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Bachelor Thesis

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The forecasting of conditional volatility of South American equity markets by back-tested single-order generalized autoregressive conditional heteroskedasticity models

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Abstract

The goal of this bachelor thesis is the development of a model for the financial returns of two indices: the Bovespa Index in Brazil and the Mercado de Valores in Argentina. These models should be able to correctly forecast the conditional volatility and should be able to capture volatility clustering, leverage effects, fat tails, and non-linear dependence. In this thesis the following models are tested: ARCH, GARCH, and EGARCH with different innovation distributions: normal, Student's T, and skewed Student's T. To evaluate model performance the in-sample fit: log-likelihood, Akaike's information criterion, and Bayesian information criterion are computed. Based on these results, the best performing models capture fat tails, skewness, and a memory factor. In order to evaluate the out of sample fit a: graphical backtest, Diebold-Mariano test on mean and median absolute error, Diebold-Mariano test based on the logarithmic scoring rule, and Value at Risk are computed. Based on these results it becomes possible for the Mercado de Valores that the normal distribution outperforms the other distributions and that models that capture fat tails and leverage effects do not always outperforms models which do not utilizes these parameters and that the Value at Risk is more precise estimated at a 5% level than at a 1% level. For the Bovespa Index it is possible to conclude that models which do capture fat tails and skewness outperforms models based on the normal distributions, that models which captures leverage effects do outperform other models, and that the Value at Risk is more precise estimated at a 1% level than at a 5% level. These unsatisfying results, more sophisticated models do not always outperform simpler models, is partly due to changed market dynamics this could the results of the fixed estimation window utilized and for further research rolling estimation window models or regime switching models are recommended.

Keywords: Conditional volatility, Mercado de Valores, Bovespa Index, ARCH, GARCH, EGARCH, Normal distribution, Student's T distribution, skewed Student's T distribution, Backtesting, Graphical backtest, Diebold-Mariano test, Mean absolute error, Median absolute error, Logarithmic scoring rule, Value at Risk, Bernoulli coverage test, Independence test, Joint test.

Contents

1 Introduction

This report is written as a finance bachelor thesis for the study Economics and Business Economics at the Vrije Universiteit Amsterdam. In this report the main aim is to investigate if autoregressive conditional heteroskedasticity models and generalized autoregressive conditional heteroskedasticity models are able to predict future conditional volatility in South American stock market indices. The forecasting of conditional volatility is for example important in financial risk management. Underneath a small introduction and the methodology of this thesis are given.

A stock market index, measures the performance of the largest companies within a country and its financial market. Thereby, this indices provide valuable information on the financial performance of this market. A example of such a index is, the Bovespa Index which consist of approximately sixty stocks traded on the Brasil Bolsa Balcão, abbreviated the B3, in Brazil. Another example, is the Mercado de Valores, abbreviated the Merval index, the most important stock index of Buenos Aires in Argentina. The Merval index consists of maximum sixteen stock exchange funds.

The main research questions are:

1. Is there evidence for volatility clustering in the South American stock market indices?
2. If there is evidence for volatility clustering is it also possible to precisely model conditional volatility of these indices and which model does predict future conditional volatility most precisely?
3. Will models with a memory factor, like the generalized autoregressive conditional heteroskedasticity model, outperform models which do not utilizes a memory factor?
4. Are leverage effects present in the indices? And if so, would a asymmetric model, like the exponential generalized autoregressive conditional heteroskedasticity model, outperform symmetric models?
5. Which innovation distribution yields the most precise forecast on the conditional volatility?

This thesis is structured in a theoretical part and a empirical part. Firstly, the concepts of financial returns and financial volatility are introduced and explained. Secondly, the method of model estimation and the model specifications are introduced and explained. Namely, the autoregressive conditional heteroskedasticity model, the generalized autoregressive conditional heteroskedasticity model, and the exponential generalized autoregressive conditional heteroskedasticity model. Thirdly, three different innovation distributions are introduced and explained. Namely, the normal distribution, the Student's T distribution, and the skewed Student's T distribution. Fourthly, in-sample performance evaluation measures and out of sample performance evaluation measures are introduced and explained. Namely, log-likelihood, AIC, BIC, mean error models, Diebold-Mariono test, and Value at Risk. All the aforementioned concepts together form the theoretical part of this thesis. The empirical part starts with introducing the data used and plotting prices, returns, auto-correlation plots, and quantile to quantile plots. Thereafter, the model estimates and results on the forecasting and model performance evaluation measures are given and reflected on. Lastly, a final conclusion is given.

2 Background

2.1 Returns

The end of day closing prices of securities are non stationary time series, so they do not have a constant mean and variance. Therefore, instead of prices daily returns are used in this report (Tsay 2010). A financial return is the relative change in the price of a financial asset over a certain time interval, which is practice expressed as a percentage. In this report, dividend payments are excluded in the computation of the financial returns.

In finance two types of returns are often applied namely simple returns and compounded returns. A simple net return is the percentage change in prices, indicated by R, and is given by:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

A continuously compounded return is the logarithm of gross return, indicated by Y_t, and is given by:

$$Y_t = \log(P_t) - \log(P_{t-1}) \tag{2}$$

In a strict sense only the use of simple returns is correct and therefore used for accounting purposes and investors are interested in simple returns over continuously compounded returns. However, the use of continuously compounded returns does have some advantages over the use of simple returns. Firstly, continuously compounded returns are symmetric while simple returns are asymmetric. Secondly, the mathematics is easier illustrated by how continuously compounded returns aggregate over time periods. Lastly, the use of continuously compounded returns is convenient for derivatives pricing and pricing models like the Black-Scholes formula in formula 5. Therefore, in this report continuously compounded returns are used given in formula 2.

2.2 Volatility

Volatility is the most common measure of market uncertainty and gives the degree of variation of trading price series over time it is given by:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(y_t - \mu)^2} \tag{3}$$

Due to the fact that the average daily returns are approximately zero and only one day volatility forecasts are computed the equation can be rewritten to this simple formula:

$$\sigma = \sqrt{y_t^2} \tag{4}$$

In finance volatility is used for many practices. For example, the pricing of a option over time by the Black-Scholes equation. The derivation of the Black-Scholes formula exceeds the scope of this report so the formula is just given in formula 5

$$\frac{\partial \mathrm{C}}{\partial \mathrm{t}} + \frac{1}{2}\sigma^2 \mathrm{S}^2 \frac{\partial^2 \mathrm{C}}{\partial \mathrm{C}^2} + \mathrm{rS}\frac{\partial \mathrm{C}}{\partial \mathrm{S}} = \mathrm{rC} \tag{5}$$

C = Call option price

S = Current stock price

K = Strike price of the option

r = risk-free interest rate

σ = volatility of the stocks return

t = time to option maturity (in years)

N = normal cumulative distribution function

(Black and Scholes 1973)

Volatility forecasting can also be crucial in financial risk management by the computation of Value at Risk and the expected shortfall. More information on these topics will be given later on in this report.

3 Model estimation and specification

3.1 Maximum likelihood

For the parameters estimation a Python package was used for univariate volatility modeling, bootstrapping, multiple comparison procedures and unit root tests (Sheppard, Khrapov, Lipták, and Capellini 2020). For the estimation of the model parameters maximum likelihood estimation is applied which is generally defined as:

$$\hat{\sigma}_T = \arg\max_{\theta \in \Theta} \mathcal{L}(y_1, ..., y_T, \theta) \tag{6}$$

The specific algorithm used is the Sequential Least Squares Programming algorithm which makes use of the Han-Powell quasi-Newton method with Broyden–Fletcher–Goldfarb–Shanno update of the B-matrix and an L1-test function in the step-length algorithm (Kraft 1988). The Sequential Least Squares Programming algorithm optimizes the log-likelihood function, $p(\cdot)$, conditional on the parameters, θ, and conditional on the observations, y_t. The estimates with the highest likelihood are selected.

Furthermore,the Sequential Least Squares Programming algorithm imposes the parameter restriction that α, β, and ω needs to be at least zero but no stationarity constraint, $\alpha + \beta < 1$, is imposed (Sheppard et al. 2020). Imposing a stationarity constraint could lead to non unique forecasts, multiple parameter estimations, and forecasts that are not stable over time.

3.2 Autoregressive conditional heteroskedasticity model:

The autoregressive conditional heteroskedasticity model, hereafter ARCH, was first introduced by Engle in 1982 and the general form is given by:

$$\begin{aligned} y_t &= \mu + \sigma_t \epsilon_t, \\ \sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2. \end{aligned} \tag{7}$$

For a single order ARCH model, which will be the focus of this report, the model specification is given by:

$$\begin{aligned} y_t &= \mu + \sigma_t \epsilon_t, \\ \sigma_t^2 &= \omega + \alpha_1 y_{t-1}^2. \end{aligned} \tag{8}$$

Where ω is a constant term and α represents the news factor. Further, to ensure that the conditional variance is always positive it is imposed that $\omega > 0$ and $\alpha \geq 0$.

The introduction of the ARCH model did result in advances in volatility modelling with more accurate results than previously used volatility models like moving average models and exponentially weighted moving average volatility models. However, the standard ARCH model did not utilizes the memory factor and, especially, for higher order ARCH models estimation could be problematic due to the non negativity constraints on the parameters. Therefore, there was still room for improvement.

3.3 Generalized autoregressive conditional heteroskedasticity model

The generalized autoregressive conditional heteroskedasticity model, hereafter GARCH, is introduced in 1986 by Bollerslev. Bollerslev extended the ARCH model of Engle by adding a memory component β and the general form is given by:

$$\begin{aligned} y_t &= \mu + \sigma_t \epsilon_t, \\ \sigma_t^2 &= \omega + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2. \end{aligned} \tag{9}$$

For a single order GARCH model, which will be the focus of this report, the model specification is given by:

$$\begin{aligned} y_t &= \mu + \sigma_t \epsilon_t, \\ \sigma_t^2 &= \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2. \end{aligned} \tag{10}$$

Where ω is a constant term, α represents the news factor, and β represents the memory factor. Further, to ensure that the conditional variance is always positive it is imposed that $\omega > 0$, $\alpha \geq 0$, and $\beta \geq 0$. The size of $(\alpha + \beta)$ determines how quickly the predictability of the volatility process dies out.

The GARCH(1, 1) model is preferred over the ARCH(p) model for several reasons. Firstly, the GARCH(1, 1) model requires less parameters and therefore the non negativity constraint of the parameters is less likely to be violated. Secondly, fewer parameters avoids over fitting and saves computational power. Moreover, by recursive substitution of σ^2_{t-1} it can be shown that the GARCH(1, 1) model is actually restricted ARCH(∞) model and therefore in practice a GARCH(1, 1) is preferred over a ARCH(p) model.

However, a disadvantage of both the ARCH model and the GARCH model is that it assumes that positive and negative shocks does have a symmetric effect on the volatility and therefore its forecast. To put it differently, good news and bad news do have the same effect on the volatility estimates of these models. In practice this symmetric effect on volatility does not seem to hold and therefore in 1976 Black introduced the term leverage effect. A quote from a article of Black was

> "a drop in the value of the firm will cause a negative return on its stock, and will usually increase the leverage of the stock. [...] That rise in the debt-equity ratio will surely mean a rise in the volatility of the stock" (Franke, Härdle, and Hafner 2015).

Normally one would assume that the leverage effect would be present in the index markets, consisting of large corporations. This is because negative shocks will result a larger proportion of corporate debt due though reduced firm value which is amplified by financial leverage resulting in higher volatility. To investigate if leverage effects are also present in South American index markets a asymmetric model will be applied, explained in the following subsection.

3.4 Exponential generalized autoregressive conditional heteroskedasticity model

The exponential generalized autoregressive conditional heteroskedasticity model, hereafter EGARCH, was firstly introduced in 1991 by Nelson and the general model specification is given by:

$$y_t = \mu + \sigma_t \epsilon_t,$$
$$\ln \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \left(|e_{t-i}| - \sqrt{2/\pi} \right) + \sum_{j=1}^{o} \gamma_j e_{t-j} + \sum_{k=1}^{q} \beta_k \ln \sigma^2_{t-k}. \tag{11}$$

For a single order EGARCH model, which will be the focus of this report, the model specification is given by:

$$y_t = \mu + \sigma_t \epsilon_t,$$
$$\ln \sigma_t^2 = \omega + \alpha \left(|e_{t-1}| - \sqrt{2/\pi} \right) \gamma e_{t-1} + \beta \ln \sigma^2_{t-1}. \tag{12}$$

Where $e_t = \epsilon_t / \sigma_t$, ω is a constant term, α represents the news factor, β represents the memory factor, and γ represents the leverage effect. Positive shocks have a $\alpha + \gamma$ effect on the variance forecast and negative shocks have a $\alpha - \gamma$ effect on the variance forecast.

