GSPMD 是一个自动化，compiler-based 的并行系统，允许 users 通过少量指示如何 distribute tensors 的标注，像单机那样编程。GSPMD 的 OP 划分表示简单有效，可表示不同或混合类别的并行，包括 DP，in-layer MP，spatial parallelism 和 weight update sharding，并通过一个 wrapper library 将 pipeline reduce 为一个 tensor/OP 划分问题

GSPMD: General and Scalable Parallelization for ML Computation Graphs

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Abstract

We present GSPMD, an automatic, compiler-based parallelization system for common machine learning computations. It allows users to write programs in the same way as for a single device, then give hints through a few annotations on how to distribute tensors, based on which GSPMD will parallelize the computation. Its representation of partitioning is simple yet general, allowing it to express different or mixed paradigms of parallelism on a wide variety of models.

GSPMD infers the partitioning for every operator based on limited user annotations, making it convenient to scale existing single-device programs. It solves several technical challenges for production usage, allowing GSPMD to achieve 50% to 62% compute utilization on up to 2048 Cloud TPUv3 cores for models with up to one trillion parameters.

1 Introduction

Recent development of neural networks has shown dramatic benefit from model scaling, creating a demand to parallelize computation in terms of both training data and model parameters. Parallelism may be introduced in several ways: data parallelism [22] partitions training data, pipeline parallelism [18, 25, 27] partitions the computation graph, and within-layer model parallelism [36] partitions the weight and computation of each model layer.

We present GSPMD, a system that uses simple *tensor sharding annotations* to achieve different parallelism paradigms in a unified way, including data parallelism, in-layer model parallelism, and novel strategies like image spatial partitioning [11] and weight-update/optimizer-state sharding [30, 40]. Although pipeline parallelism partitions the graph instead of individual operators/tensors, GSPMD could still achieve it with the help of a simple wrapper library that reduces pipelining to a tensor/operator partitioning problem. GSPMD is flexible enough to express combinations of these approaches, e.g., different layers could be partitioned in different manners, and different approaches could be combined in the same layer.

GSPMD 从 GShard 的后端发展而来

GSPMD is generalized from the backend of GShard [23] based on our experiences of model scaling beyond the mixture-of-expert (MoE) use case, and it has helped Google to scale many deep learning models across several domains, including language (e.g., LamBDA [1], GShard-M4 [24]), image (e.g., MetNet-2 [14]), and speech (e.g., BigSSL[41]). GSPMD as a shared, robust mechanism for different parallelism patterns is particularly relevant moving forward because the ML community is increasingly investing into multimodality, where text, image and audio are combined into a single model [31].

GSPMD separates the concerns of machine learning model programming and parallelism. It allows users to write programs with giant tensors as if there were a single giant device. Then the user can insert annotations in a few places that specify how tensors are distributed across devices; GSPMD will run compiler passes that complete the sharding specification on the entire computation graph, and transform it into a mathematically equivalent, parallelized computation to run on each device. It allows the users to focus on model building instead of sharding implementation, and enables easy porting of existing single-device programs to run at a much larger scale. To experiment with different partitioning strategies, only the annotations need to be reconfigured.

GSPMD 运行 compiler passes 来完成整个计算图的 sharding specification，转换为数学上等价的并行计算图。GSPMD 允许用户将现有的单机程序 port 为更大 scale 的版本，且仅需重配置标注。

GSPMD addresses several practical issues when applying automatic partitioning to production models:

- Generating one program for each partition would increase compilation time significantly, so GSPMD instead produces a single program for all partitions. This property is called Single Program Multiple Data (SPMD), and is crucial for scaling to thousands of partitions.
- GSPMD supports unevenly partitioned dimensions, allowing any tensor to be partitioned on arbitrary device meshes. It is often a practical constraint for accelerators to require statically known shapes at compile time in order to ease development. Despite supporting uneven partitions, GSPMD is compatible with such constraints.
- We implemented GSPMD as an extension to our production ML compiler, XLA [3]. The implementation covers the full set of operators in XLA, including those with complicated semantics like `Convolution` [2]. XLA is a unifying abstraction for multiple frameworks (TensorFlow [5], Jax [7], Pytorch [29] and Julia [6]) and hardware platforms (CPUs, GPUs and Cloud TPUs [16]), making GSPMD reusable.
- GSPMD supports nested patterns of parallelism; at per-operator level, that means different types of dimensions could be partitioned across orthogonal subgroups of devices. We have developed a recursive method for such nested patterns, maximizing the generality of GSPMD without requiring excessive handwritten partitioning rules.

1) GSPMD 为所有划分方案生成单个程序，即 Single Program Multiple Data (SPMD)，这对 scale 到更多划分很重要；2) GSPMD 支持非均衡划分的维度，允许任何 tensor 在任意 device meshes 上划分；3) GSPMD 被使纤维 XLA 编译器的扩展，后者是一个涵盖多架构和多硬件平台的统一抽象，这使得 GSPMD 是可复用的；4) GSPMD 支持嵌套模式的并行：在 OP level，不同类别的维度可以在正交的 devices 组间划分。GSPMD 使用一个递归方法来实现上述嵌套模式，在无需手写额外划分规则的同时最大化通用性。为啥是 nested?

We demonstrate the capability of GSPMD by applying it on several categories of ML model, and measuring the performance and memory scaling of model training on thousands of Cloud TPUv3 [16] devices. The use cases include image models, speech models, and sparse and dense language models. By choosing intuitive initial annotations, we can achieve close-to-linear memory and performance scaling with respect to the number of devices.

2 Background

Modern ML models are typically defined as dataflow graphs of connected layers (subgraphs). Each layer has its model weights/parameters, and produces outputs that are referred to as "activations". Model training requires first computing the model's final output (forward pass), then computing the gradients of each layer weight (backward pass), which happens in the reverse layer order due to backward data dependencies. Model serving requires only the forward pass.

2.1 Common parallelism patterns in ML workloads

Below are a few typical parallelism patterns used in modern ML workloads.

Data parallelism is a technique for parallelizing training [22]. Different devices (replicas) have the same copy of the model, but compute on different training data to produce local gradients. They collect and sum their gradients to keep in sync with each other. Such synchronization is typically implemented as an MPI-style `AllReduce` operator [26].

Within-layer model parallelism partitions the weight tensor of a layer across multiple devices [36]. It may also require communication across devices, e.g., `AllReduce` when the layer sums over a partitioned dimension. Compared to data parallelism, this technique can help building larger models by sharding the weights.

Spatial partitioning is a technique to shard image input data along spatial dimensions [11], which helps fitting large image data on devices with limited memory capacity.

Weight-update sharding or optimizer-state sharding is an enhancement to data parallelism where the computation to apply combined gradients onto weights is sharded across replica devices [30, 40]. It is an optimization especially for expensive optimizers like ADAM [21].

Pipeline parallelism partitions the training graph into multiple stages that run on different devices [18] to help build large models. Each stage sends its result to its downstream stage in the forward pass, and to its upstream stage in the backward pass. Due to data dependencies between the forward and backward passes, devices can be idle during part of the training step, which is known as a "bubble". The overhead of bubbles can be amortized with larger global training batch size. Pipelining can also help build large models.

GSPMD is designed to be a general solution for all of the above types of parallelism. It natively supports **in-operator parallelism**, which includes everything above except pipelining. Although pipeline parallelism is not in-operator partitioning, it could be **reduced to an operator partitioning** problem with the help of a wrapper library over existing code (Section 3.3) in some common cases, and GSPMD could still be used to shard individual stages in combination with other pipelining implementations for unsupported cases.

More importantly, GSPMD can easily express **nested patterns** of the above techniques. Section 5 shows examples of combining them in the Transformer [38] model.

2.2 XLA and TensorFlow

The automatic partitioner in GSPMD is implemented as transformation passes in the XLA compiler [3]. XLA defines a backend-agnostic intermediate representation (IR) calld HLO, which models a computation as a dataflow graph where nodes are operators and edges are tensors.

Multiple front-end frameworks including TensorFlow [5], JAX [7], PyTorch [29] and Julia [6] already have lowering logic to transform their graph representation to XLA HLO graph, which can then be compiled into target executables if the accelerator has a compiler backend for XLA. XLA has a much smaller set of operators than front-end frameworks like TensorFlow. This reduces the burden of implementing a partitioner without harming generality.

