Advanced_Derivatives_Coursework_final

December 23, 2021

[1]:
```
#pip install financepy
```

Advanced Derivatives Final Assignment

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[17]:
```
import pandas as pd
import numpy as np
import scipy.stats as si
import matplotlib.pyplot as plt
import math

from financepy.utils import *
from financepy.products.equity import *
```

```
####################################################################
# FINANCEPY BETA Version 0.204 - This build:  02 Nov 2021 at 10:17 #
#      This software is distributed FREE & WITHOUT ANY WARRANTY    #
#  Report bugs as issues at https://github.com/domokane/FinancePy  #
####################################################################
```

Question 1 - Black-Scholes-Merton Hedging

1a.

We can determine the value of an option by applying the Black Scholes Option Formula:

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-rt}$$

Where:

$$d_1 = \frac{ln\frac{S_t}{K} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Using financepy's function 'EquityVanillaOption_' we can estimate a plain vanilla Option's price and risk measures. Considering that we are pricing a call, we can assume the following inputs for our computation based on a Call option on the VIXY with a strike option of $22.00.

Inputs for Call Price:

1. The current value of the asset is
$$S_0 = \$19.15$$
2. The option tenor is
$$T = 2year$$
3. The risk free rate is
$$r_f = 0.828\%$$
4. The strike price is
$$K = \$22.00$$
5. The stock's volatility is
$$sigma = 132.609\%$$
6. The expected future dividend yield is

$$d = 0.85\%$$

[3]:
```
# We will first establish the expire date as a function of T, which is the time␣
↪to expiry

valuation_date = Date(30, 11, 2021)

expiry_date = valuation_date.add_years(2)

expiry_date
```

[3]: 30-NOV-2023

[4]:
```
# Then, we will assign the value to our inputs required to create our Call␣
↪option.
# The inputs we will be using are the ones corresponding to a Call on the VIXY␣
↪ETF.

stock_price = 19.15
strike_price = 22
volatility = 1.32609
interest_rate = 0.00828
```

```
dividend_yield = 0.00850
```

[5]:
```
# Now, we can creat a Call option object by using EquityVanillaOption function

call_option = EquityVanillaOption(expiry_date, strike_price, OptionTypes.
 ↪EUROPEAN_CALL)
```

[6]:
```
# We need to define the discount curve and the Dividends curve as well

discount_curve = DiscountCurveFlat(valuation_date, interest_rate)
dividend_curve = DiscountCurveFlat(valuation_date, dividend_yield)
```

[7]:
```
# We will use the Black Scholes model as our standard model

model = BlackScholes(volatility)
```

[8]:
```
# Now, we can compute the call value at any chosen date before the time to␣
 ↪expiry

call_value = call_option.value(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)
print(call_value)
```

```
11.80473279145044
```

1b.

[9]:
```
# At the same time, we can compute the corresponding call delta for the created␣
 ↪option

call_delta = call_option.delta(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)
print(call_delta)
```

```
0.79247405614197
```

1c.

Now, we will test the so called "Delta Dynamic Hedging" strategy. To do so, we will create a hedged position by constructing the following portfolio:

$$\pi = \Delta_t S_t - C(S_t, t) + Cash$$

To test the efficiency of this hedge, we will create a function that simulates the path of a stock and that replicates a dynamic hedge strategy. At the end of the strategy, the value of the hedged portfolio should be close to zero. This means that the sum of the cash and the stock position should offset as closely as possible the payoff at expiration. Given that we will be writting the call, the delta of the option will be negative. To offset it we will need to go long the stock, which we will finance by borrowing money at the risk free rate. This can be seen as follow:

$$HedgePosition = \Delta S - Borrowing = CallValue$$

At expiration, the Call Value should be equal to the option Payoff.

[10]:
```
def OptionSim(n_years_, steps_per_year_, stock_price_, strike_price_, mu_,␣
↪volatility_, interest_rate_, dividend_yield_):
    """

    """
    # We wlll define a new dataFrame to store the important features of the␣
↪strategy implementation

    columns=('Period','Spot','CallValue','Delta','Shares_bought','Cost',
             'Interest','Cash','Shares','Shares_$','TotalValue','Error')
    df = pd.DataFrame(columns=columns)

    # Here we define the initial inputs for loop

    periods = n_years_ * steps_per_year_ # Computing total number of porfolio␣
↪rebalances
    valuation_date_f = Date(30, 11, 2021)
    expiry_date_f = valuation_date_f.add_years(n_years_)
    results=[]
    cash = 0
    shares = 0
    stocks_b=0
    stock_pos_m=0
    cost=0
    roll=0
    spot=0
    dt = 1/steps_per_year_

    # Calling on financepy, we create an Call object

    call_option_i = EquityVanillaOption(expiry_date_f, strike_price_,␣
↪OptionTypes.EUROPEAN_CALL)

    # With a Loop we will simulate the evolution of the Stock price and the␣
↪heddging rebalance of the Hedged Portfolio
    for rebal in range(0,periods+1):

        # We modify the valuation date one jump ahead at a time.
        # The number of periods is equal to the number of years times the␣
↪number of rebalances per year
```

```
        new_val_date_f = valuation_date_f.add_years(rebal*dt)
        p = rebal*dt

        # We simulate the evolution of the stock price at each jump

        if p>0:
            spot=spot*exp((mu_ - 0.5*(volatility_**2))*dt +␣
↪volatility_*sqrt(dt)*np.random.randn())
        else:
            spot=stock_price_ # We set the spot equal to S_0 at initiation of␣
↪the count

        # We compute the option value and the option delta at each point in time

        discount_curve_ = DiscountCurveFlat(new_val_date_f, interest_rate_)
        dividend_curve_ = DiscountCurveFlat(new_val_date_f, dividend_yield_)
        model_ = BlackScholes(volatility_)

        call_value_=call_option_i.value(new_val_date_f, spot, discount_curve_,␣
↪dividend_curve_, model_)
        call_delta_=-call_option_i.delta(new_val_date_f, spot, discount_curve_,␣
↪dividend_curve_, model_) # Delta is negative as we are short the call

        # We perform the delta hedge by buying or selling the required number␣
↪of shares to have a
        # position equal to delta*shares

        roll=cash*((exp(interest_rate_*dt))-1) # This is the amount of interest␣
↪generated in per period
        cash=cash*exp(interest_rate_*dt) # Cash is rolled over one period
        if new_val_date_f==valuation_date_f:
            cash=call_value_ # Sell the call at t=0
        shares_b = -(call_delta_ + shares) # Computes number of shares bought,␣
↪positive number means negative cash

        # Updating the variables

        shares= -call_delta_
        cost= -shares_b*spot
        cash-= shares_b*spot
        stock_pos_m=shares*spot
        HedgeValue = cash + shares*spot
        error = HedgeValue - call_value_
```

```
        # Feeding dataFrame
        L = [p,spot, call_value_, call_delta_, shares_b, cost, roll, cash,␣
↪shares,stock_pos_m, HedgeValue, error]

        df.loc[len(df)] = L

    # Preparing tuple with final results
    records = df[['Spot', 'CallValue', 'Shares', 'Cash', 'TotalValue',␣
↪'Error']].to_records(index=False)
    results = records[len(records)-1]

    return results
```

These are the results of our function in which the results of the tuple are: 1. Spot 2. Call Value 3. Shares held in the hedging portfolio 4. Cash held in the hedging portfolio 5. Total value of Cash and Shares held in the portfolio 6. Total value against option payoff

[11]:
```
results = OptionSim(1, 52, 19.15, 22, 0.10, 1.32609, 0.00828, 0.00850)
results
```

[11]:
```
(3.67677014, 0., 0., 0.34623453, 0.34623453, 0.34623453)
```

1d.

Now, we will create a function to simulate 1,000 different paths for a stock with the following characteristics:

1. The current value of the asset is
$$S_0 = \$100.00 = Strike$$
2. The option tenor is
$$T = 1 year$$
3. The risk free rate and stock's expected return are
$$r_f = \mu = 5.00\%$$
4. The strike price is
$$K = \$100.00$$
5. The stock's volatility is
$$sigma = 20.00\%$$
6. The expected future dividend yield is
$$d = 0.00\%$$

[12]:
```
def IterateSim(n_scenarios, n_years, steps_per_year, stock_price, strike_price,␣
↪mu, volatility, interest_rate, dividend_yield):
    """
```

```
    """
    errors = []
    for x in range (0, n_scenarios):
        results = OptionSim(n_years_=n_years, steps_per_year_=steps_per_year,␣
↪stock_price_=stock_price, strike_price_=strike_price, mu_=mu,␣
↪volatility_=volatility, interest_rate_=interest_rate,␣
↪dividend_yield_=dividend_yield)
        errors.append(results[-1]) # Here we create a vector with the error at␣
↪the final date of each path
    mean_error = np.average(errors)
    print ("N Scenarios: ", n_scenarios)
    print ("Mean Error", mean_error)
```

After creating the function, we will look for the mean error of the dynamic hedging strategy

[13]:
```
IterateSim(1000, 1, 12, 100, 100, 0.05, 0.20, 0.05, 0)
```

```
N Scenarios:  1000
Mean Error 0.08357313603422138
```

As we can see, on average the error is fairly close to zero, which means that the hedging strategy does pretty well on average. Next we will review the impact of doing rebalances more often during the same time frame.

1e.

We will modify slightly the previous fucntion in order to get as an output a dataframe containing the final prices, the corresponding error, the total value of the hedging portfolio and the option payoff for simulations in which the rebalacing occurs 12, 52 and 252 times per year.

[14]:
```
# Creating a new function that outputs prices and errors as a dataFrame

def IterateSim_2(n_scenarios_i, n_years_i, steps_per_year_i, stock_price_i,␣
↪strike_price_i, mu_i, volatility_i, interest_rate_i, dividend_yield_i):
    """


    """
    prices = [] # We create an empty vector to store the final price of the␣
↪stock
    errors = [] # We create an empty vector to store the error at the final␣
↪stage of each path
    payoff = [] # We create an empty vector to store the final payoff of the␣
↪option
    TotalV = [] # We create an empty vector to store the final total value of␣
↪the cash held and the stock position
    for x in range (0, n_scenarios_i):
```

```
        results = OptionSim(n_years_=n_years_i,␣
↪steps_per_year_=steps_per_year_i, stock_price_=stock_price_i,␣
↪strike_price_=strike_price_i, mu_=mu_i, volatility_=volatility_i,␣
↪interest_rate_=interest_rate_i, dividend_yield_=dividend_yield_i)
        prices.append(results[0])
        errors.append(results[-1])
        payoff.append(results[1])
        TotalV.append(results[-2])
    df = pd.DataFrame(
    {'Prices': prices,
     'Errors': errors,
     'Payoff': payoff,
     'TotalV': TotalV
    })
    return df
```

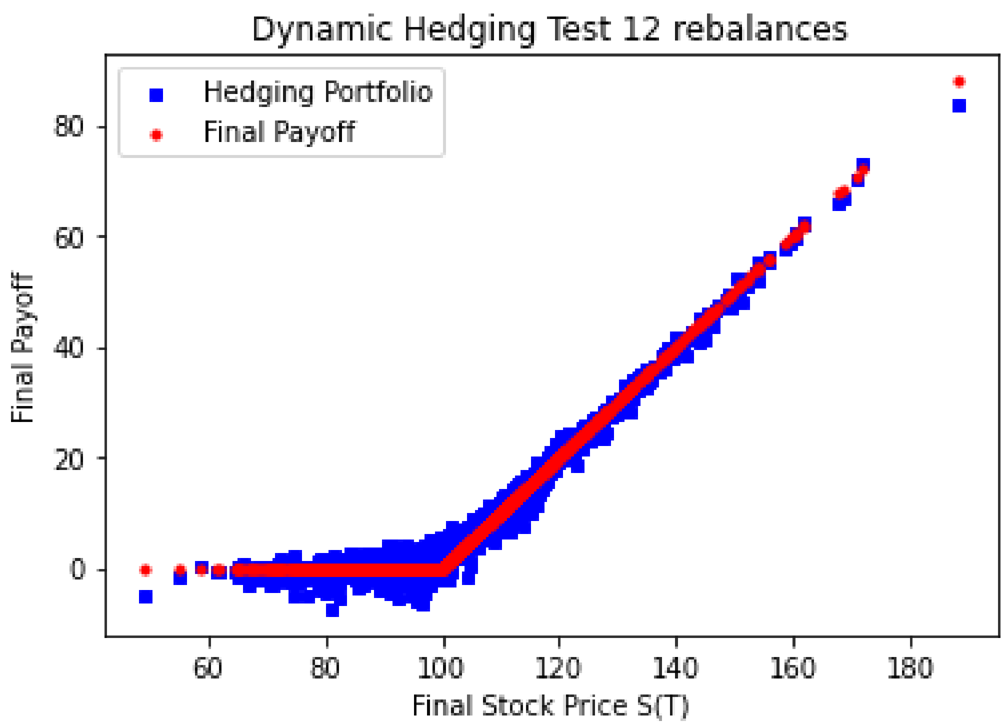
We will now perform the corresponding simulations

[15]:
```
# 1000 simulations for monthly rebalances

df_12 = IterateSim_2(1000, 1, 12, 100, 100, 0.05, 0.20, 0.05, 0)
```

[16]:
```
# Mean error for the hedging stragey with monthly rebalances

np.average(df_12['Errors'])
```

[16]: -0.001347441443013409

[17]:
```
import matplotlib.pyplot as plt

df_12['Payoffs_e_12']=df_12['Errors']+df_12['Payoff']
fig = plt.figure()
ax1 = fig.add_subplot(111)

ax1.scatter(df_12['Prices'], df_12['Payoffs_e_12'], s=10, c='b', marker="s",␣
↪label='Hedging Portfolio')
ax1.scatter(df_12['Prices'],df_12['Payoff'], s=10, c='r', marker="o",␣
↪label='Final Payoff')
plt.legend(loc='upper left');
plt.title('Dynamic Hedging Test 12 rebalances')
plt.xlabel('Final Stock Price S(T)')
plt.ylabel('Final Payoff')
plt.show()
```

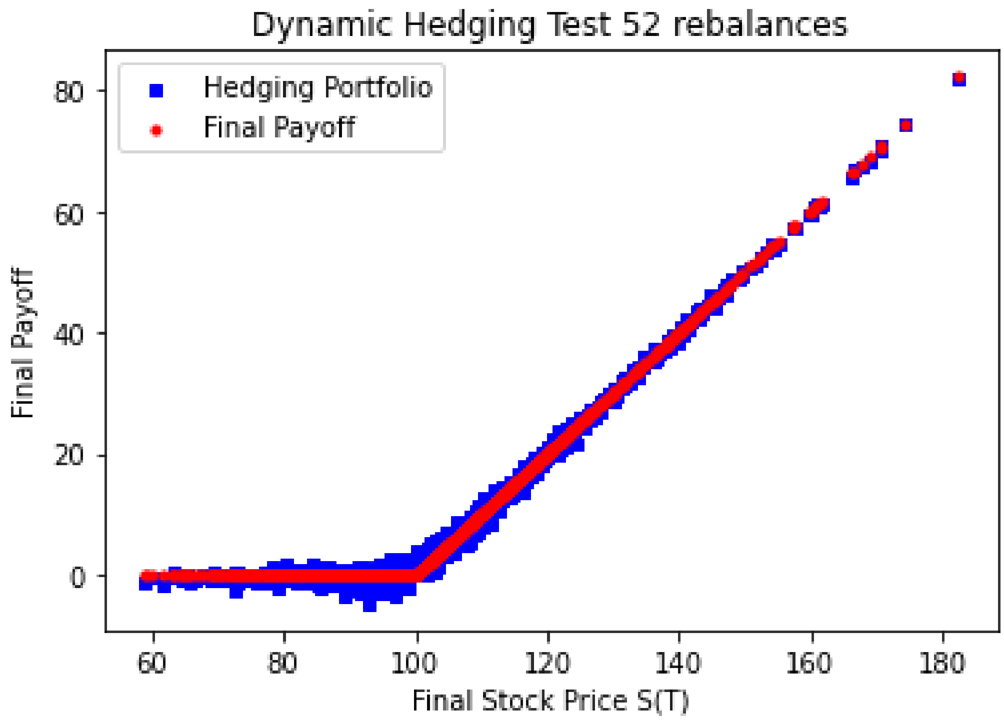


[18]:
```
# 1000 simulations for weekly realances

df_52 = IterateSim_2(1000, 1, 52, 100, 100, 0.05, 0.2, 0.05, 0)
```

[19]:
```
# Mean error for the hedging stragey with weekly rebalances

np.average(df_52['Errors'])
```

[19]: 0.002005111562484025

[20]:
```
df_52['Payoffs_e_52']=df_52['Errors']+df_52['Payoff']
fig = plt.figure()
ax1 = fig.add_subplot(111)

ax1.scatter(df_52['Prices'], df_52['Payoffs_e_52'], s=10, c='b', marker="s",␣
 ↪label='Hedging Portfolio')
ax1.scatter(df_52['Prices'],df_52['Payoff'], s=10, c='r', marker="o",␣
 ↪label='Final Payoff')
plt.legend(loc='upper left');
plt.title('Dynamic Hedging Test 52 rebalances')
plt.xlabel('Final Stock Price S(T)')
plt.ylabel('Final Payoff')
plt.show()
```

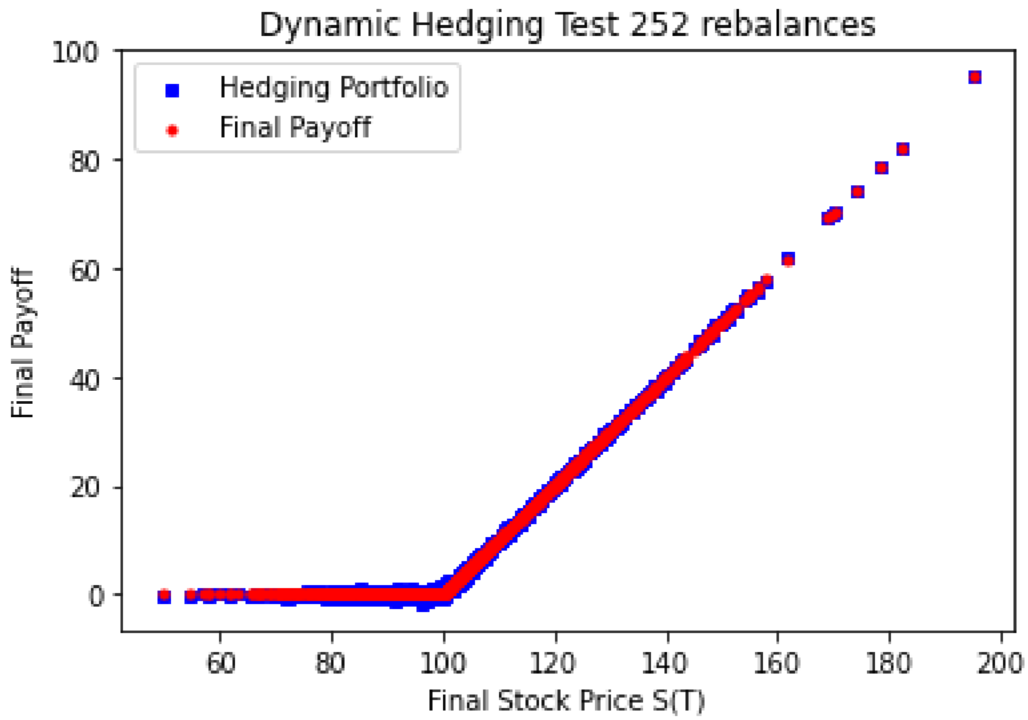


[21]:
```
# 1000 simulations for daily rebalances

df_252 = IterateSim_2(1000, 1, 252, 100, 100, 0.05, 0.2, 0.05, 0)
```

[22]:
```
# Mean error for the hedging stragey with daily rebalances

np.average(df_252['Errors'])
```

[22]: 0.0004178050380292961

[23]:
```
df_252['Payoffs_e_252']=df_252['Errors']+df_252['Payoff']
fig = plt.figure()
ax1 = fig.add_subplot(111)

ax1.scatter(df_252['Prices'], df_252['Payoffs_e_252'], s=10, c='b', marker="s",␣
 ↪label='Hedging Portfolio')
ax1.scatter(df_252['Prices'],df_252['Payoff'], s=10, c='r', marker="o",␣
 ↪label='Final Payoff')
plt.legend(loc='upper left');
plt.title('Dynamic Hedging Test 252 rebalances')
plt.xlabel('Final Stock Price S(T)')
plt.ylabel('Final Payoff')
plt.show()
```



As we can see in the graphs above, the Dynamic Hedging strategy works pretty well and its' accuracy increases as the number of rebalances goes up. This makes sense as the delta measure reflects the change in value of the option when the stock changes by small amounts. If the rebalances take long, the changes in the stock prices can be large enough to impact negatively in the hedging strategy perfrormance.