By adding both the sign component and the memory factor it is likely that the EGARCH model outperforms the symmetric models if leverage effects are present in the data. Moreover, another advantage of the EGARCH model is because the EGARCH model is specified in logs there is no need for additional restrictions to prevent negative parameters.

4 Tail distributions

4.1 Normal distribution

When normally distributed innovations are assumed, which in the past have resulted in underestimation of financial risk, the following conditions and equations are able to describe the innovation distributions of the models. The parameter vector is given by $\theta = (\mu, \omega, \alpha, \beta)^T$. The past of the conditional distribution of y_t is by $Y^{t-1} = y_{t-1}, y_{t-2}, ...$ which is given by $y_t|Y^{t-1}$. The innovations are given by $\epsilon_t = z_t\sigma_t$ with $z_t \sim N(0,1)$ i.i.d.

The conditional probability density function of single observation y_t is given by:

$$p\left(y_t|Y^{t-1};\theta\right) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(-\frac{(y_t-\mu)^2}{2\sigma_t^2}\right). \tag{13}$$

The log-density of a single observation y_t is given by:

$$\log p\left(y_t|Y^{t-1};\theta\right) = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma_t^2) - \frac{1}{2}\frac{(y_t-\mu)^2}{\sigma_t^2}. \tag{14}$$

The log-likelihood function $\mathcal{L}(y_1, ..., y_T; \theta)$ is given by the sum of the conditional log-densities.

$$\mathcal{L}\left(y_1, ..., y_T; \theta\right) = \sum_{t=q+1}^{T}\left(-\frac{1}{2}\log(\sigma_t^2) - \frac{1}{2}\frac{(y_t-\mu)^2}{\sigma_t^2}\right) \tag{15}$$

Where q is the order of the model updating equation. Moreover, the term $\frac{1}{2}\log(2\pi)$ in the log-density equation is constant and thus is dropped in the log-likelihood function.

The normal density has a skewness of 0 and a kurtosis 3. However, financial returns of stocks often exhibits fat tails compared to a normal distribution (Webb 2016). This means that if the returns and its innovations are not normally distributed, the distribution exhibits more extreme events and less normal events. In order to test for non-normality of the distribution one could compute the Jarque-Bera (JB) statistic which is given in the summary table, table 1, and calculated by (Hayashi 2011):

$$\text{JB} = \frac{T+1}{6}\left(\hat{\mu}_3^2 + \frac{1}{4}(\hat{\mu}_4 - 3)^2\right). \tag{16}$$

Where $\hat{\mu}_3$ is sample skewness and $\hat{\mu}_4$ is sample kurtosis. The null-hypothesis is 'the data is normal' and the alternative hypothesis is 'the data is not normal'. Under the null-hypothesis the the Jarque-Bera statistic follows a Chi-square distribution with 2 degrees of freedom, $JB \sim \chi^2_{(2)}$.

Another way to investigate the normality of a data-set is to execute graphical test like a quantile-to-quantile plot. The quantile-to-quantile plot compares the quantile of the sample data with quantiles of the normal distribution.

4.2 Student's T distribution

The Student's T distribution is named after the publication of a paper by Gosset, under the pseudonym 'Student', in Biometrika in the year 1908 . The Student's T distribution was introduced into the world of financial modeling in the year 1987 by Bollerslev. Unlike the normal distribution, the Student's T distribution is able to capture the presence of fat tails.

When Student's T distributed innovations are assumed, the following conditions and equations are able to describe the innovation distributions of the models. The parameter vector is given by $\theta = (\mu, \omega, \alpha, \beta)^T$. The past of the conditional distribution of y_t is given by $Y^{t-1} = y_{t-1}, y_{t-2}, ...$ which is $y_t|Y^{t-1}$. The innovations are given by $\epsilon_t = z_t\sigma_t$ with $z_t \sim$ Student's-t(0,1) i.i.d.

The conditional probability density function of single observation y_t is given by (Tsay 2010):

$$p\left(y_t|Y^{t-1};\theta,\nu\right) \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2\sqrt{(v-2)\pi\sigma_t^2}}\left(1 + \frac{(y_t-\mu)^2}{(\nu-2)\sigma_t^2}\right)^{-(\nu+1)/2} \tag{17}$$

The log-density of a single observation y_t is given by (Tsay 2010):

$$\log p\left(y_t|Y^{t-1};\theta,\nu\right) = \log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log(\pi\left(\nu-2\right)\sigma_t^2) - \frac{\nu+1}{2}\log(1 + (y_t-\mu)^2/(\sigma_t^2(\nu-2))) \tag{18}$$

The log-likelihood function $\mathcal{L}(y_1, ..., y_T; \theta)$ is given by the sum of the conditional log-densities (Tsay 2010):

$$\mathcal{L}\left(y_1, ..., y_T; \theta, \nu\right) = \sum_{t=q+1}^{T} \left(\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2}\log(\pi\left(\nu-2\right)\sigma^{2_t}) - \frac{\nu+1}{2}\log(1+(y_t-\mu)^2/(\sigma_t^2(\nu-2))) \right) \tag{19}$$

Where ν is the shape parameter which gives the degrees of freedom, if ν tends to infinity the Student's T distribution will almost converge to the normal distribution. ν is restricted to be larger than 2 and the tails are said to be fat when ν is approximately 4. Γ is the gamma function.

Kurtosis is a measure of tailedness of a distribution, the kurtosis of the Student's T distribution is for $2 < \nu \leq 4$ ∞. For ν larger than 4, the kurtosis is given by the following formula (Tsay 2010):

$$3\frac{(\nu-2)}{(\nu-4)} \tag{20}$$

4.3 Skewed Student's T distribution

Hansen introduced in 1994 the skewed Student's T distribution which according to him allowed for a richer set of behaviors by modelling for skewness . If skewness is present in the data one would expect that a skewed model outperforms the normal distribution and the student-T distribution by taking the skewness into account. When skewed Student's T distributed innovations are assumed, the following conditions and equations are able to describe the innovation distributions of the models. The parameter vector is given by $\theta = (\mu, \omega, \alpha, \eta, \lambda)^T$. The past of the conditional distribution of y_t is by $Y^{t-1} = y_{t-1}, y_{t-2}, ...$ which is given by $y_t|Y^{t-1}$. The innovations are given by $\epsilon_t = z_t\sigma_t$ with $z_t \sim$ skewed Student's T(0,1) i.i.d.

$$p(y_t|Y^{t-1};\theta) = \frac{bc}{\sigma}\left(1+\frac{1}{\eta-2}\left(\frac{a+b(y_t-\eta)\sigma}{1+sgn(y_t-\eta)/\sigma+a/b)\lambda}\right)^2\right)^{-(\eta+1)/2}. \tag{21}$$

The log-density of a single observation y_t is given by

$$\log p(y_t|Y^{t-1};\theta) = \log\left[\frac{bc}{\sigma}\left(1+\frac{1}{\eta-2}\left(\frac{a+b(y_t-\mu)/\sigma}{1+sgn((y_t-\mu)/\sigma+a/b)\lambda}\right)^2\right)^{-(\eta+1)/2}\right]. \tag{22}$$

The log-likelihood function $\mathcal{L}(y_1, ..., y_T; \theta)$ is given by the sum of the conditional log-densities.

$$\mathcal{L}(y_1, ..., y_T; \theta) = \sum_{t=q+1}^{T}\left(\log\left[\frac{bc}{\sigma}\left(1+\frac{1}{\eta-2}\left(\frac{a+b(y_t-\mu)/\sigma}{1+sgn((y_t-\mu)/\sigma+a/b)\lambda}\right)^2\right)^{-(\eta+1)/2}\right]\right) \tag{23}$$

The constants a, b, c are given by:

$$a = 4\lambda c\frac{\eta-2}{\eta-1}, \quad b^2 = 1+3\lambda^2-a^2, \quad c = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi\left(\eta-2\right)}\Gamma\left(\frac{\eta}{2}\right)}, \tag{24}$$

Where $2 < \eta < \infty$, $-1 < \lambda < 1$, Γ is the gamma function, and q is the order of the model updating equation.

The skewed Student's T distribution utilizes two parameters η and λ. Where η controls for the tail shape, and is equal to ν in the Student's t-distribution. λ controls for skewness and service as a asymmetry parameter. When $\lambda = 1$, there is no asymmetry. When $\lambda < 1$, the density is skewed to the left. When $\lambda > 1$, the density is skewed to the right.

5 Forecasting and model performance evaluation measures

In the previous sections the forecasting models and its distributions were defined for the conditional volatility. In order to investigate which models best forecast the conditional volatility, in this section several measures, and classes, are introduced. The reason to utilize multiple performance measures is to increase the robustness of the findings.

5.1 In-sample performance evaluation measures

5.1.1 Log-likelihood

The log-likelihood, as defined in the innovation distribution section, offers a natural comparison term for models. In general, the higher the negative log-likelihood the better the fit of the model is.

However, model selection by merely comparing log-likelihoods leads to over-fitting. Over-fitting occurs due to the fact that larger nested models always increase log-likelihood. Thus, sample log-likelihood of the specified model increases not because the model is superior but simply because the model is able to over-fit the data (Dougherty 2011). Therefore, the solution is to penalize the number of parameters in a specified model. The penalization on the number of parameters is the main idea behind the majority of information criteria models.

5.1.2 Akaike's information criterion

The Akaike information criterion (AIC) was firstly introduced by Akiake in 1974 in his the paper "A New Look at the Statistical Model Identification". The criterion is given by the following equation:

$$\text{AIC} = 2k - 2\log\mathcal{L}(y_1, ..., y_T; \hat{\theta}_T). \tag{25}$$

Where k is the number of independently adjusted parameters within the model to get $\hat{\phi}$.

The Akaike information criterion is a more reasonable basis for model selection in comparison to the log-likelihood criterion. Lastly, the Akaike information criterion is based on a negative log-likelihood and in general a lower value of the Akaike information criterion indicates a better model.

5.1.3 Bayesian information criterion

The Bayesian, or Schwarz, information criteria (BIC) was firstly introduced by Schwarz in 1978 in his paper "Estimating the dimension of a model" . The criterion is give by the following equation:

$$\text{BIC} = \log(T)k - 2\log\mathcal{L}(y_1, ..., y_T; \hat{\theta}_T) \tag{26}$$

Where T is the length of vector y_T and k is the number of independently adjusted parameters within the model to get $\hat{\theta}_T$.

The Bayesian information criterion is closely related to the Akaike information criterion and is also a more reasonable basis for model selection in comparison to the log-likelihood criterion. Lastly, the Bayesian information criterion is also based on a negative log-likelihood and in general a lower value of the Bayesian information criterion indicates a better model.

5.2 Out of sample performance evaluation measures

Out of sample performance evaluation measures give the out of sample fit of the model estimates. Thus this yields more reasonable measures to reflect on the actual performance of the volatility forecasting models in comparison to the in-sample performance measures.

5.2.1 Diebold-Mariano test, mean error models, and median error models

Firstly, the class mean error models is specified. This class compares the out of sample observations of the conditional volatility with the forecasted out of sample predictions of conditional volatility. Due to the fact that the actual variance σ_t^2 is unobserved it is assumed that the actual squared returns y_t^2 is a good approximation of the actual variance. This could be justified due to the fact that average daily returns are approximately 0. In general for all mean error models, the lower the error the better the model. The mean absolute error takes the average absolute distance of the out-of-sample observations of the conditional variance and the forecasted out-of-sample predictions of conditional variance. If large outliers are present in the forecast evaluation, than they could heavily affect the mean error measure. Therefore, one could use a median error measure. The median absolute error takes the median absolute distance of the out-of-sample observations of the conditional variance and the forecasted out-of-sample predictions of conditional variance. The Diebold-Mariano test is useful for comparing the performance of different forecast models (Diebold and Mariano 1995).

Diebold-Mariano test, mean absolute error, and median absolute error (Diebold et al. 1995)

$$H_0\colon\ \hat{d} = 0$$

The Diebold-Mariano statistic for the mean absolute error is given by:

$$DM = \sqrt{N}\frac{\bar{d} - 0}{\sqrt{Var(d)}} \sim student's - t(N-1) \tag{27}$$

The Diebold-Mariano statistic for the median absolute error is given by:

$$DM = \sqrt{N}\frac{Inv(f_{med}(d) - 0}{\sqrt{Var(d)}} \sim student's - t(N-1) \tag{28}$$

The loss differential for the mean and median absolute error is given by:

$$d_t = |\sigma_t^2 - \hat{\sigma}^2_{t,model1}| - |\sigma_t^2 - \hat{\sigma}^2_{t,model2}| \tag{29}$$

The variance for the mean absolute error is given by:

$$Var(d) = \frac{1}{N-1}\sum_{t=1}^{N}(d_t - \overline{d})^2 \tag{30}$$

The variance for the median absolute error is given by:

$$Var(d) = \frac{1}{N-1}\sum_{t=1}^{N}(d_t - Inv(f_{med}(d))^2 \tag{31}$$

Where $N =$ the number of out of sample observations.