In this paper we demonstrate the use cases with models written in TensorFlow. TensorFlow offers a Python API for defining and running ML models. A component called the *TF2XLA bridge* transforms the TensorFlow model into an equivalent XLA graph, so that XLA can compile it into a device executable. GSPMD is integrated to JAX with a slightly different API, but it is mapped to the same XLA abstraction.

GSPMD 中的自动划分被实现为 XLA compiler 中的 transformation passes。XLA 定义了一个无视后端的中间表示，HLO，将计算建模为数据流图。许多前端框架 (TF, JAX, PyTorch) 将图表示转换为 XLA HLO 图，可以被编译进目标可执行文件（如果 accelerator 有 XLA 的编译后端）。XLA 支持的 OP 远少于前端框架，这保证了 GSPMD 的通用性。本工作中，模型通过 TF 实现，TF2XLA bridge (Python) 可以将 TF 模型转换为等价的 XLA 图，并编译进 device executable；GSPMD 集成到 JAX 中，API 略有不同，被映射到相同的 XLA 抽象。

3 Tensor Sharding and Auto Completion

GSPMD defines an intuitive and general representation of tensor sharding. Following the philosophy of separation of concern, GSPMD has two independent compiler transformations: sharding completion and per-operator partitioning.

3.1 Representation of tensor sharding

In GSPMD, each tensor will be assigned a **sharding** property, either explicitly by the user as initial annotations, or by the sharding completion pass. The sharding property specifies how the data is distributed across devices. GSPMD defines three types of sharding (see also Figure 1).

- Replicated. All devices have the same full data.
- Tiled. A tiled sharding contains a multi-dimensional tensor consisting of device IDs (e.g., [[0,2],[1,3]] in Figure 1), which must have the same rank as the data tensor. Each data dimension is sharded across devices along the same dimension in the device tensor, and each device occupies the corresponding tile in the data tensor that matches its location in the device tensor. There is zero data duplication.

GSPMD 给每个 tensor 分配一个 sharding 属性 (by user or sharding completion pass)，进而将 sharding 分为三类：1) Replicated；2) Tiled；3) Partially tiled。

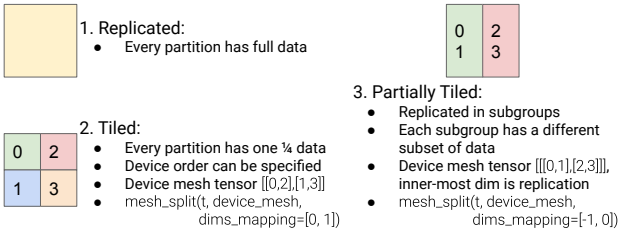


Figure 1. Examples of the three types of sharding on a data tensor, and the `mesh_split` API calls to represent them.

- Partially tiled (an extension to GShard [23]). The devices are first divided into equally sized subgroups, and the data tensor is replicated across devices in each subgroup but tiled across subgroups. Internally, it is also represented as a tensor of device IDs, with an additional dimension at the end for replication subgroups.

This representation is simple yet general, because it treats all tensor dimensions in the same way and does not specialize for batch dimensions (data parallelism [22]), weight dimensions (model-parallelism or weight-update sharding [40]), or image dimensions (spatial partitioning [11]).

GSPMD 提供了一个 sharding 表示的顶层抽象，devices 被组织为多维 tensor，称为 device mesh。GSPMD 定义了 mesh_split 作为基础 API，生成 tensor 的 sharding 标注，以 device mesh 和从 data dim 到 device dim 的映射为输入。在生成通信 OP 时，user 可以配置 device mesh 以优化通信，一般仅需定义 device mesh 一次，就可以重点关注 tensor dim 的映射，但也支持不同 tensor 对应不同的 device mesh。

GSPMD provides a convenient abstraction on top of the above low-level sharding representation. Devices can be organized in a logical multi-dimensional tensor called a **device mesh**, which can be used with an intuitive API.

mesh_split(tensor, device_mesh, dims_mapping) is the primary API GSPMD provides for users. It generates a sharding annotation for `tensor`, based on the device mesh and a mapping from each data tensor dimension (`i`) to an optional device mesh dimension (`dims_mapping[i]`). It uses -1 to represent a non-existing mapping in `dims_mapping`. Each device mesh dimension should appear at most once in `dims_mapping`. This simple API is general enough to express all types of sharding: depending on whether `dims_mapping` contains all, part, or none of the mesh dimensions, it can represent tiled, partially tiled, and replicated sharding.

When generating collective communication operators, GSPMD preserves the order of devices in `device_mesh`. Therefore, `device_mesh` can be configured by the user in order to optimize communication based on the topology of the device network. For example, it could model the actual topology of device network, or reorder devices to avoid long links.

In typical use cases, the user only needs to define the device mesh once, and then focus on the mapping of tensor dimensions. However, GSPMD does not have any restrictions if the user wants to use a different device mesh for each tensor. This could be useful when different parts of the model have very different parallelism patterns.

3.2 Examples of expressing in-operator parallelism

We explain a useful and succinct operator in TensorFlow (and other libraries like numpy), the *Einstein Summation* or `Einsum`. The equivalent operator in XLA is `Dot`. It is a generalized matrix multiply, where the user can define arbitrary numbers of dimensions of different types. An `Einsum` can be expressed as a string equation, e.g., $ABC, ACD \rightarrow ABD$, where A is an embarrassingly parallel *batch* dimension in both operands and the output, C is a *contracting* dimension that only exist in the operands and will be sum-reduced, while B and D are *non-contracting* dimensions that exist in one operand and are inherited by the output.

With GSPMD, the user can annotate the operands and the output to combine different parallelism modes. For a typical fully connected projection layer, $BD, DF \rightarrow BF$, the user can combine data- and model-parallelism by annotating

```
bd = mesh_split(bd, mesh, [0, -1])
df = mesh_split(df, mesh, [-1, 1])
```

and GSPMD will auto-complete the output sharading with `mesh_split(bf, mesh, [0,1])` so that the input and output are partitioned on the batch dimension (data-parallelism) across mesh dimension 0, while the layer weight `df` and the output are partitioned on the feature dimension F (model-parallelism) on mesh dimension 1.

If the user further partitions the weights along the other mesh dimension, i.e.,

```
df = mesh_split(df, mesh, [0, 1])
```

若 contracting dimension 也被标注为 partition，则自动触发 weight update sharding 优化 (ZeRO)，通过额外通信来维持未划分的状态

it will additionally trigger the weight-update sharding optimization [30, 40] where the weight will be unsharded on demand on the D dimension right before this layer in the forward pass to reduce peak memory usage, and gradients will be communicated with `ReduceScatter` instead of `AllReduce` in the backward pass and applied on a sharded optimizer.

If the layer has a sparsely activated mixture-of-expert (MoE) architecture [34], it can additionally have an expert E dimension in the `Einsum` equation on both operands and the output, i.e., $EBD, EDF \rightarrow EBF$. To parallelize the experts across different devices, the user only needs to annotate the E dimension on these tensors, e.g.,

```
ebd = mesh_split(ebd, mesh, [0, -1, -1])
edf = mesh_split(edf, mesh, [0, -1, 1])
ebf = mesh_split(ebf, mesh, [0, -1, 1])
```

实际上，激励的标注可以被忽略，因为可以从 weights 和上下游 OPs 的 sharding spec 中推导，除非上下游 OPs 的并行模式不同（应该是指 device mesh 不同）

In practice, the annotations on the activations ebd and ebf can be omitted and GSPMD can infer them from the weights, unless the upstream or downstream layers have a different pattern of parallelism.

3.3 Pipeline parallelism reduced to tensor sharding

Pipelining does not partition individual operators or tensors, but partitions the graph into *pipeline stages*. We consider a constrained scenario of pipelining: all stages are the same subcomputation except for having different weight values. This constraint is practical because a common way to scale up models is to stack layers of the same pattern [8, 18].

