STA457TUT5

For an AR(2) process, we have that:

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + w_t$$

In our case, $\phi_1 = 1.5$, $\phi_2 = -0.75$.

The difference equation for the ACF of the process is:

$$\rho(h) - \phi_1\rho(h-1) - \phi_2\rho(h-2) = 0$$

.

So we have a difference equation:

$$\rho(h) - 1.5\rho(h-1) + 0.75\rho(h-2) = 0$$

Solve for general expression of $\rho(h)$. Let $z_1$and z_2 be the roots of the associated polynomial:

$$\phi(z) = 1 - \phi_1 z - \phi_2 z^2$$

$$\phi(z) = 1 - 1.5z + 0.75z^2$$

where $z_1 = 1 + i\sqrt{3}/3$, $z_2 = 1 - i\sqrt{3}/3$.

The general solutions are:

$$\rho(h) = c_1 z_1^{-h} + c_2 z_2^{-h}$$

For initial conditions,

$$\rho(0) = c_1 + c_2 = 1$$
$$\rho(-1) = c_1 z_1 + c_2 z_2 = \phi_1/(1-\phi_2) = 6/7$$

We solve that : $c_1 = 1/2 + (\sqrt{3}/14)i$, $c_2 = 1/2 - (\sqrt{3}/14)i$

The general solution is given by:

$$\rho(h) = (1/2 + (\sqrt{3}/14)i)(1 + (\sqrt{3}/3)i)^{-h} + (1/2 - (\sqrt{3}/14)i)(1 - (\sqrt{3}/3)i)^{-h}$$

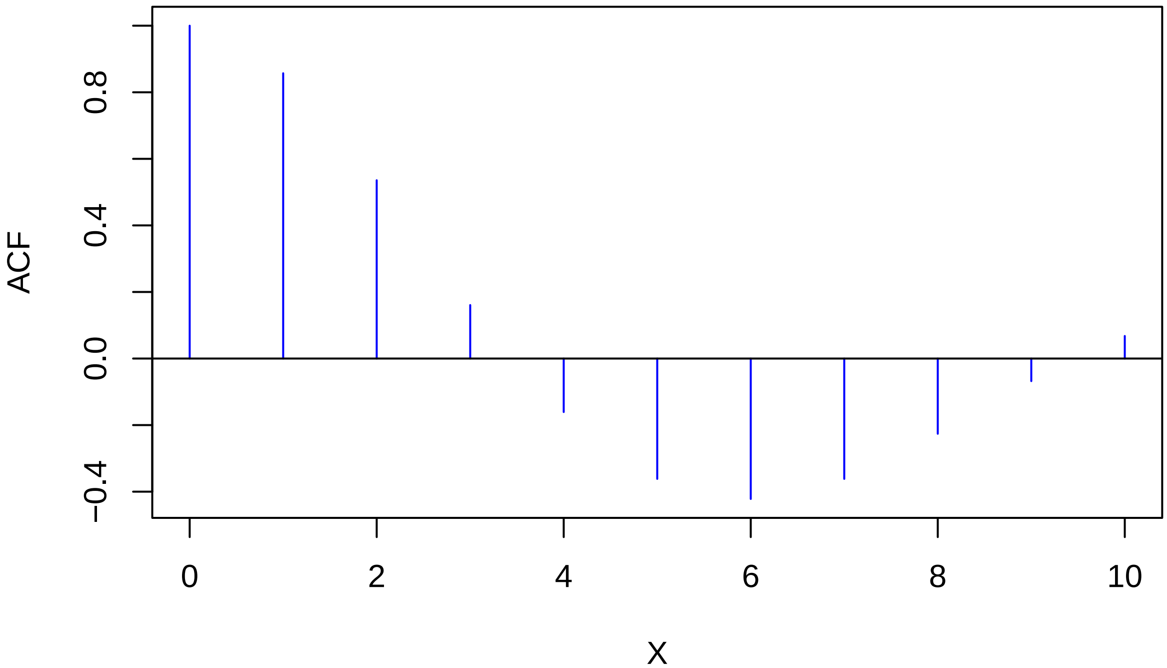
We use a loop to calculate the given recurrence relation.

```
loop <- function(n){

  a <- 1
  b <- 6/7
  for (i in 2:n){
    a_new <- b
```

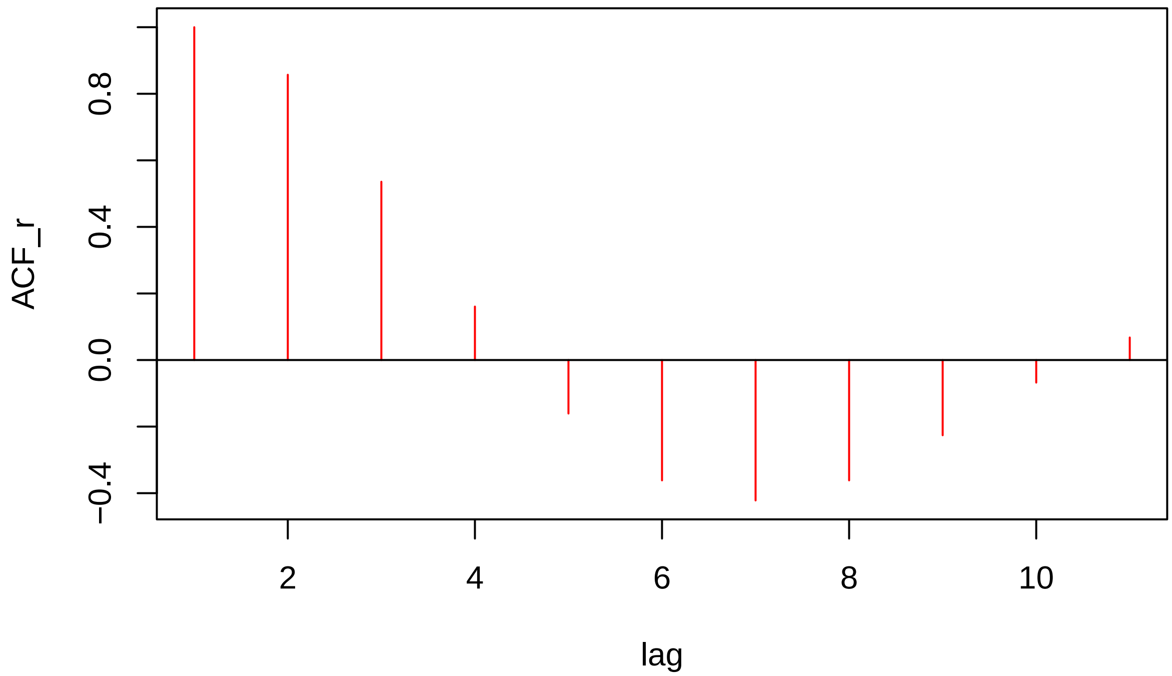
```
    b <- 1.5*b - 0.75*a
    a <- a_new
  }
  return(b)
}
```

```
ACF<-c(1,6/7,loop(2),loop(3),loop(4),loop(5),loop(6),loop(7),loop(8),loop(9),loop(10))
X<-c(0:10)
plot(X,ACF,type = "h",col = "blue")
abline(h=0)
```



We use ARMAacf function to check the value we calculated.

```
ACF_r = ARMAacf(ar= c(1.5,-0.75),ma=0,10)
plot(ACF_r,type = "h",xlab = "lag",col = "red")
abline(h=0)
```



ACF_r
0.8
0.4
0.0
−0.4
2
4
6
8
10
lag