5.2.2 Diebold-Mariano test and logarithmic scoring

Logarithmic scoring compares the out of sample log-likelihood of two models, this log-likelihood is computed on the estimated model parameters and the innovation distribution. Logarithmic scoring is mathematically given by:

$$\text{LSR}(\hat{p}; y_{t+1}) = \log f(y_{t+1}) \tag{32}$$

Where LSR is the logarithmic scoring rule and $p(\cdot)$ is the innovation distribution.

Diebold-Mariano test & logarithmic scoring (Diebold et al. 1995)

$$H_0\colon\ \hat{d} = 0$$

The Diebold-Mariono statistic is given by:

$$DM = \sqrt{N}\frac{\bar{d} - 0}{\sqrt{Var(d)}} \sim student's - t(N-1) \tag{33}$$

The loss differential is given by:

$$d_t = LSR_{t,model\ 1} - LSR_{t,model\ 2} \tag{34}$$

The variance is given by:

$$Var(d) = \frac{1}{N-1}\sum_{t=1}^{N}(d_t - \overline{d})^2 \tag{35}$$

And where $N =$ the number of out of sample observations.

If the loss differential does significantly differ from zero there is evidence that one model is able to more precisely predict the conditional volatility forecast based on the Diebold-Mariano test in combination with the chosen test statistic.

5.2.3 Value at Risk:

Value at Risk, or VaR, is a widely used risk measure in the sector of financial risk management. VaR measures the severity of a loss over a certain time interval of an investment with a certain probability, α. The Value at Risk measure is given by:

$$\mathrm{VaR}_{t+1}(\mathrm{p}) = \mu_t + \hat{\sigma}^2_{t+1} CDF_R^{-1}(p). \tag{36}$$

Where p is the probability of losses exceeding the Value at Risk. μ_t is the time-varying mean of returns, in this report set to zero. $\hat{\sigma}_{t+1}$ is the time-varying conditional volatility estimate for a certain point in time obtained by a combination of volatility model and innovation distribution. CDF_R^{-1} is the inverse of the selected volatility model and innovation distribution cumulative density function (Danielsson 2011).

In order to the backtest the quality of the Value at Risk forecasts the hit sequence, η_t, needs to be defined. A Value at Risk violation occurs when the observed loss of a asset is smaller than the estimated Value at Risk for that asset.

$$\eta_t = \begin{cases} 1, & \text{if } y_t \leq \mathrm{VaR}_t \\ 0, & \text{if } y_t > \mathrm{VaR}_t \end{cases} \tag{37}$$

A property of a good Value at Risk forecast is that it should not yield statistically significant total number of violations than the expected number violations based on the confidence level p, in this report 5% and 1%. The violation ratio compares the actual number of Value at Risk violations with the expected number and is given by:

$$\mathrm{VR} = \frac{\text{Observed number of violations}}{\text{Expected number of violations}} = \frac{v_1}{p * W_T} \tag{38}$$

If the violation ratio is greater than one the Value at Risk model underforecasts risk and if it is smaller than one the model over-forecasts risk. Ideally, a violation ratio of 1 is expected but any other value might be statistically significant. Therefore, as a rule of thumb if $VR \in [0.8, 1.2]$ it is a good forecast and if $VR < 0.5$ or $VR > 1.5$ the model is imprecise (Danielsson 2011). Another way of testing the observed number of violations is the Bernoulli coverage test. The Bernoulli coverage test ensures that the empirical probability of violations matches the confidence level p.

Bernoulli coverage test (Danielsson 2011)

$$H_0\text{: } \eta \sim B(p).$$

Where B stands for the Bernoulli distribution.

The Bernoulli density is given by:

$$(1-p)^{1-\eta_t}(p)^{\eta_t}, \eta_t = 0, 1. \tag{39}$$

Probability, p, can be estimated by:

$$\hat{p} = \frac{v_1}{W_T}. \tag{40}$$

The likelihood function is expressed by:

$$\mathcal{L}_{\mathcal{U}}(\hat{p}) = \prod_{t=W_E+1}^{T} (1-\hat{p})^{1-\eta_t}(\hat{p})^{\eta_t} = (1-\hat{p})^{v_0}\hat{p}^{v_1}. \tag{41}$$

Under H_0, $p = \hat{p}$, the restricted likelihood function is given by:

$$\mathcal{L}_{\mathcal{R}}(p) = \prod_{t=W_E+1}^{T} (1-p)^{1-\eta_t}(p)^{\eta_t} = (1-p)^{v_0}(p)^{v_1}. \tag{42}$$

To test whether $\mathcal{L}_{\mathcal{R}} = \mathcal{L}_{\mathcal{U}}$ an LR Test is conducted:

$$\mathrm{LR}_{bern} = 2(\log \mathcal{L}_{\mathcal{U}}(\hat{p}) - \log \mathcal{L}_{\mathcal{R}}(p)) \tag{43}$$

$$= 2 \log \frac{(1-\hat{p})^{v_0}(\hat{p})^{v_1}}{(1-p)^{v_0}(p)^{v_1}} \overset{\text{asymptotic}}{\sim} \chi^2_{(1).}$$

At 5% significance level for the Bernoulli coverage test, the null-hypothesis is rejected if LR is larger than 3.84.

Besides the coverage property a good Value at Risk forecast model should also have independently distributed violations, so there should not be any auto-correlation in the hit sequence. One way to test this is the independence test. This test requires that sequentially violations in the hit sequence to be distributed independently (Danielsson 2011). In order to test this the probability of two sequentially hits and the probability of a hit if there was no hit at $t-1$ needs to be defined:

$$p_{i,j} = Pr(\eta_t = j | \eta_{t1} = i), \tag{44}$$

where i and j are either 0 or 1.

Independence test (Danielsson 2011)

$$H_0\colon\ p_{01} = p_{11} = p$$

The first order probability transition matrix is expressed by:

$$\Pi_1 = \begin{pmatrix} 1 - p_{01} & p_{01} \\ 1 - p_{11} & p_{11} \end{pmatrix} \tag{45}$$

The unrestricted likelihood function is expressed by:

$$\mathcal{L}_{\mathcal{U}}(\Pi_1) = (1 - p_{01})^{v_{00}} p_{01}^{v_{01}} (1 - p_{11})^{v_{10}} p_{11}^{v11} \tag{46}$$

Where v_{ij} is the number of observations where j follows i.

By optimizing $\mathcal{L}_{\mathcal{U}}(\hat{\Pi}_1)$ the maximum likelihood estimates are obtained :

$$\hat{\Pi}_1 = \begin{pmatrix} \frac{v_{00}}{v_{00}+v_{01}} & \frac{v_{01}}{v_{00}+v_{01}} \\ \frac{v_{10}}{v_{10}+v_{11}} & \frac{v_{11}}{v_{10}+v_{11}} \end{pmatrix} \tag{47}$$

Given that there is no volatility clustering under H_0:

$$\hat{\Pi}_0 = \begin{pmatrix} 1 - \hat{p} & \hat{p} \\ 1 - \hat{p} & \hat{p} \end{pmatrix} \tag{48}$$

$$\hat{p} = \frac{v_{01} + v_{11}}{v_{00} + v_{10} + v_{01} + v_{11}}$$

The restricted likelihood function under H_0 is given by:

$$\mathcal{L}_{\mathcal{R}}(\hat{\Pi}_0) = (1 - \hat{p})^{v_{00}+v_{10}} \hat{p}^{v_{01}+v_{11}} \tag{49}$$

The likelihood ratio test is expressed by:

$$\mathrm{LR}_{ind} = 2\left(\log \mathcal{L}_U(\hat{\Pi}_1) - \log \mathcal{L}_R(\hat{\Pi}_0)\right) \overset{\text{asymptotic}}{\sim} \chi^2_{(1)} \tag{50}$$

The main weakness of the independence test is that it only test the independence of two direct sequential lags and not any other order of lags. A good Value at Risk forecast should not only have sequentially independently distributed violations but there should also not be any auto-correlation in any other lags in the hit sequence.

The joint test combines the Bernoulli coverage test and the independence test into one test. This test jointly tests if the number of Value at Risk violations are significantly different than the expected number of violations based on the confidence level p and if the violations on the hit sequence are clustering together. The test statistic is given by:

$$\mathrm{LR}_{joint} = \mathrm{LR}_{uc} + \mathrm{LR}_{ind} \sim \chi^2_{(2)} \tag{51}$$

The total quality of the Value at Risk forecast can be tested by the joint test, the test has less power to reject a model only satisfying one of the two properties (Danielsson 2011).

6 Data

The time period selected for this report is from 1 January 2000 up until 15 April 2020 so almost 20 years of data is utilized in this report. For the parameter estimation 3000 observations are used which is in correspondence with (Kruse, Berthold, Moewes, Gil, Grzegorzewski, and Hryniewicz 2012) and (Levy 2004) in order to model extreme events more accurate.

	y_t whole sample	p_t whole sample	y_t in-sample	p_t in-sample	y_t out of sample	p_t out of sample
Argentina						
Observations	4956	4957	3000	3000	1956	1956
Mean	0.00082	7602.176	0.00054	1485.46	0.00125	16971.14
Median	0.00124	2257.28	0.00094	1492.08	0.00178	13116.056
Minimum	-0.47692	200.86	-0.12952	200.86	-0.47692	2121.52
Maximum	0.16117	44355.09	0.16117	3664.82	0.09754	44355.09
Standard deviation	0.02332	10610.98	0.02174	881.55	0.02556	11776.23
Skewness	-0.00198	1.67747	-0.00012	0.41577	-0.00371	0.4431
Kurtosis	0.03892	1.57651	0.00491	-0.68256	0.0645	-1.09798
JB-statistic	316037.26	2838.09	3025.69	144.67	3025.69	144.67
JB P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
Brazil						
Observations	5018	5019	3000	3000	2018	2018
Mean	0.00031	48338.52	0.00044	36348.47	0.0001	66148.05
Median	0.00072	51848.00	0.00115	33725.00	0.00015	59090.50
Minimum	-0.15993	8371.00	-0.12096	8371.00	-0.15993	37497.00
Maximum	0.13677	119528.00	0.13677	73517.00	0.13022	119528.00
Standard deviation	0.01823	24719.36	0.01936	20710.87	0.01641	18727.39
Skewness	-0.00037	0.24667	-0.00011	0.27724	-0.00101	1.02069
Kurtosis	0.00665	-0.36237	0.00366	-1.44972	0.01436	0.04195
JB-statistic	9368.21	78.36	1676.69	301.14	1676.69	301.14
JB P-value	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Table 1: Summary statistic of the index returns. Y_t denotes the log returns (not expressed as %). P_t denotes the adjusted closing price of the respective index, in countries own currency. The Jarque-Bera statistic test for normality of the data and the critical value for $\alpha = 0.05$ is approximately 5.970. Therefore, it is possible to conclude that all index returns are not normally distributed.

