Bitswap Analysis

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In this document, we analyze 3 strategies for a simple 2-player Bitswap infinitely repeated game.

TODO: finish this intro. Might want to mention the paragraph below, but have to be clear that the first section is more general than the specific case done in the analysis

In a given round, a player's utility is dependent on the actions each of the players took in the previous round – in other words, the debt ratio only considers the immediately preceding round, rather than all previous rounds.

Actions and Utility Functions

A player has two possible actions: reciprocate (R) or defect (D). The utility function for player i at time t u_i^t:

$$u_i^t = \sum_{j \in \mathcal{N}_i} \delta_{a_j^t R} S_j(d_{ji}, \mathbf{d}_j^{-i}) B - \delta_{a_i^t R} B$$

where

- $\mathcal{N}_i$ is the neighborhood of user i (i.e. the set of peers i is connected to)
- $a_i^t \in \{R, D\}$ is the action user i takes in round t
- δij is the kronecker delta function
- d_{ji}^t is the reputation of user i as viewed by peer j (also referred to as the *debt ratio* from i to j) in round t
- $S_j(d_{ij}, \mathbf{d}_j^{-i}) \in \{0, 1\}$ is the *strategy function* of user j. This function considers the relative reputation of peer i to the rest of j's peers, and returns a weight for peer i. This weight is used to determine what proportion of j's bandwidth to give to peer i.
- B is the (constant) amount of bandwidth that a user has to offer in a given round. We make the simplifying assumption that the users are homogeneous in this value, so they all have the same amount of bandwidth to offer.

The terms *strategy* and *strategy function* are defined as:

- A *strategy* is meant in the standard game-theoretical sense, which is a predetermined set of actions that a user will take in a game (potentially dependent on that user's previous payoffs, the actions of its peers, etc.).
- A *strategy function* is a term used to specify the weighting function that a user uses when running the Bitswap protocol to determine how much bandwidth it wants to allocate to each of its peers whenever it's playing the R strategy.

Putting this all together, we see that the utility of peer i in round t is the total amount of bandwidth that i is allocated by its neighboring peers, minus the amount of bandwidth that i provides to its peers. If i reciprocates, then we say that they provide a total of B bandwidth to the network; otherwise (when i defects), i provides 0 bandwidth in that round.

We can write the debt ratio d_{ij} in terms of the number of bits exchanged between peers i and j:

$$d_{ji}^t = \frac{b_{ji}^{t-1}}{b_{ij}^{t-1} + 1}$$

where b_{ij}^{t-1} is the total number of bits sent from i to j from round 0 through round $t-1$ (so, all rounds prior to round t).

We can define b_{ij}^{t+1} in terms of b_{ij}^t and $\delta_{a_i^t R}$ as follows:

$$b_{ij}^{t+1} = b_{ij}^t + \delta_{a_i^t R} S_i(d_{ij}, \mathbf{d}_i^{-j}) B$$

So, the total number of bits sent from i to j increases by $S_i(d_{ij}, \mathbf{d}_i^{-j})B$ (the proportion of i's total bandwidth that i allocates to j) if and only if peer i reciprocated in round t (i.e., $a_i^t = R \implies \delta_{a_i^t R} = 1$).

Now we can write d_{ij}^{t+1} in terms of values from round t.

$$d_{ij}^{t+1} = \frac{b_{ij}^t + \delta_{a_i^t R} S_i(d_{ij}, \mathbf{d}_i^{-j}) B}{b_{ji}^t + \delta_{a_j^t R} S_j(d_{ji}, \mathbf{d}_j^{-i}) B + 1}$$

Analysis

The strategy function that user j will use to weight some peer i is:

$$S_j(d_{ji}, \mathbf{d}_j^{-i}) = \frac{d_{ji}}{\sum_{k \in \mathcal{N}_j} d_{jk}}$$

$$u_i^t = \sum_{j \in \mathcal{N}_i} \frac{\delta_{a_j^t R} d_{ji}^t}{\sum_{k \in \mathcal{N}_j} d_{jk}^t} B - \delta_{a_i^t R} B$$

TODO: Mention that d_{ij}^t only considers previous round (and ensure explanation is consistent with analysis below)

We now consider 3 strategies: tit-for-tat, pavlov, and grim-trigger. For each of these, we'll determine whether the strategy is an subgame-perfect Nash equilibrium (SPNE) for the 2-player infinitely repeated game.

Tit-for-Tat

We start by analyzing the well-studed tit-for-tat (TFT) strategy. A peer that uses this strategy always takes the strategy that their peer took in the previous round. So, if player 1 plays action R (D) in round t, then player 2 will play action R in round $t+1$ (D), and vice-versa.

To determine whether TFT is an SPNE for the 2-player infinitely repeated (simplified) Bitswap game, we will do the following:

- Consider an initial pair of actions at $t = 0$, (a_1^0, a_2^0).
- Assume player 1 plays TFT for all rounds.
- Consider two cases:
  1. Player 2 never deviates from TFT.
  2. Player 2 deviates from TFT for a single round, at $t = 1$, then plays TFT for all future rounds.
- Compare player 2's payoff for the infinitely-repeated game. If player 2's payoff in case 2 (when they deviate from TFT) is less than or equal to their payoff in case 1 (when they don't) for all initial pairs of actions, then TFT is an SPNE.

Let's look at the case where the initial pair of actions is (D, D).

(D, D) **at** $t = 0$

Here, both players start by playing the D strategy. We'll first consider the case where player 2 does not deviate from TFT. The strategies at each round then follow:

t	0	1	2	3	4	...
a_1^t	D	R	D	R	D	...
a_2^t	R	D	R	D	R	...

TODO: describe the resulting pattern and payoff in each case

TODO: show total payoff calculating

TODO: summarize results for each other pair of initial conditions

Grim-Trigger

TODO

Pavlov

TODO

Discussion

TODO: compare results to that of prisoner's dilemma

TODO: discuss next steps (more peers)