1f.

To take the analysis one step further, we will compare the variance of the error in addition to the mean error as the number of rebalances increase. We can already expect the variance to diminish given the graph observed above.

[24]:
```
results_df = pd.DataFrame(columns = ['Number of rebalances', 'Mean Error',␣
 ↪'Error Variance'])

L12 = [12, np.average(df_12['Errors']), np.var(df_12['Errors'])]
L52 = [52, np.average(df_52['Errors']), np.var(df_52['Errors'])]
L252 = [252, np.average(df_252['Errors']), np.var(df_252['Errors'])]

results_df.loc[len(results_df)] = L12
results_df.loc[len(results_df)] = L52
results_df.loc[len(results_df)] = L252

results_df
```

[24]:

	Number of rebalances	Mean Error	Error Variance
0	12.0	-0.001347	3.475879
1	52.0	0.002005	1.004891
2	252.0	0.000418	0.176771

As we expected, the variance of the errors decreases, similar to the mean error, when the number of rebalances goes up.

1g.

Finally, we will see what happens to the error and the variance of the error when we fix the number of rebalances per year and change the drift of the stock. Let us remember that when we started the simulations we assumed that the risk free rate was equal to the drift of the option, which is not usually the case in practice.

[25]:
```
mu_25 = IterateSim_2(1000, 1, 52, 100, 100, 0.025, 0.2, 0.05, 0)
```

[26]:
```
mu_50 = IterateSim_2(1000, 1, 52, 100, 100, 0.05, 0.2, 0.05, 0)
```

[27]:
```
mu_75 = IterateSim_2(1000, 1, 52, 100, 100, 0.075, 0.2, 0.05, 0)
```

[28]:
```
mu_100 = IterateSim_2(1000, 1, 52, 100, 100, 0.10, 0.2, 0.05, 0)
```

[29]:
```
results_df = pd.DataFrame(columns = ['Mu', 'Mean Error', 'Error Variance'])

L25 = [2.5, np.average(mu_25['Errors']), np.var(mu_25['Errors'])]
L50 = [5.0, np.average(mu_50['Errors']), np.var(mu_50['Errors'])]
L75 = [7.5, np.average(mu_75['Errors']), np.var(mu_75['Errors'])]
L100 = [10, np.average(mu_100['Errors']), np.var(mu_100['Errors'])]


results_df.loc[len(results_df)] = L25
results_df.loc[len(results_df)] = L50
results_df.loc[len(results_df)] = L75
results_df.loc[len(results_df)] = L100

results_df
```

[29]:

	Mu	Mean Error	Error Variance
0	2.5	0.016494	0.911191
1	5.0	0.045134	0.838204
2	7.5	0.029610	0.852560
3	10.0	0.006429	0.856287

It is interesting to see how the change in the value of mu doesn't seem to impact much on the absolute value of the mean error nor of the the error variance. This is somehow expected given that the Black and Scholes model doesn't depend on the real drift of the stock but allow us to price the option based on a risk-neutral model. This result shows that this characteristic of the model actually holds and, therefore, the price of our option will not depend upon the stock's drift or the real probabilites embedded in the evolution of the stock price

Question 2 - Transaction Costs

2a.

We will now consider the impact of transaction cost in our dynamic hedging strategy. We will set a bid and ask price based on the observed spot price as follows:

$$Ask = Spot * (1 + \frac{t}{2})$$

And

$$Bid = Spot * (1 - \frac{t}{2})$$

Where t% represents our transaction cost for doing a round trip. This means that you pay t/2 when you buy and you pay t/2 when you sell.

[30]:
```
def OptionSim_transactioncost(n_years_, steps_per_year_, stock_price_,␣
↪strike_price_, mu_, volatility_, interest_rate_, dividend_yield_, t):
    """

    """
    # We wlll define a new dataFrame to store the important features of the␣
↪strategy implementation

    columns=('Period','Spot','Bid','Ask','CallValue','Delta','Shares_bought',
             'Transac_Cost','Cost','Interest','Cash','Shares','Shares_$',
             'TotalValue','Error')
    df = pd.DataFrame(columns=columns)

    # Here we define the initial inputs for loop

    periods = n_years_ * steps_per_year_ # Computing total number of porfolio␣
↪rebalances
    valuation_date_f = Date(30, 11, 2021)
    expiry_date_f = valuation_date_f.add_years(n_years_)
    results=[]
    cash = 0
    shares = 0
    stocks_b=0
    stock_pos_m=0
    cost=0
    roll=0
    spot=0
```

```
    ask=0
    bid=0
    transaction_cost=0
    dt = 1/steps_per_year_

    # Calling on financepy, we create a Call object

    call_option_i = EquityVanillaOption(expiry_date_f, strike_price_,␣
↪OptionTypes.EUROPEAN_CALL)

    # With a Loop we will simulate the evolution of the Stock price and the␣
↪heddging rebalance of the Hedged Portfolio
    for rebal in range(0,periods+1):

        # We modify the valuation date one jump ahead at a time.
        # The number of periods is equal to the number of years times the␣
↪number of rebalances per year

        new_val_date_f = valuation_date_f.add_years(rebal*dt)
        p = rebal*dt

        # We simulate the evolution of the stock price at each jump

        if p>0:
            spot=spot*exp((mu_ - 0.5*(volatility_**2))*dt +␣
↪volatility_*sqrt(dt)*np.random.randn())
        else:
            spot=stock_price_ # We set the spot equal to S_0 at initiation of␣
↪the country

        # Based on the observed spot price we establish a bid and ask price

        bid=spot*(1-0.5*t)
        ask=spot*(1+0.5*t)

        # We compute the option value and the option delta at each point in time

        discount_curve_ = DiscountCurveFlat(new_val_date_f, interest_rate_)
        dividend_curve_ = DiscountCurveFlat(new_val_date_f, dividend_yield_)
        model_ = BlackScholes(volatility_)

        call_value_=call_option_i.value(new_val_date_f, spot, discount_curve_,␣
↪dividend_curve_, model_)
        call_delta_=-call_option_i.delta(new_val_date_f, spot, discount_curve_,␣
↪dividend_curve_, model_) # Delta is negative as we are short the call
```

```
        # We perform the delta hedge by buying or selling the required number␣
↪of shares to have a
        # position equal to delta*shares

        roll=cash*((exp(interest_rate_*dt))-1) # This is the amount of interest␣
↪generated per period
        cash=cash*exp(interest_rate_*dt) # Cash is rolled over one period
        if new_val_date_f==valuation_date_f:
            cash=call_value_ # Sell the call at t=0
        shares_b = -(call_delta_ + shares) # Computes number of shares bought,␣
↪positive number means negative cash

        # We will store the cummulative transaction costs

        transaction_cost = transaction_cost +  abs(shares_b*t/2)

        # Updating the variables

        shares= -call_delta_

        # If we buy shares we pay the ask, if we sell we receive the bid

        if shares_b > 0:
            cost= -shares_b*ask
            cash-= shares_b*ask
        else:
            cost= -shares_b*bid
            cash-= shares_b*bid

        # Regardless of the price paid or received, the stock position is␣
↪valued at spot

        stock_pos_m=shares*spot
        HedgeValue = cash + shares*spot
        error = HedgeValue - call_value_


        # Feeding dataFrame
        L = [p,spot, bid, ask, call_value_, call_delta_, shares_b,␣
↪transaction_cost, cost, roll, cash, shares,stock_pos_m, HedgeValue, error]

        df.loc[len(df)] = L

    # Preparing tuple with final results
    records = df[['Spot', 'CallValue', 'Shares', 'Cash', 'TotalValue',␣
↪'Error']].to_records(index=False)
```

```
    results = records[len(records)-1]

    return results
```

[31]:
```
results = OptionSim_transactioncost(1, 52, 19.15, 22, 0.10, 1.32609, 0.00828, 0.
 ↪00850, 0.005)
results
```

[31]:
```
(20.42024365, 0., 0., -2.82300866, -2.82300866, -2.82300866)
```

In this case, the results of our function are represented as a tuple too containing the following data: 1. Spot 2. Call Value 3. Shares held in the hedging portfolio 4. Cash held in the hedging portfolio 5. Total value of Cash and Shares held in the portfolio 6. Total value against option payoff

Until this point, it would seem that transaction costs don't add much to the amount of the error. To test this idea we will now run different simulations by changing the value of t.

2b.

We will now create a similar function to the one created in the previous question that will allow us to run a simulation for 1,000 paths. This time we will change the transaction costs in addition to the number of rebalances per year.

[32]:
```
def IterateSim_transactioncost(n_scenarios, n_years, steps_per_year,␣
 ↪stock_price, strike_price, mu, volatility, interest_rate, dividend_yield, t):
    """

    """
    prices = [] # We create an empty vector to store the final price of the␣
 ↪stock
    errors = [] # We create an empty vector to store the error at the final␣
 ↪stage of each path
    payoff = [] # We create an empty vector to store the final payoff of the␣
 ↪option
    TotalV = [] # We create an empty vector to store the final total value of␣
 ↪the cash held and the stock position
    for x in range (0, n_scenarios):
        results = OptionSim_transactioncost(n_years_=n_years,␣
 ↪steps_per_year_=steps_per_year, stock_price_=stock_price,␣
 ↪strike_price_=strike_price, mu_=mu, volatility_=volatility,␣
 ↪interest_rate_=interest_rate, dividend_yield_=dividend_yield, t=t)
        prices.append(results[0])
        errors.append(results[-1])
        payoff.append(results[1])
        TotalV.append(results[-2])
    df = pd.DataFrame(
    {'Prices': prices,
     'Errors': errors,
```

```
        'Payoff': payoff,
        'TotalV': TotalV
    })
    return df
```

[33]:
```
# Here, we will run an exercise of 1000 simulations for 12, 52 and 252␣
↪rebalances per year and with t = 0.5%, 1.0% and 2.0%

# Simulations for t=0.5%
df_12_0005 = IterateSim_transactioncost(1000, 1, 12, 100, 100, 0.05, 0.2, 0.05,␣
↪0, 0.005)
df_52_0005 = IterateSim_transactioncost(1000, 1, 52, 100, 100, 0.05, 0.2, 0.05,␣
↪0, 0.005)
df_252_0005 = IterateSim_transactioncost(1000, 1, 252, 100, 100, 0.05, 0.2, 0.
↪05, 0, 0.005)

# Simulations for t=1.0%
df_12_001 = IterateSim_transactioncost(1000, 1, 12, 100, 100, 0.05, 0.2, 0.05,␣
↪0, 0.01)
df_52_001 = IterateSim_transactioncost(1000, 1, 52, 100, 100, 0.05, 0.2, 0.05,␣
↪0, 0.01)
df_252_001 = IterateSim_transactioncost(1000, 1, 252, 100, 100, 0.05, 0.2, 0.
↪05, 0, 0.01)

# Simulations for t=2.0%
df_12_002 = IterateSim_transactioncost(1000, 1, 12, 100, 100, 0.05, 0.2, 0.05,␣
↪0, 0.02)
df_52_002 = IterateSim_transactioncost(1000, 1, 52, 100, 100, 0.05, 0.2, 0.05,␣
↪0, 0.02)
df_252_002 = IterateSim_transactioncost(1000, 1, 252, 100, 100, 0.05, 0.2, 0.
↪05, 0, 0.02)
```

[34]:
```
results_df = pd.DataFrame(columns = ['N', 'Mean Error - t = 0.5%', 'Mean Error␣
↪- t = 1%', 'Mean Error - t = 2%'])

L12 = [12, np.average(df_12_0005['Errors']), np.average(df_12_001['Errors']),␣
↪np.average(df_12_002['Errors'])]
L52 = [52, np.average(df_52_0005['Errors']), np.average(df_52_001['Errors']),␣
↪np.average(df_52_002['Errors'])]
L252 = [252, np.average(df_252_0005['Errors']), np.
↪average(df_252_001['Errors']), np.average(df_252_002['Errors'])]

results_df.loc[len(results_df)] = L12
results_df.loc[len(results_df)] = L52
results_df.loc[len(results_df)] = L252
```

```
results_df
```

[34]:

	N	Mean Error - t = 0.5%	Mean Error - t = 1%	Mean Error - t = 2%
0	12.0	-0.445641	-0.936813	-1.820501
1	52.0	-0.712696	-1.473253	-3.016700
2	252.0	-1.421441	-2.860512	-5.640406

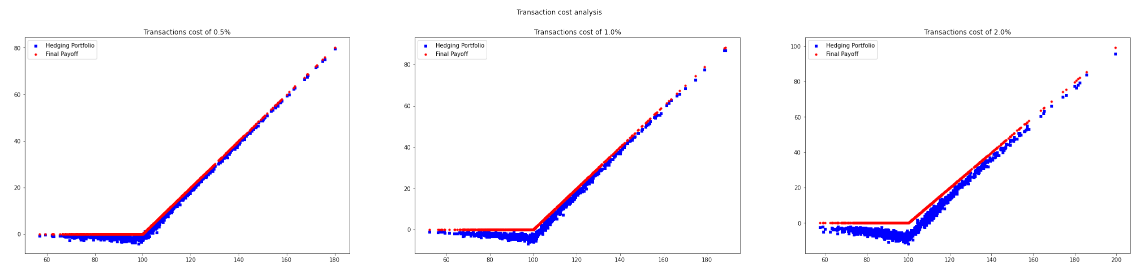
From the table above, we can see that the mean error values increase when transaction costs increase. It makes sense since the difference of the terminal portfolio value and the payoff will be greater because the cash account is affected by those costs.

Another important result is that hedging errors now go up as the number of rebalances increase even in the presence of transaction costs. This is also somehow expected as more transaction costs paid given that there are more transactions, which more than offset the benefit shown in the previous question from more frequent rebalncing dates.

[35]:

```
df_252_0005['Payoffs_e_252_0.5']=df_252_0005['Errors']+df_252_0005['Payoff']
df_252_001['Payoffs_e_252_0.1']=df_252_001['Errors']+df_252_001['Payoff']
df_252_002['Payoffs_e_252_0.2']=df_252_002['Errors']+df_252_002['Payoff']



fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(35,7))
fig.suptitle('Transaction cost analysis')
ax1.scatter(df_252_0005['Prices'], df_252_0005['Payoffs_e_252_0.5'], s=10,␣
 ↪c='b', marker="s", label='Hedging Portfolio')
ax1.scatter(df_252_0005['Prices'],df_252_0005['Payoff'], s=10, c='r',␣
 ↪marker="o", label='Final Payoff')
ax1.legend(loc='upper left');
ax1.set_title('Transactions cost of 0.5%')
ax2.scatter(df_252_001['Prices'], df_252_001['Payoffs_e_252_0.1'], s=10, c='b',␣
 ↪marker="s", label='Hedging Portfolio')
ax2.scatter(df_252_001['Prices'],df_252_001['Payoff'], s=10, c='r', marker="o",␣
 ↪label='Final Payoff')
ax2.legend(loc='upper left');
ax2.set_title('Transactions cost of 1.0%')
ax3.scatter(df_252_002['Prices'], df_252_002['Payoffs_e_252_0.2'], s=10, c='b',␣
 ↪marker="s", label='Hedging Portfolio')
ax3.scatter(df_252_002['Prices'],df_252_002['Payoff'], s=10, c='r', marker="o",␣
 ↪label='Final Payoff')
ax3.legend(loc='upper left');
ax3.set_title('Transactions cost of 2.0%')
```

[35]: Text(0.5, 1.0, 'Transactions cost of 2.0%')



Finally, the transaction cost impact always pushes the error into the negative side of the pay off graph which suggests that the trader will always suffer a negative impact from transaction cost.

2c.

Now, there is a model suggested by Leland (1985), which suggest that the impact of transaction costs can be incorporated into the option price by changing the volatility used to compute its price. The suggested change is the following:

$$\sigma^{Adjusted} = \sigma\left(1 + \sqrt{\frac{2}{\pi}\frac{t}{\sigma}}\sqrt{N}\right)^{\frac{1}{2}}$$

This augmented volatility should derive into a higher option price. This additional premium should offset the transanction cost impact. In this part, we will compare the call values using both the original and the adjusted volatilities for the call option used above for the simulations with a set of transaction costs of 0.5%, 1% and 2%.

