

# Logistic regression

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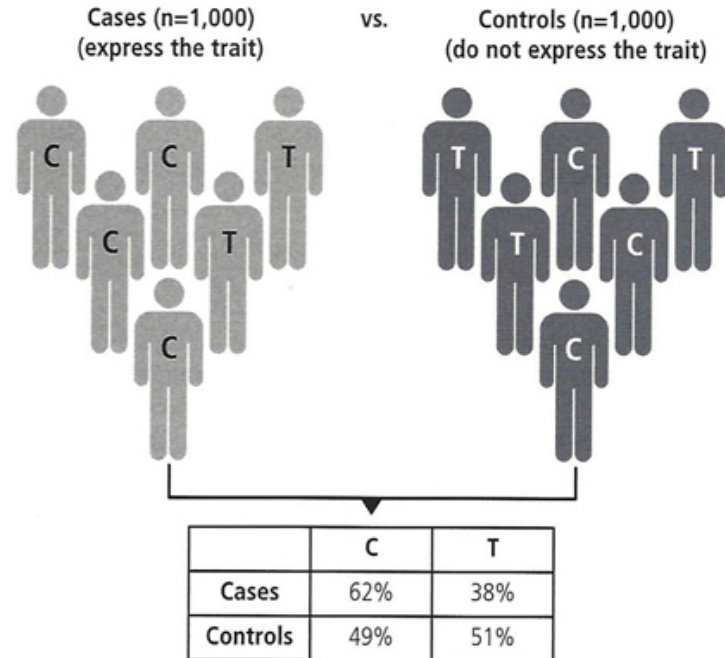
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**Data aren't always "Normal"**  
**Case/control status is binary**

## Case-control study for genetic association




$$C_i = b_0 + b_1 G_i + e_i$$

$C = 1$  if case,  $0$  if control

$G = 0$  if  $C$ ,  $1$  if  $T$

Not continuous



$$C_i = b_0 + b_1 G_i + e_i$$

$C = 1$  if case,  $0$  if control

$G = 0$  if  $C$ ,  $1$  if  $T$

Between 0 and 1



$$\Pr(C_i = 1) = b_0 + b_1 G_i + e_i$$

$C = 1$  if case,  $0$  if control

$G = 0$  if  $C$ ,  $1$  if  $T$

Always less than 0



$$\log(p) = b_0 + b_1 G_i + e_i$$

C = 1 if case, 0 if control

G = 0 if C, 1 if T

p = Pr(C = 1)

