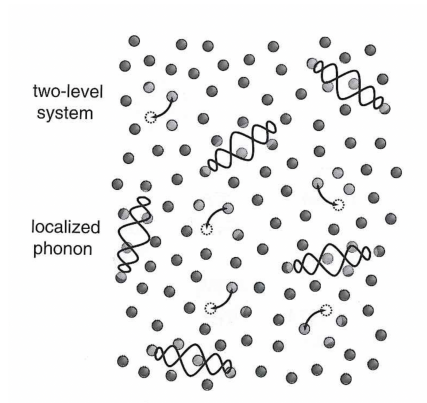


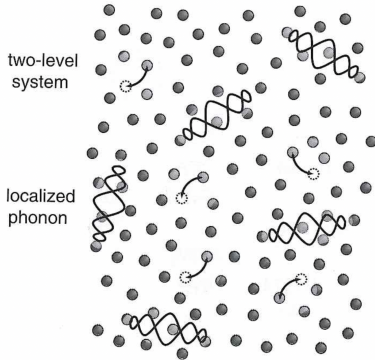
*Electronic effects in disordered systems

Weak localization and conductivity at low temperatures; interpretation of the experimental conductivity trends as a function of the magnetic field

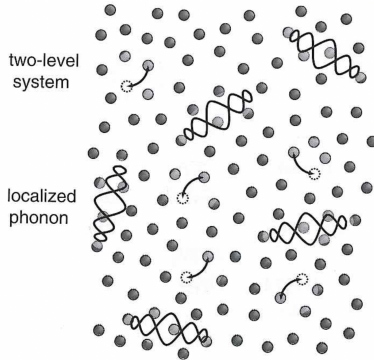
July 1, 2021



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- ✦ For each atom there is a probability to move to the next one and a probability to return back to the previous one

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- ✦ This will affect the electrical conductivity.

Band conduction as an interference effect

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- ✱ This is perhaps easiest to see in a tight binding model where the Hamiltonian explicitly contain terms which corresponds to jumps between neighbouring atoms like:

$$H = E \sum_n |n\rangle\langle n| + T \sum_n (|n\rangle\langle n+1| + |n+1\rangle\langle n|)$$

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- * Solving the time-Independent Schrödinger $H|\psi\rangle = \hbar\omega|\psi\rangle$ equation we yields with the dispersion relation:

$$\omega_k = \frac{2T}{\hbar} \cos ka$$

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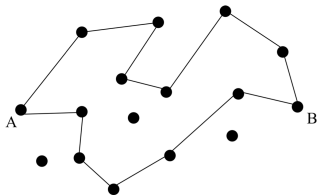
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- * In a crystal the probability for an electron to jump to another atom is independent of the considered atom in the chain.
- * If the perfect symmetry of the crystal lattice is destroyed, the interference pattern is lost, and it is not easy to guess what will happen. The answer is localization: The electrons will not propagate far from the starting point.

- ✱ There is one situation where the analysis of the interference pattern of all the possible alternative paths is simplified. This is when the probability to return to the same site is small, so repeated returns can be ignored. This is far from the insulating state, and only is seen as a small reduction in the conductivity, hence the name weak localization

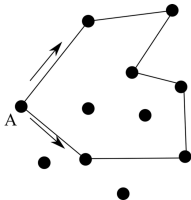
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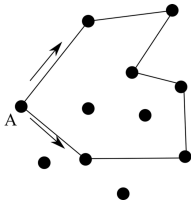
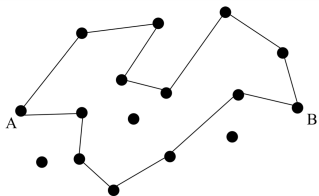
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- ✦ Assume that the electrons are moving on classical trajectories between scattering events.
- ✦ This will be a good approximation if the mean free path, Λ is much larger than the Fermi wavelength, λ_F .