[36]:
```python
# We define the common variables that we will be using to create the different options

valuation_date = Date(30, 11, 2021)
expiry_date = valuation_date.add_years(1)

stock_price = 100
strike_price = 100
volatility = 0.20
interest_rate = 0.05
dividend_yield = 0.0

steps_per_year = 52
volatility = 0.20

# We create the Call object and the Discount curves to value the Call option

call_option = EquityVanillaOption(expiry_date, strike_price, OptionTypes.EUROPEAN_CALL)
discount_curve = DiscountCurveFlat(valuation_date, interest_rate)
```

```
dividend_curve = DiscountCurveFlat(valuation_date, dividend_yield)
```

[37]:
```
# First, we will compute the adjusted Volatility based on the original 20.0%
↪volatiltiy
# 0.5% of transaction costs and assuming 52 rebalances per year.

t=0.005
adjusted_volatility = volatility*(1+((2/np.pi)**0.5)*(t/
↪volatility)*(steps_per_year**0.5))**0.5
print ("Adjusted Volatility:", adjusted_volatility)
```

```
Adjusted Volatility: 0.21390097566806843
```

[38]:
```
# Comparison BS and BS with adjusted volatility for transaction costs equal to
↪0.5%

# BS model with original volatility
model = BlackScholes(volatility)
call_value = call_option.value(valuation_date, stock_price, discount_curve,
↪dividend_curve, model)

# BS model with adjusted volatility
model_adjusted = BlackScholes(adjusted_volatility)
call_value_adjusted = call_option.value(valuation_date, stock_price,
↪discount_curve, dividend_curve, model_adjusted)

print ("Call value: ", call_value)
print ("Call value using Adjusted Volatility:", call_value_adjusted)
print ("Price increase:", (call_value_adjusted - call_value))
print ("Price increase as %:", ((call_value_adjusted - call_value) /
↪call_value)*100)
```

```
Call value:  10.450575619322274
Call value using Adjusted Volatility: 10.973069085705152
Price increase: 0.522493466382878
Price increase as %: 4.999662080018153
```

[39]:
```
# Now we compute the adjusted Volatility for transaction costs of 1.0%

t=0.01
adjusted_volatility = volatility*(1+((2/np.pi)**0.5)*(t/
↪volatility)*(steps_per_year**0.5))**0.5
print ("Adjusted Volatility:", adjusted_volatility)
```

```
Adjusted Volatility: 0.22695209799317384
```

[40]:
```
# Comparison BS and BS with adjusted volatility for transaction costs equal to
↪1.0%
```

```
# BS model with original volatility
model = BlackScholes(volatility)
call_value = call_option.value(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)

# BS model with adjusted volatility
model_adjusted = BlackScholes(adjusted_volatility)
call_value_adjusted = call_option.value(valuation_date, stock_price,␣
 ↪discount_curve, dividend_curve, model_adjusted)

print ("Call value: ", call_value)
print ("Call value using Adjusted Volatility:", call_value_adjusted)
print ("Price increase:", (call_value_adjusted - call_value))
print ("Price increase as %:", ((call_value_adjusted - call_value) /␣
 ↪call_value)*100)
```

```
Call value:  10.450575619322274
Call value using Adjusted Volatility: 11.464970583491281
Price increase: 1.014394964169007
Price increase as %: 9.706594173563726
```

[41]:
```
# Now we compute the adjusted Volatility for transaction costs of 1.0%

t=0.02
adjusted_volatility = volatility*(1+((2/np.pi)**0.5)*(t/
 ↪volatility)*(steps_per_year**0.5))**0.5
print ("Adjusted Volatility:", adjusted_volatility)
```

```
Adjusted Volatility: 0.2510269100455295
```

[42]:
```
# Comparison BS and BS with adjusted volatility for transaction costs equal to␣
 ↪2.0%

# BS model with original volatility
model = BlackScholes(volatility)
call_value = call_option.value(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)

# BS model with adjusted volatility
model_adjusted = BlackScholes(adjusted_volatility)
call_value_adjusted = call_option.value(valuation_date, stock_price,␣
 ↪discount_curve, dividend_curve, model_adjusted)

print ("Call value: ", call_value)
print ("Call value using Adjusted Volatility:", call_value_adjusted)
print ("Price increase:", (call_value_adjusted - call_value))
```

```
print ("Price increase as %:", ((call_value_adjusted - call_value) /␣
↪call_value)*100)
```

```
Call value:  10.450575619322274
Call value using Adjusted Volatility: 12.374848083156243
Price increase: 1.9242724638339688
Price increase as %: 18.413076312046808
```

Interpretation of results:

We can see that the adjusted volatility increases when the transaction costs increase. This translates into a higher premium for the call option reflecting the fact that the hedging is now more expensive. Therefore, increasing the volatility with the adjusted volatility permits to incorporate into the call price part of the expected negative impact from transaction costs.

Question 3 - Implied density of Terminal Stock Price and Volatility Skew

3a.

For this question, we will be trying to derive the implied stock's price distribution at maturity. The idea will be that starting from the implied volatility given market prices we can get the price distribution using the Breedon-Litzenberg forumula.

The firs step will be to assume that we observed the volatility smile for an equity function, which we managed to fit using the following equation:

$$\sigma(x) = ax^2 + bx + c$$

Where:

$$a = 0.025$$

$$b = -0.225$$

$$c = 0.50$$

Now, we will define the "moneyness" of the stock as $x = \frac{K}{S(0)}$. In this case, the stock price at inception is going to be set at $S(0) = 100$. Finally, we will get the implied distribution of the stock's price at maturity using the Breedon-Litzenberg forumula. The idea is that the price of a call option can be expressed as:

$$V(t) = Z(t,T)\int_0^\infty g(S)Max(S-K,0)\mathrm{d}S$$

The idea is to approximate the probability function of S(T). $g(S)$, as follows:

$$g(K) \approx \frac{1}{Z(t,T)}\frac{V(K+dK) - 2V(K) + V(K-dK)}{dK^2}$$

[43]:
```python
# We will first define the initial inputs

S = 100
r = 0.05
T = 1
a = 0.025
b = -0.225
c = 0.5
d=0
df = exp(-r*T)
```

[44]:
```python
# Now, we will create a list containing different values for the strike which␣
↪will go from 1 to 401

l = []
for sk in range(1, 401):
    l.append(sk)

# Next, we will compute the corresponding moneyness to each of those strikes

mn =[]
for k in l:
    mn.append(k / S)

# Finally, we will use the equation that fits the volatility smile to recover␣
↪the implied volatilityes for each strike

vols = []
for vol in mn:
    vols.append(a*vol**2 + b*vol + c)
```

[45]:
```python
valuation_date = Date(30, 11, 2021)
expiry_date = valuation_date.add_years(T)

# Now, we will generate the discount curves for our options, which are going to␣
↪be the same for every option
discount_curve = DiscountCurveFlat(valuation_date, r)
dividend_curve = DiscountCurveFlat(valuation_date, d)

# Now, we will create a new option for each strike we have and compute its price
BS = []
Strike=1
for bs in vols:
    call_option = EquityVanillaOption(expiry_date, Strike, OptionTypes.
↪EUROPEAN_CALL)
    model = BlackScholes(bs)
```

```
    BS.append(call_option.value(valuation_date, S, discount_curve,␣
↪dividend_curve, model))
    Strike = Strike + 1
```

[46]:
```
# Next, we will estimate the probability function with the equation suggested␣
↪above

g = [0]
for G in range(1, len(l)-1):
    g.append((1/df)*(BS[G + 1] - 2*BS[G] + BS[G - 1])/1)
g.append(0)
```

[47]:
```
# Here, we will present a Dataframe with the results

df_smile = pd.DataFrame(columns=['Option Strike', 'Inverse Moneyness', 'Option␣
↪Vol', 'Black Scholes', 'Density'])
df_smile['Option Strike'] = l
df_smile['Inverse Moneyness'] = mn
df_smile['Option Vol'] = vols
df_smile['Black Scholes'] = BS
df_smile['Density'] = g

df_smile
```

[47]:

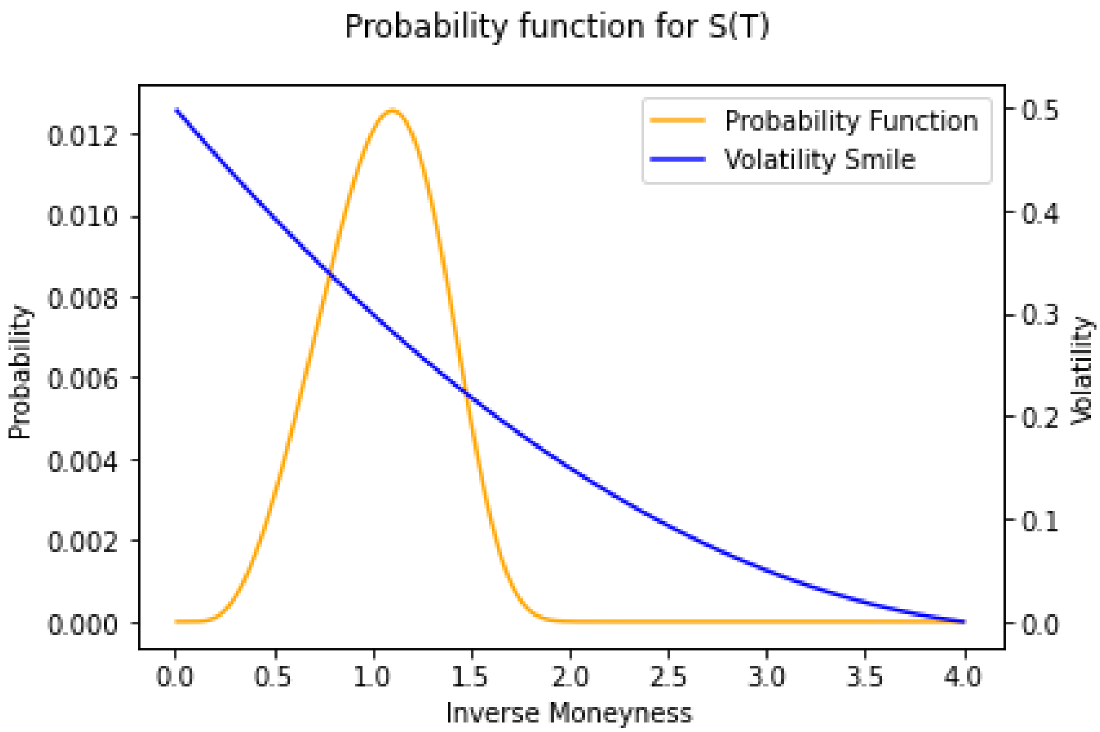
	Option Strike	Inverse Moneyness	Option Vol	Black Scholes	Density
0	1	0.01	0.497752	99.048771	0.000000e+00
1	2	0.02	0.495510	98.097541	3.286681e-13
2	3	0.03	0.493273	97.146312	1.874902e-11
3	4	0.04	0.491040	96.195082	3.431893e-10
4	5	0.05	0.488812	95.243853	3.035340e-09
..	…	…	…	…	…
395	396	3.96	0.001040	0.000000	0.000000e+00
396	397	3.97	0.000772	0.000000	0.000000e+00
397	398	3.98	0.000510	0.000000	0.000000e+00
398	399	3.99	0.000252	0.000000	0.000000e+00
399	400	4.00	0.000000	0.000000	0.000000e+00

[400 rows x 5 columns]

[48]:
```
# This is how the derived probability function looks like

import matplotlib.pyplot as plt

fig = plt.figure()
ax1 = fig.add_subplot(111)
ax2 = ax1.twinx()
plt1=ax1.plot(mn,g,c='orange',label='Probability Function')
```

```
plt2=ax2.plot(mn,vols,c='b',label='Volatility Smile')
plts = plt1+plt2
labs = [l.get_label() for l in plts]
ax1.legend(plts, labs, loc='upper right')
fig.suptitle('Probability function for S(T)')
ax1.set_xlabel('Inverse Moneyness')
ax1.set_ylabel('Probability')
ax2.set_ylabel('Volatility')
```

[48]: Text(0, 0.5, 'Volatility')



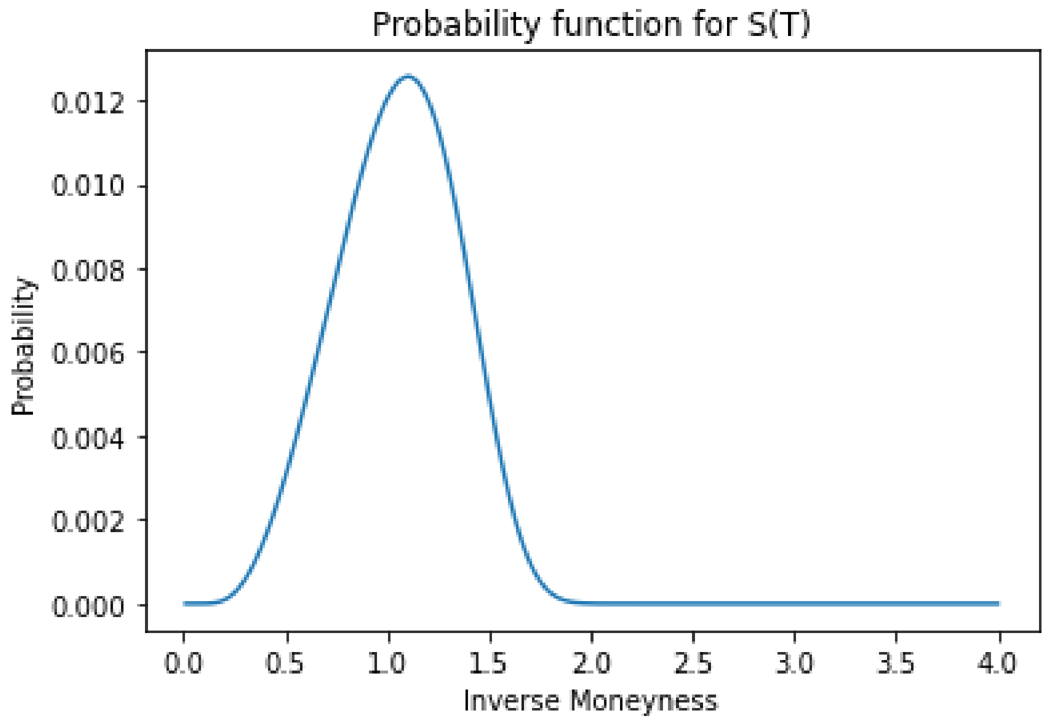
Here we will define a function that makes the same process as we did before and directly plots the implicit probability function for S

[49]:
```
def Get_Dist(spot, strike, step, r, T, a_, b_, c_, graph=""):
    """
    spot = spot price
    strike = maximum strike
    step = desired step
    r = risk free rate
    T = number of years
    a,b,c are the parameters that fit the volatility smile given the following␣
↪equation:
```

```
    sigma(x)=ax^2+bx+c
    graph=graph if you want to plot the curve. It will give the DataFrame␣
↪otherwise
    """
    # Create the strikes list

    strikes = []
    for sk in range(1, strike+1, step):
        strikes.append(sk)
    # Computing discount factor

    disc_f = 1/exp(r*T)

    # We add 2 values to the strike list: one in the beggining and one in the␣
↪end

    first_v = strikes[0] - step
    last_v = strikes[-1] + step
    strikes.insert(0, first_v)
    strikes.append(last_v)
    n = len(strikes)

    # Then, we create the moneyness list

    mn = []
    for x in strikes:
        mn.append(x / spot)

    # Next, we create the volatility list

    vols = []
    for vol in mn:
        vols.append(a_*vol**2 +b_*vol + c_)

    # Call financepy functions for BS pricing

    valuation_date = Date(30, 11, 2021)
    expiry_date = valuation_date.add_years(T)
    discount_curve = DiscountCurveFlat(valuation_date, r)
    dividend_curve = DiscountCurveFlat(valuation_date, 0)

    # We, then, create the BS option price list

    BS = []
    _ = 0
    for bs in vols:
        Strike = strikes[_]
```

```
        call_option = EquityVanillaOption(expiry_date, Strike, OptionTypes.
↪EUROPEAN_CALL)
        model = BlackScholes(bs)
        BS.append(call_option.value(valuation_date, S, discount_curve,␣
↪dividend_curve, model))
        _ = _ + 1

    # Create probabilities list

    g = [0]

    for G in range(1, n-1):
        g.append((1/disc_f)*(BS[G + 1] - 2*BS[G] + BS[G - 1])/(step**2))
    g.append(0)

    # We create a dataframe with values
    df_smile = pd.DataFrame(columns=['Option Strike', 'Inverse Moneyness',␣
↪'Option Vol', 'Black Scholes', 'Density'])
    df_smile['Option Strike'] = strikes
    df_smile['Inverse Moneyness'] = mn
    df_smile['Option Vol'] = vols
    df_smile['Black Scholes'] = BS
    df_smile['Density'] = g

    df_smile.drop(df_smile.index[[0,-1]], inplace=True)

    # We readjust strikes list

    strikes.pop(0)
    strikes.pop(-1)

    # Finally, we plot density
    if graph=="graph":
        plt.plot(df_smile['Inverse Moneyness'],df_smile['Density'])
        plt.title('Probability function for S(T)')
        plt.xlabel('Inverse Moneyness')
        plt.ylabel('Probability')
    else:
        return df_smile
```

[50]:
```
Get_Dist(100, 400, 1, 0.05, 1, 0.025, -0.225, 0.5, "graph")
```



3b.

Now, using the derived probability function we will try to compute the price of a digital option. A digital option pays \$1.00 if $S(T) > K$. First, we well get the dataframe with the probability function we got above

[51]:
```
df_density = Get_Dist(100, 400, 1, 0.05, 1, 0.025, -0.225, 0.5)

df_density
```

[51]:

	Option Strike	Inverse Moneyness	Option Vol	Black Scholes	Density
1	1	0.01	0.497752	99.048771	-1.015883e-12
2	2	0.02	0.495510	98.097541	3.286681e-13
3	3	0.03	0.493273	97.146312	1.874902e-11
4	4	0.04	0.491040	96.195082	3.431893e-10
5	5	0.05	0.488812	95.243853	3.035340e-09
..	…	…	…	…	…
396	396	3.96	0.001040	0.000000	0.000000e+00
397	397	3.97	0.000772	0.000000	0.000000e+00
398	398	3.98	0.000510	0.000000	0.000000e+00
399	399	3.99	0.000252	0.000000	0.000000e+00
400	400	4.00	0.000000	0.000000	0.000000e+00

[400 rows x 5 columns]

With the probability density function we can compute the implied probabilities such that $Pr(S(T) < K)$. We can then create a table of probabilities for the stikes [60, 80, 100, 120, 140]

To compute the digital option price for a certain strike, we simply multiply the payoffs (1 or zero) by their respective probabilities. And then we multiply this expected payoff by the discount factor considering r_f and T. This can be better understood by looking at the next equation for the value of a digital option:

$$V(S(0), t) = \mathrm{e}^{-r_f T} E[1_{S(T)>K}]$$

We will create a dataframe with the 'Strikes', 'Volatilities' and 'Digital Prices' for strikes [60, 80, 100, 120, 140]

[52]:
```
# Creating list with digital option strike prices

digital_strikes = [60, 80, 100, 120, 140]
```

[53]:
```
# We feed a list with the cummulative probabilities of each strike price

probabilities = []
for x in digital_strikes:
    probabilities.append(df_density.loc[df_density['Option Strike'] >= x,␣
 ↪'Density'].sum())
```

[54]:
```
# We will then store the implied volatilities of each each strike price

volatilities = []

for z in digital_strikes:
    volatilities.append(df_density.loc[z, "Option Vol"])
```

[55]:
```
# We will then create a dataframe with the results

df_digital = pd.DataFrame(columns = ['Strike', 'Volatility','Probabilities'])
df_digital['Strike'] = digital_strikes
df_digital['Volatility'] = volatilities
df_digital['Probabilities'] = probabilities
```

[56]:
```
# We compute digital prices per strike price and add them as a new column

r=0.05
T=1
digital_prices = []

for z in df_digital.index:
    digital_prices.append(df_digital.iloc[z, 2]*exp(-r*T))
```

```
df_digital['Digital Price'] = digital_prices

df_digital
```

[56]:

	Strike	Volatility	Probabilities	Digital Price
0	60	0.374	0.927436	0.882205
1	80	0.336	0.792861	0.754193
2	100	0.300	0.582446	0.554040
3	120	0.266	0.334978	0.318641
4	140	0.234	0.129079	0.122784

3c.

