2) $V = 3 - 8x + 6y$

$x^2 + y^2 = 36$

$L(x, y, \lambda) = 3 - 8x + 6y + \lambda(x^2 + y^2 - 36)$

$L'_x = -8 + 2\lambda x = 0 \qquad x = \dfrac{4}{\lambda}$

$L'_y = 6 + 2\lambda y = 0 \qquad y = -\dfrac{3}{\lambda}$

$L'_\lambda = x^2 + y^2 - 36 = 0$

$\dfrac{16}{\lambda^2} + \dfrac{9}{\lambda^2} = 36$

$\lambda = \pm \dfrac{5}{6}$

$\left(\dfrac{24}{5}; -\dfrac{18}{5}; \dfrac{5}{6} \right) \qquad \left(-\dfrac{24}{5}; \dfrac{18}{5}; -\dfrac{5}{6} \right)$

$L''_{xx} = 8 + 2\lambda \qquad\qquad L''_{xy} = \emptyset$

$L''_{yy} = 2\lambda \qquad\qquad\qquad L''_{x\lambda} = 2x$

$L''_{x\lambda} = \emptyset \qquad\qquad\qquad L''_{y\lambda} = 2y$

$\begin{pmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & \emptyset \\ 2y & \emptyset & 2\lambda \end{pmatrix}$

$\Delta = -8\lambda(x^2 + y^2) = -8 \cdot 36\lambda = -288\lambda$

$\lambda = \dfrac{5}{6} \quad \Delta < 0 \quad$ точка мин $\left(\dfrac{24}{5}; -\dfrac{18}{5}; \dfrac{5}{6} \right)$

$\lambda = -\dfrac{5}{6} \quad \Delta > 0 \quad$ точка max $\left(-\dfrac{24}{5}; \dfrac{18}{5}; -\dfrac{5}{6} \right)$

2) $U = 2x^2 + 12xy + 32y^2 + 15$

$x^2 + 16y^2 = 64$

$L(x, y, \lambda) = 2x^2 + 12xy + 32y^2 + 15 + \lambda(x^2 + 16y^2 - 64)$

$L'_x = 4x + 12y + 2\lambda x = 0 \qquad x = \dfrac{-6 \cdot y}{2 + \lambda}$

$L'_y = 12x + 64y + 32\lambda y = 0 \Rightarrow$

$L'_\lambda = x^2 + 16y^2 - 64 = 0 \qquad y = \dfrac{-6x}{32 + 16\lambda} = \dfrac{-3x}{16 + 8\lambda}$

$y = -\dfrac{3}{16 + 8\lambda} \cdot \dfrac{-6y}{2 + \lambda}$

$(2 + \lambda) \cdot (\overset{8}{\cancel{16}} + \overset{4}{\cancel{8}}\lambda) = \overset{9}{\cancel{18}}$

$16 + 8\lambda + 8\lambda + 4\lambda^2 = 9$

$4\lambda^2 + 16\lambda + \cancel{7} = 0$

$\lambda_1 = \dfrac{-16 \pm \sqrt{16^2 - \cancel{112}}}{8}$

$\lambda = \dfrac{-16 \pm \sqrt{144}}{8} = \dfrac{-16 \pm 12}{8}$

$\lambda_1 = -\dfrac{1}{2} \qquad \lambda_2 = -\dfrac{28}{8} = -\dfrac{14}{4} = -\dfrac{7}{2}$

$(0; 0)$ — не является решением

$\begin{cases} x_1 = \dfrac{-6y_1}{2 \cancel{+} \frac{1}{2}} = \dfrac{-\cancel{12}y_1}{\cancel{5}} = \dfrac{6y_1 \cdot 2}{3} = -4y_1 \\[4mm] y_1 = \dfrac{-3x_1}{16 - \frac{8}{2}} = \dfrac{-3x_1}{12} = -\dfrac{x_1}{4} \end{cases}$

$16y_1^2 + 16y_1^2 = 64$

$y_1^2 = 32$

$y_1 = \pm\sqrt{32} \qquad x_1 = \pm 4\sqrt{32}$

$x_2 = -\dfrac{6y_2}{2 - \frac{7}{2}} = -\dfrac{12y_2}{-3} = 4y_2 \qquad 4y_2 \qquad y_2 = \pm 4\sqrt{32}$

$x_2 = \pm 4\sqrt{32} \cdot 4$

$$L''_{xx} = 4 + 2\lambda \qquad L''_{xy} = 12$$

$$L''_{yy} = 64 + 32\lambda \qquad L''_{x\lambda} = 2x$$

$$L''_{\lambda\lambda} = \emptyset \qquad L''_{y\lambda} = 32y$$

$$\begin{pmatrix} \cancel{64+32\lambda}\,0 & 2x & 32y \\ 2x & 4+2\lambda & 12 \\ 32y & 12 & 64+32\lambda \end{pmatrix}$$

$$\Delta = -2x \begin{vmatrix} 2x & 12 \\ 32y & 64+32\lambda \end{vmatrix} + 32y \begin{vmatrix} 2x & 4+2\lambda \\ 32y & 12 \end{vmatrix}$$

$$= -2x \left[2x(64+32\lambda) - 12 \cdot 32y \right]$$

$$+ 32y \left[2x \cdot 12 - 32y(4+2\lambda) \right]$$

$$= -2x^2(64+32\lambda) - 2x \cdot 12 \cdot 32y \; + \; 32y12 \cdot 2x \; -$$

$$- 32^2 y^2 (4+2\lambda) = -64x^2(2+\lambda) - \cancel{}$$

$$- 32^2 \cdot 2 y^2(2+\lambda) = -64(2+\lambda)(x^2 + 32y^2)$$

$$\lambda = -\frac{1}{2} \quad \Delta > 0 \quad \text{точки max} \quad \left(-4\sqrt{2}; \sqrt{2}; -\frac{1}{2}\right) \left(4\sqrt{2}; -\sqrt{2}; -\frac{1}{2}\right)$$

$$\lambda = -\frac{7}{2} \quad \not{A} \; \Delta < 0 \quad \text{точки min} \quad \left(4\sqrt{2}; \sqrt{2}; -\frac{7}{2}\right) \left(-4\sqrt{2}; -\sqrt{2}; -\frac{7}{2}\right)$$