- ✱ the amplitude to go from point A to point B is:

$$A_{A \rightarrow B} = \sum_i A_i e^{iS_i} \quad (1)$$



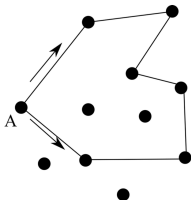
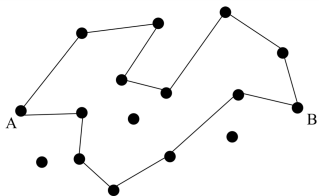


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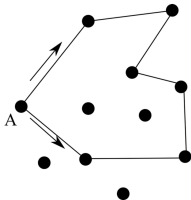
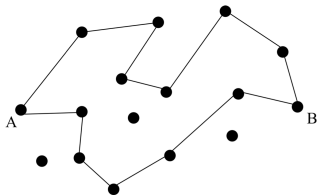
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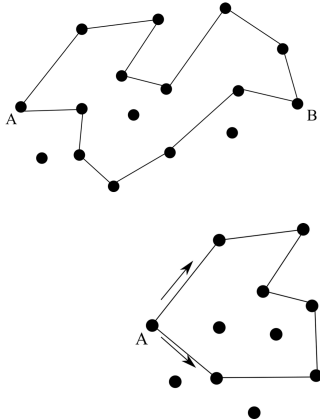
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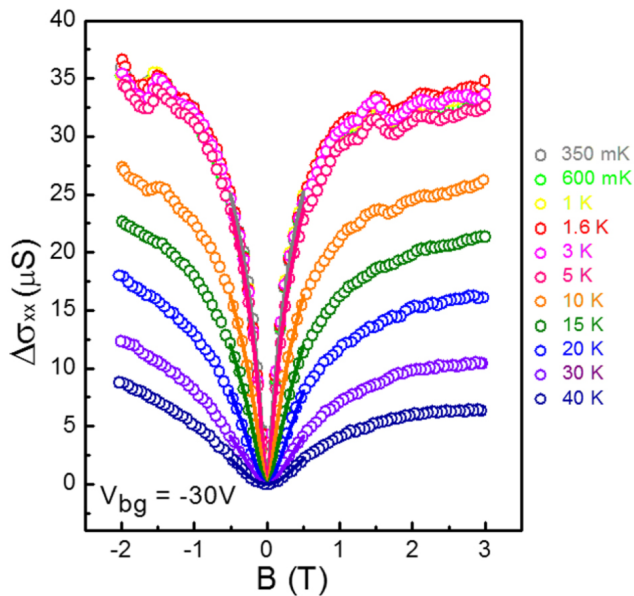
- ✱ The terms in the second sum will then average to zero because of their randomly distributed phases. Thus, we are left with independent contributions from each path and no interference.



- ✦ However, there is a special case where the quantum corrections are much more important. This is the case where the path returns to the starting point. That is, we consider the probability for the particle to remain at the point where it started.



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- ✦ To each geometrical closed path there will correspond two quantum paths (clockwise and counterclockwise directions) As long as time reversal symmetry is not broken these will have the same amplitude and the same phase. This means that the two path interfere constructively, so the electrons will move less than expected classically, this will reduce the conductivity.



- ✱ However, if an electron runs across a closed path, and there is a magnetic field, the electron gain a phase (Berry phase , look at Aharonov-Bohm effect). Taking into account this phase we expect the quantum correction to vanish, and the classical conductivity to be restored.
- ✱ All curves show that the conductivity increases when a magnetic field is applied because the weak localization is removed.
- ✱ The effect is more pronounced at low temperatures. This is because at higher temperatures, the thermal motion of the defects will give random phase changes to the electron waves, and thereby reduce or destroy the phase correlation between the two directions of transversing a loop. Quantum interference is lost by dephasing, and weak localization disappears.
- ✱ When the B increase (in modulus) the electrons gain the berry phase, and the quantum correction disappear.
- ✱ The interference term acquires a phase that bring the average to zero when all paths are taken into account. Thus, we expect the quantum correction to vanish, and the classical conductivity to be restored.