Now, we will compare this prices with the ones computed using the Black and Scholes formula. To compute the digital options BS price we will use financepy's EquityDigitalOption class and the formula used in class, which states that the value of a digital option can be computed as:

$$V(S(0),t) = \mathrm{e}^{-r_f T} N(d_2)$$

Where:

$$d_2 = -\left[\frac{ln(\frac{K}{S(0)}) - (r - \frac{\sigma^2}{2})T}{\sigma\sqrt(T)}\right]$$

[57]:
```
from scipy.stats import norm
```

[58]:
```
stock_price=100
r=0.05
T=1
digital_strikes = [60, 80, 100, 120, 140]

d_2=[]
BS_1=[]
for z in df_digital.index:
    k=df_digital.iloc[z, 0]
    vol=df_digital.iloc[z, 1]
    d_2.append(-(np.log(k/stock_price)-(r-0.5*vol**2)*T)/(vol*sqrt(T)))
    BS_1.append(norm.cdf(d_2[z])*exp(-r*T))

df_digital['d_2'] = d_2
df_digital['BS_1'] = BS_1

df_digital
```

[58]:

	Strike	Volatility	Probabilities	Digital Price	d_2	BS_1
0	60	0.374	0.927436	0.882205	1.312534	0.861176

```
1      80        0.336        0.792861        0.754193  0.644927  0.704398
2     100        0.300        0.582446        0.554040  0.016667  0.481939
3     120        0.266        0.334978        0.318641 -0.630449  0.251315
4     140        0.234        0.129079        0.122784 -1.341240  0.085536
```

Now, we will compute the price with the built-in function in financepy

[59]:
```python
valuation_date = Date(30, 11, 2021)
expiry_date = valuation_date.add_years(T)
discount_curve = DiscountCurveFlat(valuation_date, r)
dividend_curve = DiscountCurveFlat(valuation_date, 0)
underlying_type = FinDigitalOptionTypes.CASH_OR_NOTHING
```

[60]:
```python
# We compute prices for each strike and store them in a list

BS_prices = []

for z in df_digital.index:
    model = BlackScholes(df_digital.iloc[z, 1])
    digital_call = EquityDigitalOption(expiry_date, df_digital.iloc[z,0].
 ↪astype(np.float64),OptionTypes.EUROPEAN_CALL, underlying_type)
    BS_prices.append(digital_call.value(valuation_date, stock_price,␣
 ↪discount_curve, dividend_curve, model))

# Finally,we append the BS prices to our previous dataframe

df_digital['BS Prices'] = BS_prices

df_digital
```

[60]:
```
   Strike  Volatility  Probabilities  Digital Price       d_2      BS_1  \
0      60       0.374       0.927436       0.882205  1.312534  0.861176
1      80       0.336       0.792861       0.754193  0.644927  0.704398
2     100       0.300       0.582446       0.554040  0.016667  0.481939
3     120       0.266       0.334978       0.318641 -0.630449  0.251315
4     140       0.234       0.129079       0.122784 -1.341240  0.085536

   BS Prices
0   0.861176
1   0.704398
2   0.481939
3   0.251315
4   0.085536
```

3d.

As can be observed in the data frame, the digital price calculated with the implied PDF from smile is always larger than the BS price. Since the implied stock price distribution is not exactly lognormal, as it is assumed under the BS framework, we cannot expect that the implied PDF prices the digital option correctly. We can understand this as if we were overstating the probability that the stock's price at maturity will end above the established strike.

But even if the BS price is correct, the risk measures might not be as the volatility is assumed to not depend on the stock price. To better estiamate the risk measures, we would need to account for the change in value of the option as the volatility varies, since at different levels of $S(t)$ we have different volatilities, which should not be a constant function of the moneyness of the option or of the strike more specifically.

Considering the points mentioned above and that one of the advantages of the BS model is that it is not dependent on the real PDF of the stock price, we consider that the BS Prices are more correct.

3e.

Finally, we will compute the price of a put option with $Strike = 100$ but that only pays if the stock price falls below \$ \$60.00\$ at maturity. To do this we can define the value of the option as follows:

$$V(S(0), t) = \mathrm{e}^{-r_f T} max[K - S(T), 0]E[1_{S(T) > \$60.00}]$$

This option can be understood as a Down-In option wiht strike of \$100 and barrier of \$60.00. We will price this using, first, our implied density function and then with the built-in function in financepy.

[61]:
```
# We will create a new dataframe in which we will keep only the probability of␣
↪the stock price
# going below the barrier level and we will compute the corresponding payoff
# We will, then, compute the present value of the expected payoff

Strike=100
probabilities_2 = []
stock_p = []
pay_off = []
for x in l:
    price=df_density.iloc[x-1,0]
    if df_density.iloc[x-1,0] <= 60:
        prob = df_density.iloc[x-1,4] # Filter the probabilities when the S(T)␣
↪is above $60
        payoff_2 = Strike - price # Compute the payoff as the difference␣
↪between the Strike and S(T)
    else:
        prob = 0
        payoff_2 = 0
    probabilities_2.append(prob)
    stock_p.append(price)
    pay_off.append(payoff_2)
```

```
df_put = pd.DataFrame(columns=['Stock Price (T)', 'Pay Off', 'Probability'])
df_put['Stock Price (T)'] = stock_p
df_put['Pay Off'] = pay_off
df_put['Probability'] = probabilities_2

df_put
```

[61]:

```
     Stock Price (T)  Pay Off   Probability
0                  1       99 -1.015883e-12
1                  2       98  3.286681e-13
2                  3       97  1.874902e-11
3                  4       96  3.431893e-10
4                  5       95  3.035340e-09
..               …        …             …
395              396        0  0.000000e+00
396              397        0  0.000000e+00
397              398        0  0.000000e+00
398              399        0  0.000000e+00
399              400        0  0.000000e+00

[400 rows x 3 columns]
```

[62]:

```
# Compute the present value of the expected payoff

exp_payoff = []
for x in l:
    expect_po = df_put.iloc[x-1,1]*df_put.iloc[x-1,2]
    exp_payoff.append(expect_po)

put_price=exp(-r*T)*sum(exp_payoff)

put_price
```

[62]: 3.7631470428952696

[63]:

```
# We define the common variables that we will be using to create the different␣
↪options

valuation_date = Date(30, 11, 2021)
expiry_date = valuation_date.add_years(1)
num_observations=expiry_date-valuation_date
stock_price=100
r=0.05
T=1
stock_price = 100
strike_price = 100
volatility = df_density.iloc[strike_price-1,2]
```

```
interest_rate = 0.05
dividend_yield = 0.0
barrier_price = 60

model = BlackScholes(volatility)
discount_curve = DiscountCurveFlat(valuation_date, interest_rate)
dividend_curve = DiscountCurveFlat(valuation_date, dividend_yield)

# We create three option objects, a Put, a Down-In Put and a Down-Out Put, and␣
 ↪the Discount curves to value them

put_option = EquityVanillaOption(expiry_date, strike_price, OptionTypes.
 ↪EUROPEAN_PUT)
put_value = put_option.value(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)

barrierType = EquityBarrierTypes.DOWN_AND_IN_PUT
barrierOpt = EquityBarrierOption(expiry_date, strike_price, barrierType,␣
 ↪barrier_price, num_observations)
barrier_put_in=barrierOpt.value(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)

barrierType = EquityBarrierTypes.DOWN_AND_OUT_PUT
barrier_price = 60
barrierOpt = EquityBarrierOption(expiry_date, strike_price, barrierType,␣
 ↪barrier_price, num_observations)
barrier_put_out=barrierOpt.value(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)

# We create a Dataframe to show the results

Puts=[]
Puts = pd.DataFrame(columns=['Barrier DO Put Price', 'Barrier DO Put Price -␣
 ↪BS', 'Barrier DI Put Price - BS', 'Put Price - BS'])
Puts.loc[0,'Barrier DO Put Price'] = put_price
Puts.loc[0,'Barrier DO Put Price - BS'] = barrier_put_in
Puts.loc[0,'Barrier DI Put Price - BS'] = barrier_put_out
Puts.loc[0,'Put Price - BS'] = put_value

Puts
```

[63]:
```
  Barrier DO Put Price Barrier DO Put Price - BS Barrier DI Put Price - BS  \
0             3.763147                  3.045652                  6.308535

  Put Price - BS
0       9.354187
```

With these values we can show that the values of the Down Out Put barrier option calculated with the implied density function are higher that the one suggested with the built-in function in financepy. One explanation for this is the volatiliy smile from where we got the implied density function, as it shifted the probability mass down. This means that given the market prices, the market is thinking that downard movements are more probable, probably overstating the probabilities of the stock's price being below the barrier level.

[64]:
```
put_value - barrier_put_in - barrier_put_out
```

[64]:
```
3.019806626980426e-14
```

Finally, we can conclude that the sum of the values for a Down-In Put and a Down-Out Put add exatly to the value of a regular Put.

Question 4 - Modelling Volatility Skew

4a.

As we mentioned before, there is a problem with the Black and Scholes model regarding the risk measures as it does not account for the different levels for the volatility of the stock and its relationship with its price. To overcome this problem we can change the volatility in the model and substitute it with a stochastic volatility. One that depends on the stock. To show this, it might be good to start with a local model, which means that it depend on itself only.

$$\frac{dS(t)}{S(t)} = (r - q)dt + \sigma(S)dW_t$$

Where $\sigma(S)$ is a determenistic function. We can compare this with the regular Black and Shcoles framework in which the relationship is as follows:

$$\frac{dS(t)}{S(t)} = (r - q)dt + \sigma_{BS}dW_t$$

One popular model is the Constant Elasticity of Variance which stablishes the following relationship:

$$\frac{dS(t)}{S(t)} = (r - q)dt + \sigma\left(\frac{S(t)}{S(0)}\right)^{\beta} dW_t$$

Then, in this case the deterministic function is:

$$\sigma(S) = \sigma\left(\frac{S(t)}{S(0)}\right)^{\beta}$$

This implies that when we have $\beta = 0$, we recover the regular Black and Scholes framework. In this question, we will create a code to price a T-year european option based on Monte Carlo simulation so we can introduce this model. We will set $dt = 0.02$ and implement antithetic variables to diminish the variance of our results. After solving our equation using Itto's lemma, we found that the solution for the stocasthic differential equation suggested first is:

$$S_{t+1} = S_t \mathrm{e}^{\left[(\mu - \frac{\sigma_{BSnew}^2}{2})dt + \sigma_{BSnew} dW_t\right]}$$

Where:

$$\sigma_{BSnew} = \sigma_{CEV} \left(\frac{S_t}{S_0}\right)^{\beta}$$

[65]:
```
def mc_cev(stock_p, K, r, q, vol, t, number_steps_p_y, num_paths, seed, beta,␣
↪opt_type="Call"):

    num_paths = int(num_paths)
    np.random.seed(seed)
    mu_2 = r - q
    dt = 1/number_steps_p_y
    vol_cev_1=[]
    FinalP_1 = []
    FinalP_2 = []
    Payoffs_1 = []
    Payoffs_2 = []

    # We recognize if we will be pricing a call or a put

    if opt_type=="Call":
        o=1
    else:
        o=-1

    # We generate the estimated final prices

    for i in range(0,num_paths):
        s_t_1=stock_p
        s_t_2=stock_p
        for j in range(0, int(number_steps_p_y*t)):
            g=np.random.randn()

            if j==0:
                vol_cev_1=vol
            else:
                vol_cev_1=vol*(s_t_1/stock_p)**beta
            s_t_1=s_t_1*exp((mu_2 - 0.5*(vol_cev_1**2))*dt +␣
↪vol_cev_1*sqrt(dt)*g)

            # We generate an alternative path to use as an antithetic variable

            if j==0:
                vol_cev_2=vol
```

```
            else:
                vol_cev_2=vol*(s_t_2/stock_p)**beta
            s_t_2=s_t_2*exp((mu_2 - 0.5*(vol_cev_2**2))*dt -␣
↪vol_cev_2*sqrt(dt)*g)

        FinalP_1.append(s_t_1)
        FinalP_2.append(s_t_2)

    # We compute the estimated payoffs

    for k in range(0,num_paths):
        pay_o=o*(FinalP_1[k]-K)
        if pay_o>0:
             Payoffs_1.append(pay_o)
        else:
             Payoffs_1.append(0)

    for k in range(0,num_paths):
        pay_o=o*(FinalP_2[k]-K)
        if pay_o>0:
             Payoffs_2.append(pay_o)
        else:
             Payoffs_2.append(0)

    vol_cev_1=pd.DataFrame({'Final Prices_1':FinalP_1,
                      'Final Prices_2':FinalP_2,
                      'PayOff_1':Payoffs_1,
                             'PayOff_2':Payoffs_2})

    # We value the option as the discounted average expected payoff

    discounted_payoff = np.mean([np.mean(vol_cev_1['PayOff_1']),np.
↪mean(vol_cev_1['PayOff_2'])]) * np.exp(-r * t)

    return discounted_payoff
```

[66]:
```
def mc_BS(stock_p, K, r, q, vol, t, number_steps_p_y, num_paths, seed,␣
↪opt_type="Call"):

    num_paths = int(num_paths)
    np.random.seed(seed)
    mu_2 = r - q
    bs_m=[]
    FinalP_1 = []
    FinalP_2 = []
    Payoffs_1 = []
    Payoffs_2 = []
```

```
dt = 1/number_steps_p_y

# We recognize if we will be pricing a call or a put

if opt_type=="Call":
    o=1
else:
    o=-1

# We generate the estimated final prices

for i in range(0,num_paths):
    s_t_1=stock_p
    s_t_2=stock_p
    for j in range(0, int(number_steps_p_y*t)):
        g=np.random.randn()

        s_t_1=s_t_1*exp((mu_2 - 0.5*(vol**2))*dt + vol*sqrt(dt)*g)

        # We generate an alternative path to use as an antithetic variable

        s_t_2=s_t_2*exp((mu_2 - 0.5*(vol**2))*dt - vol*sqrt(dt)*g)

    FinalP_1.append(s_t_1)
    FinalP_2.append(s_t_2)

# We compute the estimated payoffs

for k in range(0,num_paths):
    pay_o=o*(FinalP_1[k]-K)
    if pay_o>0:
         Payoffs_1.append(pay_o)
    else:
         Payoffs_1.append(0)

for k in range(0,num_paths):
    pay_o=o*(FinalP_2[k]-K)
    if pay_o>0:
         Payoffs_2.append(pay_o)
    else:
         Payoffs_2.append(0)

bs_m=pd.DataFrame({'Final Prices_1':FinalP_1,
                   'Final Prices_2':FinalP_2,
                   'PayOff_1':Payoffs_1,
                          'PayOff_2':Payoffs_2})
```

```
    # We value the option as the discounted average expected payoff

    discounted_payoff = np.mean([np.mean(bs_m['PayOff_1']),np.
 ↪mean(bs_m['PayOff_2'])]) * np.exp(-r * t)

    return discounted_payoff
```

[67]:
```
mc_BS(100, 100, 0.05, 0, 0.2, 1, 50, 10000, 10, "Call")
```

[67]: 10.504033406713246

[68]:
```
mc_cev(100, 100, 0.05, 0, 0.2, 1, 50, 10000, 10, 0, "Call")
```

[68]: 10.504033406713246

[69]:
```
# We will compute the price using the BS model to test the accuracy

valuation_date = Date(30, 11, 2021)
expiry_date = valuation_date.add_years(1)
stock_price = 100
strike_price = 100
volatility = 0.20
interest_rate = 0.05
dividend_yield = 0.0
call_option = EquityVanillaOption(expiry_date, strike_price, OptionTypes.
 ↪EUROPEAN_CALL)
discount_curve = DiscountCurveFlat(valuation_date, interest_rate)
dividend_curve = DiscountCurveFlat(valuation_date, dividend_yield)
model = BlackScholes(volatility)

call_value = call_option.value(valuation_date, stock_price, discount_curve,␣
 ↪dividend_curve, model)
print(call_value)
```

```
10.450575619322274
```

4b.

Now, to better analyze the effect of the parameter β, we will price options with different strikes changing the value of the parameter β.

[70]:
```
# Here, we initialize the common parameters of our options

s = 100
t = 0.5
r = 0.05
q = 0
```

```python
v = 0.2
num_paths=10000
steps_y = 50
seed=9

valuation_date = Date(30, 11, 2021)
expiry_date = valuation_date.add_years(t)
discount_curve = DiscountCurveFlat(valuation_date, r)
dividend_curve = DiscountCurveFlat(valuation_date, q)
model = BlackScholes(v)
```

[71]:
```python
# We prepare the dataframe to store the results

l = []
for sk in range(80, 125,5):
    l.append(sk)
beta = [-0.5, -0.25, 0, 0.25, 0.5]
df_cev_t=pd.DataFrame(index=['80', '85', '90', '95', '100', '105', '110',␣
↪'115', '120'],columns=['Beta = -0.5', 'Beta = -0.25', 'Beta = 0', 'Beta = 0.
↪25', 'Beta = 0.5'])
```

[72]:
```python
# Here, we run the simulations for the varying values of the strike and the␣
↪betas

m = 0
n= 0
for i in beta:
    n=0
    for j in l:
        call_option = mc_cev(s, j, r, q, v, t, steps_y, num_paths, seed, i,␣
↪"Call")
        df_cev_t.iloc[n,m]=call_option
        n=n+1
    m=m+1
```

[73]:
```python
# Finally, we compute the corresponding Call price for each given strike

BS_P = []
leng = len(df_cev_t.index)
for a in range(0,leng):
    call_option = EquityVanillaOption(expiry_date, l[a], OptionTypes.
↪EUROPEAN_CALL)
    call_value = call_option.value(valuation_date, s, discount_curve,␣
↪dividend_curve, model)
    BS_P.append(call_value)

df_cev_t['BS_P'] = BS_P
```

```
df_cev_t
```

[73]:

	Beta = -0.5	Beta = -0.25	Beta = 0	Beta = 0.25	Beta = 0.5	BS_P
80	22.267162	22.239467	22.215334	22.194542	22.177004	22.154896
85	17.797871	17.757851	17.720287	17.685097	17.652215	17.628314
90	13.676006	13.635395	13.596369	13.558859	13.522799	13.469845
95	10.041982	10.016639	9.99234	9.969017	9.946729	9.840506
100	7.004435	7.007224	7.011142	7.016185	7.022323	6.855316
105	4.631343	4.667105	4.704207	4.742656	4.782532	4.550021
110	2.897465	2.95797	3.020136	3.083836	3.149368	2.879009
115	1.705148	1.777474	1.852331	1.929819	2.010097	1.739637
120	0.938678	1.00915	1.083259	1.161176	1.243088	1.006317

Quite interestingly, we can see that the differences in the prices, as the β varies, are larger when the strike moves far from S_0. And it seems that the differences are larger when the strikes are tilted to the upside.

