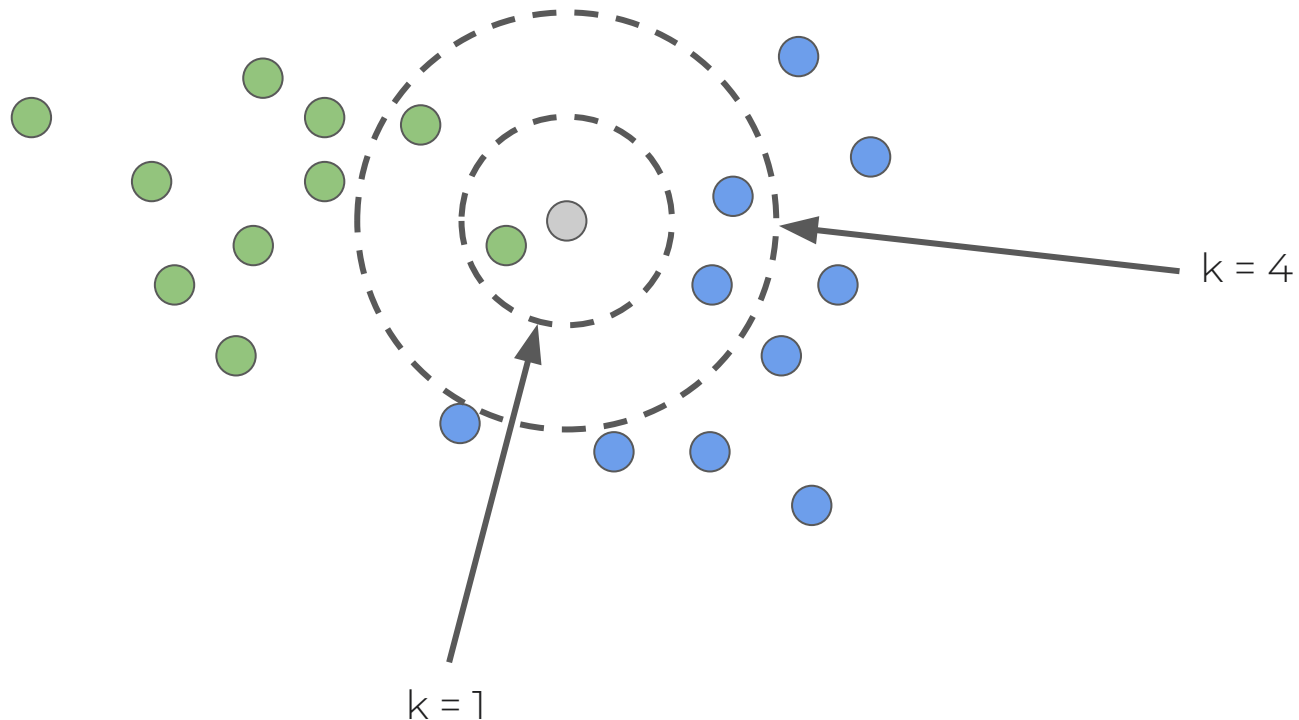
Given a new observation:

1. Calculate the distance to each of the samples in the dataset
2. Select samples from the dataset with the minimal distance to them
3. The label of the new observation will be the most frequent label among those nearest neighbors

