

$$\begin{aligned}
 1) \quad U'_x &= 3x^2 + 3y^2 - 39 & U''_{xx} &= 6x & U'_{yy} &= 6x & U''_{zz} &= 2 \\
 U'_y &= 6xy - 36 & \Rightarrow U''_{xy} &= 6y & U'_{yx} &= 6y & U''_{zy} &= 0 \\
 U'_z &= 2z + 2 & U''_{xz} &= 0 & U''_{yz} &= 0 & U''_{zx} &= 0
 \end{aligned}$$

$$\begin{aligned}
 2) \quad U &= \frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2 \\
 U'_x &= -\frac{256}{x^2} + \frac{2x}{y} & U''_{xx} &= \frac{512}{x^3} + \frac{2}{y} & U''_{yy} &= \frac{x^2 \cdot 2}{y^3} + \frac{2}{z} \\
 U'_y &= -\frac{x^2}{y^2} + \frac{2y}{z} & \Rightarrow U''_{xy} &= -\frac{2x}{y^2} & U''_{yx} &= -\frac{2x}{y^2} \\
 U'_z &= -\frac{y^2}{z^2} + 2z & U''_{xz} &= 0 & U''_{yz} &= -\frac{2y}{z^2}
 \end{aligned}$$

$$U''_{zz} = \frac{-2y^2z}{z^3}$$

$$U''_{zy} = \frac{-2y}{z^2}$$

$$U''_{zx} = 0$$

$$\begin{aligned}
 3) \quad U &= x^2 + y^2 + z^2 \\
 U'_x &= 2x, \quad U'_y = 2y, \quad U'_z = 2z \\
 \text{grad } U &= (2x, 2y, 2z)
 \end{aligned}$$

$$M(8, -12, 9) \quad \text{grad } U = (16, -24, 18)$$

$$|\vec{c}| = \sqrt{(-9)^2 + 8^2 + (-12)^2} = \sqrt{289} = 17$$

$$\vec{c}_0 = \frac{\vec{c}}{|\vec{c}|} = \left( \frac{8}{17}; -\frac{12}{17}; \frac{9}{17} \right)$$

$$\frac{8}{17} \cdot 16 - \frac{12}{17}(-24) + \frac{9}{17} \cdot 18 = \frac{128}{17} + \frac{288}{17} + \frac{162}{17} = 34$$

$$4) U = e^{x^2+y^2+z^2} \quad L(-16, 4, -13) \quad \bar{c}(4, -13, -16)$$

$$U'_x = 2x \cdot e^{x^2+y^2+z^2}$$

$$U'_y = 2y \cdot e^{x^2+y^2+z^2}$$

$$U'_z = 2z \cdot e^{x^2+y^2+z^2}$$

$$\text{grad } U = (-32 \cdot e^{441}; 8e^{441}; -26e^{441})$$

$$|\text{grad } U| = \sqrt{32^2 e^{882} + 8^2 e^{882} + 26^2 e^{882}}$$

$$|\bar{c}| = \sqrt{16 + 13^2 + 16^2} = 21$$

$$c_0 = \frac{\bar{c}}{|\bar{c}|} = \left( \frac{4}{21}; -\frac{13}{21}; -\frac{16}{21} \right)$$

$$\left[ -\frac{32 \cdot 4}{21} - \frac{13 \cdot 8}{21} + \frac{26 \cdot 16}{21} \right] \cdot e^{441} = \frac{e^{441}}{21} \cdot 184$$

$$6) U = x^2 y + \frac{1}{3} y^3 + 2x^2 + 3y^2 - 1$$

$$U'_x = 2xy + 4x = 0$$

$$U'_y = x^2 + y^2 + 6y = 0$$

$$2y + 4 = 0 \quad y = -2$$

$$x^2 + y^2 + 6y = 0 \Rightarrow x^2 + 4 - 12 = 0$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

$$a) (2\sqrt{2}; -2) \quad (2\sqrt{2}; 2)$$

$$b) x = \emptyset \quad y^2 + 6y = 0$$

$$y = 0 \quad y = -6$$

$$(0; 0) \quad (0; -6)$$

$$v''_{xx} = 2y + 4$$

$$v''_{xy} = 2x$$

$$v''_{yy} = 2y + 6$$

$$\Delta_1 = 2y + 4$$

$$(0; 0) \quad (2\sqrt{2}; 2) \Rightarrow \Delta_1 > 0$$

$$(2\sqrt{2}; -2) \Rightarrow \Delta_1 = 0$$

$$(0; -6) \Rightarrow \Delta_1 < 0$$

$$\begin{aligned} \Delta_2 &= (2y+4) \cdot (2y+6) - 4x^2 = 4y^2 + 8y + 12y + 24 - 4x^2 = \\ &= 4y^2 + 20y + 24 - 4x^2 \end{aligned}$$

$$(0; 0) \quad \Delta_2 = 24 > 0 \quad \text{min}$$

$$(0; -6) \quad \Delta_2 = 48 > 0 \quad \text{max}$$

$$(2\sqrt{2}, 2) \quad \Delta_2 > 0 \quad \text{min}$$

$$(2\sqrt{2}, -2) \quad \text{Течее не падает}$$