Given the average return of approximately 0 for both indices, the observation equation specified in section 4 is rewritten to:

$$y_t = \sigma_t \epsilon_t \tag{52}$$

Based on table 1 one can observe that the in-sample standard deviation for Brazil is higher than the out of sample standard deviation and for Argentina the in-sample standard deviation is lower than the out of sample standard deviation. This could potentially lead to overestimation of the conditional volatility of Brazil and underestimation of the conditional volatility of Argentina. To analyze the dynamics of indices, their price and return plots of the full data-sets are given below:

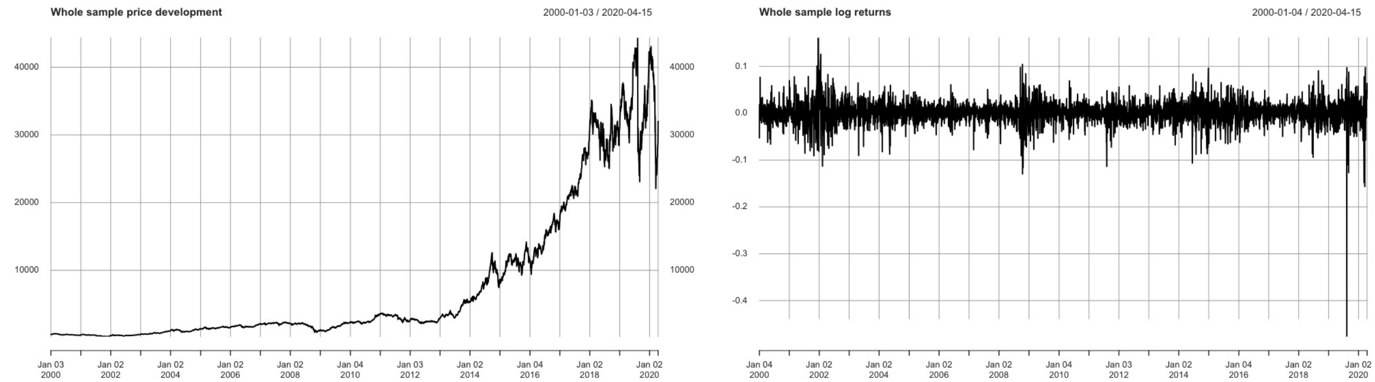


Figure 1: Price development plot and log return plot of the index of Argentina

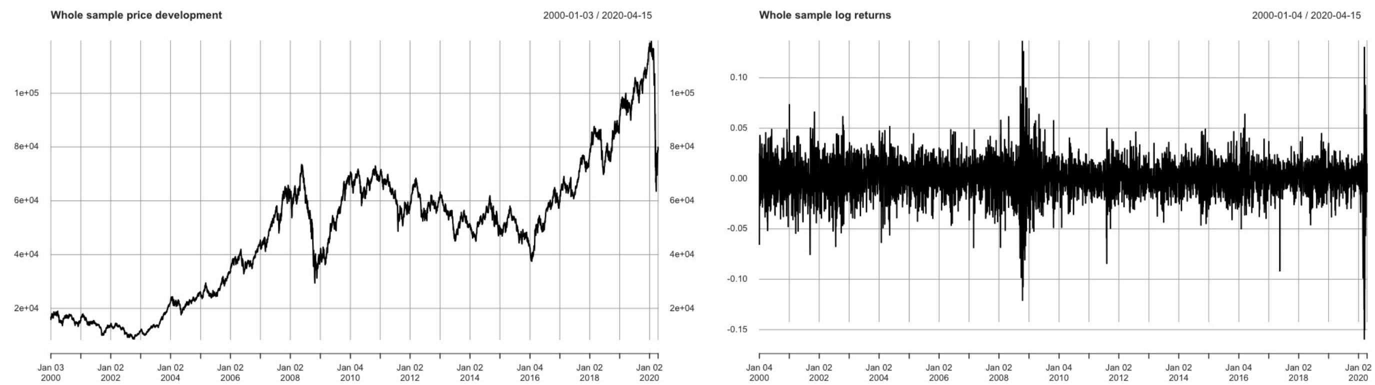
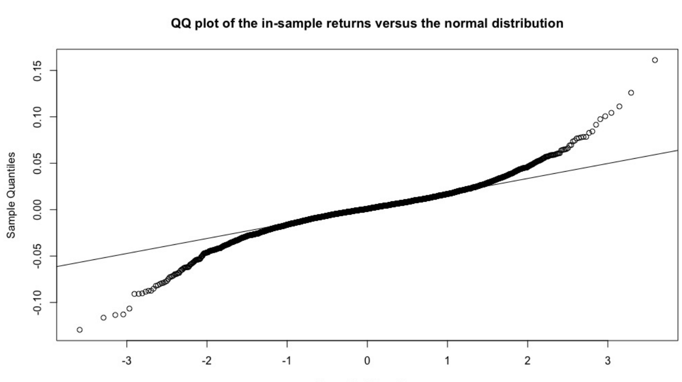


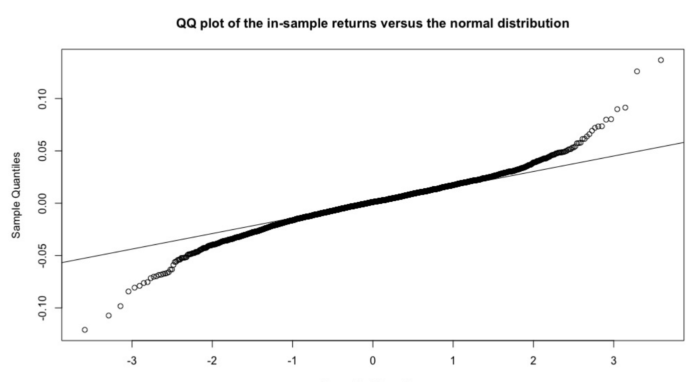
Figure 2: Price development plot and log return plot of the index of Brazil

The periods of high volatility and low volatility indicates volatility clustering, observable in the volatility plots (Stock and Watson 2015).

By plotting the quantile-to-quantile plots of the two indices, given in the following two graphs for the in-sample return data, the fatness of the tails of the return distributions becomes less abstract.



In-sample QQ-plot of the index of Argentina



In-sample QQ-plot of the index of Brazil

Figure 3: Three subfigures

Like expected given the in-sample Jarque-Bera statistic, the in-sample plots of the indices returns also indicates the presence of fat tails. Therefore, based on the in-sample data, it is highly likely that the models with a normal tail distribution do under-perform relative to models with tail distributions which do utilizes the presence of fat tails.

6.1 Auto correlation

Auto-correlation measures how a single variable is correlated with itself, with N lag this can be expressed as (Hayashi 2011):

$$\hat{\beta}_i = Corr(X_1, ..., N - i, X_{i+1}, ..., N) \tag{53}$$

If there are significant auto-correlations in the lagged returns then there is evidence for predictability in the return data (Calhoun, Khashanah, and Chatterjee 2014). If there are significant auto-correlations in the lagged squared returns then there is evidence for predictability in the variance data (Calhoun et al. 2014). In the following figures the auto-correlation plots are given for the different returns and squared returns of the indices.

For both index returns there seems to be little to no auto-correlation in the lagged values of the returns y_t. Therefore, there is not much predictability in the return data. However, in the squared returns y_t^2 there is more auto-correlation. Therefore, the variance can be predicted to some extent.

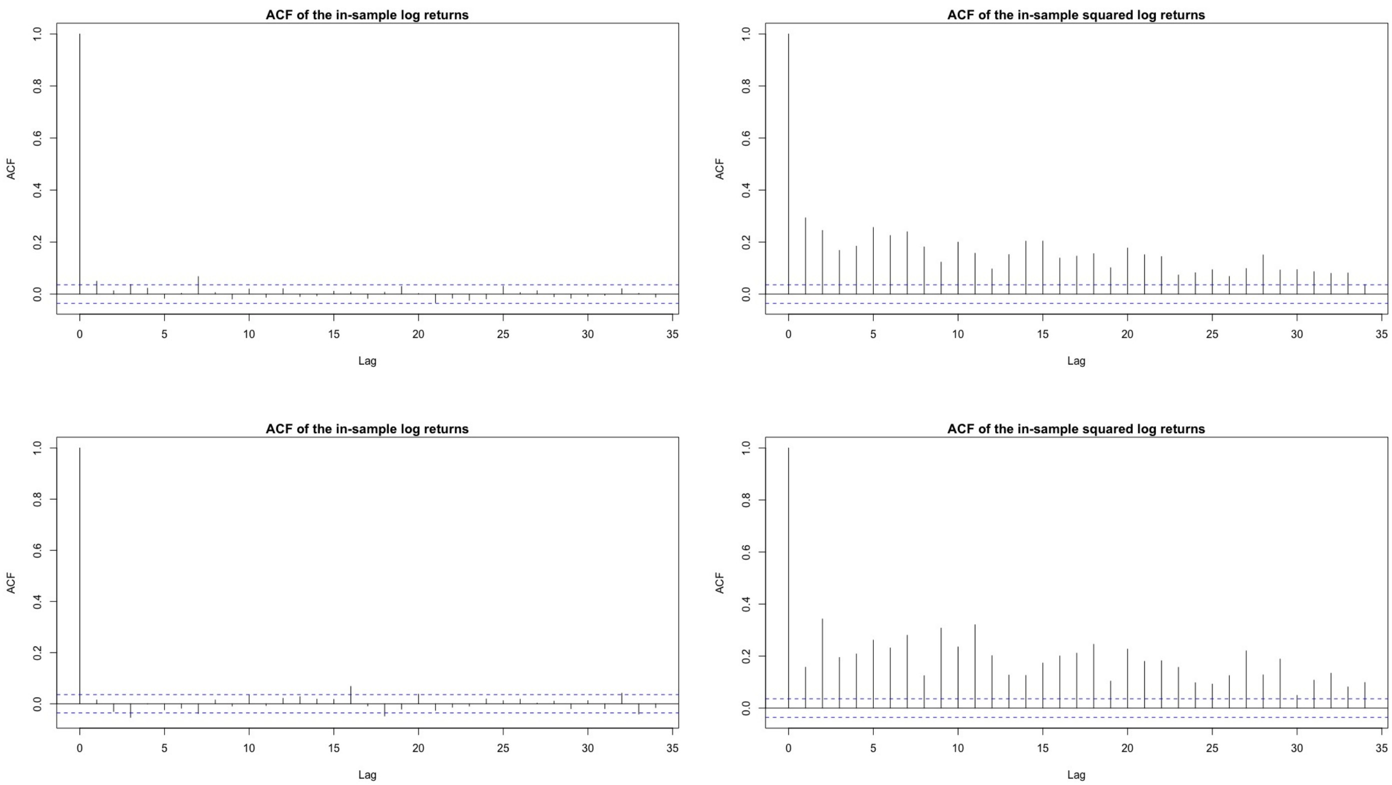


Figure 4: Auto correlation of the index log returns and squared log returns of Argentina (top) and Brazil (bottom)

7 Results

7.1 Model estimates

The estimated parameters, by maximum likelihood estimation, of the conditional volatility models explained in section 4 are given in the table below. Overall, most parameters are significant at a 1% significance level which empowers the validity of the models. Given the model parameters it becomes now possible to compute the log likelihood performance, AIC performance, BIC performance, mean error performance, median error performance, Diebold-Mariano performance, and Value at Risk performance of the several models and its innovation distributions.

Model	ω	α	β	ν/η	γ	λ
Argentina						
ARCH normal	0.00036^{***}	0.20222^{***}				
(SD)	$(1.908 \cdot 10^{-5})$	$(3.984 \cdot 10^{-2})$				
ARCH T	0.00040^{***}	0.20134^{***}		3.5120^{***}		
(SD)	$(2.724 \cdot 10^{-5})$	$(3.984 \cdot 10^{-2})$		(0.226)		
ARCH skew T	0.00041^{***}	0.20419^{***}		3.45784^{***}		-0.06469^{***}
(SD)	$(2.849 \cdot 10^{-5})$	$(4.053 \cdot 10^{-2})$		(0.222)		$(1.853 \cdot 10^{-2})$
GARCH normal	$1.193 \cdot 10^{-5***}$	0.09706^{***}	0.87570^{***}			
(SD)	$(3.489 \cdot 10^{-6})$	$(1.611 \cdot 10^{-2})$	$(1.850 \cdot 10^{-2})$			
GARCH T	$9.738 \cdot 10^{-6***}$	0.08893^{***}	0.89104^{***}	5.41848^{***}		
(SD)	$(2.652 \cdot 10^{-6})$	$(1.403 \cdot 10^{-2})$	$(1.645 \cdot 10^{-2})$	(0.522)		
GARCH skew T	$9.998 \cdot 10^{-6***}$	0.09086^{***}	0.88943^{***}	5.33619^{***}		-0.08372^{***}
(SD)	$(2.732 \cdot 10^{-6})$	$(1.443 \cdot 10^{-2})$	$(1.687 \cdot 10^{-2})$	(0.507)		$(2.118 \cdot 10^{-2})$
EGARCH normal	$2.073 \cdot 10^{-7***}$	0.18438^{***}	0.96656^{***}		-0.05542^{***}	
(SD)	$(6.664 \cdot 10^{-8})$	$(3.013 \cdot 10^{-2})$	$(1.101 \cdot 10^{-2})$		$(1.439 \cdot 10^{-2})$	
EGARCH T	$1.700 \cdot 10^{-7***}$	0.1773^{***}	0.97357^{***}	5.44835^{***}	-0.06307^{***}	
(SD)	$(4.568 \cdot 10^{-8})$	$(2.369 \cdot 10^{-2})$	$(7.511 \cdot 10^{-3})$	(0.547)	$(1.307 \cdot 10^{-2})$	
EGARCH skew T	$1.667 \cdot 10^{-7***}$	0.1781^{***}	0.97374^{***}	5.36198^{***}	-0.06033^{***}	-0.07569^{***}
(SD)	$(4.539 \cdot 10^{-8})$	$(2.394 \cdot 10^{-2})$	$(7.488 \cdot 10^{-3})$	(0.530)	$(1.296 \cdot 10^{-2})$	$(2.129 \cdot 10^{-2})$
Brazil						
ARCH normal	0.00031^{***}	0.16265^{***}				
(SD)	$(1.419 \cdot 10^{-5})$	$(3.807 \cdot 10^{-2})$				
ARCH T	0.00032^{***}	0.12584^{***}		5.77055^{***}		
(SD)	$(1.421 \cdot 10^{-5})$	$(2.674 \cdot 10^{-2})$		(0.595)		
ARCH skew T	0.00032^{***}	0.12682^{***}		5.74604^{***}		-0.0857^{***}
(SD)	$(1.434 \cdot 10^{-5})$	$(2.727 \cdot 10^{-2})$		(0.590)		$(2.324 \cdot 10^{-2})$
GARCH normal	$6.800 \cdot 10^{-6***}$	0.06904^{***}	0.91088^{***}			
(SD)	$(2.226 \cdot 10^{-6})$	$(1.176 \cdot 10^{-2})$	$(1.546 \cdot 10^{-2})$			
GARCH T	$5.489 \cdot 10^{-6***}$	0.06540^{***}	0.9186^{***}	12.67415^{***}		
(SD)	$(1.783 \cdot 10^{-6})$	$(1.032 \cdot 10^{-2})$	$(1.308 \cdot 10^{-2})$	(2.674)		
GARCH skew T	$5.1298 \cdot 10^{-6***}$	0.06569^{***}	0.91993^{***}	13.30544^{***}		-0.11543^{***}
(SD)	$(1.726 \cdot 10^{-6})$	$(1.037 \cdot 10^{-2})$	$(1.297 \cdot 10^{-2})$	(2.907)		$(2.573 \cdot 10^{-2})$
EGARCH normal	$1.560 \cdot 10^{-7***}$	0.12416^{***}	0.97322^{***}		-0.08559^{***}	
(SD)	$(4.235 \cdot 10^{-8})$	$(1.694 \cdot 10^{-2})$	$(7.316 \cdot 10^{-3})$		$(1.505 \cdot 10^{-2})$	
EGARCH T	$1.384 \cdot 10^{-7***}$	0.12309^{***}	0.97627^{***}	15.04615^{***}	-0.08436^{***}	
(SD)	$(3.934 \cdot 10^{-8})$	$(1.665 \cdot 10^{-2})$	$(6.815 \cdot 10^{-3})$	(3.723)	$(1.446 \cdot 10^{-2})$	
EGARCH skew T	$1.360 \cdot 10^{-7***}$	0.12423^{***}	0.97676^{***}	15.57013^{***}	-0.08347^{***}	-0.11705^{***}
(SD)	$(4.011 \cdot 10^{-8})$	$(1.690 \cdot 10^{-2})$	$(6.949 \cdot 10^{-3})$	(3.969)	$(1.457 \cdot 10^{-2})$	$(2.649 \cdot 10^{-2})$

