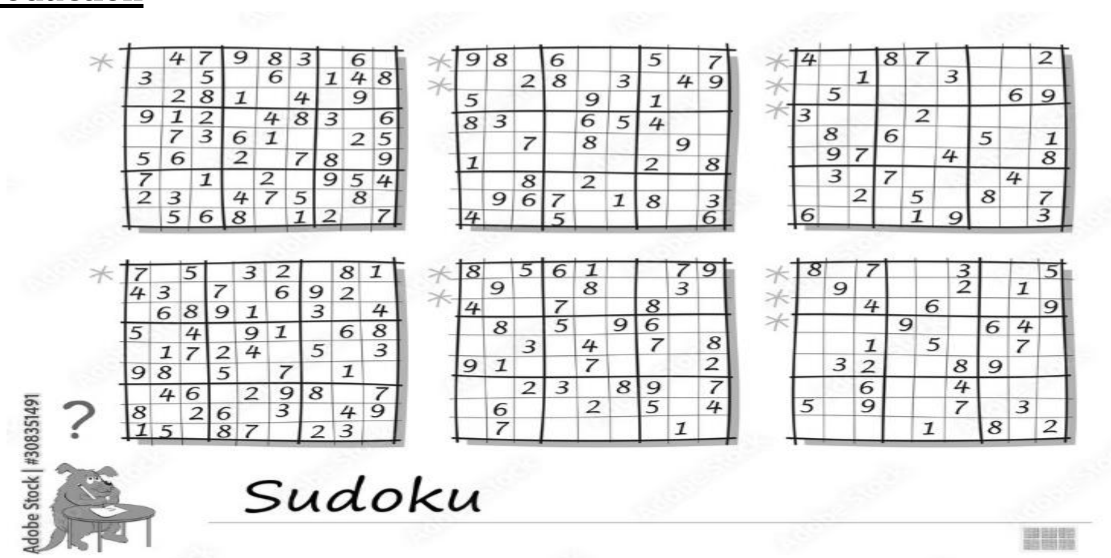
IT159: Artificial Intelligence
Lab#4 & 5: Sudoku (Constraint Satisfaction Problems)

<u>Introduction</u>



In Constraint Satisfaction Problems (CSPs), the goal is to find a complete, consistent assignment of values to a set of variables X (taken from their domains D) satisfying a set of constraints C that limit the valid combinations of variable values. In this assignment, you will have an opportunity to develop a program using CSP solution techniques to solve Sudoku puzzles.

Sudoku (Japanese meaning *number place*) puzzle is a 9x9 grid (81 variables) where each cell in the grid can take on the integer values 1-9 (the domain of each variable). A solution to a Sudoku puzzle is an assignment of values for each cell in the grid such that no two cells in the same row, column, or 3x3 square have the same value.

For example, for an initial configuration of a Sudoku puzzle, you might be given:

```
003|020|600
900|305|001
001|806|400
 -----------
008|102|900
700|000|008
006|708|200
 -----------
002|609|500
800|203|009
005|010|300
```

which has the solution:

```
483|921|657
967|345|821
251|876|493
-----------
548|132|976
729|564|138
136|798|245
-----------
372|689|514
814|253|769
695|417|382
```

Getting Started

The code you will be using can be downloaded as a zip archive on Blackboard, namely **Lab45-Sudoku**.

Assignment

Your assignment is to write a program in Python that can take a set of Sudoku puzzles as input from a file, models each puzzle as a CSP, and outputs solutions to each puzzle.

Extract the files into a directory/folder on your computer which will contain several files of Sudoku puzzles:

1. data/euler.txt, a set of Sudoku puzzles from Project Euler https://projecteuler.net/problem=96
2. data/magictour.txt, a more difficult set of Sudoku puzzles from http://magictour.free.fr/top95

Each file contains a multiple Sudoku puzzles (one per line), in the following format:

- Each line is a string of 81 characters, where characters in positions 0-8 correspond to the first row of the puzzle, characters in positions 9-17 correspond to the second row of the puzzle, etc.
- Known values are represented by the digits 1-9.
- Initially unknown values are represented by digit 0.

