

Multivariable regression

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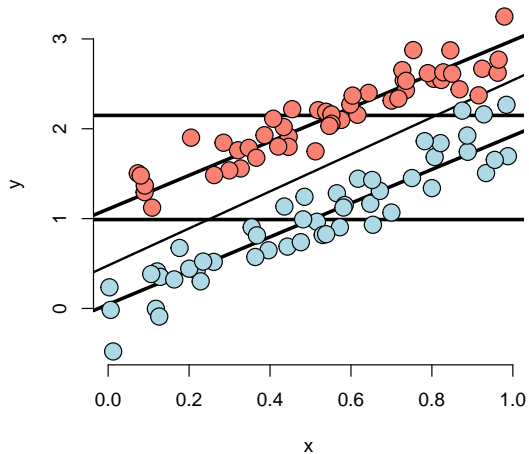
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Consider the following simulated data

Code for the first plot, rest omitted (See the git repo for the rest of the code.)

```
n <- 100; t <- rep(c(0, 1), c(n/2, n/2)); x <- c(runif(n/2))
beta0 <- 0; beta1 <- 2; tau <- 1; sigma <- .2
y <- beta0 + x * beta1 + t * tau + rnorm(n, sd = sigma)
plot(x, y, type = "n", frame = FALSE)
abline(lm(y ~ x), lwd = 2)
abline(h = mean(y[1 : (n/2)]), lwd = 3)
abline(h = mean(y[(n/2 + 1) : n]), lwd = 3)
fit <- lm(y ~ x + t)
abline(coef(fit)[1], coef(fit)[2], lwd = 3)
abline(coef(fit)[1] + coef(fit)[3], coef(fit)[2], lwd = 3)
points(x[1 : (n/2)], y[1 : (n/2)], pch = 21, col = "black")
points(x[(n/2 + 1) : n], y[(n/2 + 1) : n], pch = 21, col = "black")
```

Simulation 1

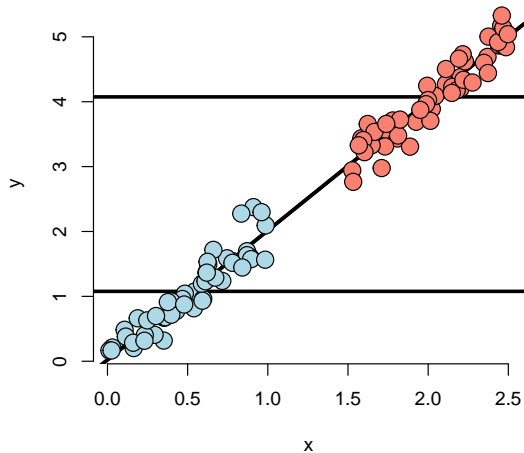


Discussion

Some things to note in this simulation

- ▶ The X variable is unrelated to group status
- ▶ The X variable is related to Y , but the intercept depends on group status.
- ▶ The group variable is related to Y .
- ▶ The relationship between group status and Y is constant depending on X .
- ▶ The relationship between group and Y disregarding X is about the same as holding X constant

Simulation 2

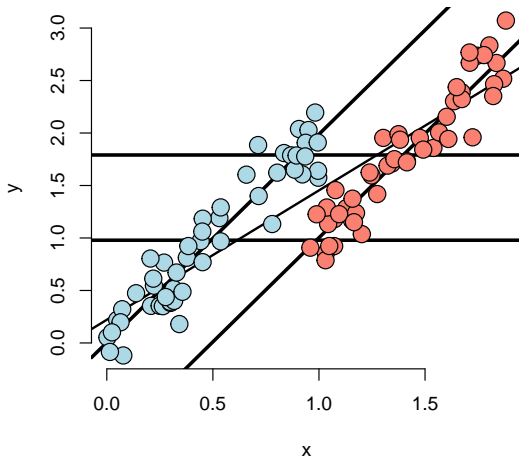


Discussion

Some things to note in this simulation

- ▶ The X variable is highly related to group status
- ▶ The X variable is related to Y, the intercept doesn't depend on the group variable.
- ▶ The X variable remains related to Y holding group status constant
- ▶ The group variable is marginally related to Y disregarding X.
- ▶ The model would estimate no adjusted effect due to group.
- ▶ There isn't any data to inform the relationship between group and Y.
- ▶ This conclusion is entirely based on the model.