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table 2: Parameter estimates of the different conditional volatility models and their distributions for the index of Argentina and Brazil. Overall, most parameter estimates are highly significant. Where ω is the long-term average variance, α is the news factor, β is the memory factor, v/η is the shape parameter, γ is the leverage factor, and λ is the skewness parameter.

7.2 In-sample model performance evaluation

7.2.1 Log-likelihood

Below, the log-likelihood scores, as explained in section 6.1.1, for the different models and innovation distributions are given:

Position	Model Argentina	Likelihood	Model Mexico	Likelihood
1	EGARCH skew T	-12989.5	EGARCH skew T	-12809
2	GARCH skew T	-12991.9	EGARCH T	-12819.8
3	EGARCH T	-12996	EGARCH normal	-12832.7
4	GARCH T	-13000.1	GARCH skew T	-12835
5	GARCH normal	-13089.2	GARCH T	-12846.4
6	EGARCH normal	-13094.7	GARCH normal	-12864.4
7	ARCH skew T	-13158.6	ARCH skew T	-12983.9
8	ARCH T	-13164.8	ARCH T	-12991.5
9	ARCH normal	-13367	ARCH normal	-13089

Table 3: Log-likelihood scores for the different models and innovations distributions.

Based on this table it becomes possible to conclude that based on log-likelihood the asymmetric EGARCH model with skewed Student's T distributed innovations outperforms the other models based on their in-sample fit for both stock indices. Moreover, the EGARCH and GARCH models seems to outperform all ARCH models. However, like aforementioned, model selection based on merely log-likelihood leads to over-fitting maybe the EGARCH model is not better than the ARCH model but is simply able to over-fit the data due to a larger number of parameters being utilized. Therefore, these results should be interpreted with caution.

7.2.2 Akaike's information criterion

Underneath, the AIC scores, as explained in section 6.1.2 for the different models and innovation distributions are given:

Position	Model Argentina	AIC	Model Brazil	AIC
1	EGARCH skew T	25991.1	EGARCH skew T	25630.1
2	GARCH skew T	25993.8	EGARCH T	25649.6
3	EGARCH T	26002.1	EGARCH normal	25673.4
4	GARCH T	26008.1	GARCH skew T	25680
5	GARCH normal	26184.5	GARCH T	25700.7
6	EGARCH normal	26197.4	GARCH normal	25734.7
7	ARCH skew T	26325.2	ARCH skew T	25975.9
8	ARCH T	26335.6	ARCH T	25989.1
9	ARCH normal	26738	ARCH normal	26182

Table 4: AIC scores for the different models and innovations distributions.

Based on this table it becomes possible to conclude that based on AIC the asymmetric EGARCH model with skewed Student's t-distributed innovations is the best model for both indices after adjusting for the number of parameters. Moreover, there seems to be a strong preference for EGARCH models for the Brazilian index. Lastly, the EGARCH and GARCH models seems to outperform all ARCH models.

7.2.3 Bayesian information criteria

On the next page, the BIC scores, as explained in section 6.1.3 for the different models and innovation distributions are given.

Based on this table it becomes possible to conclude that based on BIC the GARCH model with skewed Student's T distributed innovations is the best model for Argentina, which contradicts the results based on log-likelihood and AIC. For Brazil the best performing model based on the BIC still remains the asymmetric EGARCH model with skewed Student's t-distributed innovations. Moreover, consistent with the results based on the log-likelihood and AIC there seems to be a strong preference for EGARCH models for the Brazilian index. Lastly, the EGARCH and GARCH models seems to outperform all ARCH models.

Position	Model Argentina	Likelihood	Model Mexico	Likelihood
1	GARCH skew T	26026.3	EGARCH skew T	25669.2
2	EGARCH skew T	26030.1	EGARCH T	25682.2
3	GARCH T	26034.2	EGARCH normal	25699.5
4	EGARCH T	26034.6	GARCH skew T	25712.6
5	GARCH normal	26204	GARCH T	25726.8
6	EGARCH normal	26223.5	GARCH normal	25754.3
7	ARCH skew T	26351.2	ARCH skew T	26001.9
8	ARCH T	26355.1	ARCH T	26008.6
9	ARCH normal	26751	ARCH normal	26195

Table 5: BIC scores for the different models and innovations distributions.

7.3 Out of sample model performance evaluation

7.3.1 Graphical backtest

A first step in evaluating the accuracy of the different conditional volatility forecasts is to plot the predicted volatility against the observed out of sample realized conditional volatility, $\sqrt{y_t^2}$.

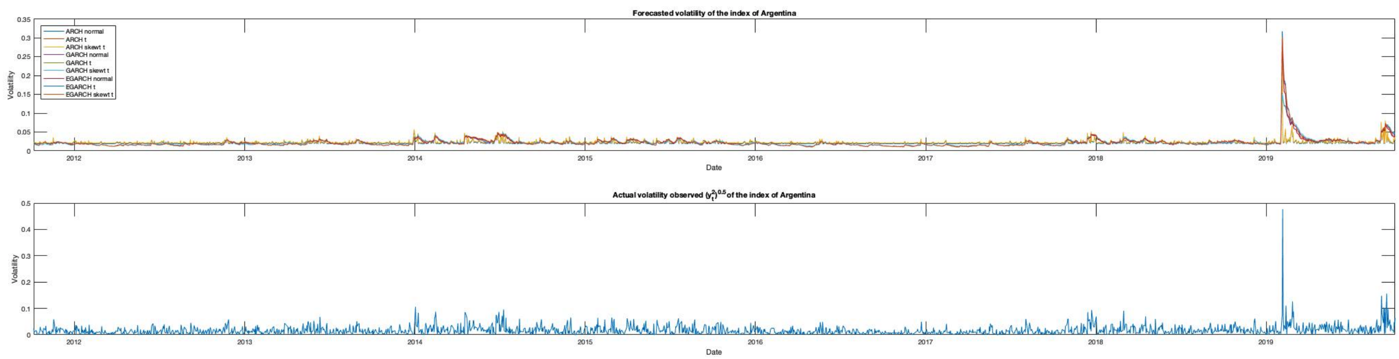


Figure 5: Graphical backtest for the index of Argentina

Due to large peaks in the forecasted conditional volatility the plot for Argentina unfortunately difficult to interpreted. Overall, the forecasted volatility estimates of the models seems to follow a same pattern compared to the observed conditional volatility. However, the models seems to slightly overestimate conditional volatility.

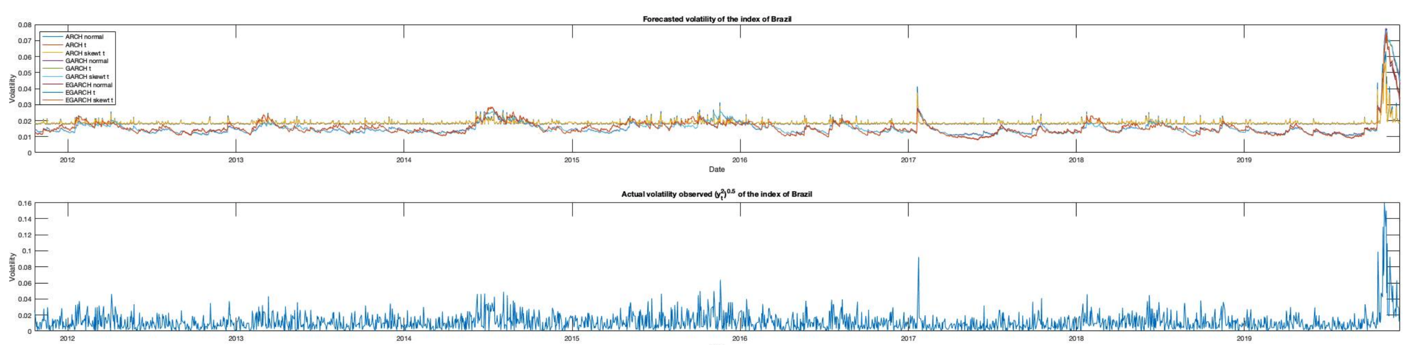


Figure 6: Graphical backtest for the index of Brazil

For the index of Brazil the dynamics of the forecasted conditional volatility are better observable compared to the index of Argentina, due to less peaks. Overall, the forecasted volatility estimates of the models seems to follow a same pattern compared to the observed realized volatility. However, the models seems to overestimate conditional volatility this is due to the fact that most volatility forecasts are not lower than 0.01 but observed volatility is often close to zero. Just like the index for Argentina, only at the very end of the testing window a period of high volatility is observed. This period of high volatility is likely due to the COVID-19 pandemic started in the end of 2019 with the highest impact on the markets in the start of the second quarter in 2020.

7.3.2 Diebold-Mariano test based on the mean absolute error

Table 6 shows the Diebold-Mariano test on the mean absolute error results as described in section 6.2.1 for the index of Argentina. By keeping the volatility model constant it becomes possible to conclude that for the ARCH, the GARCH, and the EGARCH models the normal distribution significantly outperforms the Student's T distribution and the skewed Student's T distribution based on the mean absolute error. The Student's T distribution ARCH model and GARCH model significantly outperforms the same models with a skewed Student's T distribution. Therefore, it is possible to conclude that the skewed Student's T distribution is the worst performer based on this measure. By comparing different models and distributions together it becomes possible to conclude that the ARCH normal, the ARCH T, and the ARCH skewed T significantly outperforms the EGARCH T and EGARCH skewed T model, the GARCH T and GARCH skewed T models significantly outperform the EGARCH T and EGARCH skewed T models, the EGARCH normal model significantly outperforms the EGARCH T and EGARCH skewed T models. The best performer is the GARCH normal model.