Log odds can be any number



$$\log(p/(1-p)) = b_0 + b_1 G_i + e_i$$

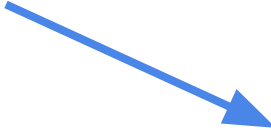
$C = 1$  if case,  $0$  if control

$G = 0$  if  $C$ ,  $1$  if  $T$

$p = \Pr(C = 1)$



Increase in log odds of case status  
given genotype


$$\log(p/(1-p)) = b_0 + b_1 G_i + e_i$$

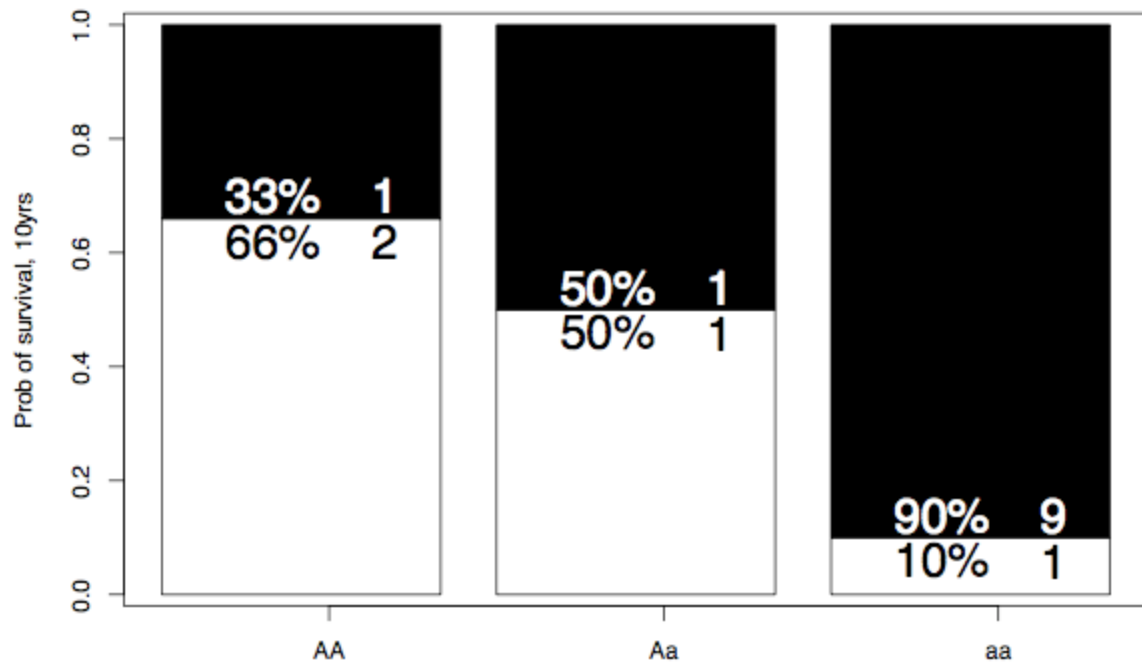
C = 1 if case, 0 if control

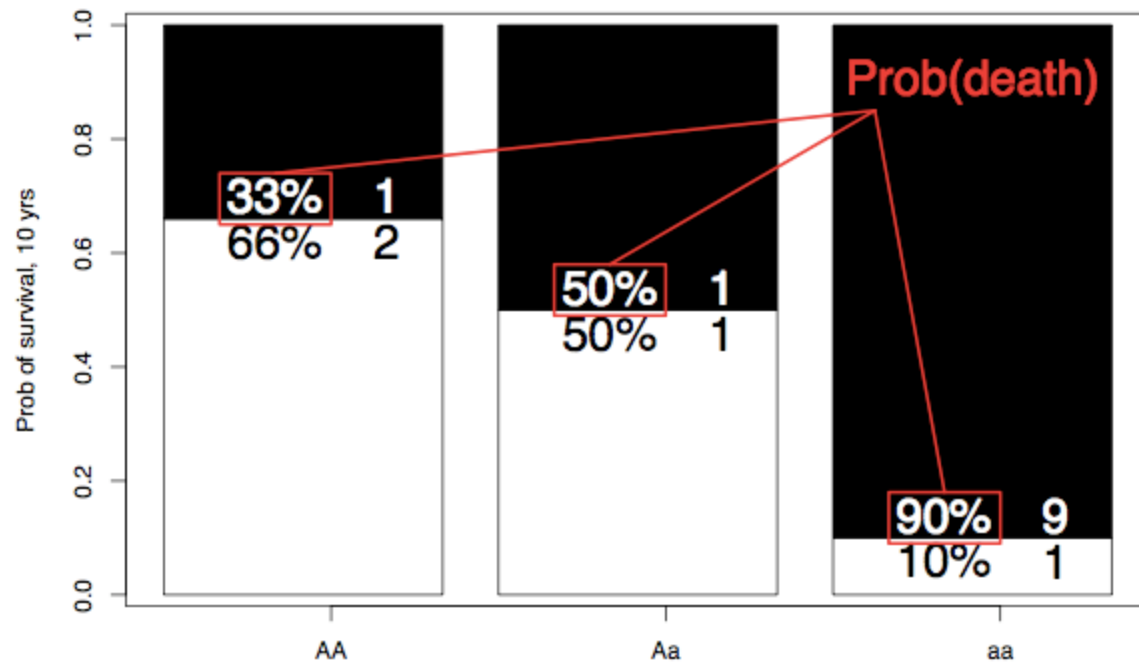
G = 0 if C, 1 if T

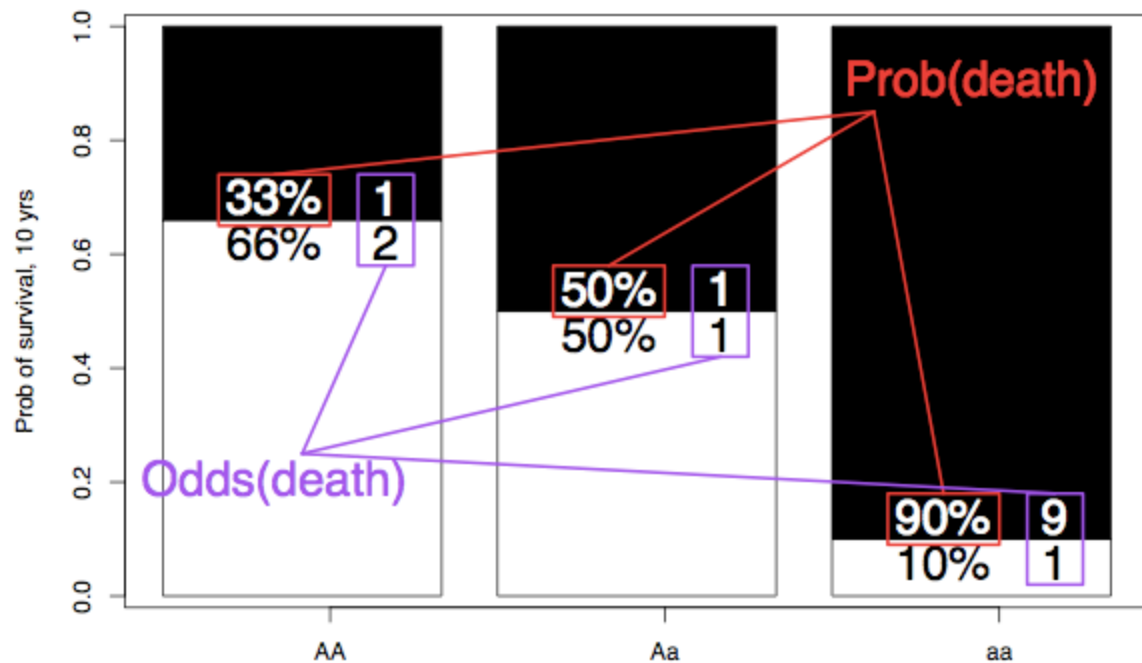
p = Pr(C = 1)