We reduce pipelining into a layer-wise sharding problem. Imagine that the layer computation is rewritten in a

GSPMD 约束 pipeline 中所有 stages 都进行相同的子计算，仅数据不同，进而将 pipelining reduce 为一个 layer-wise 的 sharding 问题（即与划分 OP/layer 不同的是，这里以 layer 为粒度，划分整个模型图），每个 tensor 均添加一个 layer/stage 维度 L，并将模型计算抽象为 L dim 上的完全并行，可通过前端向量化操作来实现，例如 JAX vmap。

stage-parallel way, where a leading layer/stage dimension L has been added to each tensor, and the computation is entirely parallel on this dimension. This transformation can be done by existing frontends' vectorization support like JAX's `vmap()` and TensorFlow's `vectorized_map()`.

使用 shifting buffer 来在 stages 间传递数据，包含一个 leading L dim（即可被 distribute 到不同 devices）

Basic GPipe schedule. Pipelining requires data to be organized into *microbatches*; each stage loops over them one at a time [18]. We use a *shifting buffer* to pass data across stages, and it also has a leading L dimension, as shown below.

state 长度为 L，分别对应每个 stage 当前的输入

更新 input，若 idx = 0，则对应新的输入；否则为上一个 stage 的输出

利用向量化操作 vmap，将 OSC 复用到 pipeline 场景中，注意提前假设了每个 stage 进行相同的子计算，进数据不同

```
# Shifting buffer.
state = zeros([L, ...])
for i in range(num_microbatches + L - 1):
  # Shift state to the right by 1.
  from_prev_stage = pad_left(state, 1)[:-1]
  stage_ids = range(L)  # [0, 1, 2, ...]
  inp = next_input()
  input = elementwise_select(
    stage_ids == 0, inp, from_prev_stage)
  state = vmap(OneStageCompute)(input, ...)
```

The above is the Python code for the forward pass of vectorized pipelining. During each iteration, the state buffer is shifted to the right by one along L, so that the previous stage's last result is passed to the next stage. The loop runs additional `stages - 1` iterations to wait for the last stage to finish processing all the microbatches. The extra iterations are equivalent to the *bubbles* in earlier work [18] that describe the idle time due to data dependency, although the waiting devices compute on padded data instead of being idle.

With the help of `vmap` or `vectorized_map`, this loop implementation can wrap a legacy single-stage implementation `OneStageCompute` and turn it to a pipelined computation.

User 需要对 L dim 进行 sharded 标注（即将 stages 分布到不同 devices 上），并将 buffer shifting 变为 cross-device 通信

This user-level loop library does not implement distributed execution, and it can run on a single device. To distribute it on multiple devices, users can simply annotate the L dimension to be sharded, and GSPMD will turn the buffer shifting into cross-device communication via `CollectivePermute`.

There are several benefits of this pipelining implementation. 1) It runs naturally when combined with other types of parallelism in GSPMD, avoiding the need for extra infrastructure. 2) It enables pipelining on part of the model, and switching to other types of parallelism in other parts (Section 5.3). 3) It benefits from the SPMD property, so that the infrastructure is very simple and does not need to maintain the interface between multiple programs.

该方法被限制在 stages 子计算相同，但对 encoder–decoder 模型算不上约束，因为可以独立运行几条 pipelines（encoder 和 decoder 各自一条）。对于异构 stages，GSPMD 无法支持。

It is limited to homogeneous pipeline stages, but this is not a constraint for encoder-decoder models in general, since we can run separate pipelines for the encoder and the decoder separately. Figure 2 shows a configuration where the encoder and decoder have their own pipelines that share the same set of devices, in combination of sharding on other dimensions.

For heterogeneous stages that cannot be supported, we recommend integrating GSPMD with other pipeline implementations [18, 25, 27] and sharding each stage.

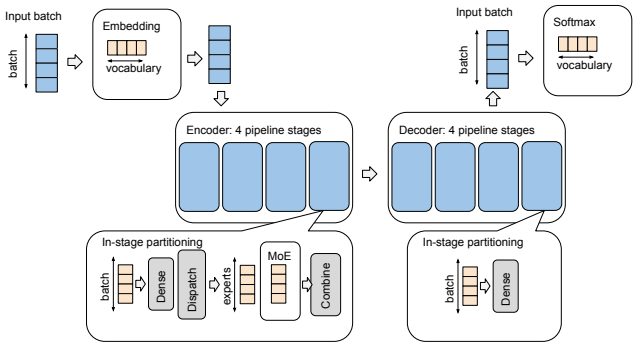


Figure 2. A partitioning strategy over 16 devices organized as a logical 4x4 mesh for an encoder-decoder model, where the encoder conains MoE layers. Blue represents partitioning along the first mesh dimension X, and yellow represents partitioning along the second mesh dimension Y. X and Y are repurposed for different model components to achieve different parallelism modes. For example, the X dimension is used for data parallelism in the embedding and softmax layers, but used for pipeline parallelism in the encoder and decoder. The Y dimension is also used in different ways to partition the vocabulary, batch or model expert dimensions.

Circular schedule. This method also allows us to implement more advanced pipelinine scheduling algorithms. For example, by assigning layers to devices in a non-contiguous manner (e.g., Layers 0, 4, 8 to Device 0, Layers 1, 5, 9 to Device 1, ...), we can reduce the bubble ratio with the same number of microbatches. It is implemented by adding an extra dimension to represent the layers within a device. We refer to this type of scheduling as **circular pipelining**, which is similar to the interleaved schedule in [28].

该方法同样支持更为先进的 pipeline 调度算法，例如 circular pipelining，以非连续的方式向 devices 分配 layers（layer 0, 4, 8 到 device 0，layer 1, 5, 9 到 device 1），这会减少相同 microbatches 下的 bubble 比例（但对带宽要求更高），通过添加表示单个 device 内 layers 的额外维度来实现。

3.4 Manually partitioned subgraphs and pipelining

GSPMD has a mechanism to allow power users to control exactly how a subgraph is partitioned, by entering a *manual partitioning* mode in the subgraph. Within this subgraph, the user writes program with shard-sized shapes; outside the subgraph, the program is still to be partitioned automatically by the compiler, and there are special conversion nodes to switch between the modes. It was originally used to work around cases where GSPMD was inefficient (see Section 3.2 in [23]), but as the software matures and advanced optimizations are added, most of the use cases are no longer needed.

On the other hand, GSPMD pipelining (Section 3.3) becomes a popular use case for manual-mode subgraphs, as an alternative to avoid `vectorized_map()` in TensorFlow since it supports only a subset of operators. Instead of doing `vectorized_map()` then partitioning the new stage dimension, we can simply convert the inputs to manual mode before `OneStageCompute` thus removing the stage dimension, and convert the outputs back to automatic mode.

GSPMD 支持部分子图的人工划分模式，需要添加额外的 conversion nodes 来切换模式 (layer i 人工，layer i+1 自动)，被用来解决 GSPMD 不够高效的 cases；同时 GSPMD 的 pipelining 也适用于人工划分，这样可以避免使用 vectorized_map()，支持更多的 OPs。

A_y 的意思是维度 A 沿 device mesh 的 dim y 划分

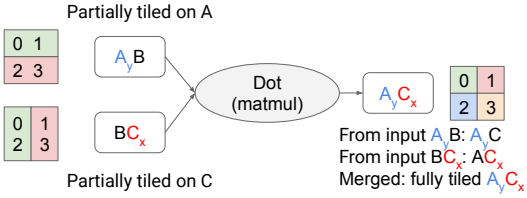


Figure 3. Sharding propagation through a Dot operator. The result is merged from both inputs. Different colors and subscripts of the letters represent sharding along different device mesh dimensions.

为了不影响其他维度的并行，将人工模式扩展为支持 subgroups，即相同 subgroup 内的 devices 进行人工划分，不同 subgroups 间的 devices 进行自动划分。感觉这里反过来了，应该是 pipeline stages 是 inter-subgroups，其他并行是 intra-subgroups，参见 alpa

To allow GSPMD to still partition other dimensions for data- or in-layer model-parallelism, we extended the **manual mode to support subgroups** similar to partial replication, i.e., devices within a subgroup are manually partitioned, while devices across subgroups are automatically partitioned. In this case, the sets of devices used as pipeline stages are the manual subgroups.

3.5 Intuitive sharding completion

This section describes how GSPMD auto-completes the sharding on every tensor based on limited user annotations. It is implemented as a compiler pass in XLA.