Model	ARCH normal	ARCH T	ARCH skew T	GARCH normal	GARCH T	GARCH skew T	EGARCH normal	EGARCH T	EGARCH skew T
ARCH normal	X	-14.1613	-18.2649	3.3557	1.0539	0.6208	-1.6667	-2.703	-2.7981
	X	[0.0000]	[0.0000]	[0.0015]	[0.2289]	[0.3289]	[0.0995]	[0.0104]	[0.0080]
ARCH T	14.1613	X	-12.1802	4.0333	1.7777	1.3498	-1.0623	-2.3003	-2.342
	[0.0000]	X	[0.0000]	[0.0001]	[0.0822]	[0.1604]	[0.2269]	[0.0284]	[0.0258]
ARCH skew T	18.2649	12.1802	X	4.2243	1.9747	1.5467	-0.9154	-2.2135	-2.2441
	[0.0000]	[0.0000]	X	[0.0001]	[0.0568]	[0.1206]	[0.2623]	[0.0345]	[0.0322]
GARCH normal	-3.3557	-4.0333	-4.2243	X	-37.9019	-34.1752	-3.3138	-3.7116	-3.8643
	[0.0015]	[0.0001]	[0.0001]	X	[0.0000]	[0.0000]	0.0017	0.0004	0.0002
GARCH T	-1.0539	-1.7777	-1.9747	37.9019	X	-22.6782	-1.9955	-2.7952	-2.8543
	[0.2289]	[0.0822]	[0.0568]	[0.0000]	X	[0.0000]	[0.0545]	[0.0081]	[0.0068]
GARCH skew T	-0.6208	-1.3498	-1.5467	34.1752	22.6782	X	-1.7386	-2.6184	-2.6591
	[0.3289]	[0.1604]	[0.1206]	[0.0000]	[0.0000]	X	[0.0880]	[0.0130]	[0.0117]
EGARCH normal	1.6667	1.0623	0.9154	3.3138	1.9955	1.7386	X	-4.4507	-5.5195
	[0.0995]	[0.2269]	[0.2623]	[0.0017]	[0.0545]	[0.088]	X	[0.0000]	[0.0000]
EGARCH T	2.703	2.3003	2.2135	3.7116	2.7952	2.6184	4.4507	X	1.976
	[0.0104]	[0.0284]	[0.0345]	[0.0004]	[0.0081]	[0.0130]	[0.0000]	X	[0.0567]
EGARCH skew T	2.7981	2.342	2.2441	3.8643	2.8543	2.6591	5.5195	-1.976	X
	[0.008]	[0.0258]	[0.0322]	[0.0002]	[0.0068]	[0.0117]	[0.0000]	[0.0567]	X

Table 6: Mean absolute error comparison based on the Diebold-Mariano test for Argentina. A positive (negative) cell value means that the the row (column) model has a higher mean absolute error. A coloured cell means that the Diebold-Mariano statistic is significantly different from zero at a 5% level. A green (red) cell means that the row (column) has a lower mean absolute error.

Table 7 shows the Diebold-Mariano test on the mean absolute error results as described in section 6.2.1 for the index of Brazil. By keeping the volatility model constant it becomes possible to conclude that for the ARCH model, and the EGARCH model the Student's T distribution significantly outperforms the normal distribution based on the mean absolute error. For the ARCH and for the GARCH models the skewed Student's T distribution significantly outperforms the same model with a a Student's T distribution based on the mean absolute error. For the EGARCH model the Student's T distribution significantly outperforms the skewed Student's T distribution. By comparing different models and distributions together it becomes possible to conclude that the ARCH normal, the ARCH T, and the ARCH skewed T significantly outperforms the GARCH normal, the GARCH T, and GARCH skewed T models. Moreover, the EGARCH normal, the EGARCH T, and the EGARCH skewed T significantly outperforms the GARCH normal, the GARCH T, and GARCH skewed T models. Lastly, it becomes possible to conclude that the GARCH skewed T model is the worst performer and that the volatility is best modeled by a ARCH model or a EGARCH model.

Model	ARCH normal	ARCH T	ARCH skew T	GARCH normal	GARCH T	GARCH skew T	EGARCH normal	EGARCH T	EGARCH skew T
ARCH normal	X	8.4159	5.1926	-9.602	-9.2979	-9.6486	0.7222	1.1476	0.936
	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.3073]	[0.2065]	[0.2574]
ARCH T	-8.4159	X	-4.4427	-9.9706	-9.6818	-10.0326	-0.3851	-0.0812	-0.345
	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.3704]	[0.3976]	[0.3758]
ARCH skew T	-5.1926	4.4427	X	-9.6954	-9.4185	-9.763	-0.1217	0.1915	-0.0455
	[0.0000]	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]	[0.396]	[0.3916]	[0.3985]
GARCH normal	9.602	9.9706	9.6954	X	0.1495	-3.1859	13.056	12.6644	12.3554
	[0.0000]	[0.0000]	[0.0000]	X	[0.3945]	[0.0025]	[0.0000]	[0.0000]	[0.0000]
GARCH T	9.2979	9.6818	9.4185	-0.1495	X	-566.0342	12.5458	12.1942	11.9089
	[0.0000]	[0.0000]	[0.0000]	[0.3945]	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]
GARCH skew T	9.6486	10.0326	9.763	3.1859	566.0342	X	13.0079	12.6381	12.3459
	[0.0000]	[0.0000]	[0.0000]	[0.0025]	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]
EGARCH normal	-0.7222	0.3851	0.1217	-13.056	-12.5458	-13.0079	X	2.8477	0.5417
	[0.3073]	[0.3704]	[0.396]	[0.0000]	[0.0000]	[0.0000]	X	[0.007]	[0.3444]
EGARCH T	-1.1476	0.0812	-0.1915	-12.6644	-12.1942	-12.6381	-2.8477	X	-4.4284
	[0.2065]	[0.3976]	[0.3916]	[0.0000]	[0.0000]	[0.0000]	[0.007]	X	[0.0000]
EGARCH skew T	-0.936	0.345	0.0455	-12.3554	-11.9089	-12.3459	-0.5417	4.4284	X
	[0.2574]	[0.3758]	[0.3985]	[0.0000]	[0.0000]	[0.0000]	[0.3444]	[0.0000]	X

Table 7: Mean absolute error comparison based on the Diebold-Mariano test for Brazil. A positive (negative) cell value means that the the row (column) model has a higher mean absolute error. A coloured cell means that the Diebold-Mariano statistic is significantly different from zero at a 5% level. A green (red) cell means that the row (column) has a lower mean absolute error.

7.3.3 Diebold-Mariano test based on the median absolute error

Table 8 shows the Diebold-Mariano test on the median absolute error results as described in section 6.2.1 for the index of Argentina. Firstly, by keeping the volatility model constant it becomes possible to conclude that for the ARCH model and the GARCH model, the normal distribution significantly outperform the Student's T distribution and the skewed Student's T distribution. Secondly, for the ARCH model and for the GARCH model the Student's T distribution significantly outperforms the skewed Student's T distribution. Thirdly, the ARCH normal model significantly outperforms the GARCH normal, the GARCH T, and the GARCH skewed T models. Fourthly, the ARCH T model and the ARCH skewed T model significantly outperforms the GARCH T model and the GARCH skewed T model. Fifthly, the ARCH T model significantly outperforms the GARCH T and GARCH skewed T models. Sixthly, the ARCH skewed T model significantly outperforms the GARCH T and GARCH skewed T models. Lastly, the EGARCH normal model significantly outperforms the GARCH T model and the GARCH skewed T model.

	ARCH normal	ARCH T	ARCH skew T	GARCH normal	GARCH T	GARCH skew T	EGARCH normal	EGARCH T	EGARCH skew T
ARCH normal	X	-5.8856	-8.6269	-2.29	-4.2046	-4.5048	0.0039	-0.0963	-0.199
	X	[0.0000]	[0.0000]	[0.0290]	[0.0001]	[0.0000]	0.3989	0.397	0.3911
ARCH T	5.8856	X	-6.5791	-1.7424	-3.6715	-3.9903	0.233	0.0747	-0.0079
	[0.0000]	X	[0.0000]	0.0874	[0.0005]	[0.0001]	0.3882	0.3978	0.3989
ARCH skew T	8.6269	6.5791	X	-1.6144	-3.5553	-3.8652	0.3281	0.1272	0.0676
	[0.0000]	[0.0000]	X	0.1084	[0.0007]	[0.0002]	0.378	0.3957	0.398
GARCH normal	2.29	1.7424	1.6144	X	-33.4997	-29.79	1.1584	0.7171	0.7004
	[0.0290]	0.0874	0.1084	X	[0.0000]	[0.0000]	0.2039	0.3084	0.3121
GARCH T	4.2046	3.6715	3.5553	33.4997	X	-18.3458	2.3988	1.5984	1.6925
	[0.0001]	[0.0005]	[0.0007]	[0.0000]	X	[0.0000]	[0.0225]	0.1112	0.0953
GARCH skew T	4.5048	3.9903	3.8652	29.79	18.3458	X	2.6137	1.7523	1.8639
	[0.0000]	[0.0001]	[0.0002]	[0.0000]	[0.0000]	X	[0.0132]	0.0859	0.0703
EGARCH normal	-0.0039	-0.233	-0.3281	-1.1584	-2.3988	-2.6137	X	-0.4065	-0.8765
	0.3989	0.3882	0.378	0.2039	[0.0225]	[0.0132]	X	0.3672	0.2716
EGARCH T	0.0963	-0.0747	-0.1272	-0.7171	-1.5984	-1.7523	0.4065	X	-0.6042
	0.397	0.3978	0.3957	0.3084	0.1112	0.0859	0.3672	X	0.3323
EGARCH skew T	0.199	0.0079	-0.0676	-0.7004	-1.6925	-1.8639	0.8765	0.6042	X
	0.3911	0.3989	0.398	0.3121	0.0953	0.0703	0.2716	0.3323	X

Table 8: Median absolute error comparison based on the Diebold-Mariano test for Argentina. A positive (negative) cell value means that the the row (column) model has a higher median absolute error. A coloured cell means that the Diebold-Mariano statistic is significantly different from zero at a 5% level. A green (red) cell means that the row (column) has a lower median absolute error.

Table 9 shows the Diebold-Mariano test on the median absolute error results as described in section 6.2.1 for the index of Brazil. Firstly, it becomes possible to conclude that for the ARCH and GARCH models the Student's T distribution and the skewed Student's T distribution significantly outperforms the normal distribution and that the skewed Student's T distribution significantly outperform the Student's T distribution. Secondly, for the GARCH model the normal distribution significantly outperforms the Student's T distribution and the skewed Student's T distribution and the Student's T distribution significantly outperforms the skewed Student's T distribution. Thirdly, the ARCH normal model significantly outperform the GARCH normal, GARCH T, GARCH skewed T, and EGARCH skewed T models. Fourthly, the ARCH T model significantly outperforms the GARCH normal, GARCH T, GARCH skewed T, EGARCH normal, EGARCH T, and EGARCH skewed T models. Fifthly, the ARCH skewed T model is significantly the best performing model outperforming all other model specifications. Lastly, the EGARCH normal, EGARCH T, and EGARCH skewed T models significantly outperforms the GARCH normal, GARCH T, and GARCH skewed T models.

	ARCH normal	ARCH T	ARCH skew T	GARCH normal	GARCH T	GARCH skew T	EGARCH normal	EGARCH T	EGARCH skew T
ARCH normal	X	9.5696	9.1508	-16.4848	-16.4066	-16.749	-2.1916	-1.3839	-1.2954
	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0362]	0.1531	0.1724
ARCH T	-9.5696	X	4.0817	-16.8101	-16.7799	-17.1234	-3.0057	-2.2984	-2.3648
	[0.0000]	X	[0.0001]	[0.0000]	[0.0000]	[0.0000]	[0.0044]	[0.0285]	[0.0244]
ARCH skew T	-9.1508	-4.0817	X	-16.5444	-16.566	-16.9031	-2.9034	-2.0911	-2.2741
	[0.0000]	[0.0001]	X	[0.0000]	[0.0000]	[0.0000]	[0.0059]	[0.0449]	[0.0301]
GARCH normal	16.4848	16.8101	16.5444	X	-7.8165	-11.0991	19.3697	19.1142	18.8772
	[0.0000]	[0.0000]	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
GARCH T	16.4066	16.7799	16.566	7.8165	X	-549.1008	19.1853	18.9505	18.7601
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]
GARCH skew T	16.749	17.1234	16.9031	11.0991	549.1008	X	19.6374	19.3892	19.1891
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]
EGARCH normal	2.1916	3.0057	2.9034	-19.3697	-19.1853	-19.6374	X	8.8038	6.2408
	[0.0362]	[0.0044]	[0.0059]	[0.0000]	[0.0000]	[0.0000]	X	[0.0000]	[0.0000]
EGARCH T	1.3839	2.2984	2.0911	-19.1142	-18.9505	-19.3892	-8.8038	X	0.9183
	0.1531	[0.0285]	[0.0449]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	X	0.2616
EGARCH skew T	1.2954	2.3648	2.2741	-18.8772	-18.7601	-19.1891	-6.2408	-0.9183	X
	0.1724	[0.0244]	[0.0301]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	0.2616	X

Table 9: Median absolute error comparison based on the Diebold-Mariano test for Brazil. A positive (negative) cell value means that the the row (column) model has a higher median absolute error. A coloured cell means that the Diebold-Mariano statistic is significantly different from zero at a 5% level. A green (red) cell means that the row (column) has a lower median absolute error.