# Odds/log odds

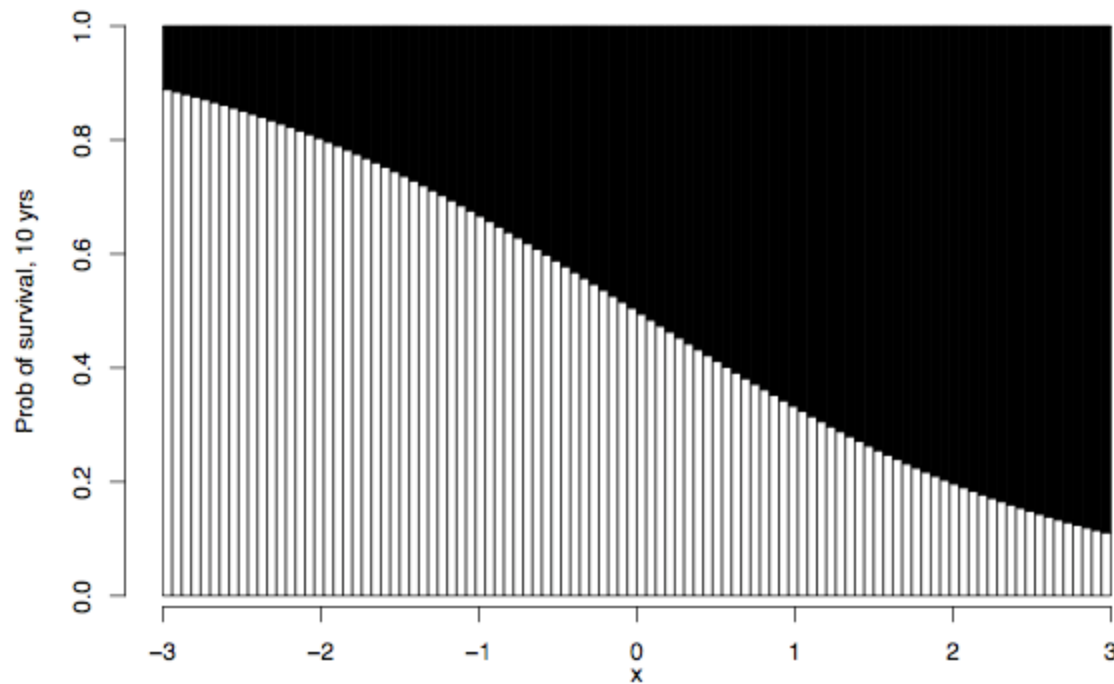
| <b>Quantity</b>                  | <b>Log Odds</b> | <b>Odds</b> |
|----------------------------------|-----------------|-------------|
| <b>Definition</b>                | $\log(p/(1-p))$ | $p$         |
| <b>In logistic regression</b>    | $b$             | $\exp(b)$   |
| <b>Definition of “no” effect</b> | 0               | 1           |

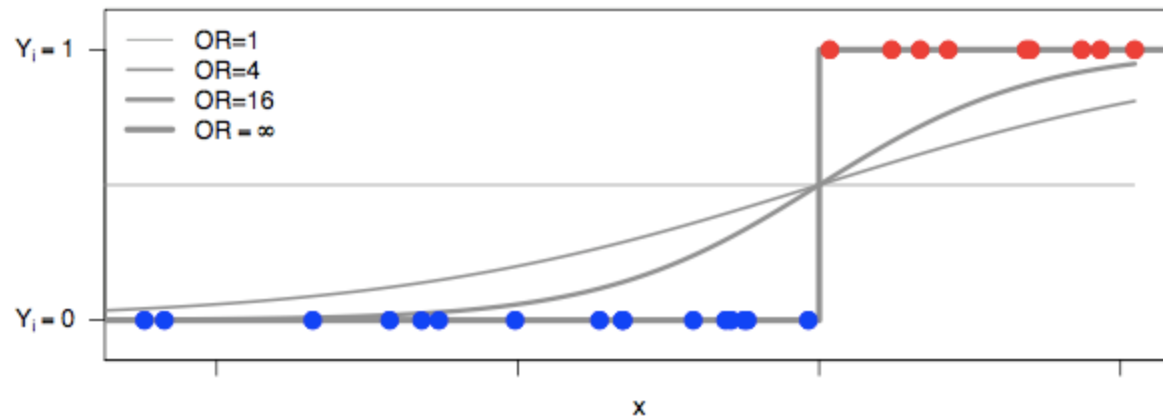






Odds ratio of 2





# Notes and further reading

- Logistic regression is a “generalized linear model”
  - [https://en.wikipedia.org/wiki/Generalized\\_linear\\_model](https://en.wikipedia.org/wiki/Generalized_linear_model)
- A nice set of lecture notes
  - <http://data.princeton.edu/wws509/notes/>
- This is again a huge topic and we have only scratched the surface.