Preserved dimensions in operators. XLA operators typically preserve some dimensions from inputs to outputs. We simply propagate sharding of such a dimension from inputs to outputs, and vice versa. For example, a sharding annotation on the input batch dimension could be propagated down to all layers' results (activations), and the same is true for image spatial dimensions.

XLA OPs 保存某些从输入到输出的维度，并广播该维度信息到中间每个 layers 的激励 sharding spec，这虽然可能不是最优，但非常直观，且 users 可以通过插入 sharding 标注来指示 GSPMD 将每个 tensor 按需划分。

We decided to keep the sharding propagation simple, so it does not try to create new sharding patterns on dimensions. The propagation result may not always be optimal, but results will be the most **intuitive** to users. Users may insert more sharding annotations to instruct GSPMD to partition each tensor as desired.

In comparison, a fully automatic approach could apply advanced algorithms to find the best partitioning strategy (e.g., [19]) beyond user annotations, but there has not been a working implementation for our production need due to different representations and incompleteness in problem formulation. GSPMD instead focuses on propagating user intentions, though it may also be useful in defining the search space for a fully automatic approach.

XLA OP 可以从不同输入中继承维度，因此 GSPMD 从每个 operand 推断该 OP 的 sharding。GSPMD 尝试将相匹配的 shardings 合并（来自不同的正交 operands），这是通过简单 user 标注实现嵌套并行模式的重点

Merging compatible shardings. The result of an XLA operator can inherit dimensions from different inputs. For example, the Dot operator is a generalized matrix multiply, and GSPMD infers sharding based on each operand; GSPMD also tries to combine shardings from different operands if they are compatible to form a more refined sharding. A typical example of compatible shardings is two orthogonal partially tiled shardings created by `mesh_split` on different tensor dimensions, as shown in Figure 3.

More formally, we use $Offset(S, d, i)$ to denote the *shard offset* of device d in dimension i according to sharding S. The shard offset describes the location of the device's data partition within the original shape before partitioning. Then, two shardings S_0 and S_1 are compatible with each other if there exists a sharding S where for each device d,

$$Offset(S, d, i) == Offset(S_0, d, i)$$

for every sharded dimension i in S_0, and

$$Offset(S, d, j) == Offset(S_1, d, j)$$

for every sharded dimension j in S_1. In this case, S is a merged sharding of S_0 and S_1.

Merging compatibile shardings is the key to support nested parallelism patterns with simple user annotations. For example, if the user wants to mix data- and model-parallelism, they could simply annotate the inputs to be sharded on the batch dimension along one device mesh dimension, while the layer weights are sharded on certain model dimensions along other mesh dimensions. This way, the result of this layer will be propagated with sharding on both dimensions.

Iterative, priority-based sharding propagation. To complete the sharding assignment on all tensors, GSPMD runs the propagation pass on the entire graph in multiple iterations, and alternates between forward propagation (input to output) and backward propagation (output to input). While it preserves initial user annotations, shardings assigned by the pass could be refined incrementally over the iterations. This means it changes the sharding on a tensor only when it finds a more fine-grained sharding (possibly by combining with existing compatible sharding). This property guarantees the pass could reach a fixed point after finite iterations.

Figure 4 illustration of the sharding propagation processes with and without priorities on a typical linear layer followed by a ReLu activation function.

In practice, some use cases would require switching the sharding on a dimension in different tensors (Section 5), and the sharding decision on each tensor will depend on the order in which the propagation pass iterates over the operators. To produce the most intuitive sharding assignment, GSPMD assigns a priority to the propagation through each operator from each direction. Elementwise operators are assigned the highest priority to propagate through in both directions, because there is no communication if their inputs/outputs are sharded consistently, and propagating through elementwise operators would be the most intuitive decision to users. Operators like `Dot`, which add or remove dimensions are assigned lower priority. We also assign different priorities to different directions; for example, the `Broadcast` operator adds dimensions to the input by duplicating the data, so we assign higher priority to the backward propagation, which helps to avoid potential communication on the larger shape due to mismatched shardings. See Figure 4.

GSPMD 在多次迭代中运行 propagation pass，并在 forward propagation (fp) 和 backward propagation (bp) 中选择。Pass 分配的 shardings 可以在多次迭代中被不断改进，仅在发现更加细粒度 sharding 时发生（例如合并相匹配的 shardings）。为了产生最直观的 sharding 分配，GSPMD 给每个 OP 不同方向的 propagation 分配一个优先级。Element-wise OPs 有最高的优先级（propagation 更直观），Dot 这类需要增删 dim 的优先级较低。同样也需要为某些 OPs 的不同方向分配不同优先级，例如 broadcast OP 通常会通过复制数据来增加 tensor 维度，因此给 bp 更高优先级，以避免需要添加额外通信，来 convert tensor shape（否则 shardings 不匹配）

这里 mismatch 了，需要添加额外 cross-device 通信

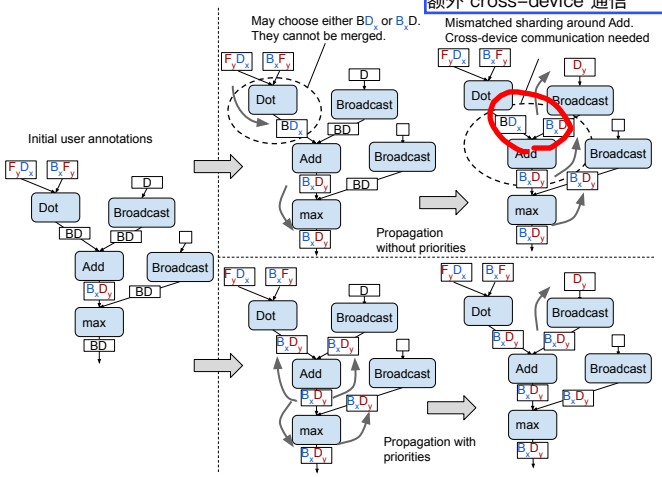


Figure 4. Comparison between sharding propagation algorithms with and without priorities. The top-right figure shows a potential propagation process without priority, where tensors are visited in a topological order; since there are multiple ways to propagate through `Dot`, it may result in mismatched sharding specifications around an elementwise operator. The bottom-right figure shows the propagation process when elementwise operators are given a higher priority, where all the BD-shaped tensors are assigned the same sharding specification.

这里的 element-wise OP 是 add

Partial specification. By default, GSPMD does not change user-provided annotations. However, in some cases the user wants to specify only a subset of tensor dimensions. For example, the wrapper library for GSPMD pipeline (Section 3.3) wants to specify sharding only for the stage and the `num_microbatches` dimensions, while letting the wrapped layers determine other dimensions of the variables and inputs. We could not use partial replication to specify such cases in the default way, because that would prevent sharding propagation from refining them. To support such cases, we extended the annotation API to have a subset of **unspecified tensor dimensions** subject to propagation changes.

GSPMD 支持用户对 tensor 部分 dim 的标注，并对其他未标注的 dim 使用 sharding propagation 来 refine

Guide for users. Sharding propagation allows the users of GSPMD to skip manual annotations on many intermediate results, such as those produced by layers like ReLu and Batch Normalization. If users want explicit control over sharding decisions, especially when tensors are sharded differently along a logical dimension, they could focus on operators that significantly change the dimensions. For example, if the inputs of a `dot` operator do not have compatible shardings, there are multiple ways for the sharding propagation to infer the output sharding; the user can explicitly annotate the output to precisely control the sharding decision.

Sharding propagation 允许 users 跳过许多中间结果的人工标注，仅需关注部分显著改变 dim 的 OPs。例如，对输入不适配的 dot OP，输出 sharding 有多种方案，可以指定输出 sharding，从而控制 sharding 决策。

3.6 API Integration in high-level frameworks

GSPMD's sharding API is essentially an annotation on unpartitioned graphs. We created a wrapper operator in TensorFlow, `XlaSharding`, to allow users to instrument GSPMD by writing regular TensorFlow programs. From the user's point of view, `XlaSharding` is semantically equivalent to an `Identity` operator that passes the input through unchanged, but the sharding annotation can be specified as an attribute. The TF2XLA bridge preserves the annotation when converting the TensorFlow program to an XLA program.

