Bitswap Analysis

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In this document, we analyze 3 strategies for a simple 2-player Bitswap infinitely repeated game. In a given round, a player's utility is dependent on the actions each of the players took in the previous round – in other words, the debt ratio only considers the immediately preceding round, rather than all previous rounds.

TODO: finish this intro

Actions and Utility Functions

A player has two possible actions: reciprocate (R) or defect (D). The utility functions for player i at times t and $t+1$ are given by u_i^t and u_i^{t+1}, resp.

$$u_i^t = \sum_{j \in \mathcal{N}_i} \delta_{a_j^t R} S_j(d_{ji}, \mathbf{d}_j^{-i}) B - \delta_{a_i^t R} B$$

where

- $\mathcal{N}_i$ is the neighborhood of user i (i.e. the set of peers i is connected to)
- $a_i^t \in \{R, D\}$ is the action user i takes in round t
- δij is the kronecker delta function
- d_{ji}^t is the reputation of user i as viewed by peer j (also referred to as the *debt ratio* from i to j) in round t
- $\mathbf{d}_j^{-i}$ is the vector of the debt ratios j has stored for each of its peers, not including peer i
- $S_j(d_{ij}, \mathbf{d}_j^{-i}) \in \{0, 1\}$ is the *strategy function* of user j. This function considers the relative reputation of peer i to the rest of j's peers, and returns a weight for peer i. This weight is used to determine what proportion of j's bandwidth to give to peer i.
- B is the (constant) amount of bandwidth that a user has to offer in a given round. We make the simplifying assumption that all users are homogeneous in this value, so they all have the same amount of bandwidth to offer.

We can write the debt ratio d_{ij} in terms of the number of bits exchanged between peers i and j:

$$d_{ij}^t = \frac{b_{ij}^{t-1}}{b_{ji}^{t-1} + 1}$$

where b_{ij}^{t-1} is the total number of bits sent from i to j from round 0 through round $t - 1$ (so, all rounds prior to round t).

Analysis

We'll analysis 3 strategies under the following conditions:

TODO: define strategy function

TODO: single-round-lookbehind condition

TODO: write denominator of equation below

$$u_i^t = \sum_{j \in \mathcal{N}_i} \frac{\delta_{a_j^t R} d_{ji}^t}{\sum_{k \in \mathcal{N}_j} d_{jk}^t} B - \delta_{a_i^t R} B$$

TODO: show debt ratio for $t + 1$

TODO: denominator

$$u_i^{t+1} = \sum_{j \in \mathcal{N}_i} \frac{\delta_{a_j^{t+1} R} \frac{b_{ij}^t + \delta_{a_i^t R} \frac{B}{|\mathcal{N}_i|}}{b_{ji}^t + \delta_{a_j^t R} \frac{B}{|\mathcal{N}_j|} + 1}}{1}$$

We now consider 3 strategies: tit-for-tat, pavlov, and grim-trigger. For each of these, we'll determine whether the strategy is an SPNE for the 2-player game.

Tit-for-Tat

Grim-Trigger

Pavlov

TODO: compare results to that of prisoner's dilemma

TODO: discuss next steps (more peers)