Homework 1 for Stat Inference

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About these slides

- These are some practice problems for Statistical Inference Quiz 1
- They were created using slidify interactive which you will learn in Creating Data Products
- Please help improve this with pull requests here (https://github.com/bcaffo/courses)

Consider influenza epidemics for two parent heterosexual families. Suppose that the probability is 15% that at least one of the parents has contracted the disease. The probability that the father has contracted influenza is 10% while that the mother contracted the disease is 9%. What is the probability that both contracted influenza expressed as a whole number percentage? Watch a video solution

1. 15%
2. 10%
3. 9%
4. *4%*

*** .hint $A = Father$, $P(A) = .10$, $B = Mother$, $P(B) = .09$ $P(A \cup B) = .15$,

*** .explanation $P(A \cup B) = P(A) + P(B) - P(AB)$ thus

$$.15 = .10 + .09 - P(AB)$$

```
.10 + .09 - .15
```

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## [1] 0.04
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A random variable, X, is uniform, a box from 0 to 1 of height 1. (So that its density is $f(x) = 1$ for $0 \leq x \leq 1$.) What is its median expressed to two decimal places? Watch a video solution.

1. 1.00
2. 0.75
3. *0.50*
4. 0.25

*** .hint The median is the point so that 50% of the density lies below it.

*** .explanation This density looks like a box. So, notice that $P(X \leq x) = width \times height = x$. We want $.5 = P(X \leq x) = x$.

You are playing a game with a friend where you flip a coin and if it comes up heads you give her X dollars and if it comes up tails she gives you Y dollars. The odds that the coin is heads is d. What is your expected earnings? Watch a video solution.

1. $-X\frac{d}{1+d} + Y\frac{1}{1+d}$ *
2. $X\frac{d}{1+d} + Y\frac{1}{1+d}$
3. $X\frac{d}{1+d} - Y\frac{1}{1+d}$

4. $-X\frac{d}{1+d} - Y\frac{1}{1+d}$

*** .hint The odds that you lose on a given round is given by $p/(1-p) = d$ which implies that $p = d/(1+d)$.

*** .explanation You lose X with probability $p = d/(1+d)$ and you win Y with probability $1-p = 1/(1+d)$. So your answer is

$$-X\frac{d}{1+d} + Y\frac{1}{1+d}$$

A random variable takes the value -4 with probability .2 and 1 with probability .8. What is the variance of this random variable? Watch a video solution.

1. 0
2. *4*
3. 8
4. 16

*** .hint This random variable has mean 0. The variance would be given by $E[X^2]$ then.

*** .explanation

$$E[X] = 0$$

$$Var(X) = E[X^2] = (-4)^2 * .2 + (1)^2 * .8$$

```
-4 * .2 + 1 * .8
```

```
## [1] 0
```

```
(-4)^2 * .2 + (1)^2 * .8
```

```
## [1] 4
```

If $\bar{X}$ and $\bar{Y}$ are comprised of n iid random variables arising from distributions having means μ_x and μ_y, respectively and common variance σ^2 what is the variance $\bar{X} - \bar{Y}$? Watch a video solution of this problem.

1. 0
2. $2\sigma^2/n$
3. $\mu_x - \mu_y$
4. $2\sigma^2$

*** .hint Remember that $Var(\bar{X}) = Var(\bar{Y}) = \sigma^2/n$.

*** .explanation

$$Var(\bar{X} - \bar{Y}) = Var(\bar{X}) + Var(\bar{Y}) = \sigma^2/n + \sigma^2/n$$

Let X be a random variable having standard deviation σ. What can be said about X/σ? Watch a video solution of this problem.

1. Nothing
2. *It must have variance 1.*
3. It must have mean 0.
4. It must have variance 0.

*** .hint $Var(aX) = a^2 Var(X)$

*** .explanation

$$Var(X/\sigma) = Var(X)/\sigma^2 = 1$$

If a continuous density that never touches the horizontal axis is symmetric about zero, can we say that its associated median is zero? Watch a video solution.

1. *Yes*
2. No.
3. It can not be determined given the information given.

*** .explanation This is a surprisingly hard problem. The easy explanation is that 50% of the probability is below 0 and 50% is above so yes. However, it is predicated on the density not being a flat line at 0 around 0. That's why the caveat that it never touches the horizontal axis is important.

Consider the following pmf given in R

```
p <- c(.1, .2, .3, .4)
x <- 2 : 5
```

What is the variance expressed to 1 decimal place? Watch a solution to this problem.

1. *1.0*
2. 4.0
3. 6.0
4. 17.0

*** .hint The variance is $E[X^2] - E[X]^2$

*** .explanation

```
sum(x ^ 2 * p) - sum(x * p) ^ 2
```

```
## [1] 1
```