4c.

Now, given the prices we obtained earlier, we will try to derive the volatility smile corresponding to each value of β

[74]:
```
from financepy.utils.global_vars import gDaysInYear
from financepy.models.black_scholes_analytic import bs_implied_volatility
```

[75]:
```
# We prepare the dataframe to store the implied volatilities

df_cev_iv=pd.DataFrame(index=['80', '85', '90', '95', '100', '105', '110',␣
↪'115', '120'],columns=['Beta = -0.5', 'Beta = -0.25', 'Beta = 0', 'Beta = 0.
↪25', 'Beta = 0.5', "B&S"])
m = 0
n= 0
beta.append('B&S')

# Using the Financepy built-in function, we derived the implied volatility for␣
↪each value of K and beta
for i in beta:
    n=0
    for j in l:
        call_option = EquityVanillaOption(expiry_date, j, OptionTypes.
↪EUROPEAN_CALL)
        Implied_volatility = call_option.implied_volatility(valuation_date, s,␣
↪discount_curve, dividend_curve,  df_cev_t.iloc[n,m])
        df_cev_iv.iloc[n,m]=Implied_volatility
        n=n+1
    m=m+1
```

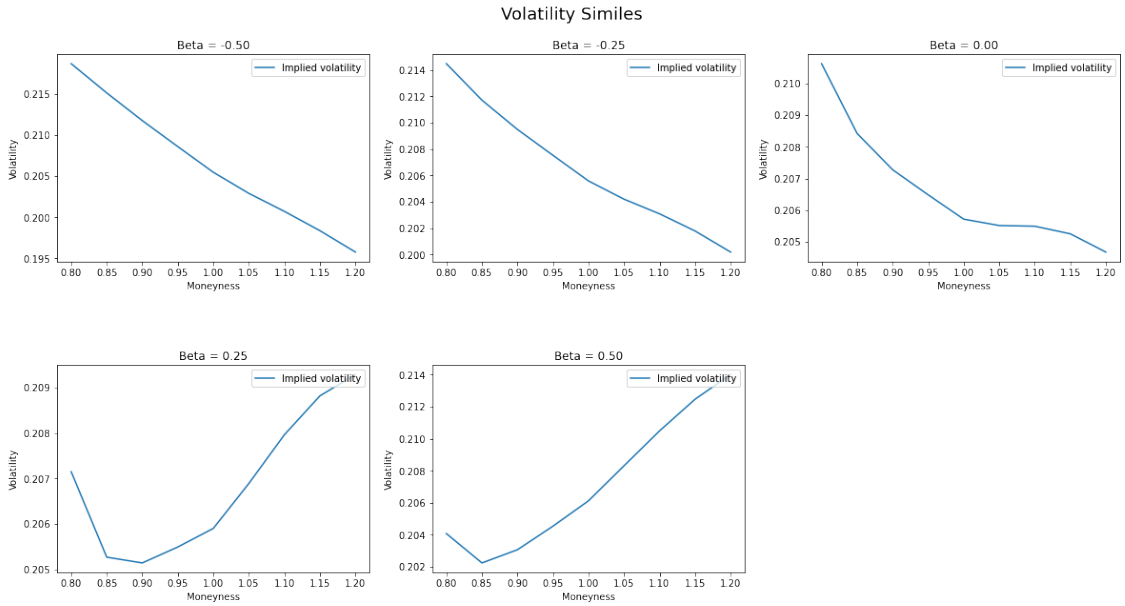
```
df_cev_iv
```

[75]:

	Beta = -0.5	Beta = -0.25	Beta = 0	Beta = 0.25	Beta = 0.5	B&S
80	0.218665	0.214469	0.210624	0.207147	0.204077	0.2
85	0.215115	0.211704	0.208421	0.205268	0.202246	0.2
90	0.211752	0.209479	0.207276	0.205141	0.203071	0.2
95	0.208591	0.207516	0.206483	0.205491	0.204542	0.2
100	0.20547	0.205572	0.205716	0.205901	0.206126	0.2
105	0.202909	0.204188	0.205514	0.206889	0.208314	0.2
110	0.20072	0.203077	0.205493	0.207961	0.210492	0.2
115	0.198374	0.201774	0.205253	0.208817	0.21247	0.2
120	0.195774	0.200175	0.204676	0.209286	0.21401	0.2

[76]:

```
# Here, we create the graphs to analyze the results

cols = ['Beta = -0.50', 'Beta = -0.25', 'Beta = 0.00', 'Beta = 0.25', 'Beta = 0.
↪50', 'B&S']
inv_mn=[]
for f in l:
    invmn = f/s
    inv_mn.append(invmn)
df_cev_iv2 = df_cev_iv
df_cev_iv2.index = inv_mn

plt.figure(figsize=(20, 10))
plt.subplots_adjust(hspace=0.5)
plt.suptitle("Volatility Similes", fontsize=18, y=0.95)

for n in range(0,len(beta)-1):
    # We add a new subplot iteratively

    ax = plt.subplot(2, 3, n + 1)
    df_cev_iv2.iloc[:,n].plot(ax=ax, label="Implied volatility")
    ax.set_title(cols[n])
    ax.legend(loc="upper right")
    ax.set_xlabel("Moneyness")
    ax.set_ylabel("Volatility")
```



As we can see, when beta is positive, the relationship between the strike and the volatility changes to an upward sloping curve. This implies that expectations for the volatility are higer at higher prices. This is consistent with our model, as with a positive β, as the stockprice goes up, so does the volatility used to feed the model. It is important to note that as β gets more negative, the curve gets more steeper. Same happens when β increases. In the case of $\beta = 0$, we should be looking at a straight line given that in that case we are under the Black and Scholes framework with the volatility being fixed. Nonetheless, as the Monte Carlo simulation adds some error to the computed prices, the implied volatility is not exactly the 0.20% that we used, but is pretty close.

4d.

Finally, we will try to estimate the deltas for our models given the different values of β and K. Given that the volatility is no longer constant, we need to add an additional adjustment to our delta estimation. This would look as follows:

$$\Delta = \frac{\delta V}{\delta S}\Big|_{\sigma_{BS}} + \frac{\delta V}{\delta \sigma_{BS}} \frac{\delta \sigma_{BS}}{\delta S}$$

Which can be rewritten as:

$$\Delta = \Delta_{BS} + \nu_{BS} \frac{\delta \sigma_{BS}}{\delta S}$$

We can compute the delta and the vega using the built-in financepy formulas. For the last term, it refers to the slope of the volatilites smiles. We can estimate it by changing the stock price by \$0.1 and recomputing the Call price. Finally, we recover the implied volatilityes for the new prices and estimate the slopes of the curves as follows:

$$\frac{\delta \sigma_{BS}}{\delta S} = \frac{\sigma_{BS}^{new} - \sigma_{BS}^{original}}{S^{new} - S^{original}}$$

[77]:

```
# We prepare the dataframe to store the results

df_cev_t_2=pd.DataFrame(index=['80', '85', '90', '95', '100', '105', '110',␣
↪'115', '120'],columns=['Beta = -0.5', 'Beta = -0.25', 'Beta = 0', 'Beta = 0.
↪25', 'Beta = 0.5'])
```

[78]:

```
# Here, we run the simulations for the varying values of the strike and the␣
↪betas and
# we change the value of S as well

s2=s+0.1
beta2 = [-0.5, -0.25, 0, 0.25, 0.5]

m = 0
n= 0
for i in beta2:
    n=0
    for j in l:
        call_option = mc_cev(s2, j, r, q, v, t, steps_y, num_paths, seed, i,␣
↪"Call")
        df_cev_t_2.iloc[n,m]=call_option
        n=n+1
    m=m+1
```

[79]:

```
# Finally, we compute the corresponding Call price for each given strike

BS_P = []
leng = len(df_cev_t.index)
for a in range(0,leng):
    call_option = EquityVanillaOption(expiry_date, l[a], OptionTypes.
↪EUROPEAN_CALL)
    call_value = call_option.value(valuation_date, s2, discount_curve,␣
↪dividend_curve, model)
    BS_P.append(call_value)

df_cev_t_2['BS_P'] = BS_P

df_cev_t_2
```

[79]:

	Beta = -0.5	Beta = -0.25	Beta = 0	Beta = 0.25	Beta = 0.5	BS_P
80	22.363039	22.335618	22.311728	22.291227	22.273975	22.251566
85	17.889119	17.849161	17.811696	17.776625	17.74392	17.720297
90	13.760063	13.719257	13.680066	13.642411	13.606221	13.553933

```
95     10.115796    10.089984  10.065288    10.041564  10.018848    9.913552
100     7.065894     7.067984   7.071282     7.075668   7.081153    6.915188
105     4.679052     4.714258   4.750738     4.788554     4.8278    4.596181
110     2.932393     2.992624   3.054373     3.117771   3.182877    2.912489
115     1.728944     1.801184   1.875949     1.953351   2.033545    1.762532
120     0.953977     1.024688    1.09899     1.177108   1.259188    1.021130
```

[80]:
```python
# We prepare the dataframe to store the implied volatilities

df_cev_iv2=pd.DataFrame(index=['80', '85', '90', '95', '100', '105', '110',␣
↪'115', '120'],columns=['Beta = -0.5', 'Beta = -0.25', 'Beta = 0', 'Beta = 0.
↪25', 'Beta = 0.5', "B&S"])
m = 0
n= 0
beta2.append('B&S')

# Using the Financepy built-in function, we derived the implied volatility for␣
↪each value of K and beta
for i in beta2:
    n=0
    for j in l:
        call_option = EquityVanillaOption(expiry_date, j, OptionTypes.
↪EUROPEAN_CALL)
        Implied_volatility = call_option.implied_volatility(valuation_date, s2,␣
↪discount_curve, dividend_curve,  df_cev_t_2.iloc[n,m])
        df_cev_iv2.iloc[n,m]=Implied_volatility
        n=n+1
    m=m+1


df_cev_iv2
```

[80]:

	Beta = -0.5	Beta = -0.25	Beta = 0	Beta = 0.25	Beta = 0.5	B&S
80	0.218729	0.214535	0.210691	0.207228	0.204178	0.2
85	0.215171	0.21174	0.20844	0.205271	0.20224	0.2
90	0.211817	0.20952	0.207295	0.20514	0.203051	0.2
95	0.208651	0.207552	0.2065	0.205487	0.204517	0.2
100	0.205532	0.205609	0.20573	0.205891	0.206092	0.2
105	0.202959	0.204216	0.205518	0.206868	0.208268	0.2
110	0.200774	0.203111	0.2055	0.207947	0.210454	0.2
115	0.198427	0.201801	0.205255	0.208793	0.212422	0.2
120	0.195841	0.200217	0.204693	0.209278	0.213977	0.2

[81]:
```python
# We prepare the dataframe to store the new deltas
```

```python
df_cev_delta=pd.DataFrame(index=['80', '85', '90', '95', '100', '105', '110',␣
↪'115', '120'],columns=['Beta = -0.5', 'Beta = -0.25', 'Beta = 0', 'Beta = 0.
↪25', 'Beta = 0.5', "B&S"])
m = 0
n= 0
beta2.append('B&S')

# Using the Financepy built-in function, we derived the implied volatility for␣
↪each value of K and beta
for i in beta:
    n=0
    for j in l:
        call_option = EquityVanillaOption(expiry_date, j, OptionTypes.
↪EUROPEAN_CALL)
        call_delta=call_option.delta(valuation_date, s, discount_curve,␣
↪dividend_curve, model)
        call_vega=call_option.vega(valuation_date, s, discount_curve,␣
↪dividend_curve, model)
        call_delta2 = call_delta+call_vega*(df_cev_iv2.iloc[n,m]-df_cev_iv.
↪iloc[n,m])/(s2-s) # We implement the adjustment suggested at the begining of␣
↪the question

        df_cev_delta.iloc[n,m]=call_delta2
        n=n+1
    m=m+1


df_cev_delta
```

[81]:

	Beta = -0.5	Beta = -0.25	Beta = 0	Beta = 0.25	Beta = 0.5	B&S
80	0.969766	0.96989	0.969972	0.970729	0.971756	0.966439
85	0.925239	0.923105	0.921282	0.919666	0.918641	0.919304
90	0.851004	0.847073	0.843414	0.839914	0.836546	0.840025
95	0.743289	0.737877	0.733196	0.72848	0.72351	0.72929
100	0.61432	0.607352	0.601185	0.594651	0.588162	0.59734
105	0.474298	0.468164	0.46134	0.454412	0.447494	0.460189
110	0.347176	0.342045	0.335495	0.330107	0.323527	0.333502
115	0.239064	0.233599	0.228182	0.222913	0.217769	0.227869
120	0.158196	0.154258	0.150023	0.146054	0.141912	0.147303

[82]:

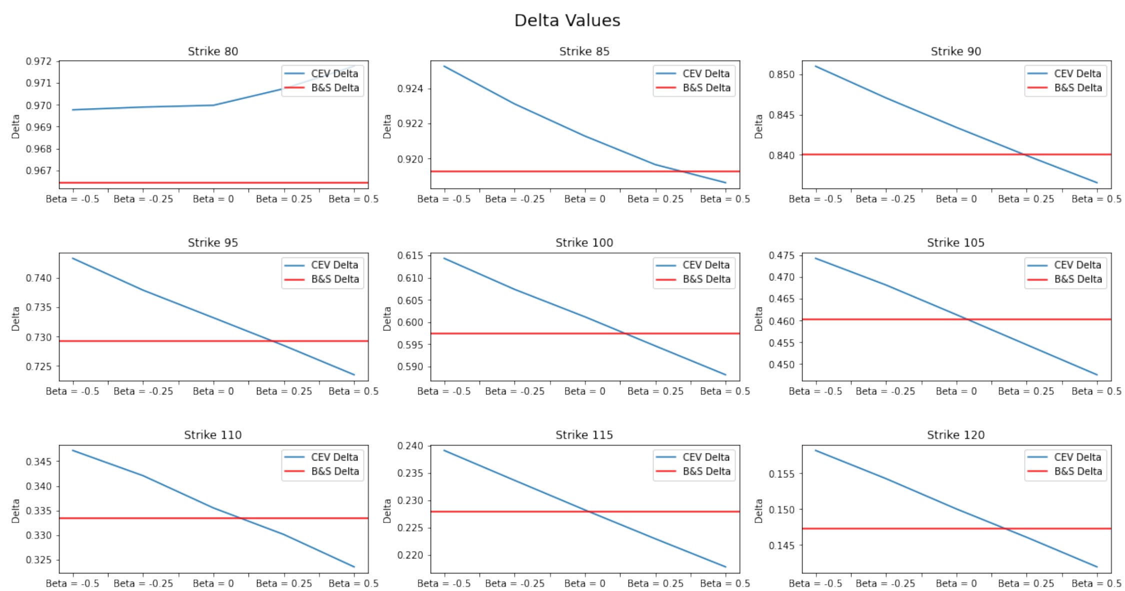
```python
df_cev_iv2
```

[82]:

	Beta = -0.5	Beta = -0.25	Beta = 0	Beta = 0.25	Beta = 0.5	B&S
80	0.218729	0.214535	0.210691	0.207228	0.204178	0.2
85	0.215171	0.21174	0.20844	0.205271	0.20224	0.2
90	0.211817	0.20952	0.207295	0.20514	0.203051	0.2
95	0.208651	0.207552	0.2065	0.205487	0.204517	0.2

100	0.205532	0.205609	0.20573	0.205891	0.206092	0.2
105	0.202959	0.204216	0.205518	0.206868	0.208268	0.2
110	0.200774	0.203111	0.2055	0.207947	0.210454	0.2
115	0.198427	0.201801	0.205255	0.208793	0.212422	0.2
120	0.195841	0.200217	0.204693	0.209278	0.213977	0.2

[83]:
```python
df_cev_delta2=df_cev_delta.drop('B&S', axis=1)

plt.figure(figsize=(20, 10))
plt.subplots_adjust(hspace=0.5)
plt.suptitle("Delta Values", fontsize=18, y=0.95)

for n in range(0,len(l)):
    # We add a new subplot iteratively

    ax = plt.subplot(3, 3, n + 1)
    m=0
    df_cev_delta2.iloc[n,0:len(beta2)].plot(ax=ax, label="CEV Delta")
    plt.axhline(y=df_cev_delta.iloc[n,5], color='r', label="B&S Delta")
    ax.set_title('Strike '+str(l[n]))
    ax.legend(loc="upper right")
    ax.set_ylabel("Delta")
```



As we can see, as the value of βs increases the option total sensitivity to changes in the stock price slightly decreases. This makes sense as the volatility skew observed above gets inverted for positive and higher βs. One could understand this as if the vega exposure of the option actually changed the sign. For a skewed volatility curve like the ones we can see when β is negative, the vega of an option is positive. So, when the curve gets inverted, it makes sense to think as if the vega now will be negative. This detail makes the adjustment made to the option delta yield lower sensitivities when the β is positive.

Question 5 - Variance Swaps

5a.

