

$$1) \arctan\left(\frac{y}{x}\right) = \ln \sqrt{x^2 + y^2}$$

$$\frac{1}{1 + \left(\frac{y}{x}\right)^2} \left[\frac{y'}{x} - \frac{y}{x^2} \right] = \frac{1}{\sqrt{x^2 + y^2}} \cdot \frac{1}{2} (2x + 2yy') (x^2 + y^2)^{-\frac{1}{2}}$$

$$\frac{x^2}{y^2 + x^2} \cdot \frac{xy' - y}{x^2} = \frac{2x + 2yy'}{(x^2 + y^2)^{\frac{3}{2}}}$$

$$xy' - y = \frac{2x + 2yy'}{x^2 + y^2}$$

$$xy' - y = 2x$$

$$y'x(x^2 + y^2) - y(x^2 + y^2) = 2x + 2yy'$$

$$y' = \frac{2x + y(x^2 + y^2)}{x(x^2 + y^2) - 2y}$$

$$2) \begin{cases} y = \frac{t^2}{t-1} \\ x = \frac{t}{t^2-1} \end{cases} \Rightarrow \begin{cases} y'_t = \frac{y'_t}{x'_t} \\ x'_t = -\frac{1}{t^2} \end{cases} \Rightarrow y'_t = \frac{1}{-t^2} = -t^2$$

$$3) y = (x^2 + 2)^5 (3x - x^3)^3$$

$$\ln y = \ln (x^2 + 2)^5 \cdot \ln (3x - x^3)^3 = 15 \ln(x^2 + 2) \ln(3x - x^3)$$

$$(\ln y)' \cdot \frac{1}{y'} = 15 \left[\frac{1}{x^2 + 2} \cdot 2x \right] \cdot \ln(3x - x^3) + \frac{3x - 3x^3}{3x - x^3} \ln(x^2 + 2)$$

$$= \frac{30x \cdot \ln(3x - x^3)}{x^2 + 2} + \frac{3(1 - x^2) \ln(x^2 + 2)}{3x - x^3}$$

$$y' = \frac{(x^2 + 2) \cdot (3x - x^3)}{(3x - x^3) \cdot 30x \cdot \ln(3x - x^3) + (x^2 + 2) \cdot 3(1 - x^2) \ln(x^2 + 2)}$$

$$4) y = x^x$$

$$y^x (\ln(x) + 1)$$

$$y'(x) = x^x (1 + \ln(x))$$

$$5) y = \frac{(2-x^2)^3 \cdot (x-1)^2}{(2x^3-3x) \cdot e^x}$$

$$\ln y = 3 \ln(2-x^2) + \ln(x-1) \cdot 2 - x - \ln(2x^3-3x)$$

$$\frac{1}{y'} = \frac{-6x}{2-x^2} + \frac{2}{x-1} - 1 - \frac{6x^2-3}{2x^3-3x} =$$

$$= \frac{-6x(x-1)+2}{(x-1)(2-x^2)} - \left[\frac{6x^2-3+2x^3-3x}{2x^3-3x} \right]$$

$$y' = - \left[\frac{(x-1)(2-x^2)(2x^3-3x)}{[6x(x-1)+2](2x^3-3x) + (2x^3+6x^2-3x-3)} \right]$$

$$6) (\arctan x)' = \frac{1}{1+x^2}$$

$$7) P=144$$

$$S=?$$

$$a=?$$

$$b=?$$

$$P = 2(a+b)$$

$$2(a+b) = 144 \Rightarrow$$

$$a+b = 72$$

$$S = a \cdot b = (72 - b) \cdot b$$

$$S(x) = 72x - x^2$$

$$S'(x) = 72 - 2x$$

$$S'(x) = 0 \Rightarrow 72 - 2x = 0$$

$$x = 36$$

$$S = 36 \cdot 36 = 1296$$