Frameworks like TensorFlow also support automatic gradient calculation. It requires each operator to have a registered gradient computation. We define the gradient of `XlaSharding` to be a copy of itself. In this way, the backward computation will be annotated automatically with the same sharding.

4 The SPMD Partitioner

There are two options when implementing the partitioner: 1) creating a customized program for each of the partitions (Multiple Programs Multiple Data, or MPMD), and 2) creating a single program that works for all partitions (Single Program Multiple Data). We choose SPMD because we aim to scale up to thousands of partitions, where compiling the many programs would be prohibitively slow in MPMD. Compilation time is an important usability concern because modern ML frameworks often include JIT optimizations and compilation, especially for those targeting custom accelerators [3, 9, 32], and parallelizing the compilation can be non-trivial because operators in different programs may need to be globally scheduled to maintain correct communication order.

选择 SPMD 的方式实现 partitioner，具有更好的扩展性，而大规模场景中 MPMD 多程序的编译是很慢的。编译时间是一个重要的实用性考量，因为现代 ML 架构多包含 JIT 优化和编译。此外，进行编译并行是不简单的，不同程序里的 OP 需要全局调度，来维持正确的通信顺序

However, implementing the partitioner in SPMD creates unique challenges for production ML compilers. This section covers the challenges for SPMD partitioning and the techniques we use to solve them.

4.1 Static constraints

Static shapes. ML accelerators get their performance edge via specialized hardware, such as vector and matrix units. In practice, the XLA compiler supports only limited degree of dynamism in tensor shapes, in order to ease the development of highly efficient operator kernels.

GSPMD is designed to work even with full static shape constraints, but static shapes create a challenge for SPMD partitioning. It's not always the case that all partitions have the same input/output shapes, because dimensions may not be evenly divisible by the number of partitions. GSPMD rounds up the size of the shape to a multiple of partition count, and the data in that padded region can be arbitrary.

When creating certain partitioned operators, data in the padding area needs to be masked off. For example, a `reduce` operator needs to prevent the padding data from affecting the result, so GSPMD first replaces them with the identity value of the reduction.

The amount of padding can vary between different partitions. Although shapes are static, several XLA operators accept dynamic offsets as operands, which allows GSPMD

XLA compiler 仅支持 tensor shapes 有限度数的动态，而 GSPMD 更是被限制为仅支持静态 shape，这可能会导致不同分块的输入/输出 shapes 不同。对此，GSPMD 进行 padding，用随机数据 padding 到分块数的倍数大小，并在 OP 时将 padding 数据替换为标记 padding 的值。XLA OPs 可接受动态 offsets 作为 operands，因此可将动态 padding 区域表示为 paritionID 的函数

to express dynamic padding area as a function of the partition ID. Masking padded data can be expressed as a `Select` according to a mask calculated by comparing `Iota` and an offset based on `PartitionId`.

Static operator configurations. XLA operators also have static configurations, like the padding, stride, and dilation defined in `Convolution`. However, different partitions may not execute with the same operator configuration. E.g., for a `Convolution`, the left-most partition applies padding to its left while the right-most partition applies padding to its right. The partitioner chooses a conservative configuration that makes some partitions to produce slightly more data than needed, then slices out the the irrelevant parts.

XLA OPs 有静态配置，但不同分块可能会执行不同的 OP 配置

4.2 Communication primitives

Since the partitioner forces all the devices to run the same program, the communication patterns are regular. We use XLA's operators that provide MPI-style collective communications [26]. **CollectivePermute** exchanges data among a list of source-destination pairs. **AllGather** concatenates tensors from all participants following a specified order. **AllReduce** performs elementwise reduction (e.g., summation) over the inputs from all participants. **AllToAll** logically splits the input of each participant along one dimension, then sends each piece to a different participant. On receiving data pieces from others, each participant concatenates the pieces to produce its result. **ReduceScatter** is semantically equivalent to an `AllReduce` followed by a `DynamicSlice` where each partition gets one slice of the fully reduced data. An efficient implementation has only half the cost of `AllReduce`.

Partitioner 强制所有 devices 运行相同程序，因此通信模式是 regular 的。XLA OPs 提供 MPI-style 的共同通信，包括 collective_permute、allgather、allreduce、all-to-all、reduce-scatter 等。

4.3 Halo exchange with dynamic bounds

Certain operators have a communication pattern which involves partial data exchange with neighboring partitions, which we call *halo exchange*. We use the `CollectivePermute` operator to exchange halo data between partitions.

Windowed operators. The most typical use case of halo exchange is for partitinoning window-based operators (e.g., `Convolution`, `ReduceWindow`), because neighboring partitions may require overlapping input data (Figure 5a). In practice, halo-exchange for these operators often needs to be coupled with proper padding, slicing, and masking due to advanced use of window configurations (dilation, stride, and padding), as well as uneven halo sizes. See Section A.2 in the Appendix for more details.

Non-constant halo size. The amount of halo data needed by different partitions are often different. In such cases, GSPMD uses maximum halo size across partitions, then uses `DynamicSlice` to remove excessive data in the halos. GSPMD supports the full set of configurations in the XLA `Convolution` operator, including arbitrary padding and dilation. These configurations add further complexity to the partitioner, but this can be ameliorated by applying careful padding and slicing.

部分 OP 有 halo exchange 的通信模式，能够在相邻分块间交换部分数据，使用 collective_permute OP 实现，主要为 windowed OP (e.g., conv)，data formatting OP (e.g., slice or pad)，以及不改变 size 的 data formatting OP (e.g., reverse, reshape)

Halo exchange for data formatting. Another use of halo exchange is for data formatting operators that change the size of the shape. For example, after a `Slice` or `Pad` operator, the shape of the tensor changes, and so do the boundaries between partitions. This requires us to realign the data on different partitions, which can be handled as a form of halo exchange (Figure 5b).

Other data formatting operators may need halo exchange even when not changing the size, because the partitions may be uneven and the shapes are constrained to be static. For example, the `Reverse` operator reverses the order of elements in a tensor, but if it is partitioned unevenly, we need to shift data across partitions to keep the padding logically to the right of the result tensor. Another example is `Reshape`. Consider reshaping a tensor from (3, 2) to (6), where the input is unevenly partitioned in 2 ways on the first dimension (partition shape (2, 2)), and the output is also partitioned in 2 ways (partition shape (3)). There is padding on the input due to uneven partitioning, but after `Reshape`, the output tensor no longer has padding; as a result, halo exchange is required in a similar way to `Slice` (Figure 5c).

4.4 Grouping and recursive partitioning

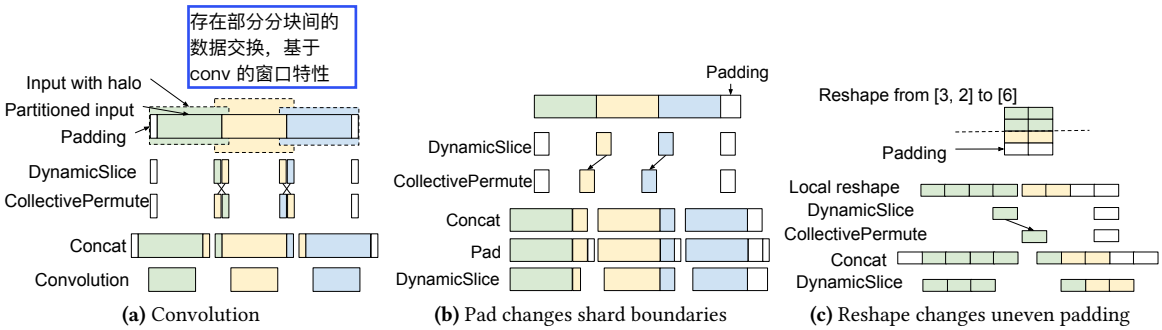
Many XLA and TensorFlow operators are rank polymorphic, where the same opcode can be used on tensors with arbitrary number of dimensions; a classic example is the `Einsum` operator as we discussed in Section 3.2. GSPMD's generic annotation API makes it convenient to combine different parallelism modes by sharding multiple dimensions, so there is a need for the partitioner to recognize not only fixed patterns but also nested cases. In order to avoid manually writing partitioning rules on all combinations of patterns, we developed a framework (Figure 6) to recursively pattern-match each set of sharded dimensions, and partition nested cases.