In this question we will explore how Variance Swaps work and how can we hedge a Variance Swap. The first thing to notice is that the variance in this context is computed as follows:

$$\sigma^2 = \frac{252}{N}\sum_{i=1}^{N}\Big(ln\Big(\frac{S_i}{S_{i-1}}\Big)\Big)^2$$

Next, we will introduce the general idea behind a variance swap. A variance swap is a pure volatility trade whose payoff is the difference between the strike and the realized volatility. So:

$$V_{Swap} = Notional \cdot (\sigma^2_{Realized} - \sigma^2_{Strike})$$

It is useful to notice that the realised variance of an underlying through some period of time can be replicated with a portfolio consisting of two parts. First, a dynamic portfolio which we will be calling the Dynamic Hedge and will consist on $\frac{2}{T}$ times $\frac{1}{S_t}$ shares of the stock:

$$DynamicHedge = \frac{2}{T}\frac{1}{S_t}$$

And a static portfolio, which will be called Static Hedge, such that:

$$StaticHedge = \frac{2}{T}logcontract$$

Where the logcontract payoff is based on the total appreciation or depreciation of the stock. Interestingly, it can be shown that forming a portfolio of a long Dynamic Hedge and short the Static Hedge allow us to replicate rather closely the realized variance of the underlying. Along some period of time, the value of the realized variance is:

$$V = \frac{2}{T}\Big[\int_0^T \frac{dS_t}{S_t} - ln\frac{S_t}{S_0}\Big]$$

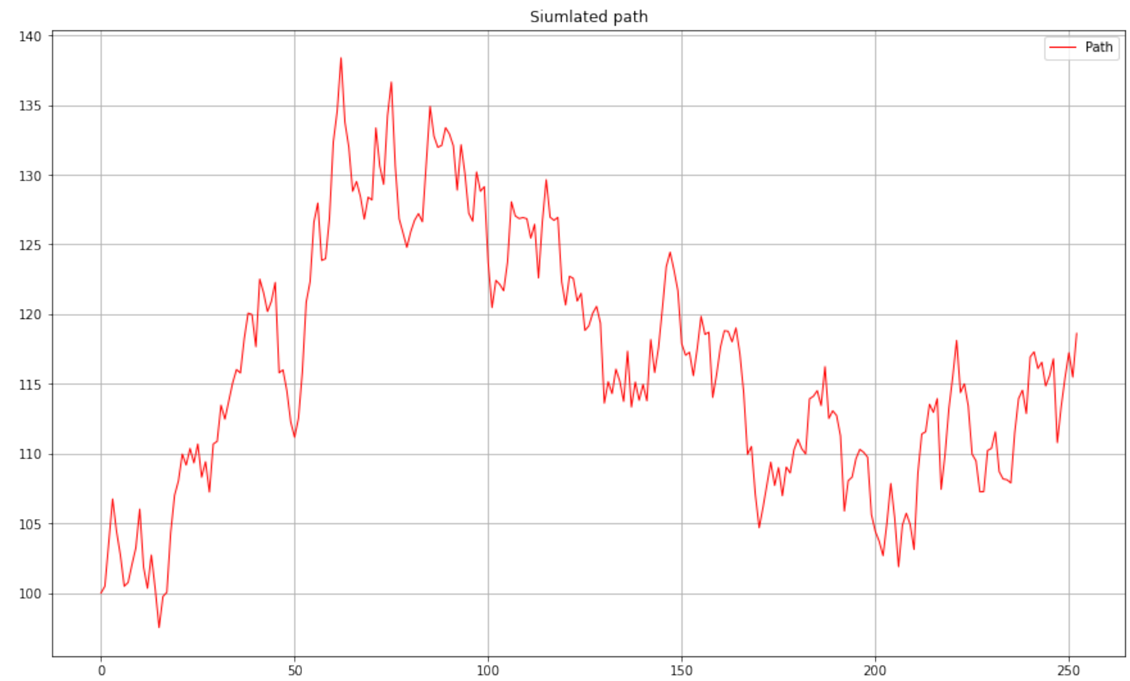
We, will try to show this in this question. First, we will create a function to simulate the evolution of the Dynamic Hedge portfolio and, next, we will combine this with a short position in the logcontract described above. The first step into our analysis will be to create a function to simulate the one path of the evolution of a stock assuming it follows a lognormal process. This means that:

$$S_{t+1} = S_t \mathrm{e}^{\Big[(\mu - \frac{\sigma^2}{2})dt + \sigma dW_t\Big]}$$

[84]:
```
# This function will allow us to simulate one path for the stock

def logNormal_path(S0, mu, sigma, T, M):
    dt = float(T) / M
    rn = np.random.standard_normal(M + 1)
    path = S0 * np.exp(np.cumsum((mu - 0.5 * sigma ** 2) * dt + sigma * np.
 ↪sqrt(dt) * rn))
    path = pd.DataFrame(path, columns=['Path'])
    path=path.shift(1)
    path.Path[0] = S0
    return path
```

[85]:
```
# Here, we will define the default values for our simulation and obtain the␣
 ↪simulated value

S0 = 100
mu = 0.05
sigma = 0.3
M = 252
T = 1

S = logNormal_path(S0, mu, sigma, T, M)
```

[86]:
```
# Here, we present a graph with the simulated path

S.plot(figsize=(15, 9), title='Siumlated path', color = 'red', linewidth=1.0,␣
 ↪grid=True)
plt.legend()
plt.show()
```



[87]:
```
# Here, we create a function to compute the total proceeds of the Dynamic␣
↪Hedging porftolio. At each day you have
# 1/s(t) stocks and you buy at s(t) and sell at s(t+1). The total proceeds per␣
↪day of such strategy is (s(t+1)-s(t))/s(t)

def dynamic_hedge(path):
    dh = ((path - path.shift(1))/path.shift(1))
    dh.dropna(inplace = True)
    return (2/T) * dh['Path'].sum()
```

[88]:
```
# Here, we compute the total proceeds of the Static Heding portfolio. This␣
↪component is based on a logcontract that pays the log of
# the total appreciation of the underlying.

def static_hedge(path):
    St = path['Path'].values.tolist()
    return (2/T) * np.log(St[-1]/St[0])
```

[89]:
```
# This is a function to calculate the realised variance for every path generated

def realized_var(path):
    log_ret = np.log(path/path.shift(1))
    log_ret.dropna(inplace = True)
    return (252/log_ret.shape[0]) * (log_ret['Path'].pow(2)).sum()
```

Now, we will compare the value of the portfolio comprising of a long position on the Dynamic Hedge and short the Static Hedge to see how well our portfolio replicates the realized volatility.

[90]:
```
Hedge = dynamic_hedge(S)-static_hedge(S)
Real_Var = realized_var(S)

Hedge**0.5
```

[90]: 0.3294704495294486

[91]:
```
Real_Var**0.5
```

[91]: 0.329631300432506

[92]:
```
error=Hedge**0.5-Real_Var**0.5
error
```

[92]: -0.00016085090305739688

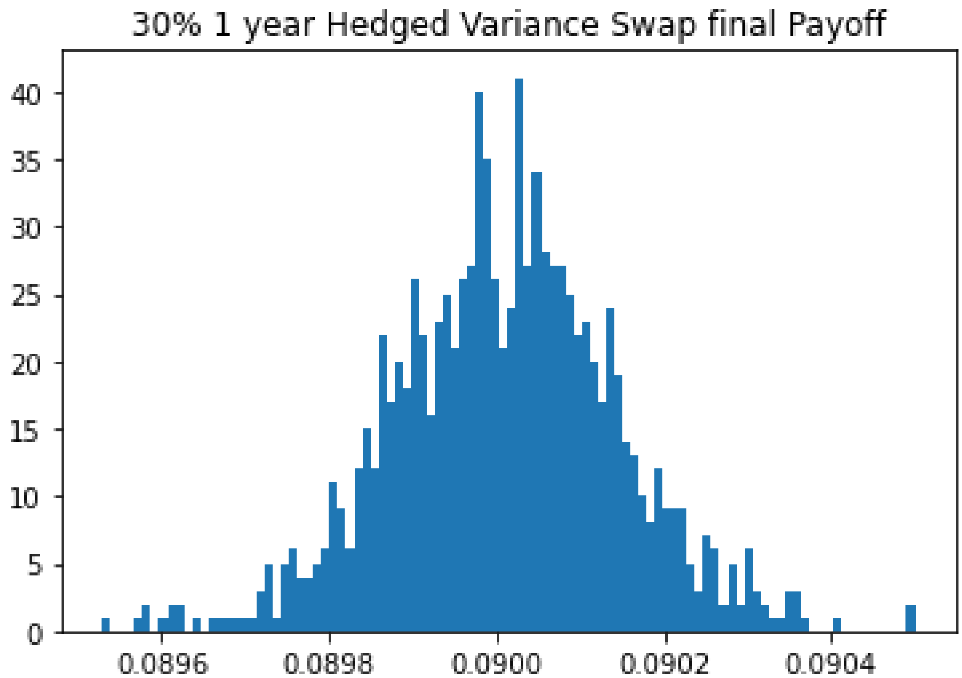
As we can see, our portfolio does a pretty good job in replicating the realized volatility. Now, we will simulate 1,000 differents paths to simulate the final Payoff of a variance 1 year swap with strike of 30% that has been hedged dynamically. This means that we will sell the swap and try to hedge the position by going long the replicating portfolio. On average, the final payoff should be equal to the square of the strike, as the payoffs are based on the variance and not on the volatility.

5b.

Next, we will test how well does the portfolio consisting on the Dynamic Hedge position and the Statig Hedge position can help use to hedge a Variance Swap. In theory, a hedged swap payoff should by pretty close to the variance swap payoff as the realized variance exposure is hedged away.

[93]:
```
def simulations(N, S0, mu, sigma, T, M):
    PLs = []
    for i in range(N):
        S = logNormal_path(S0, mu, sigma, T, M)
        V = dynamic_hedge(S) - static_hedge(S)
        PL = - realized_var(S) + sigma**2 + V
        #print(PL)
        PLs.append(PL)
    PLs = np.array(PLs)
    plt.figure(1)
    plt.hist(PLs, bins = 100, density=False)
    plt.title('30% 1 year Hedged Variance Swap final Payoff')
    #plt.legend()
    plt.show()
    #return PLs
```

[94]:
```
simulations(1000, S0, mu, sigma, T, M)
```



[95]:
```
Strike=0.30
Strike**2
```

[95]: 0.09

As we can see, the final payoff is very close to the strike in all the simulations, which suggest that the hedge works very well not only on average but all along all the paths.

Question 6 - Swaption Pricing

In this question, we will go through the process to compute the value of a swaption. We will explain all the process behind and show the results we obtained in excel.

[1]:
```
from PIL import Image
```

[2]:
```
# We assign a variable to each of the images we will be using

img_a_1 = Image.open('Q6_images/ex_a_1.png')
img_a = Image.open('Q6_images/ex_a.png')
img_b = Image.open('Q6_images/ex_b.png')
img_c_d_cashflow = Image.open('Q6_images/ex_c_d_cashflow.png')
img_c_d_results = Image.open('Q6_images/ex_c_d_results.png')
img_e_results = Image.open('Q6_images/ex_e_results.png')
```

6a.

The first step we need to value a swaption is to have the discount factors that fit exactly the fixed swap rates observed in the market. To to this we follow the next reasoning. We can think as each swap rate to be the coupon rate of a bond paying semiannual payments.

As a swap rate is valued zero at inception, this is the same as saying that the hypothetical bond is priced at par. This means that the discount rates are such that, for the given coupon rates (swap rates), all the possible bonds are priced at par. In an equation, this means that:

$$S(t,T)\sum_{i=1}^{N}\Delta_i Z(t,t_i) + Z(t,T) = 1$$

This can be represented as a matrix operation as follows:

$$S \cdot Z = \begin{pmatrix} 1 \\ 1 \\ . \\ . \\ 1 \end{pmatrix}$$

Where S is a matrix containing all the cashflows for the hypotetical bonds and Z is the column vector of discount factors. Given, that we now have all the cashflows, we can solve for the discount factors. This is what we did in this first part of the question. First, we replicated the excel sheet we viewed in class.

[3]:
```
img_a_1
```

[3]:

	Today	20/06/2013											
	Date	20/12/2013	20/06/2014	20/12/2014	20/06/2015	20/12/2015	20/06/2016	20/12/2016	20/06/2017	20/12/2017	20/06/2018		
Term	Swap Rate:	0.501369863	0.498630137	0.501369863	0.498630137	0.501369863	0.501369863	0.501369863	0.498630137	0.501369863	0.498630137	Final Payoff	Discount Factor
0.5	2.46%	1.01233	-	-	-	-	-	-	-	-	-	1	0.987821
1	2.50%	0.01253	1.01247	-	-	-	-	-	-	-	-	1	0.975459
1.5	2.85%	0.01429	0.01421	1.01429	-	-	-	-	-	-	-	1	0.958329
2	3.20%	0.01604	0.01596	0.01604	1.01596	-	-	-	-	-	-	1	0.938241
2.5	3.40%	0.01705	0.01695	0.01705	0.01695	1.01705	-	-	-	-	-	1	0.918720
3	3.60%	0.01805	0.01795	0.01805	0.01795	0.01805	1.01805	-	-	-	-	1	0.897735
3.5	3.70%	0.01855	0.01845	0.01855	0.01845	0.01855	0.01855	1.01855	-	-	-	1	0.878596
4	3.80%	0.01905	0.01895	0.01905	0.01895	0.01905	0.01905	0.01905	1.01895	-	-	1	0.859038
4.5	3.90%	0.01955	0.01945	0.01955	0.01945	0.01955	0.01955	0.01955	0.01945	1.01955	-	1	0.838925
5	4.00%	0.02005	0.01995	0.02005	0.01995	0.02005	0.02005	0.02005	0.01995	0.02005	1.01995	1	0.818470

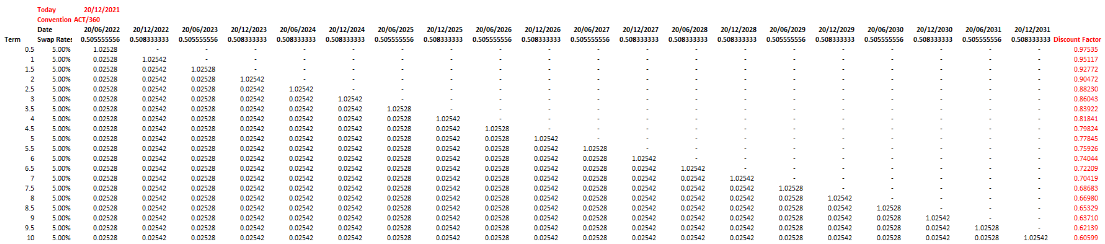
Then we modified this to create a template for a 10 years swap curve. You will find below the Swap sheet screenshot that we built in excel, with defined rates for 1, 2, 3, 5, 7, 10 years tenor. We used the following assumptions to construct the cashflow shcedule:

- The coupon payment conventions for the fixed leg is Act / 360
- The Swap Rate is flat at 5% - which leads to interpolated rates of 5% between defined maturities
- We assumed t_0 as 08/12/2021

By performing the a matrix operation we derived the following discount factors

```
[4]: img_a
```

[4]:



Today 20/12/2021
Convention ACT/360

	Date	20/06/2022	20/12/2022	20/06/2023	20/12/2023	20/06/2024	20/12/2024	20/06/2025	20/12/2025	20/06/2026	20/12/2026	20/06/2027	20/12/2027	20/06/2028	20/12/2028	20/06/2029	20/12/2029	20/06/2030	20/12/2030	20/06/2031	20/12/2031	
Term	Swap Rates	0.505555556	0.508333333	0.505555556	0.508333333	0.508333333	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	0.508333333	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	Discount Factor
0.5	5.00%	1.02528	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.97535
1	5.00%	0.02528	1.02542	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.95117
1.5	5.00%	0.02528	0.02542	1.02528	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.92772
2	5.00%	0.02528	0.02542	0.02528	1.02542	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.90472
2.5	5.00%	0.02528	0.02542	0.02528	0.02542	1.02542	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.88230
3	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	1.02542	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.86043
3.5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	1.02528	-	-	-	-	-	-	-	-	-	-	-	-	-	0.83922
4	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	1.02542	-	-	-	-	-	-	-	-	-	-	-	-	0.81841
4.5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	1.02528	-	-	-	-	-	-	-	-	-	-	-	0.79824
5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	1.02542	-	-	-	-	-	-	-	-	-	-	0.77845
5.5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	1.02528	-	-	-	-	-	-	-	-	-	0.75926
6	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	1.02542	-	-	-	-	-	-	-	-	0.74044
6.5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	1.02542	-	-	-	-	-	-	-	0.72209
7	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	0.02542	1.02542	-	-	-	-	-	-	0.70419
7.5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	1.02528	-	-	-	-	-	0.68683
8	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	1.02542	-	-	-	-	0.66980
8.5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	1.02528	-	-	-	0.65329
9	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	1.02542	-	-	0.63710
9.5	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	1.02528	-	0.62139
10	5.00%	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	0.02542	0.02542	0.02542	0.02528	0.02542	0.02528	0.02542	0.02528	1.02542	0.60599

6b.

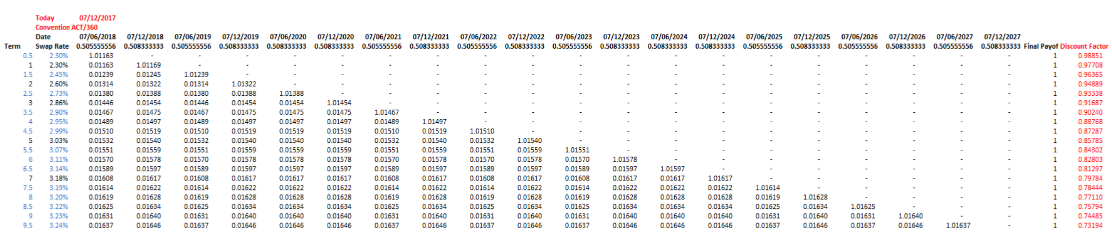
Next, we will use the above templeate to retrieve the discount factors that fit the following swap curve:

- 1Y: 2.30%
- 2Y: 2.60%
- 3Y: 2.86%
- 5Y: 3.03%
- 7Y: 3.18%
- 10Y: 3.25%

Below you will find the screenshot of the Excel Swap sheet with the requested inputs. The rates and terms in blue are interpolated values. The ones in black are pre-defined.You can see the final discount factor curve in the last column of the image.

```
[5]: img_b
```

[5]:



Today 07/12/2017
Convention ACT/360

	Date	07/06/2018	07/12/2018	07/06/2019	07/12/2019	07/06/2020	07/12/2020	07/06/2021	07/12/2021	07/06/2022	07/12/2022	07/06/2023	07/12/2023	07/06/2024	07/12/2024	07/06/2025	07/12/2025	07/06/2026	07/12/2026	07/06/2027	07/12/2027		
Term	Swap Rate	0.505555556	0.508333333	0.505555556	0.508333333	0.508333333	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	0.508333333	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	0.505555556	0.508333333	Final Payof	Discount Factor
0.5	2.30%	1.01163	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.98851
1	2.30%	0.01163	1.01169	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.97708
1.5	2.45%	0.01239	0.01245	1.01239	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.96365
2	2.60%	0.01314	0.01322	0.01314	1.01322	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.94889
2.5	2.73%	0.01380	0.01388	0.01380	0.01388	1.01388	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.93338
3	2.86%	0.01446	0.01454	0.01446	0.01454	0.01454	1.01454	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.91687
3.5	2.90%	0.01467	0.01475	0.01467	0.01475	0.01475	0.01475	1.01467	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.90240
4	2.95%	0.01489	0.01497	0.01489	0.01497	0.01497	0.01497	0.01489	1.01497	-	-	-	-	-	-	-	-	-	-	-	-	1	0.88768
4.5	2.99%	0.01510	0.01519	0.01510	0.01519	0.01519	0.01519	0.01510	0.01519	1.01510	-	-	-	-	-	-	-	-	-	-	-	1	0.87287
5	3.03%	0.01532	0.01540	0.01532	0.01540	0.01540	0.01540	0.01532	0.01540	0.01532	1.01540	-	-	-	-	-	-	-	-	-	-	1	0.85785
5.5	3.07%	0.01551	0.01559	0.01551	0.01559	0.01559	0.01559	0.01551	0.01559	0.01551	0.01559	1.01551	-	-	-	-	-	-	-	-	-	1	0.84302
6	3.11%	0.01570	0.01578	0.01570	0.01578	0.01578	0.01578	0.01570	0.01578	0.01570	0.01578	0.01570	1.01578	-	-	-	-	-	-	-	-	1	0.82803
6.5	3.14%	0.01589	0.01597	0.01589	0.01597	0.01597	0.01597	0.01589	0.01597	0.01589	0.01597	0.01589	0.01597	1.01597	-	-	-	-	-	-	-	1	0.81297
7	3.18%	0.01608	0.01617	0.01608	0.01617	0.01617	0.01617	0.01608	0.01617	0.01608	0.01617	0.01608	0.01617	0.01617	1.01617	-	-	-	-	-	-	1	0.79784
7.5	3.19%	0.01614	0.01622	0.01614	0.01622	0.01622	0.01622	0.01614	0.01622	0.01614	0.01622	0.01614	0.01622	0.01622	0.01622	1.01614	-	-	-	-	-	1	0.78444
8	3.20%	0.01619	0.01628	0.01619	0.01628	0.01628	0.01628	0.01619	0.01628	0.01619	0.01628	0.01619	0.01628	0.01628	0.01628	0.01619	1.01628	-	-	-	-	1	0.77110
8.5	3.22%	0.01625	0.01634	0.01625	0.01634	0.01634	0.01634	0.01625	0.01634	0.01625	0.01634	0.01625	0.01634	0.01634	0.01634	0.01625	0.01634	1.01625	-	-	-	1	0.75794
9	3.23%	0.01631	0.01640	0.01631	0.01640	0.01640	0.01640	0.01631	0.01640	0.01631	0.01640	0.01631	0.01640	0.01640	0.01640	0.01631	0.01640	0.01631	1.01640	-	-	1	0.74485
9.5	3.24%	0.01637	0.01646	0.01637	0.01646	0.01646	0.01646	0.01637	0.01646	0.01637	0.01646	0.01637	0.01646	0.01646	0.01646	0.01637	0.01646	0.01637	0.01646	1.01637	-	1	0.73194
10	3.25%	0.01643	0.01652	0.01643	0.01652	0.01652	0.01652	0.01643	0.01652	0.01643	0.01652	0.01643	0.01652	0.01652	0.01652	0.01643	0.01652	0.01643	0.01652	0.01643	1.01652	1	0.71910

6c. and 6d.