And other files that you can and cannot modify as below

Files you'll edit:

`search.py`	Where all your search algorithms will reside.

`csp.py` Class description for constraint satisfaction problem.

Files you should look at but NOT edit:

`util.py` Useful data structures for implementing search algorithms.

`sudoku.py` The main file that runs to solve all Sudoku problems.

Exercise 1: Implement the constraint satisfaction problem in the initialize function in the `csp.py`. How did you represent the Sudoku puzzle a CSP? What design options did you consider, and how did you decide on this implementation?

```python
# INITIALIZING THE CSP
    def __init__(self, domain=digits, grid=""):
        """
        Unitlist consists of the 27 lists of peers
        Units is a dictionary consisting of the keys and the corresponding lists of peers
        Peers is a dictionary consisting of the 81 keys and the corresponding set of 27 peers
        Constraints denote the various all-different constraints between the variables
        """

        self.variables = [row + str(col) for row in 'ABCDEFGHI' for col in range(1, 10)]
        self.digits = "123456789"
        self.units = self.getUnits()
        self.peers = self.getPeers()
        self.values = self.getDict(grid)

    def getUnits(self):
        """
        Defines the 27 units (rows, columns, and 3x3 subgrids).
        Each unit is a list of variables.
        """
        units = []
        units += [[row + str(col) for col in range(1, 10)] for row in 'ABCDEFGHI']
        units += [[chr(65 + row) + str(col) for row in range(9)] for col in range(1, 10)]
        units += [[chr(65 + r) + str(c) for r in range(i, i+3) for c in range(j, j+3)]
                  for i in range(0, 9, 3) for j in range(1, 10, 3)]
        return units


    def getPeers(self):
        """
        Returns a dictionary mapping each variable to its peers (other variables in the same row, column, or 3x3 box).
        """
        peers = {var: set() for var in self.variables}
        for unit in self.units:
            for v in unit:
                peers[v] |= set(unit) - {v}
        return peers

```

A **Constraint Satisfaction Problem (CSP)** consists of:

- **Variables** → 81 grid cells labeled using row-column notation (e.g., A1, B3).
- **Domains** → Each variable (cell) has a domain {1-9}, except for pre-filled cells which have a fixed value.

- **Constraints** → Each row, column, and 3×3 subgrid must contain **all different numbers** (no duplicates).

In **csp.py**, I initialized:

- variables: The set of 81 grid positions {A1, A2, ..., I9}.
- domains: A dictionary mapping each variable to {1-9} or a fixed value if pre-filled.
- constraints: Ensured that **each variable shares constraints with its row, column, and 3×3 box peers**.

Option 1: Direct 2D Array Representation

- Store the Sudoku grid as a **9×9 array** (grid[row][col]).
- Use row/col indices to enforce constraints.
  Pros: Simple to implement.
  Cons: Harder to track dependencies across constraints.

Option 2: Dictionary-Based CSP Representation (Chosen Approach)

- Represent variables as {A1, A2, ..., I9}.
- Store domains as {A1: "123456789", B3: "5", ...}.
- Maintain **peer relationships** (row, column, and 3×3 box constraints).
  Pros: More flexible, easy constraint enforcement.
  Pros: Scalable for constraint propagation (AC-3, Forward Checking).
  Cons: Slightly more complex data structure.