First, we introduce a device context object for the partitioner, which defines how cross-partition collective operators are created based on given subgroups of devices, and how the partition ID is calculated. This context generalizes the concept of devices as virtualized *logical partitions* via custom factory methods of collective operators and partition IDs.

Second, we introduce partition grouping. We can divide the devices into equally sized groups, and treat each group as a logical partition. Once we have such grouping, we can create a new partitioner context, where each logical partition is mapped to a group of devices. Within this context, whenever a collective operator is created, the logical partition IDs are rewritten to subgroups of original device IDs.

With this approach, GSPMD can perform recursive pattern matching on an `Einsum` op. For example, the partitioner detects whether there is a matching pattern on how the batch dimensions are partitioned in the inputs and the output.

简单来说就是俩输入不匹配时，把其中一个输入的中间维度给 all-gather (group)，从而 reduce shape sizes。然后递归调用 partitioner 去处理其他维度。



(a) Convolution **(b)** Pad changes shard boundaries **(c)** Reshape changes uneven padding

Figure 5. Halo exchange examples. Different colors represent data from different partitions.

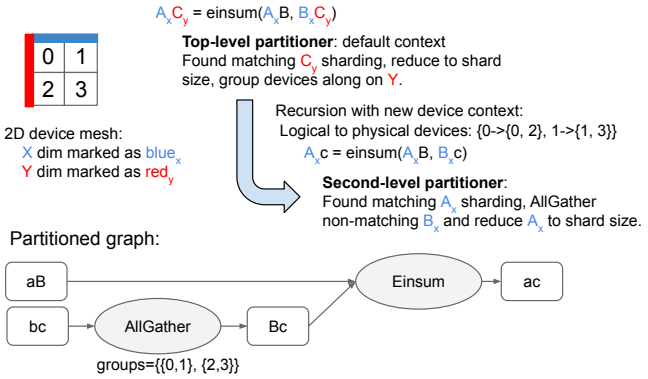


Figure 6. Recursive partitioning of $AC = einsum(AB, BC)$. Letters in blue indicate dimensions sharded on the X dimension of the device mesh, while letters in red indicate dimensions sharded on the Y dimension. Lower case letters indicate sharded size. `AllGather` is created by the inner partitioner, where logical subgroups {0,1} are rewritten to {{0,1}, {2,3}} according to the context.

If such a batch-partitioned pattern exists, it could create a nested partitioner context by grouping the partitions across the batch dimensions and reducing the shape sizes; then it recursively calls the partitioner to handle other dimensions.

This technique greatly simplifies the partitioner's implementation of operators with rank polymorphism and complicated dimension configurations. For example, we applied it to `Convolution`, which allows the user to combine spatial partitioning and feature partitioning in the same operator.

4.5 Resharding

GSPMD always produces a valid partitioned graph regardless of what sharding annotations are provided. If the provided shardings are not the typical supported cases, the partitioner will perform *resharding* on the inputs and/or outputs. Resharding can involve cross-device communications. It uses `AllGather` to replicate data along sharded dimensions, `AllToAll` to switch sharded dimensions, `CollectivePermute` to change device order, and `DynamicSlice` to shard replicated dimensions. GSPMD could use multiple steps of resharding to reach the desired sharding.

为了保证划分图的合法性，若 user 提供的 sharding 标注非法，则通过 cross-device 通信进行 resharding。

4.6 Compiler optimizations for data formatting

Pre-processing. Certain transformations may help produce faster programs simply by rearranging the data. For example, a data rotation pattern `Concat(a[k:], a[:k])` that moves the first k elements to the end can be recognized to avoid multiple rounds of halo exchanges in `Slice` and `Concat`. XLA does not define such an operator, but we define a custom `SPMD_Rotate` within the partitioner. Similar optimizations include merging a sequence of `Pad` and `Slice` operators, which is important to partition the pipelined program in Section 3.3 where the shifting using `Pad` and `Slice` can be done with a single `CollectivePermute`.

Post-partitioning optimizations. The partitioner creates various data formatting operators in order to perform slicing, padding, concatenation, masking and halo exchange. We leverage XLA's fusion capabilities, as well as new code motion optimizations for slicing and padding, to largely hide the overhead of data formatting. As a result, the run-time overhead is typically small.

5 Case Study and Evaluation

This section demonstrates a few cases where we apply GSPMD to widely-used language, speech and image models.

We measure the model performance with GSPMD on the Cloud TPUv3 platform [16], which has an XLA compiler backend so that it can execute the partitioned graphs produced by GSPMD. Each TPUv3 core has 16GB on-device memory, and the platform has high-speed homogeneous device-to-device links across many cores even if they are hosted on different machines, and these links form a 2D mesh. Such a platform is ideal to study the scalability of GSPMD, which achieves high compute utilization with operator sharding alone in many workloads. We also study pipeline parallelism that works well in certain models and can be combined with operator sharding, which could be more useful for GPU platforms [1] where high-speed links typically exist only within a server host.

[1]We have enabled GSPMD in XLA's GPU backend and verified its correctness, but do not have large-scale measurements for this paper.

5.1 Dense Transformer language model

Transformer [38] is a widely-deployed model for tasks including translation, text generation and understanding. Its core consists of two alternating types of layers. Suppose the input to such layers is a tensor of shape (B, S, M), where B is the sample batch size, S is the sequence length per batch, and M is the model dimension for features.

The *attention* layer can be described as:

$$y = \text{Attention}(W_Q \times x, W_K \times x, W_V \times x) \times W_O$$

where each of W_Q, W_K, W_V is a weight matrix that projects x into a tensor of shape (B, S, N, D). The N dimension is called the "attention heads", a parallel dimension during the computation of Attention(). The W_O weight matrix projects the attention result back to shape (B, S, M).

The *feed-forward layer* can be described as

$$y = \text{Relu}(W_{in} \times x) \times W_{out}$$

where W_{in} is a weight matrix that projects x into a tensor of shape (B, S, H), Relu() is elementwise, and the W_{out} weight matrix projects the result back to shape (B, S, M). In the rest of the section, we refer to the combination of one attention layer and one feed-forward layers as one *Transformer layer*.

Recent research has shown the benefit of scaling up the Transformer model [1, 8, 13, 18, 33] by increasing the number of layers and the sizes of dimensions M, H and N, which could reach hundreds of billions of parameters. We show that GSPMD allows these models to be trained efficiently by annotating just 7 tensors per Transformer layer (roughly 0.7% of all tensors in the entire XLA graph). Internal attention computation and other layers like normalization do not need annotations, since GSPMD can find reasonable shardings automatically.

2D sharding. One main goal of sharding is to make sure the model weights could fit into accelerator device memory. We study a 2-dimensional sharding strategy that aims to scale to very large model sizes. We define the device mesh as a matrix of shape (X, Y), and annotate the tensors as specified in Table 1. We choose 2D mesh for 2 reasons: 1) it maps to the 2D topology of the TPU platform's device network, and 2) sharding a single dimension to a very small size would affect the compute efficiency on TPUs. We study 3 types of sharding configurations.

In a vanilla approach (2D Attempt 1 in Table 1), we shard H and N along Y, and M along X. The sharding annotations are consistent on all weight and activation tensors, avoiding any resharding. GSPMD adds subgrouped `AllReduce` to the graph. However, 1) the activations are only partially sharded, so that activation memory during training becomes the bottlenneck for scalability; 2) the per-device weight is so small that it affects compute efficiency on TPUs.

