Bitswap Analysis

David Grisham

In this document, we analyze 3 strategies for a simple 2-player Bitswap infinitely repeated game. In a given round, a player's utility is dependent on the actions each of the players took in the previous round – in other words, the debt ratio only considers the immediately preceding round, rather than all previous rounds.

TODO: finish this intro

Actions and Utility Functions

A player has two possible actions: reciprocate (R) or defect (D). The utility function for player i at time t u_i^t:

$$u_i^t = \sum_{j \in \mathcal{N}_i} \delta_{a_j^t R} S_j(d_{ji}, \mathbf{d}_j^{-i}) B - \delta_{a_i^t R} B$$

where

- $\mathcal{N}_i$ is the neighborhood of user i (i.e. the set of peers i is connected to)
- $a_i^t \in \{R, D\}$ is the action user i takes in round t
- δij is the kronecker delta function
- d_{ji}^t is the reputation of user i as viewed by peer j (also referred to as the *debt ratio* from i to j) in round t
- $S_j(d_{ij}, \mathbf{d}_j^{-i}) \in \{0, 1\}$ is the *strategy function* of user j. This function considers the relative reputation of peer i to the rest of j's peers, and returns a weight for peer i. This weight is used to determine what proportion of j's bandwidth to give to peer i.
- B is the (constant) amount of bandwidth that a user has to offer in a given round. We make the simplifying assumption that the users are homogeneous in this value, so they all have the same amount of bandwidth to offer.

TODO: explain u_i^t in plain english

The terms *strategy* and *strategy function* are defined as:

- A *strategy* is meant in the standard game-theoretical sense, which is a predetermined set of actions that a user will take in a game (potentially dependent on that user's previous payoffs, the actions of its peers, etc.).
- A *strategy function* is a term used to specify the weighting function that a user uses when running the Bitswap protocol to determine how much bandwidth it wants to allocate to each of its peers whenever it's playing the R strategy.

We can write the debt ratio d_{ij} in terms of the number of bits exchanged between peers i and j:

$$d_{ji}^{t} = \frac{b_{ji}^{t-1}}{b_{ij}^{t-1} + 1}$$

where b_{ij}^{t-1} is the total number of bits sent from i to j from round 0 through round $t-1$ (so, all rounds prior to round t).

We can define b_{ij}^{t+1} in terms of b_{ij}^{t} and $\delta_{a_i^t R}$ as follows:

$$b_{ij}^{t+1} = b_{ij}^{t} + \delta_{a_i^t R} S_i(d_{ij}, \mathbf{d}_i^{-j}) B$$

So, the total number of bits sent from i to j increases by $S_i(d_{ij}, \mathbf{d}_i^{-j})B$ (the proportion of i's total bandwidth that i allocates to j) if and only if peer i reciprocated in round t (i.e., $a_i^t = R \implies \delta_{a_i^t R} = 1$).

We can use this to write out d_{ij}^{t+1} in terms of values from round t.

$$d_{ij}^{t+1} = \frac{b_{ij}^{t} + \delta_{a_i^t R} S_i(d_{ij}, \mathbf{d}_i^{-j}) B}{b_{ji}^{t} + \delta_{a_j^t R} S_j(d_{ji}, \mathbf{d}_j^{-i}) B + 1}$$

Analysis

The **strategy function** that user j will use to weight some peer i is:

$$S_j(d_{ji}, \mathbf{d}_j^{-i}) = \frac{d_{ji}}{\sum_{k \in \mathcal{N}_j} d_{jk}}$$

$$u_i^t = \sum_{j \in \mathcal{N}_i} \frac{\delta_{a_j^t R} d_{ji}^t}{\sum_{k \in \mathcal{N}_j} d_{jk}^t} B - \delta_{a_i^t R} B$$

TODO: Mention that d_{ij}^t only considers previous round (and ensure explanation is consistent with analysis below)

We now consider 3 strategies: tit-for-tat, pavlov, and grim-trigger. For each of these, we'll determine whether the strategy is an SPNE for the 2-player game.

Tit-for-Tat

TODO

Grim-Trigger

TODO

Pavlov

TODO

Discussion

TODO: compare results to that of prisoner's dilemma

TODO: discuss next steps (more peers)