Simulation 3

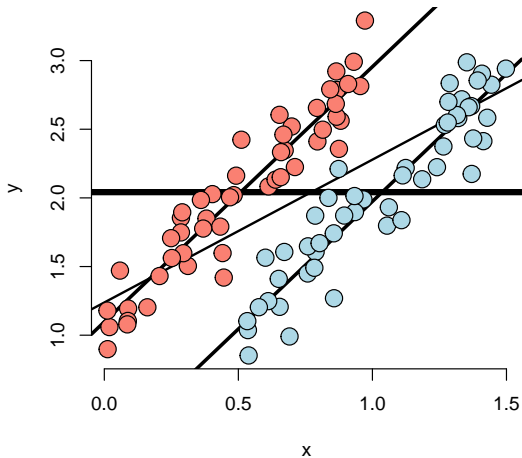


Discussion

Some things to note in this simulation

- ▶ Marginal association has red group higher than blue.
- ▶ Adjusted relationship has blue group higher than red.
- ▶ Group status related to X .
- ▶ There is some direct evidence for comparing red and blue holding X fixed.

Simulation 4

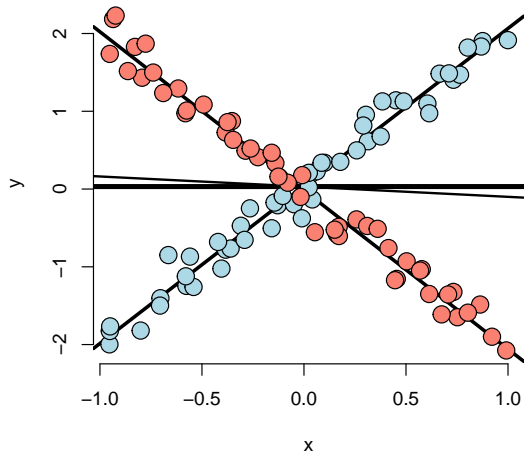


Discussion

Some things to note in this simulation

- ▶ No marginal association between group status and Y .
- ▶ Strong adjusted relationship.
- ▶ Group status not related to X .
- ▶ There is lots of direct evidence for comparing red and blue holding X fixed.

Simulation 5

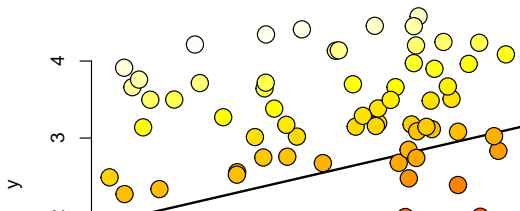


Discussion

Some things to note from this simulation

- ▶ There is no such thing as a group effect here.
- ▶ The impact of group reverses itself depending on X .
- ▶ Both intercept and slope depends on group.
- ▶ Group status and X unrelated.
- ▶ There's lots of information about group effects holding X fixed.

Simulation 6



Discussion

Some things to note from this simulation

- ▶ X_1 unrelated to X_2
- ▶ X_2 strongly related to Y
- ▶ Adjusted relationship between X_1 and Y largely unchanged by considering X_2 .
- ▶ Almost no residual variability after accounting for X_2 .

Some final thoughts

- ▶ Modeling multivariate relationships is difficult.
- ▶ Play around with simulations to see how the inclusion or exclusion of another variable can change analyses.
- ▶ The results of these analyses deal with the impact of variables on associations.
- ▶ Ascertaining mechanisms or cause are difficult subjects to be added on top of difficulty in understanding multivariate associations.