Another approach (2D Attempt 2 in Table 1) is to use the same weight shardings, but switch the activations' X sharding to the batch dimension. Weights and activations

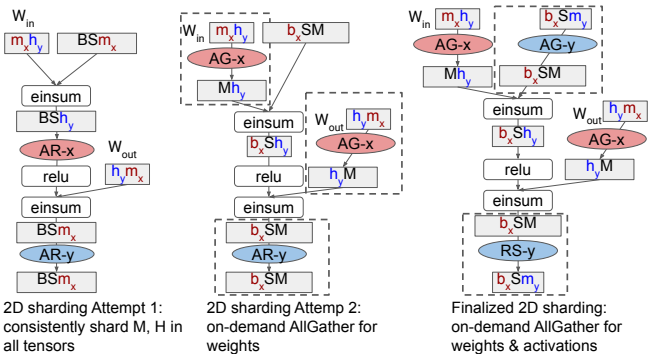


Figure 7. Partitioned graphs for a Transformer feed-forward layer produced by GSPMD with the sharding annotations in Table 1. Lower-case letters indicate sharded dimensions, where different colors and subscripts denote different mesh dimensions. Collective operators: AR is `AllReduce`, AG is `AllGather`, and RS is `ReduceScatter`.

have mismatching sharding along X, so GSPMD will perform subgrouped `AllGather` to unshard weights along X before each layer, but this avoids the need for `AllReduce` on BSH. This `AllGather` will not increase peak memory usage significantly since it is short-lived and the buffer will be freed as soon as the layer completes. In the backward pass, GSPMD will add a `ReduceScatter` on gradients due to batch and weight sharding on X. This behavior along X is conceptually the same as the weight-update/optimizer-state sharding technique [30, 40]. This approach solves the compute efficency problem, but it still suffers from an activation memory problem since the BSM tensor is still partially sharded.

In our finalized sharding configurations (2D finalized in Table 1), we enhance Attempt 2 by further sharding the BSM activation's M dimension along Y. GSPMD will add a subgrouped `AllGather` for BSM to unshard M at the beginning of each layer, and replace the `AllReduce` on the output BSM with a `ReduceScatter`. The two new collective operators combined have comparable performance to the original `AllReduce`. In this approach, all long-lived tensors are fully sharded across all devices, so that peak memory can scale linearly when we increase the number of devices. We are now also able to use a relatively large batch size, which further helps TPU compute efficiency. This sharding configuration combines the benefit of data parallelism, weight-update sharding [30, 40] and in-layer model parallelism [36], and only requires 7 annotations in the model. Figure 7 shows the partitioned graphs for the 3 approaches.

Performance experiments. To evaluate the scalability of GSPMD, we measure the performance of training Transformer models that have many layers of large weights. We choose the following dimension sizes: M is 8192, H is 65536, N is 128, D is 256, and the vocabulary size is 32000. These dimensions are comparable to GPT-3 [8]. We use a sequence

		W_Q, W_K, W_V	W_O	W_{in}	W_{out}	Activations			Memory usage	Communication
Shardings	Shape	MND	NDM	MH	HM	BSM	BSND	BSH	weight + activation	weight + activation
	2D Attempt 1	X,Y,_	Y,_,X	X,Y	Y,X	_,_,X	_,_,Y,_	_,_,Y	O(1/(XY)) + (O(1/Y)+O(1/X))	0 + (O(1/Y)+O(1/X))
	2D Attempt 2	X,Y,_	Y,_,X	X,Y	Y,X	X,_,_	X,_,Y,_	X,_,Y	O(1/(XY)) + O(1/X)	O(1/Y) + O(1/X)
	2D finalized	X,Y,_	Y,_,X	X,Y	Y,X	X,_,Y	X,_,Y,_	X,_,Y	O(1/(XY)) + O(1/(XY))	O(1/Y) + O(1/X)

Table 1. Dense Transformer sharding annotations. X and Y are the two mesh dimensions.

Parameter count	64B			128B	256B	512B	1T
Layer count	32			64	128	256	128
M dim	8192						16384
H dim	65536						131072
Total devices	128	512	2048	2048			
Device mesh	(8,16)	(16,32)	(32,64)	(32,64)			
Batch size	64	256	1024	512	256	128	128
Peak memory	15.3GB	13.3GB	15.56GB	14.0GB	12.9GB	13.6GB	15.8GB
Step time	5.74s	6.25s	6.30s	6.31s	6.71s	7.64s	12.66s
FLOPS util	62.7%	57.1%	56.9%	56.5%	54.1%	47.5%	55.6%

Table 2. Benchmarks for dense Transformer with wide layers. The last column has 4x wider layers compared to others.

length of 1024 for each input sample. Each Transformer layer (attention + feed-foward) has 2 billion parameters. We use 32-bit floating-point parameters, 16-bit floating-point activations, and Adafactor optimizer [35].

Scalability is evaluated by 2 sets of experiments: 1) a fixed model size (32 layers, 64B parameters) on different device topologies, and 2) different model sizes (32, 64, 128 and 256 layers) on a fixed device topology. Making the model deeper is more difficult than making it wider, because both compute and memory scale linearly with the model depth; in contrast, if we increase per-layer size to 4x by doubling the size of M, H and N, the activation memory only increases by 2x.

Table 2 shows the training performance of the above model configurations on the TPUv3 platform. GSPMD achieves close to linear scaling for such models, in terms of both memory and performance. For the 32-layer model, when we increase the number of TPU devices by 2x, we can roughly double the maximum input batch size, while maintaining similar step time (within 10%). On the same 2048-core device mesh, when the model depth increases by 2x, the maximum input batch size is roughly halved, while the step time increases only 7% from 64B to 256B. When the model size reaches 512B, the batch size becomes small and the step time increases by 13% over 256B. The last column shows a configuration with 1 trillion parameters, which is shallower than the 512B configuration, but each layer is 4x wider; as expected, the shallower 1T model can fit the same batch size as the 512B deeper model, while achieving higher efficiency due to wider layers. GSPMD achieves high overall FLOPS utilization (from 54% to 62% for most cases and 47.5% for the 512B configuration) of the TPU devices.

Total devices	64	128	256	64	128	256
Device mesh	(4,16)	(8,16)	(8,32)	(16,4)	(16,8)	(32,8)
Batch size	48	96	192	48	96	192
Peak memory	12.4GB	14.6GB	14.8GB	13.8GB	12.7GB	14.1GB
Step time	3.56s	3.43s	5.71s	3.10s	3.37s	3.36s
FLOPS util	39.4%	40.5%	27.1%	45.7%	41.7%	41.3%

Table 3. Benchmarks for 2D-sharded narrower dense Transformer. The model has 64 Transformer layers and 16 billion parameters.

Narrower dense Transformer. Due to the relatively large batch size, activation communication is typically much more significant than weight/gradient communication. In Transformer layers, the amount of compute is $O(MH + MND)$, while the amount of activation communication is $O(M)$; therefore, with a fixed 2D device mesh, narrower models with smaller dimensions cannot utilize the TPU's compute power as well as wider models due to higher percentage of communication time. One mitigation is to reduce the Y dimension size of the mesh, while increasing the X dimension size accordingly.

We measure the performance of a 8x narrower model which is still too big to fit on a single device: M is 4096, H is 16384, N is 64, and D is 128. It has 64 layers and 16 billion parameters in total. The results are shown in Table 3, which are consistent with our analysis above. While memory scaling is still roughly linear to the number of devices, having a smaller Y mesh dimension helps to achieve higher efficiency (with the same number of devices). With the same Y mesh dimension size, increasing the size of the X dimension allows to increase the batch size linearly with relatively constant step time.

5.2 Combining pipelining and in-layer sharding

This section studies the performance of GSPMD pipelining described in Section 3.3. We choose the same narrower model in Table 3, because activation communication is expensive in 2D-sharded narrower models if the Y mesh dimension is large, and another level of parallelism could be helpful.

We use a 3D device mesh (L, X, Y) where the leading L is used to shard pipeline stages. X and Y are used in a similar way to 2D sharding (Table 1), except that weights are not sharded along X; this is because pipelining requires the input to be divided into microbatches and weight sharding on X would incur expensive per-microbatch `AllGather`.

Device mesh	(2,16,8)	(4,16,4)	(4,16,4)	(8,16,2)	(8,8,4)
Pipeline stages	2	4	4	8	8
Batch size	16×64	16×64	32×32	32×32	32×32
Peak memory	14.9GB	13.3GB	13.1GB	15.0GB	11.5GB
Step time	24.0s	22.3s	22.2s	23.4s	23.7s
Raw FLOPS util	46.2%	58.0%	51.8%	54.8%	55.5%
Bubbles	5.6%	14.8%	8.0%	16.5%	16.1%
Recompute	22.3%	21.3%	21.7%	22.2%	20.6%

Table 4. Benchmarks for pipelining on the same model in Table 3. The device mesh has shape (L, X, Y), where L is used to shard pipeline stages, X is used for data parallelism, and Y is used for in-layer model parallelism. Raw FLOPS util is the reading from the profiler which counts padded compute (bubbles) as valid compute. The batch size is described as num_microbatches × microbatch_size.