For the next to question, we used the above excersice to value a 1x7 swaption for both the receiver and the payer leg. To simplify the visualisation of the results, we merged the answer from 6c and 6d into the same spreadsheet. To valuate the swaption, we used the same inputs from questions a and b in both questions. Besides we also took the following main assumptions:

- Rates:
  - 1Y: 2.3%

- 2Y: 2.6%
- 3Y: 2.86%
- 5Y: 3.03%
- 7Y: 3.18%
- 10Y: 3.25%

- Interpolation method: Linear
- Day count convention: Act / 360
- Valuation date: 07/12/2017
- Notional: 1.000.000,00 euros
- 1 year optionality
- 7 years maturity starting today
- Fixed leg payments: semi-annual

[6]:
```
# Cashflow and discount factor curve:

img_c_d_cashflow
```

[6]:

	Today	07/12/2017									
	Type	2	ACT/360								
	Date	07/06/2018	07/12/2018	07/06/2019	07/12/2019	07/06/2020	07/12/2020	07/06/2021	07/12/2021	07/06/2022	07/12/2022
Term	Swap Rates	0.50556	0.50833	0.50556	0.50833	0.50833	0.50833	0.50556	0.50833	0.50556	0.50833
0.5	2.30%	1.01163	-	-	-	-	-	-	-	-	-
1	2.30%	0.01163	1.01169	-	-	-	-	-	-	-	-
1.5	2.45%	0.01239	0.01245	1.01239	-	-	-	-	-	-	-
2	2.60%	0.01314	0.01322	0.01314	1.01322	-	-	-	-	-	-
2.5	2.73%	0.01380	0.01388	0.01380	0.01388	1.01388	-	-	-	-	-
3	2.86%	0.01446	0.01454	0.01446	0.01454	0.01454	1.01454	-	-	-	-
3.5	2.90%	0.01467	0.01475	0.01467	0.01475	0.01475	0.01475	1.01467	-	-	-
4	2.95%	0.01489	0.01497	0.01489	0.01497	0.01497	0.01497	0.01489	1.01497	-	-
4.5	2.99%	0.01510	0.01519	0.01510	0.01519	0.01519	0.01519	0.01510	0.01519	1.01510	-
5	3.03%	0.01532	0.01540	0.01532	0.01540	0.01540	0.01540	0.01532	0.01540	0.01532	1.01540
5.5	3.07%	0.01551	0.01559	0.01551	0.01559	0.01559	0.01559	0.01551	0.01559	0.01551	0.01559
6	3.11%	0.01570	0.01578	0.01570	0.01578	0.01578	0.01578	0.01570	0.01578	0.01570	0.01578
6.5	3.14%	0.01589	0.01597	0.01589	0.01597	0.01597	0.01597	0.01589	0.01597	0.01589	0.01597
7	3.18%	0.01608	0.01617	0.01608	0.01617	0.01617	0.01617	0.01608	0.01617	0.01608	0.01617
7.5	3.19%	0.01614	0.01622	0.01614	0.01622	0.01622	0.01622	0.01614	0.01622	0.01614	0.01622
8	3.20%	0.01619	0.01628	0.01619	0.01628	0.01628	0.01628	0.01619	0.01628	0.01619	0.01628
8.5	3.22%	0.01625	0.01634	0.01625	0.01634	0.01634	0.01634	0.01625	0.01634	0.01625	0.01634
9	3.23%	0.01631	0.01640	0.01631	0.01640	0.01640	0.01640	0.01631	0.01640	0.01631	0.01640
9.5	3.24%	0.01637	0.01646	0.01637	0.01646	0.01646	0.01646	0.01637	0.01646	0.01637	0.01646
10	3.25%	0.01643	0.01652	0.01643	0.01652	0.01652	0.01652	0.01643	0.01652	0.01643	0.01652

Date	07/06/2023	07/12/2023	07/06/2024	07/12/2024	07/06/2025	07/12/2025	07/06/2026	07/12/2026	07/06/2027	07/12/2027	
Term	0.50556	0.50833	0.50833	0.50833	0.50556	0.50833	0.50556	0.50833	0.50556	0.50833	D_Factor
0.5	-	-	-	-	-	-	-	-	-	-	0.98851
1	-	-	-	-	-	-	-	-	-	-	0.97708
1.5	-	-	-	-	-	-	-	-	-	-	0.96365
2	-	-	-	-	-	-	-	-	-	-	0.94889
2.5	-	-	-	-	-	-	-	-	-	-	0.93338
3	-	-	-	-	-	-	-	-	-	-	0.91687
3.5	-	-	-	-	-	-	-	-	-	-	0.90240
4	-	-	-	-	-	-	-	-	-	-	0.88768
4.5	-	-	-	-	-	-	-	-	-	-	0.87287
5	-	-	-	-	-	-	-	-	-	-	0.85785
5.5	1.01551	-	-	-	-	-	-	-	-	-	0.84302
6	0.01570	1.01578	-	-	-	-	-	-	-	-	0.82803
6.5	0.01589	0.01597	1.01597	-	-	-	-	-	-	-	0.81297
7	0.01608	0.01617	0.01617	1.01617	-	-	-	-	-	-	0.79784
7.5	0.01614	0.01622	0.01622	0.01622	1.01614	-	-	-	-	-	0.78444
8	0.01619	0.01628	0.01628	0.01628	0.01619	1.01628	-	-	-	-	0.77110
8.5	0.01625	0.01634	0.01634	0.01634	0.01625	0.01634	1.01625	-	-	-	0.75794
9	0.01631	0.01640	0.01640	0.01640	0.01631	0.01640	0.01631	1.01640	-	-	0.74485
9.5	0.01637	0.01646	0.01646	0.01646	0.01637	0.01646	0.01637	0.01646	1.01637	-	0.73194
10	0.01643	0.01652	0.01652	0.01652	0.01643	0.01652	0.01643	0.01652	0.01643	1.01652	0.71910

To value the swaption, the first thing we need to find is the forward swap rate. This rate is the rate that will make the future swap to be worth zero at inception. It is defined as:

$$F = \frac{Z(0,T) - Z(0,t_n)}{A(0,T,t_n}$$

Where $Z(0,T)$ and $Z(0,t_n)$ are the discount factors for the date in which the swap begins and at which it matures. On the other hand, $A(0,T,t_n)$ is the sum of discount factors, adjusted by their corresponding year fraction, between both dates. Once we have the forward swap rate, we can find the value of both legs of the swap as follows:

$$V_{Payer} = A(0,T,t_n) \times E[F_0\Phi(d_1) - K\Phi(d_2)]$$

$$V_{Receiver} = A(0,T,t_n) \times E[K\Phi(-d_2) - F_0\Phi(-d_2)]$$

Where:

$$d_1 = \frac{ln\left(\frac{F_0}{K}\right) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

[7]:

```
# Results:

img_c_d_results
```

[7]:

Swaption Calculation

Inputs	
Today	07/12/2017
Notional	1,000,000
t_n	1
T	7
Volatility	20%
Strike	2.15%
Intermediary	
Z(0, tn)	0.9771
Z(0, T)	0.7711
d1	2.31703
d2	2.11703
Outputs	
Annuity(T, t_n)	6.1494
Forward	3.350%
Receiver Value	153.03
Payer Value	73,927.50

Comentaries on answers for C and D:

1. Z(0, t_n) is the discount factor until the expiry date of the option.
2. Z(0, T) is the discount factor untiy the maturity of the swap.
3. The forward rate is referred as 'Forward' in the image.
4. The swap PV01 is referred as Annuity(T, tn).
5. 'Receiver Value' and 'Payer Value' are self-explanatoy. They are the result considering all the inputs shown in the image.

6e.

Finally, we will look at the exposure of both the receiver and the payer at each point of the yield curve that we were given initially. To do this, we changed by 1bps each swap rate at a time and we recorded the value changes in the table below.

[8]:
```
# In the image below you will see the pricing results when we add 1bp to every␣
↪pre-defined swap rate:

img_e_results
```

[8]:

	Payer Value	%	Receiver Value	%
1Y	73,828.05	-0.13%	154.06	0.67%
2Y	73,925.77	0.00%	153.03	0.00%
3Y	73,923.09	-0.01%	153.02	-0.01%
5Y	73,918.47	-0.01%	153.00	-0.02%
7Y	74,386.07	0.62%	148.18	-3.17%
10Y	74,161.42	0.32%	150.60	-1.59%

As we can see in the image above, the swap risk are concentrated in the 1, 7 and 10 years market swap rate. This makes sense as the expiration of the option on the swap is on the first year and the maturity of the embedded swap is on the 8th year, whose swap rate was found by interpolating the 7 and 10 years swap rates. In reality, a swap is exposed to all the swap curve before expiry as the discount factors will change. But, given the formula of the forward swap rate that we illustrated above it makes sense to expect a higher exposure to those discount factors coinciding with the expiry of the option and the future swap contract. In this regard, one could hedge this risk by entering in an offseting IRS position on those specifics maturities. The amount of each hedging swap will be definded by the ratio of sensitivites of the hedging swap and the original swap to that key swap rate. In the case of the 1 year hedging swap, the hedge might need to be done dynamically given the price dynamics of the embedded option.

Question 7 - CDS Valuation and Risk

Now, in this question, we will go through the process on how to value a Credit Default Swap. We will explain all the process behind and show the results we obtained in excel.

[9]:
```
# We assign a variable to each of the images we will be using

img_7a = Image.open('Q7_images/CDS model calibration.png')
img_7b = Image.open('Q7_images/CDS Problem b.png')
img_7c = Image.open('Q7_images/CDS Problem c.png')
img_7d = Image.open('Q7_images/CDS Problem d.png')
```

```python
img_7e = Image.open('Q7_images/CDS Problem e.png')
img_7f = Image.open('Q7_images/CDS Problem f.png')
```

7a.

The first step to value a CDS is to recover the hazard rates that fit exactly the CDS spreads quoted in the market. To do this we need to go through several concepts. First, we need to understand that in a CDS there is a leg offering the protection and a leg paying a premium to get that protection. The value of each leg are degined as follow:

$$PremiumLegPV = C\sum_{n=1}^{N}\Delta_n Q(t_n)Z(t_n)$$

$$ProtectionLegPV = (1-R)\int_0^T Z(t)(-dQ(t))$$

Where:

- C is the premium paid for the protection
- Δ_n is the fraction of the year for that coupon payment
- $Q(t_n)$ is the probability of no default until time t_n
- $Z(t_n)$ is the discount factor
- R is the recovery rate (usually assumed to be equal to 40%)
- $-dQ(t)$ is the probability of default between two small points of time t_n and t_{n+1}

At any given point in time, it must hold that:

$$PVofProtectionLeg = C \cdot RPV01$$

Where:

$$RPV01 = \sum_{n=1}^{N}\Delta_n Q(t_n)Z(t_n)$$

Here, $RPV01$ represents the Risky Present Value of \$1.00. This means that it is the the Present Value of \$1.00 accounting for the probability that its payment is contingent on the reference entity not defaulting. Then, we can find the CDS Spread as:

$$C = \frac{PVofProtectionLeg}{RPV01}$$

Knowing this, we can calibrate a model to construct a CDS spread curve and from there get the value at any moment of any CDS. The only thing that we need to get is the probability of not defaulting. The way to do this is to define such probability as follows:

$$Q(0,T) = exp^{-\int_0^T h(s)ds}$$

Where $h(s)$ is defined as the hazard rate. A market convention is to set the hazard rate to be piecewise flat. It means that it is constant between the quoted CDS spread. This allow us to compute the probability of default at any given point and, therefore, to construct the CDS spread term. Folowing this idea, we replicated the excel sheet we viewed in class. The calibration is done by using Solver

[10]:
```
img_7a
```

[10]:

Value Date	3-Feb-11		CONTRACT VALUATION			
			Long/Short Protection	L	Protention Leg PV	3.346742%
Market inputs		Calibrate	Maturity Date	20-Mar-16	Risky PV01	4.781060
LIBOR Zero Rate	3.00%		Contractual Spread	100	Market CDS Spread	70.000000
Recovery Rate	40%		Notional	10,000,000.00	Contract MTM	-143,432

FITTING

Term	Hazzrd Rate	MKT	Model=Protection/PV01	Protection PV	RPV01	Diff=Market-Moder	DIF^2
1Y	1.12952937%	60	60.000000	0.740771%	1.234618	0.000000	0.000000
2Y	1.20798503%	65	65.000000	1.415398%	2.177535	0.000000	0.000000
3Y	1.21254392%	67	67.000000	2.064670%	3.081598	0.000000	0.000000
4Y	1.20812447%	68	68.000000	2.684922%	3.948415	0.000000	0.000000
5Y	1.34176565%	70	70.000000	3.346742%	4.781060	0.000000	0.000000
						MINIMISE->	0.00000000

PCD: 20-Dec-10

Payment dates also used for protection leg	Yearfrac Function ACT 360	Piecewise flat Hazard Rate Curve	Time in years Days/365.242	Q(t)	-dQ(t)	Z(0,t)
04-Feb-11			0	1		1
20-Mar-11	0.250000000	1.13%	0.120468	0.99864	0.001360	0.996392
20-Jun-11	0.255555556	1.13%	0.372356	0.99580	0.002837	0.988891
20-Sep-11	0.255555556	1.13%	0.624244	0.99297	0.002829	0.981447
20-Dec-11	0.252777778	1.13%	0.873394	0.99018	0.002791	0.974138
20-Mar-12	0.252777778	1.13%	1.122543	0.98740	0.002783	0.966884
20-Jun-12	0.255555556	1.21%	1.374431	0.98440	0.003000	0.959606
20-Sep-12	0.255555556	1.21%	1.626319	0.98141	0.002991	0.952382
20-Dec-12	0.252777778	1.21%	1.875469	0.97846	0.002949	0.945289
20-Mar-13	0.250000000	1.21%	2.121881	0.97555	0.002908	0.938327
20-Jun-13	0.255555556	1.21%	2.373769	0.97258	0.002975	0.931263
20-Sep-13	0.255555556	1.21%	2.625656	0.96961	0.002966	0.924253
20-Dec-13	0.252777778	1.21%	2.874806	0.96669	0.002925	0.917370
20-Mar-14	0.250000000	1.21%	3.121218	0.96380	0.002884	0.910614
20-Jun-14	0.255555556	1.21%	3.373106	0.96087	0.002929	0.903758
20-Sep-14	0.255555556	1.21%	3.624994	0.95795	0.002920	0.896955
20-Dec-14	0.252777778	1.21%	3.874144	0.95508	0.002879	0.890276
20-Mar-15	0.250000000	1.21%	4.120556	0.95224	0.002839	0.883719
20-Jun-15	0.255555556	1.34%	4.372443	0.94902	0.003213	0.877066
20-Sep-15	0.255555556	1.34%	4.624331	0.94582	0.003202	0.870463
20-Dec-15	0.252777778	1.34%	4.873481	0.94266	0.003157	0.863981
20-Mar-16	0.252777778	1.34%	5.122631	0.93952	0.003146	0.857547

Then we modified this to create a template for a 10 years swap curve. You will find below the Swap sheet screenshot that we built in excel, with defined rates for 1, 2, 3, 5, 7, 10 years tenor. We used the following assumptions to construct the cashflow shcedule:

- The coupon payment conventions for the fixed leg is Act / 360
- The Swap Rate is flat at 5% - which leads to interpolated rates of 5% between defined maturities
- We assumed t0 as 08/12/2021

By performing the a matrix operation we derived the following discount factors

7b.

Next, we will use the above templeate to retrieve the term strucutre of hazard rates and survival probabilites given the following points on the CDS curve on the 8th December 2017:

- 1Y: 65bps
- 2Y: 69bps

- 3Y: 75bps
- 4Y: 80bps
- 5Y: 83bps

Below you will find the screenshot of the results we obtained in excel.