<u>**Exercise 2**</u>: Implement Backtracking Search algorithm in the `search.py`.

```python
def Backtracking_Search(csp):
    """
    Backtracking search initialize the initial assignment
    and calls the recursive backtrack function
    """
    "***YOUR CODE HERE ***"
    if not AC3(csp):  # Optional preprocessing
        return None

    # Initial assignment: variables with a single value
    assignment = dict((v, csp.values[v]) for v in csp.variables if len(csp.values[v]) == 1)
    return Recursive_Backtracking(assignment, csp)

def Recursive_Backtracking(assignment, csp):
    """
    The recursive function which assigns value using backtracking
    """
    "***YOUR CODE HERE ***"
    if isComplete(assignment):
        return assignment

    var = Select_Unassigned_Variables(assignment, csp)
    for value in Order_Domain_Values(var, assignment, csp):
        if isConsistent(var, value, assignment, csp):
            local_assignment = assignment.copy()
            local_assignment[var] = value

            # Optimized deepcopy
            local_csp = deepcopy(csp)
            local_csp.peers = csp.peers  # Avoid deepcopying large static structure

            inferences = {}

            if Inference(local_assignment, inferences, local_csp, var, value) != "FAILURE":
                local_assignment.update(inferences)
                result = Recursive_Backtracking(local_assignment, local_csp)
                if result:
                    return result
    return None
```

<u>Exercise 3</u>: Implement AC-3 search algorithm.

```python
def AC3(csp):
    """
    AC-3 algorithm for constraint propagation.
    Returns True if arc consistency is enforced throughout the CSP.
    """
    queue = deque([(Xi, Xj) for Xi in csp.variables for Xj in csp.peers[Xi]])

    while queue:
        Xi, Xj = queue.popleft()
        if Revise(csp, Xi, Xj):
            if len(csp.values[Xi]) == 0:
                return False  # Failure
            for Xk in csp.peers[Xi] - {Xj}:
                queue.append((Xk, Xi))
    return True


def Revise(csp, Xi, Xj):
    """
    Revise the domain of Xi to enforce arc consistency with Xj.
    Returns True if domain of Xi was revised.
    """
    revised = False
    to_remove = []

    for x in csp.values[Xi]:
        # If no value y in Xj allows (x, y) to satisfy the constraint, remove x
        if all(x == y for y in csp.values[Xj]):
            to_remove.append(x)

    if to_remove:
        for val in to_remove:
            csp.values[Xi] = csp.values[Xi].replace(val, '')
        revised = True

    return revised
```

Your program should be able to read in these puzzles, solve them, then output the solutions in the same format (a string of 81 digits, followed by a newline character) in the same order they were read in from file, so that it is called as follow:

```
python3 sudoku.py --inputFile <PuzzleFile>
```

```
Number of problems solved is:  50
Time taken to solve the puzzles is:  2.141550064086914
```