# How to make it better?



1. The number of neighbors  $k$





# How to make it better?

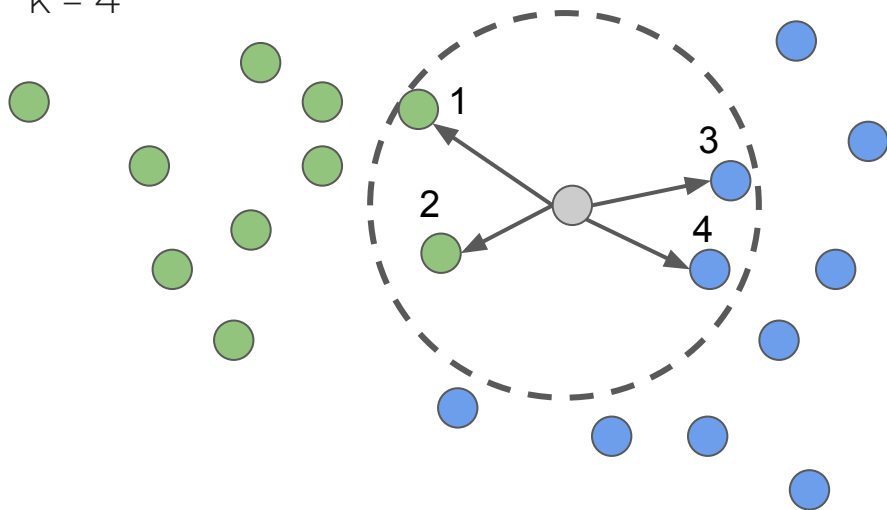
1. The number of neighbors  $k$
2. The distance measure between samples
  - a. Euclidean
  - b. Minkowski distances
  - c. cosine
  - d. Hamming
  - e. etc.
3. Weighted neighbours

They are **hyperparameters** for kNN model.



# Weighted kNN

$k = 4$



- Weights can be adjusted according to the neighbors order

$$w(\mathbf{x}_{(i)}) = w_i$$

- or on the distance itself

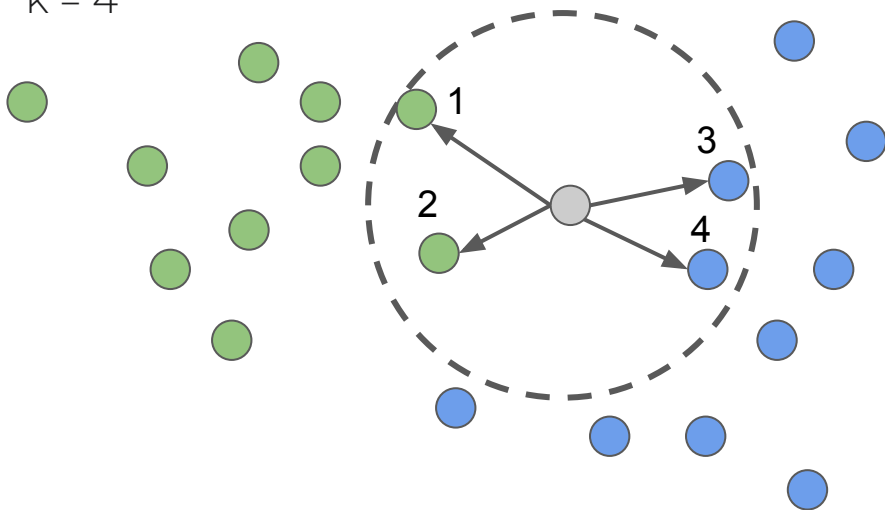
$$w(\mathbf{x}_{(i)}) = w(d(\mathbf{x}, \mathbf{x}_{(i)}))$$

$$p_{\text{green}} = \frac{w(\mathbf{x}_1) + w(\mathbf{x}_2)}{w(\mathbf{x}_1) + w(\mathbf{x}_2) + w(\mathbf{x}_3) + w(\mathbf{x}_4)}$$



# Weighted kNN

$k = 4$



- Weights can be adjusted according to the neighbors order,

$$w(\mathbf{x}_{(i)}) = w_i$$

- or on the distance itself

$$w(\mathbf{x}_{(i)}) = w(d(\mathbf{x}, \mathbf{x}_{(i)}))$$

$$p_{\text{blue}} = \frac{w(\mathbf{x}_3) + w(\mathbf{x}_4)}{w(\mathbf{x}_1) + w(\mathbf{x}_2) + w(\mathbf{x}_3) + w(\mathbf{x}_4)}$$

# Takeouts



- Remember the i.i.d. property
- Usually the first dimension corresponds to the batch size, the second (and so on) to the features/time/...
- Even the naïve assumptions may be suitable in some cases
- Simple models provide great baselines

# Revise

1. ML and AI overview
2. Thesaurus and notation
3. Maximum Likelihood Estimation
4. Some Machine Learning problems
  - a. Classification
  - b. Regression
  - c. Dimensionality reduction
5. Naïve Bayes classifier
6. k Nearest Neighbours (kNN)



# Thanks for attention!

Questions?