[11]:
```
img_7b
```

[11]:

Value Date	8-Dec-17		CONTRACT VALUATION				
			Long/Short Protection	L	Protention Leg PV	3.963048%	
Market inputs		Calibrate	Maturity Date	20-Mar-16	Risky PV01	4.774757	
LIBOR Zero Rate	3.00%		Contractual Spread	100	Market CDS Spread	83.000000	
Recovery Rate	40%		Notional	10,000,000.00	Contract MTM	-81,171	

FITTING

Term	Hazzrd Rate	MKT	Model=Protection/PV01	Protection PV	RPV01	Diff=Market-Moder	DIF^2
1Y	1.33699547%	65	65.000000	0.804716%	1.238025	0.000000	0.000000
2Y	1.25346000%	69	69.000000	1.505779%	2.182288	0.000000	0.000000
3Y	1.50973857%	75	75.000000	2.316166%	3.088222	0.000000	0.000000
4Y	1.65193780%	80	80.000000	3.160852%	3.951065	0.000000	0.000000
5Y	1.64342860%	83	83.000000	3.963048%	4.774757	0.000000	0.000000
						MINIMISE->	0.00000000

PCD Payment dates also used for protection leg	20-Sep-17 Yearfrac Function ACT 360	Piecewise flat Hazard Rate Curve	Time in years Days/365.242	Q(t)	-dQ(t)	Z(0,t)
09-Dec-17			0	1		1
20-Dec-17	0.252777778	1.34%	0.030117	0.99960	0.000403	0.999097
20-Mar-18	0.250000000	1.34%	0.276529	0.99631	0.003288	0.991738
20-Jun-18	0.255555556	1.34%	0.528417	0.99296	0.003350	0.984272
20-Sep-18	0.255555556	1.34%	0.780305	0.98962	0.003338	0.976863
20-Dec-18	0.252777778	1.34%	1.029454	0.98633	0.003291	0.969588
20-Mar-19	0.250000000	1.25%	1.275866	0.98329	0.003042	0.962447
20-Jun-19	0.255555556	1.25%	1.527754	0.98019	0.003100	0.955202
20-Sep-19	0.255555556	1.25%	1.779642	0.97710	0.003090	0.948011
20-Dec-19	0.252777778	1.25%	2.028792	0.97405	0.003047	0.940951
20-Mar-20	0.252777778	1.51%	2.277942	0.97040	0.003657	0.933945
20-Jun-20	0.255555556	1.51%	2.529830	0.96671	0.003683	0.926914
20-Sep-20	0.255555556	1.51%	2.781717	0.96304	0.003669	0.919936
20-Dec-20	0.252777778	1.51%	3.030867	0.95943	0.003616	0.913085
20-Mar-21	0.250000000	1.65%	3.277279	0.95553	0.003897	0.906360
20-Jun-21	0.255555556	1.65%	3.529167	0.95156	0.003968	0.899537
20-Sep-21	0.255555556	1.65%	3.781055	0.94761	0.003951	0.892765
20-Dec-21	0.252777778	1.65%	4.030205	0.94372	0.003892	0.886117
20-Mar-22	0.250000000	1.64%	4.276617	0.93990	0.003814	0.879591
20-Jun-22	0.255555556	1.64%	4.528504	0.93602	0.003883	0.872969
20-Sep-22	0.255555556	1.64%	4.780392	0.93216	0.003867	0.866397
20-Dec-22	0.252777778	1.64%	5.029542	0.92835	0.003809	0.859946

7c.

Now that we constructed the term strucutre of hazard rates and survival probabilities we can find the market spread at any given point in time. We will get the 3.5 years CDS market spread. To do this we need to use the following formula:

$$C = \frac{PV of Protection Leg}{RPV01}$$

As we previously stated, to get these values we need to use the following:

$$RPV01 = \sum_{n=1}^{N} \Delta_n Q(t_n) Z(t_n)$$

On the excel sheet, this is done by applying a sumproduct function to the columns Yearfrac Function, Q(t) and Z(0,t). Yearfrac is the fraction of a year that represents each coupon payment. Q(t) is found by using the fact that $Q(0,T) = exp^{-\int_0^T h(s)ds}$ and that h(s) is piecewise flat. Finally, Z(0,t) are simply the discount factors.

$$ProtectionLegPV = (1 - R)\int_0^T Z(t)(-dQ(t))$$

On the excel sheet, this is done by multiplying (1-Recovery rate) times the sumproduct of the columns Z(0,t) and -dQ(t). Where -dQ(t) is simply the difference between Q(t) at any given point and the precedent Q(t-1). The obtained the 3.5 years CDS market spread is 77.828 basis points.

[12]:
```
img_7c
```

[12]:

Value Date	8-Dec-17
Market inputs	
LIBOR Zero Rate	3.00%
Recovery Rate	40%

Calibrate

CONTRACT VALUATION

Long/Short Protection	L	Protention Leg PV	3.963048%
Maturity Date	20-Mar-16	Risky PV01	4.774757
Contractual Spread	100	Market CDS Spread	83.000000
Notional	10,000,000.00	Contract MTM	-81,171

FITTING

Term	Hazzrd Rate	MKT	Model=Protection/PV01	Protection PV	RPV01	Diff=Market-Moder	DIF^2
1Y	1.33699547%	65	65.000000	0.804716%	1.238025	0.000000	0.000000
2Y	1.25346000%	69	69.000000	1.505779%	2.182288	0.000000	0.000000
3Y	1.50973857%	75	75.000000	2.316166%	3.088222	0.000000	0.000000
4Y	1.65193780%	80	80.000000	3.160852%	3.951065	0.000000	0.000000
5Y	1.64342860%	83	83.000000	3.963048%	4.774757	0.000000	0.000000
						MINIMISE->	0.00000000

PCD	20-Sep-17					
Payment dates also used for protection le	Yearfrac Function ACT 360	Piecewise flat Hazard Rate Curve	Time in years Days/365.242	Q(t)	-dQ(t)	Z(0,t)
09-Dec-17			0	1		1
20-Dec-17	0.252777778	1.34%	0.030117	0.99960	0.000403	0.999097
20-Mar-18	0.250000000	1.34%	0.276529	0.99631	0.003288	0.991738
20-Jun-18	0.255555556	1.34%	0.528417	0.99296	0.003350	0.984272
20-Sep-18	0.255555556	1.34%	0.780305	0.98962	0.003338	0.976863
20-Dec-18	0.252777778	1.34%	1.029454	0.98633	0.003291	0.969588
20-Mar-19	0.250000000	1.25%	1.275866	0.98329	0.003042	0.962447
20-Jun-19	0.255555556	1.25%	1.527754	0.98019	0.003100	0.955202
20-Sep-19	0.255555556	1.25%	1.779642	0.97710	0.003090	0.948011
20-Dec-19	0.252777778	1.25%	2.028792	0.97405	0.003047	0.940951
20-Mar-20	0.252777778	1.51%	2.277942	0.97040	0.003657	0.933945
20-Jun-20	0.255555556	1.51%	2.529830	0.96671	0.003683	0.926914
20-Sep-20	0.255555556	1.51%	2.781717	0.96304	0.003669	0.919936
20-Dec-20	0.252777778	1.51%	3.030867	0.95943	0.003616	0.913085
20-Mar-21	0.250000000	1.65%	3.277279	0.95553	0.003897	0.906360
20-Jun-21	0.255555556	1.65%	3.529167	0.95156	0.003968	0.899537
20-Sep-21	0.255555556	1.65%	3.781055	0.94761	0.003951	0.892765
20-Dec-21	0.252777778	1.65%	4.030205	0.94372	0.003892	0.886117
20-Mar-22	0.250000000	1.64%	4.276617	0.93990	0.003814	0.879591
20-Jun-22	0.255555556	1.64%	4.528504	0.93602	0.003883	0.872969
20-Sep-22	0.255555556	1.64%	4.780392	0.93216	0.003867	0.866397
20-Dec-22	0.252777778	1.64%	5.029542	0.92835	0.003809	0.859946

3.5 CDS Market Spread	
Protection PV	2.742265%
RPV01	3.523482
Model=Protection/PV01	77.828

[

7d.

Next, we will use the above excercie to value a long protection CDS contract traded with a contractual spread of 115bps maturing the 20th June 2021 with a notional of $10 millions. To do this we need to define the value of the protection leg as:

$$V(t) = (S(t,T) - C) \cdot RPV01(T,t)$$

This can be understood as the difference the protection buyer should pay to buy at the market quoted CDS spread at somewhat equal protection to the one he initially bought. The value we obtained was $\$130,974$.

[13]:
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img_7d
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[13]:

Value Date	8-Dec-17
Market inputs	
LIBOR Zero Rate	3.00%
Recovery Rate	40%

Calibrate

CONTRACT VALUATION

Long/Short Protection	L	Protention Leg PV	2.742265%
Maturity Date	20-Jun-21	Risky PV01	3.523482
Contractual Spread	115	Market CDS Spread	77.828257
Notional	10,000,000.00	Contract MTM	-130,974

FITTING

Term	Hazzrd Rate	MKT	Model=Protection/PV01	Protection PV	RPV01	Diff=Market-Moder	DIF^2
1Y	1.33699547%	65	65.000000	0.804716%	1.238025	0.000000	0.000000
2Y	1.25346000%	69	69.000000	1.505779%	2.182288	0.000000	0.000000
3Y	1.50973857%	75	75.000000	2.316166%	3.088222	0.000000	0.000000
4Y	1.65193780%	80	80.000000	3.160852%	3.951065	0.000000	0.000000
5Y	1.64342860%	83	83.000000	3.963048%	4.774757	0.000000	0.000000
						MINIMISE->	0.00000000

PCD	20-Sep-17					
Payment dates also used for protection leg	Yearfrac Function ACT 360	Piecewise flat Hazard Rate Curve	Time in years Days/365.242	Q(t)	-dQ(t)	Z(0,t)
09-Dec-17			0	1		1
20-Dec-17	0.252777778	1.34%	0.030117	0.99960	0.000403	0.999097
20-Mar-18	0.250000000	1.34%	0.276529	0.99631	0.003288	0.991738
20-Jun-18	0.255555556	1.34%	0.528417	0.99296	0.003350	0.984272
20-Sep-18	0.255555556	1.34%	0.780305	0.98962	0.003338	0.976863
20-Dec-18	0.252777778	1.34%	1.029454	0.98633	0.003291	0.969588
20-Mar-19	0.250000000	1.25%	1.275866	0.98329	0.003042	0.962447
20-Jun-19	0.255555556	1.25%	1.527754	0.98019	0.003100	0.955202
20-Sep-19	0.255555556	1.25%	1.779642	0.97710	0.003090	0.948011
20-Dec-19	0.252777778	1.25%	2.028792	0.97405	0.003047	0.940951
20-Mar-20	0.252777778	1.51%	2.277942	0.97040	0.003657	0.933945
20-Jun-20	0.255555556	1.51%	2.529830	0.96671	0.003683	0.926914
20-Sep-20	0.255555556	1.51%	2.781717	0.96304	0.003669	0.919936
20-Dec-20	0.252777778	1.51%	3.030867	0.95943	0.003616	0.913085
20-Mar-21	0.250000000	1.65%	3.277279	0.95553	0.003897	0.906360
20-Jun-21	0.255555556	1.65%	3.529167	0.95156	0.003968	0.899537
20-Sep-21	0.255555556	1.65%	3.781055	0.94761	0.003951	0.892765
20-Dec-21	0.252777778	1.65%	4.030205	0.94372	0.003892	0.886117
20-Mar-22	0.250000000	1.64%	4.276617	0.93990	0.003814	0.879591
20-Jun-22	0.255555556	1.64%	4.528504	0.93602	0.003883	0.872969
20-Sep-22	0.255555556	1.64%	4.780392	0.93216	0.003867	0.866397
20-Dec-22	0.252777778	1.64%	5.029542	0.92835	0.003809	0.859946

7e.

Now, we will look at how the computed value varies as we change the recovery rate.

[14]:
```
img_7e
```

[14]:

Value Date	8-Dec-17
Market inputs	
LIBOR Zero Rate	3.00%
Recovery Rate	40%

Calibrate

CONTRACT VALUATION

Long/Short Protection	L	Protention Leg PV	2.742265%
Maturity Date	20-Jun-21	Risky PV01	3.523482
Contractual Spread	115	Market CDS Spread	77.828257
Notional	10,000,000.00	Contract MTM	-130,974

FITTING

Term	Hazzrd Rate	MKT	Model=Protection/PV01	Protection PV	RPV01	Diff=Market-Moder	DIF^2
1Y	1.33699547%	65	65.000000	0.804716%	1.238025	0.000000	0.000000
2Y	1.25346000%	69	69.000000	1.505779%	2.182288	0.000000	0.000000
3Y	1.50973857%	75	75.000000	2.316166%	3.088222	0.000000	0.000000
4Y	1.65193780%	80	80.000000	3.160852%	3.951065	0.000000	0.000000
5Y	1.64342860%	83	83.000000	3.963048%	4.774757	0.000000	0.000000
						MINIMISE->	0.00000000

PCD	20-Sep-17					
Payment dates also used for protection leg	Yearfrac Function ACT 360	Piecewise flat Hazard Rate Curve	Time in years Days/365.242	Q(t)	-dQ(t)	Z(0,t)
09-Dec-17			0	1		1
20-Dec-17	0.252777778	1.34%	0.030117	0.99960	0.000403	0.999097
20-Mar-18	0.250000000	1.34%	0.276529	0.99631	0.003288	0.991738
20-Jun-18	0.255555556	1.34%	0.528417	0.99296	0.003350	0.984272
20-Sep-18	0.255555556	1.34%	0.780305	0.98962	0.003338	0.976863
20-Dec-18	0.252777778	1.34%	1.029454	0.98633	0.003291	0.969588
20-Mar-19	0.250000000	1.25%	1.275866	0.98329	0.003042	0.962447
20-Jun-19	0.255555556	1.25%	1.527754	0.98019	0.003100	0.955202
20-Sep-19	0.255555556	1.25%	1.779642	0.97710	0.003090	0.948011
20-Dec-19	0.252777778	1.25%	2.028792	0.97405	0.003047	0.940951
20-Mar-20	0.252777778	1.51%	2.277942	0.97040	0.003657	0.933945
20-Jun-20	0.255555556	1.51%	2.529830	0.96671	0.003683	0.926914
20-Sep-20	0.255555556	1.51%	2.781717	0.96304	0.003669	0.919936
20-Dec-20	0.252777778	1.51%	3.030867	0.95943	0.003616	0.913085
20-Mar-21	0.250000000	1.65%	3.277279	0.95553	0.003897	0.906360
20-Jun-21	0.255555556	1.65%	3.529167	0.95156	0.003968	0.899537
20-Sep-21	0.255555556	1.65%	3.781055	0.94761	0.003951	0.892765
20-Dec-21	0.252777778	1.65%	4.030205	0.94372	0.003892	0.886117
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20-Sep-22	0.255555556	1.64%	4.780392	0.93216	0.003867	0.866397
20-Dec-22	0.252777778	1.64%	5.029542	0.92835	0.003809	0.859946

3.5 CDS Market Spread	
Protection PV	2.742265%
RPV01	3.523482
Model=Protection/PV01	77.828

CONTRACT VALUATION

Recovery Rate	0.00%	-132,205	0.94%
Recovery Rate	10.00%	-131,999	0.78%
Recovery Rate	20.00%	-131,741	0.59%
Recovery Rate	30.00%	-131,412	0.33%

As we can see from the image above, even when we change the recovery rate by a lot, the value of the CDS does not change by a lot. This makes sense as, at every change of R, the hazard rates are recomputed so that the probabilities of survival balance themselves to fit the market spreads. In practice, all the changes are almost exactly offset. This give us a hint that the most sensible part of the model is not the recovery rate but the market spreads.

7f.

Finally, as we suggested above, we will see how the value of the CDS changes as each of the market spreads move by 1bps. The results are shown in the table below.

[15]:
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img_7f
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[15]:

Value Date	8-Dec-17

Market inputs

LIBOR Zero Rate	3.00%
Recovery Rate	40%

Calibrate

CONTRACT VALUATION

Long/Short Protectio	L	Protention Leg PV	2.742265%
Maturity Date	20-Jun-21	Risky PV01	3.523482
Contractual Spread	115	Market CDS Spread	77.828257
Notional	10,000,000.00	Contract MTM	-130,974

FITTING

Term	Hazzrd Rate	MKT	Model=Protection/PV01	Protection PV	RPV01	Diff=Market-Moder	DIF^2
1Y	1.33699547%	65	65.000000	0.804716%	1.238025	0.000000	0.000000
2Y	1.25346000%	69	69.000000	1.505779%	2.182288	0.000000	0.000000
3Y	1.50973857%	75	75.000000	2.316166%	3.088222	0.000000	0.000000
4Y	1.65193780%	80	80.000000	3.160852%	3.951065	0.000000	0.000000
5Y	1.64342860%	83	83.000000	3.963048%	4.774757	0.000000	0.000000
						MINIMISE->	0.00000000

PCD	20-Sep-17						
Payment dates also used for protection leg	Yearfrac Function ACT 360	Piecewise flat Hazard Rate Curve	Time in years Days/365.242	Q(t)	-dQ(t)	Z(0,t)
09-Dec-17			0	1		1
20-Dec-17	0.252777778	1.34%	0.030117	0.99960	0.000403	0.999097
20-Mar-18	0.250000000	1.34%	0.276529	0.99631	0.003288	0.991738
20-Jun-18	0.255555556	1.34%	0.528417	0.99296	0.003350	0.984272
20-Sep-18	0.255555556	1.34%	0.780305	0.98962	0.003338	0.976863
20-Dec-18	0.252777778	1.34%	1.029454	0.98633	0.003291	0.969588
20-Mar-19	0.250000000	1.25%	1.275866	0.98329	0.003042	0.962447
20-Jun-19	0.255555556	1.25%	1.527754	0.98019	0.003100	0.955202
20-Sep-19	0.255555556	1.25%	1.779642	0.97710	0.003090	0.948011
20-Dec-19	0.252777778	1.25%	2.028792	0.97405	0.003047	0.940951
20-Mar-20	0.252777778	1.51%	2.277942	0.97040	0.003657	0.933945
20-Jun-20	0.255555556	1.51%	2.529830	0.96671	0.003683	0.926914
20-Sep-20	0.255555556	1.51%	2.781717	0.96304	0.003669	0.919936
20-Dec-20	0.252777778	1.51%	3.030867	0.95943	0.003616	0.913085
20-Mar-21	0.250000000	1.65%	3.277279	0.95553	0.003897	0.906360
20-Jun-21	0.255555556	1.65%	3.529167	0.95156	0.003968	0.899537
20-Sep-21	0.255555556	1.65%	3.781055	0.94761	0.003951	0.892765
20-Dec-21	0.252777778	1.65%	4.030205	0.94372	0.003892	0.886117
20-Mar-22	0.250000000	1.64%	4.276617	0.93990	0.003814	0.879591
20-Jun-22	0.255555556	1.64%	4.528504	0.93602	0.003883	0.872969
20-Sep-22	0.255555556	1.64%	4.780392	0.93216	0.003867	0.866397
20-Dec-22	0.252777778	1.64%	5.029542	0.92835	0.003809	0.859946

3.5 CDS Market Spread	
Protection PV	2.742265%
RPV01	3.523482
Model=Protection/PV01	77.828

CONTRACT VALUATION		
Change of 1bp on the 1Y spread	-130,967	-0.01%
Change of 1bp on the 2Y spread	-130,961	-0.01%
Change of 1bp on the 3Y spread	-129,427	-1.18%
Change of 1bp on the 4Y spread	-128,975	-1.53%
Change of 1bp on the 5Y spread	-130,974	0.00%

As we can see from the image above, the main change comes from the changes in the 3 and 4 years market spreads. This makes sense as they are the nearest points of our 3.5 years CDS contract. Therefore to hedge the position, one could use two CDS maturing exactly in 3 and 4 years. The amount to have in each node will be a ratio of the changes in both the original contract and the hedging contrat given the change of the heding contract spread. This ratio will denote the sensitivity of our CDS to that specific spread.