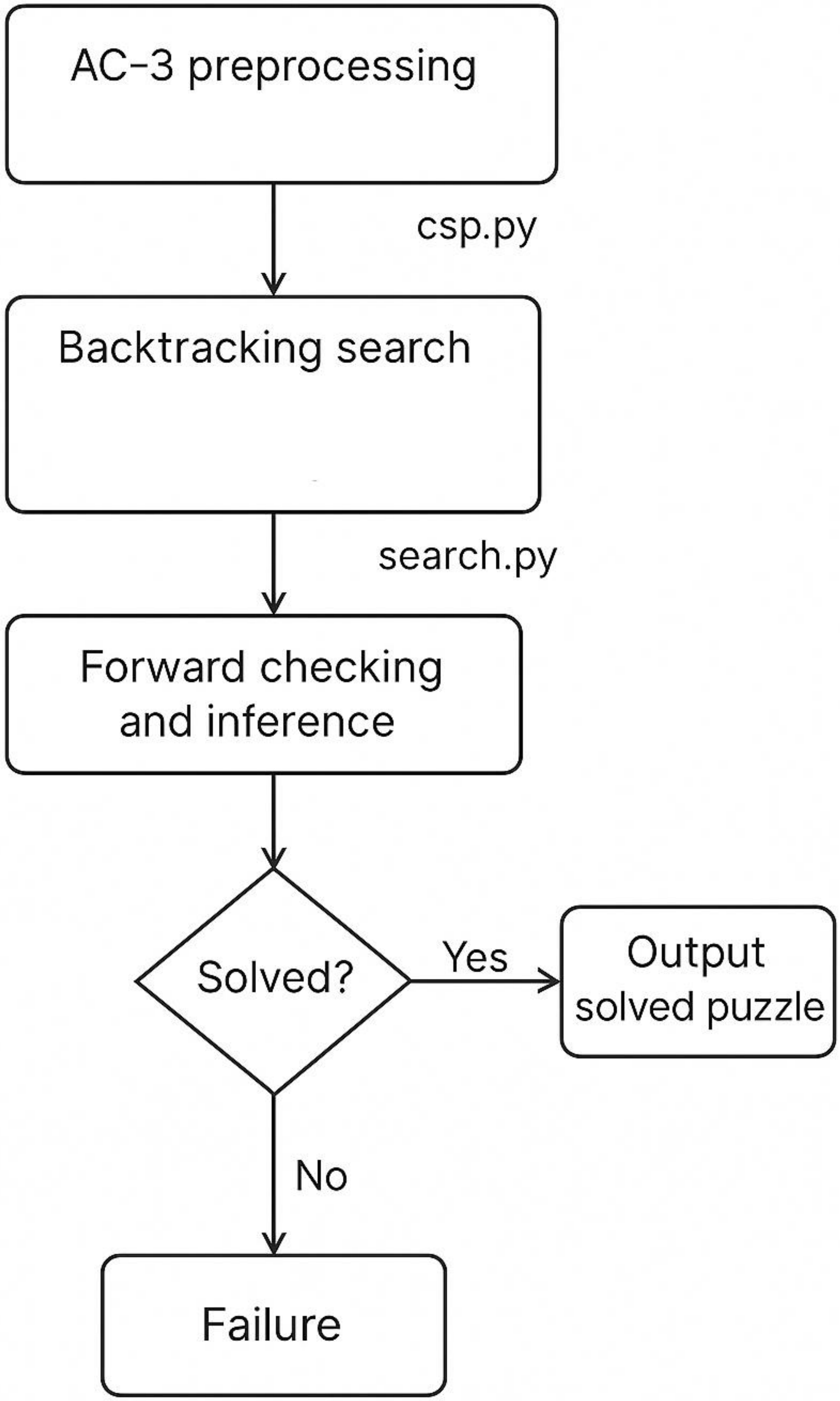
```
Number of problems solved is:  95
Time taken to solve the puzzles is:  522.6215591430664
```

1. In file sudoku_solution.txt

2.
During the assignment, I have struggled with optimizing the algorithm to efficiently handle large problems, as well as converting the Sudoku problem into CSP form for solving. In particular, I had some difficulty working with data files and exporting the results in the required format. However, after some research and experimentation, I was able to complete the course and feel proud of the results.

3.
It took me 2-3 days to complete the assignment.

4. Pipline Analysis


AC-3 preprocessing
csp.py
Backtracking search
search.py
Forward checking
and inference
Solved?
Yes
Output
solved puzzle
No
Failure

First, **AC-3** preprocessing ensures arc consistency by reducing variable domains, removing inconsistent values early. Next, the **Backtracking Search** algorithm assigns values to variables using heuristics like Minimum Remaining Values (MRV) and the Least Constraining Value to prune the search space, and backtracks when a conflict occurs. **Inference** is applied via forward checking, dynamically reducing domains of neighboring variables as values are assigned, further pruning possibilities. The search terminates when all variables are assigned consistent values that satisfy the Sudoku constraints or when no solution is found. Finally, the pipeline is analyzed for correctness, performance, and scalability, with improvements like heuristic tuning or parallelization considered to optimize the solution.

What to submit

1. The solutions to all 145 puzzles in the same format as the input files (please put all of the euler.txt solutions under a header called “Euler” and the magictour.txt solutions under a header called “Magic Tour”)
2. A short paragraph describing your experience during the assignment (what did you enjoy, what was difficult, etc.)
3. An estimation of how much time you spent on the assignment.
4. Source code + README (how to compile and run your code).
5. Please create a folder called "yourname_StudentID_Lab45" that includes all the required files and generate a zip file called "yourname_StudentID_Lab45.zip".
6. Please submit your work (.zip) to Blackboard.