7.3.4 Diebold-Mariano test based on the logarithmic scoring rule

Table 10 shows the DM-test based on logarithmic scoring, described in section 6.2.2, for Argentina. Based on all significant results it is possible to conclude the following. Firstly, for the ARCH model and the EGARCH model the normal distribution outperforms the Student's T distribution and its skewed version. Secondly, for the GARCH model the normal distribution is outperformed by both the Student's T distribution and its skewed version; the skewed Student's T distribution outperforms the Student's T distribution. Thirdly, the ARCH normal model is the best performer. Fourthly, the ARCH T model outperforms the GARCH normal model. Fifthly, the ARCH skewed T model outperforms the GARCH normal and GARCH T models. Sixthly, the GARCH T model outperforms the ARCH T model. Seventhly, the GARCH skewed T model outperforms the ARCH T model. Eightly, the EGARCH normal model outperforms the ARCH T model and all GARCH models. Ninthly, the EGARCH T model outperforms the ARCH T and GARCH normal models. Lastly, the EGARCH skewed T model outperforms the ARCH T model and the GARCH normal model.

	ARCH normal	ARCH T	ARCH skew T	GARCH normal	GARCH T	GARCH skew T	EGARCH normal	EGARCH T	EGARCH skew T
ARCH normal	X	7.76414	5.69842	28.0836	6.80826	6.23074	5.57897	5.77228	5.60606
	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ARCH T	-7.76414	X	-7.00256	13.6029	-7.64666	-8.00015	-5.55362	-7.76391	-7.30596
	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ARCH skew T	-5.69842	7.00256	X	17.6226	2.22798	1.58639	-0.17107	1.35489	1.3516
	[0.0000]	[0.0000]	X	[0.0000]	[0.0334]	0.11336	0.39309	0.1593	0.16001
GARCH normal	-28.0836	-13.6029	-17.6226	X	-17.2092	-17.3814	-19.8967	-17.1074	-16.9665
	[0.0000]	[0.0000]	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
GARCH T	-6.80826	7.64666	-2.22798	17.2092	X	-2.68975	-3.0329	-1.71005	-1.42354
	[0.0000]	[0.0000]	[0.0334]	[0.0000]	X	[0.01076]	[0.00405]	0.09247	0.14481
GARCH skew T	-6.23074	8.00015	-1.58639	17.3814	2.68975	X	-2.22398	-0.24514	-0.24979
	[0.0000]	[0.0000]	0.11336	[0.0000]	[0.01076]	X	[0.0337]	0.38708	0.38663
EGARCH normal	-5.57897	5.55362	0.17107	19.8967	3.0329	2.22398	X	2.42343	2.30333
	[0.0000]	[0.0000]	0.39309	[0.0000]	[0.00405]	[0.0337]	X	[0.02122]	[0.02817]
EGARCH T	-5.77228	7.76391	-1.35489	17.1074	1.71005	0.24514	-2.42343	X	0.02691
	[0.0000]	[0.0000]	0.1593	[0.0000]	0.09247	0.38708	[0.02122]	X	0.39875
EGARCH skew T	-5.60606	7.30596	-1.3516	16.9665	1.42354	0.24979	-2.30333	-0.02691	X
	[0.0000]	[0.0000]	0.16001	[0.0000]	0.14481	0.38663	[0.02817]	0.39875	X

Table 10: Logarithmic scoring rule comparison based on the Diebold-Mariano test for Argentina. A positive (negative) cell value means the row (column) model has a higher log likelihood. A coloured cell means that the Diebold-Mariano statistic is significantly different from zero at a 5% level. A green (red) cell means that the row (column) has a higher log likelihood.

Table 11 shows the DM-test based on logarithmic scoring, described in section 6.2.2, for Brazil. Based on all significant results it is possible to conclude the following. Firstly, for the ARCH model and the GARCH model the Student's T distribution and its skewed version outperforms the normal distribution. Secondly, for the ARCH model the Student's T distribution outperforms its skewed version. Thirdly, the ARCH T model outperforms the GARCH normal model. Fourthly, the ARCH skewed T model outperforms the GARCH normal model. Fifthly, the GARCH normal model is the worst performer. Sixthly, the GARCH T model outperforms the ARCH normal, ARCH skewed T, EGARCH normal, and the EGARCH skewed T model. Seventhly, the GARCH skewed T model outperforms the ARCH normal, ARCH skewed T, EGARCH normal, and the EGARCH skewed T models. Eightly, the EGARCH normal model outperforms the ARCH normal, ARCH skewed T and the GARCH normal models. Ninthly, the EGARCH T model outperfoms the ARCH normal, ARCH skewed T, GARCH normal, EGARCH normal, and the EGARCH skewed T model. Lastly, the EGARCH skewed T model outperforms the ARCH normal, ARCH skewed T, and the GARCH normal models.

	ARCH normal	ARCH T	ARCH skew T	GARCH normal	GARCH T	GARCH skew T	EGARCH normal	EGARCH T	EGARCH skew T
ARCH normal	X	-9.78867	-5.65196	80.0932	-12.6741	-12.285	-10.4751	-10.6791	-9.17687
	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ARCH T	9.78867	X	10.1765	58.7774	-1.89724	-1.29251	1.08266	0.0846	1.37182
	[0.0000]	X	[0.0000]	[0.0000]	0.066	0.17301	0.22196	0.39747	0.15567
ARCH skew T	5.65196	-10.1765	X	68.1831	-13.2147	-13.4043	-9.25343	-10.1876	-8.67454
	[0.0000]0	0	X	[0.0000]	0	0	0	0	0
GARCH normal	-80.0932	-58.7774	-68.1831	X	-63.0676	-62.9536	-62.8503	-61.5169	-60.292
	[0.0000]	[0.0000]	[0.0000]	X	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
GARCH T	12.6741	1.89724	13.2147	63.0676	X	0.36531	2.46469	1.36836	2.52343
	[0.0000]	0.066	[0.0000]	[0.0000]	X	0.37313	[0.01919]	0.15641	[0.01658]
GARCH skew T	12.285	1.29251	13.4043	62.9536	-0.36531	X	2.3366	1.17378	2.82208
	[0.0000]	0.17301	[0.0000]	[0.0000]	0.37313	X	[0.02608]	0.20028	[0.00748]
EGARCH normal	10.4751	-1.08266	9.25343	62.8503	-2.46469	-2.3366	X	-3.51019	0.42021
	[0.0000]	0.22196	[0.0000]	[0.0000]	[0.01919]	[0.02608]	X	[0.00086]	0.36517
EGARCH T	10.6791	-0.0846	10.1876	61.5169	-1.36836	-1.17378	3.51019	X	3.53108
	[0.0000]	0.39747	[0.0000]	[0.0000]	0.15641	0.20028	[0.0009]	X	[0.0008]
EGARCH skew T	9.17687	-1.37182	8.67454	60.292	-2.52343	-2.82208	-0.42021	-3.53108	X
	[0.0000]	0.15567	[0.0000]	[0.0000]	[0.01658]	[0.00748]	0.36517	[0.00079]	X

Table 11: Logarithmic scoring rule comparison based on the Diebold-Mariano test for Brazil. A positive (negative) cell value means the row (column) model has a higher log likelihood. A coloured cell means that the Diebold-Mariano statistic is significantly different from zero at a 5% level. A green (red) cell means that the row (column) has a higher log likelihood.

7.3.5 Value at Risk

Below the observed Value at Risk violations are given, on the next page they will be furtherly discussed.

Model	Expected Violations	Observed Violations	Violation Ratio	Estimated Probability ($\hat{p}$)
Argentina				
Normal				
ARCH 5% VaR	98	91	0.93	4.65%
ARCH 1% VaR	20	37	1.85	1.89%
GARCH 5% VaR	98	81	0.83	4.14%
GARCH 1% VaR	20	8	0.40	0.41%
EGARCH 5% VaR	98	67	0.68	3.43%
EGARCH 1% VaR	20	7	0.35	0.36%
Student's T				
ARCH 5% VaR	98	104	1.06	5.32%
ARCH 1% VaR	20	14	0.70	0.72%
GARCH 5% VaR	98	83	0.85	4.24%
GARCH 1% VaR	20	3	0.15	0.15%
EGARCH 5% VaR	98	68	0.69	3.48%
EGARCH 1% VaR	20	2	0.10	0.10%
Skewed Student's T				
ARCH 5% VaR	98	97	0.99	4.96%
ARCH 1% VaR	20	10	0.50	0.51%
GARCH 5% VaR	98	75	0.77	3.83%
GARCH 1% VaR	20	2	0.10	0.10%
EGARCH 5% VaR	98	60	0.61	3.07%
EGARCH 1% VaR	20	2	0.10	0.1%
Brazil				
Normal				
ARCH 5% VaR	101	42	0.42	2.08%
ARCH 1% VaR	20	12	0.60	0.59%
GARCH 5% VaR	101	64	0.63	3.17%
GARCH 1% VaR	20	7	0.35	0.35%
EGARCH 5% VaR	101	48	0.48	2.38%
EGARCH 1% VaR	20	7	0.35	0.35%
Student's T				
ARCH 5% VaR	101	49	0.49	2.43%
ARCH 1% VaR	20	7	0.35	0.35%
GARCH 5% VaR	101	69	0.68	3.42%
GARCH 1% VaR	20	6	0.30	0.30%
EGARCH 5% VaR	101	54	0.53	2.68%
EGARCH 1% VaR	20	6	0.30	0.30%
Skewed Student's T				
ARCH 5% VaR	101	39	0.39	1.93%
ARCH 1% VaR	20	7	0.35	0.35%
GARCH 5% VaR	101	56	0.55	2.78%
GARCH 1% VaR	20	3	0.15	0.15%
EGARCH 5% VaR	101	45	0.45	2.23%
EGARCH 1% VaR	20	3	0.15	0.15%

Table 12: Value at Risk violations table. The expected violations is defined as the number of violations expected based on the selected p, the observed violations is the total sum of η_t, violation ratio is the ratio of observed violations and expected violations, and the estimated probability is the probability corresponding to the observed number of violations with the length of the vector η_t.

Based on the results in table 12, it becomes possible to conclude that most models over-forecast the Value at Risk. For Argentina the ARCH normal 5% VaR model, the GARCH normal 5% VaR model, the ARCH student's t 5% VaR model, the GARCH Student's T 5% VaR model, and the the ARCH skewed Student's T 5% VaR model are inside $VR \in [0.8, 1.2]$ which classifies them as a good Value at Risk forecast. For Brazil no model is inside $VR \in [0.8, 1.2]$ and thereby risk is highly overestimated for the Brazilian index. Moreover, for both the index of Argentina and the index of Brazil the violation ratio of the 1% forecasts are in general further away from $VR \in [0.8, 1.2]$. Therefore, it could be concluded that the Value at Risk is more precisely estimated at a 5% level than at a 1% level.

Below the results on the Bernoulli coverage, independence, and joint backtest are given:

Model	Test	Normal	T	Skew T	Normal	T	Skew. T
		Argentina			**Brazil**		
ARCH 5% VaR	LR_{bern}	0.5017	0.4192	0.0053	45.9082	34.2994	51.5061
		[0.4788]	[0.5174]	[0.9425]	[0.0000]	[0.0000]	[0.0000]
ARCH 1% VaR	LR_{bern}	12.4656	1.7605	5.7294	3.9104	11.5975	11.5975
		[0.0004]	[0.1846]	[0.0167]	[0.0480]	[0.0007]	[0.0007]
GARCH 5% VaR	LR_{bern}	3.2004	2.4480	6.0144	16.1960	11.8187	24.8078
		[0.0736]	[0.1177]	[0.0142]	[0.0001]	[0.0006]	[0.0000]
GARCH 1% VaR	LR_{bern}	8.8722	21.9779	26.1213	11.6107	13.8769	23.0366
		[0.0029]	[0.0000]	[0.0000]	[0.0007]	[0.0002]	[0.0000]
EGARCH 5% VaR	LR_{bern}	11.3918	10.5871	17.6537	35.8709	27.3257	40.6274
		[0.0007]	[0.0011]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
EGARCH 1% VaR	LR_{bern}	10.8023	26.1213	26.1213	13.8911	19.5112	19.5112
		[0.0010]	[0.0000]	[0.0000]	[0.0002]	[0.0000]	[0.0000]
ARCH 5% VaR	LR_{ind}	0.1432	6.5254	8.7167	1.7874	2.0635	0.0754
		[0.7051]	[0.0106]	[0.0032]	[0.1813]	[0.1509]	[0.7838]
ARCH 1% VaR	LR_{ind}	1.4284	0.2022	0.1029	0.1437	0.0488	0.0488
		[0.2320]	[0.6530]	[0.7484]	[0.7046]	[0.8252]	[0.8252]
GARCH 5% VaR	LR_{ind}	0.1266	1.5881	0.4235	0.0005	1.0622	0.2395
		[0.7221]	[0.2076]	[0.5152]	[0.9822]	[0.3027]	[0.6246]
GARCH 1% VaR	LR_{ind}	0.0658	0.0092	0.0041	0.0488	0.0358	0.0089
		[0.7977]	[0.9236]	[0.9489]	[0.8253]	[0.8499]	[0.9248]
EGARCH 5% VaR	LR_{ind}	0.2114	2.4286	0.6560	0.0195	0.2003	0.0000
		[0.6457]	[0.1191]	[0.4180]	[0.8889]	[0.6545]	[1.0000]
EGARCH 1% VaR	LR_{ind}	0.0503	0.0041	0.0041	0.0358	0.0159	0.0159
		[0.8225]	[0.9489]	[0.9489]	[0.8499]	[0.8997]	[0.8997]
ARCH 5% VaR	LR_{joint}	0.6450	6.9446	8.7219	47.6956	36.3629	51.5814
		[0.7244]	[0.0310]	[0.0128]	[0.0000]	[0.0000]	[0.0000]
ARCH 1% VaR	LR_{joint}	13.8939	1.9626	5.8323	4.0541	11.6463	11.6463
		[0.0001]	[0.3748]	[0.0541]	[0.1317]	[0.0030]	[0.0030]
GARCH 5% VaR	LR_{joint}	3.3270	4.0361	6.4380	16.1966	12.8809	25.0473
		[0.1895]	[0.1329]	[0.0400]	[0.0003]	[0.0016]	[0.0000]
GARCH 1% VaR	LR_{joint}	8.9380	21.9871	26.1254	11.6594	13.9128	23.0455
		[0.0115]	[0.0000]	[0.0000]	[0.0029]	[0.0010]	[0.0000]
EGARCH 5% VaR	LR_{joint}	11.6032	13.0157	18.3096	35.8904	27.5261	40.6274
		[0.0030]	[0.0015]	[0.0001]	[0.0000]	[0.0000]	[0.0000]
EGARCH 1% VaR	LR_{joint}	10.8526	26.1254	26.1254	13.9269	19.5272	19.5272
		[0.0044]	[0.0000]	[0.0000]	[0.0009]	[0.0001]	[0.0001]

Table 13: Value at Risk (VaR) backtests, for 5% and 1%. The probability of the likelihood ratio test statistic is in brackets. Where LR_{bern} stands for the LR statistic of the Bernoulli coverage test, LR_{ind} is the LR statistic of the independence test and LR_{joint} refers to the joint test statistic. Probabilities are calculated based on their corresponding degrees of freedom on the chi-square distribution.

Based on the Bernoulli unconditional coverage test the ARCH normal 5% VaR model, the GARCH normal 5% VaR model, the ARCH Student's T 5% VaR model, the ARCH Student's T 1% VaR model, the GARCH Student's T 5% VaR model, and the ARCH skewed Student's T 5% VaR model are significant for the index of Argentina. The overall higher likelihood ratio for the 1% VaR models in comparison to the 5% VaR models indicates that the VaR is more precise estimated for the

5% VaR. Based on the independence test all VaR models are significant except for the ARCH student's t VaR 5% model and the ARCH skewed student's t VaR 5% model for the index of Argentina. However, the results of the independence test should be interpreted with caution due to the low number of violations for some models. Based on the joint test the ARCH normal 5% VaR model, the GARCH normal 5% VaR model, the ARCH student's t 1% VaR model, the GARCH student's t 5% VaR model, and the ARCH skewed student's t 1% VaR model are significant for the index of Argentina.

Based on the Bernoulli unconditional coverage test no VaR model is significant for the index of Brazil. The overall lower likelihood ratio for the 1% VaR models in comparison to the 5% VaR models indicates that the VaR is more precise estimated for the 1% VaR. Based on the independence test all VaR models are significant for the index of Brazil. However, the results of the independence test should be interpreted with caution due to the low number of violations for some models. Based on the joint test only the ARCH normal 1% VaR model is significant for the index of Brazil.

7.4 Results evaluation

Based on the summary statistics given in table 1 it was already observable that for the index of Argentina the in sample standard deviation was slightly smaller than the out of sample standard deviation. Therefore, it was likely that for this index the conditional volatility would be underestimated. Moreover, the return distribution for both in and out of sample was just slightly negatively skewed with a more negative skew in the out of sample period. Thereby, it was likely to observe that the conditional volatility would be slightly underestimated and it could be concluded that skewed distribution might not significantly outperform other not skewed distributions.

Based on the summary statistics given in table 1 it was already observable that for the index of Brazil the in sample standard deviation was higher than the out of sample standard deviation. Therefore, it was likely that for this index the conditional volatility would be overestimated. Moreover, the return distribution for both in and out of sample was just slightly negatively skewed with a more negative skew in the out of sample period. Thereby, it was likely to observe that the conditional volatility would be slightly underestimated and it could be concluded that skewed distribution might not significantly outperform other not skewed distributions.

In order to answer why more sophisticated models like the GARCH and EGARCH model did not always outperform the standard ARCH model the out of sample parameters are also computed, so the best fit for the forecasted period. In table 14 these results are given this could be compared to the in sample parameter estimates in table 2 to analyze the obtained results.

Model	ω	α	β	ν/η	γ	λ
Argentina						
ARCH normal	0.00033^{***}	0.37239^{***}				
ARCH T	0.00037^{***}	0.32693^{***}		3.74677^{***}		
ARCH skew T	0.00038^{***}	0.33969^{***}		3.64837^{***}		-0.09779^{***}
GARCH normal	$2.8525 \cdot 10^{-5***}$	0.22847^{***}	0.73999^{***}			
GARCH T	$1.2859 \cdot 10^{-5***}$	0.11366^{***}	0.86427^{***}	5.5064^{***}		
GARCH skew T	$1.2939 \cdot 10^{-5***}$	0.11588^{***}	0.86483^{***}	5.37343^{***}		-0.11558^{***}
EGARCH normal	$6.7499 \cdot 10^{-7***}$	0.49623^{***}	0.8928^{***}		-0.00884	
EGARCH T	$2.3457 \cdot 10^{-7***}$	0.19935^{***}	0.96299^{***}	5.47813^{***}	-0.06413^{***}	
EGARCH skew T	$2.2570 \cdot 10^{-7***}$	0.19898^{***}	0.96481^{***}	5.34392^{***}	-0.05903^{***}	-0.11199^{***}
Brazil						
ARCH normal	0.00023^{***}	0.26948				
ARCH T	0.00024^{***}	0.20198		4.43742^{***}		
ARCH skew T	0.00024^{***}	0.20246		4.44203^{***}		-0.067^{***}
GARCH normal	$7.4468 \cdot 10^{-6***}$	0.08336^{***}	0.89023^{***}			
GARCH T	$6.0195 \cdot 10^{-6***}$	0.07534^{***}	0.90293^{***}	8.36808^{***}		
GARCH skew T	$5.8557 \cdot 10^{-6***}$	0.07444^{***}	0.90468^{***}	8.4317^{***}		-0.07097^{***}
EGARCH normal	$1.7599 \cdot 10^{-7***}$	0.15483^{***}	0.96892^{***}		-0.09544^{***}	
EGARCH T	$1.4861 \cdot 10^{-7***}$	0.14203^{***}	0.97378^{***}	8.6471^{***}	-0.09636^{***}	
EGARCH skew T	$1.4774 \cdot 10^{-7***}$	0.14043^{***}	0.97396^{***}	8.74564^{***}	-0.09589^{***}	-0.07528^{***}

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

Table 14: Out of sample parameter estimates of the index of Argentina and Brazil.

Based on the results in table 14 it becomes possible to conclude the following for the index of Argentina. Firstly, the constant term, ω, has increased for the GARCH and EGARCH model. Secondly, the news factor, α, has increased and thus became more important for all models. Thirdly, the memory factor, β, has decreased and thus became less important for all models utilizing this parameter. Thirdly, the shape parameters, ν/η stayed approximately the same. Fourthly, leverage effects has decreased for the EGARCH model and thus became less important. Lastly, the skewness parameter, λ, has also changed.

Based on the results in table 14 it becomes possible to conclude the following for the index of Brazil. Firstly, the constant term, ω, has increased for the GARCH and EGARCH model. Secondly, the news factor, α, has decreases and thus became less for the ARCH models and has increased for the GARCH and EGARCH models. Thirdly, the memory factor, β, has increased and thus became more important for all models utilizing this parameter. Thirdly, the shape parameters, ν/η sharply decreased thus models utilizing a shape parameter are vulnerable for forecasting errors. the same. Fourthly, leverage effects has increased for the EGARCH model and thus became more important. Lastly, the skewness parameter, λ, also changed.

8 Conclusion

Based on the empirical analysis it becomes possible to conclude the following.

Firstly, in the two South American stock market indices auto-correlation is observed in the squared returns and thereby volatility could be forecasted to some extent.

Secondly, based on the in-sample performance measures models which do utilizes are models which do model for memory effects, like the GARCH model, outperforms models which do not utilizes such a parameter, like the ARCH model. Moreover, models which do utilizes leverage effects, like the EGARCH model, does also outperform models which do not captures these effects, like the ARCH model and the GARCH model.

However, based on the out of sample performance measures this is contradicted. For the stock market of Argentina it is even observed that the basic ARCH model does often outperform the more sophisticated GARCH and EGARCH models. For the index of Brazil the ARCH and EGARCH model are performing relatively well.

In addition, the Value at Risk results shows for the index of Argentina that only the ARCH models sometimes is above the violation ratio, but the estimated probability is very close to the benchmark probability. Overall, for the index of Argentina the more sophisticated a model becomes the more it overestimates the Value at Risk. For the index of Brazil the Value at Risk is best modeled by a GARCH model followed by a EGARCH model and lastly the ARCH model.

Lastly, the in-sample test on model fit shows that the skewed Student's T is the best performing model followed by the Student's T distribution, and lastly the normal distribution. However, the out of sample results show that for the index of Argentina the normal distribution often outperforms the Student's T distribution and the skewed Student's T distribution and that the Student's T distribution often outperforms the skewed Student's T distribution. For the index of Brazil it is possible to conclude that the normal distribution is often outperformed by the Student's T distribution and the skewed Student's T distribution. Between the last two mentioned distributions the results on which one performs the best differs depending on the model chosen. Based on the Value at Risk results it possible to conclude that for the index of Argentina the Student's T distribution performs best, followed by the normal distribution, and lastly the skewed Student's T distribution. For the index of Brazil, the normal distribution performs the best on forecasting Value at Risk, followed by the Student's T distribution, and lastly the skewed Student's T distribution.

The unsatisfying results that more sophisticated models do not always performs better is due to the fact that the indices dynamics has changed over time as given in table 14 and given in the in and out sample statistics in table 1. This service as proof that the modelling of extreme volatile financial data, like the index of Argentina, is difficult and shows that precisely modelling of emerging markets may not be possible. In order to solve this one could implement rolling window estimation models, which are now not implemented due to computational restrictions, or regime-changing models. These models are more capable to capture current dynamics in financial data.

9 Suggestions for further research

Besides the aforementioned rolling window estimation and regime-changing models to handle changing index dynamics, the following topics are also suggested for further research.

Firstly, more models could be tested like the GJR-GARCH which also captures leverage effects, the AVARCH model which is a asymmetric ARCH model, the AVGARCH model which is a assymetric GARCH model, the TGARCH model which is a threshold GARCH model, the GARCH-M model which is a GARCH in mean model, and IGARCH models which is a GARCH model that applies both an moving average structure and an autoregressive structure to the variance, σ^2.

Secondly, other innovations distributions could be tested like the generalized error distribution which can both deal with thinner and fatter tails than the normal distribution. Alternatively, the generalized logistic distribution or its alternative form the the exponential generalized beta distribution or the second kind exponential generalized beta distribution could be implemented. Given the results the exponential generalized beta distribution of the second kind is preferred because it is capable of modeling both skewness and peakedness, extreme peakedness is observed in the index data of both indices.

Thirdly, for parameter estimation the frequentist method is applied as a alternative approach it would be interesting to apply a alternative approach like Bayesian estimation to see if this method improves the conditional volatility forecasts.

Fourthly, for higher-order model structures it could be investigated if the outperform single-order models.

Lastly, the Value at Risk computed gives the best of worst case scenarios and is not capable of fully describing the loss function tail dynamics. Given the very large peaks at the end of the data-sets the Value at Risk measures as utilized in this thesis only are classified as a violation and not the severity of the violation is taken into account. Therefore, one could extent the Value at Risk analysis by computing the Expected Shortfall which do takes the severity of the potential loss into account.

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