Parameter count	6.47B			12.95B		
Layer count	32			64		
Pipeline stages	8			16		
Schedule	GPipe	GPipe	Circular	GPipe	GPipe	Circular
Batch size	64×1	16×1	16×1	128×1	32×1	32×1
Peak memory	13.8GB	12.2GB	14.1GB	15.4GB	12.8GB	14.8GB
Step time	8.40s	2.80s	2.30s	18.37s	5.58s	4.42s
Raw FLOPS util	59.4%	57.9%	53.9%	60.5%	59.0%	50.6%
Bubbles	9.6%	29.9%	9.0%	10.4%	31.0%	10.0%
Recompute	22.9%	22.3%	21.3%	29.9%	22.8%	22.4%

Table 5. Benchmarks for pipelining on Conformer models. The batch size is num_microbatches × microbatch_size. The circular schedule assigns layers to the stages in a round-robin manner.

Nonetheless, per-device weights are sufficiently small due to L sharding.

We follow GPipe's approach [18], where a key difference from non-pipelined configurations is that we recompute forward pass intermediate results during the backward pass, a technique known as rematerialization [10] to lower peak memory usage. Rematerialization enables us to use larger number of microbatches to reduce pipeline bubbles.

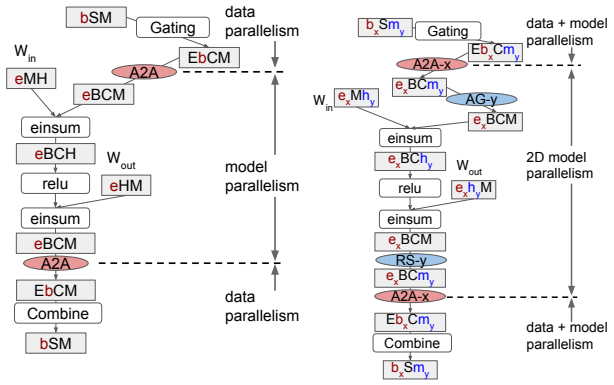
Table 4 summarizes the performance results for configurations with 2, 4 and 8 stages. There are two major observations. 1) It is beneficial to balance the number of pipeline stages (L) and the number of in-layer model-parallel shards (Y), and the fastest configuration has 4 stages and 4 model-parallel shards. 2) Although these configurations have higher raw FLOPS utilization as reported by the profiler, the best one is still 24% slower than 2D sharding with (X=32, Y=8) (Table 3), because bubbles (compute on padded data) and recompute are overheads but counted as useful compute by the profiler.

5.3 Pipelined Conformer models

Conformer [17] is a speech model, and its backbone is stack of convolution-enhanced Transformer layers. It also has a convolutional subsampling layer before the backbone. We study scaling up this model by adding more layers, while each layer has dimensions M=3072, H=12288, N=16. It is even narrower than the one in Section 5.2, and we find it is easier to scale using pipelining on the backbone, and switching to data parallelism on the layers before and after. Table 5 shows the results of using GSPMD pipelining on two model configurations, with various batch sizes and both GPipe and circular schedules. The circular schedule is especially useful when the batch size is small, achieving similar bubble ratio compared to GPipe with much larger batch sizes. This pipelining approach has been used in BigSSL [41].

5.4 Sparse Transformer language model

Mixture-of-experts (MoE) is a recently explored research direction in scaling up the Transformer model [13, 24, 34], where the feed-forward layer provides many independent



(a) MoE layer partitioning **(b)** Hybrid MoE layer partitioning

Figure 8. Partitioned graphs for MoE feed-forward layer and hybrid sparse and dense MoE layer. Lower-case letters indicate sharded dimensions, where different colors and subscripts denote different mesh dimensions. Collective operators: A2A is `AllToAll`, AG is `AllGather`, and RS is `ReduceScatter`.

model weights (called "experts"), and each input token is operated on by only a couple of experts. Such models usually consist of both MoE and non-MoE layers in an alternating way.

The per-expert matrix multiplies in the MoE feed-forward layer can be expressed as `Einsum` operators with an extra parallel dimension, E (for experts), e.g., $EBCM, EMH \rightarrow EBCH$ and $EBCH, EHM \rightarrow EBCM$, where C is the per-expert *capacity*, i.e., the number of tokens to process per batch. Before each MoE layer, a *gating* layer computes which experts each token goes to, transforming a (B, S, M) tensor to (E, B, C, M).

To scale up the model, we can increase the number of experts E, as well as the input batch size so that each expert still has a reasonable amount of data to process. We found a simple yet efficient way to partition such a model: use a 1D devicde mesh to shard the MoE layer's expert dimension, and switch to batch partitioning (data parallelism) in other layers.

Parameter count	10B	37B	145B	577B
Experts per layer	32	128	512	2048
Device mesh	(32)	(128)	(512)	(2048)
Batch size	128	512	2048	8192
Peak memory	10.8GB	11.2GB	11.2GB	11.6GB
Step time	0.98s	1.01s	1.10s	1.51s
FLOPS util	58.2%	49.8%	49.8%	46.80%
AllToAll time	2%	6%	9%	11%

Table 6. Benchmarks for sparse MoE Transformer.

Parameter count	33B	57B	420B	804B
Experts per layer	8	16	32	64
H dim	32768		131072	
N dim	128		512	
Device mesh	(8,4)	(16,8)	(32,16)	(64,32)
Batch size	32	128	128	512
Peak memory	12.3GB	12.9GB	11.9GB	8.5GB
Step time	2.12s	2.19s	2.08s	1.92s
FLOPS util	55.3%	50.2%	50.8%	53.8%

Table 7. Benchmarks for hybrid sparse/dense MoE Transformer.

For this type of sharding, GSPMD will insert `AllToAll` operators to switch the sharded dimension between E and B. In the backward pass, GSPMD inserts gradient `AllReduce` only for non-MoE layers because there is no data parallelism for MoE layers. Figure 8a shows the partitioned forward pass.

Performance experiments. Except for the gating logic, the per-sample compute of this model is constant regardless of the number of experts. We study GSPMD's scalability on this model by fixing the size of each expert, while scaling the number of experts and the batch size. Table 6 shows the performance and memory results when we set the per-device expert count to 1 (could be any constant). The step time increases only from 0.98s to 1.10s from 32 to 512 experts, as expected; when the model further increases to 2048 experts, step time increases to 1.51s. The `AllToAll` communication time is roughly $O(\sqrt{\text{num\_devices}})$ on the 2D TPU device mesh, which contributes to the overhead with 2048 devices, but the main difference is that the gating compute becomes more significant with 2048 experts.

5.5 Hybrid sparse and dense Transformer

We consider a configuration of Transformer that combines the characteristics of sparse and dense models. There are still MoE layers, but each expert is sufficiently large such that we still need to shard each expert to fit the model weights. The largest model configuration, "64B64E", in GLaM [13] falls into this category. The non-MoE layers have the same size as a single expert in an MoE layer. We use a 2D device mesh of shape (X, Y), and the non-MoE layers are sharded the same way as the dense Transformer (Table 1). The MoE layers' H and N dimensions are sharded on Y, while the E dimension is sharded on X. The partitioned forward pass graph is shown in Figure 8b.

Performance experiments. We scale the number of experts proportionally to X, and scale (batch size × per-expert weight size) proportionally to the total number of devices. Table 7 shows the performance results of different configurations of the model using GSPMD. These configurations have roughly the same theoretical per-device compute and memory usage. As expected, the measured step time and peak memory stay relatively constant as we scale the model; the variance is much smaller than pure sparse MoE configurations (Table 6) because the hybrid configuration has fewer experts and much smaller `AllToAll` and gating overhead compared to the per-expert compute.

Device mesh	(1,1)	(1,2)	(1,4)	(1,8)	(1,16)	(4,16)	(2,32)
Image size	128x128x128					256x256x256	
Batch size	4					8	
Peak memory	14.3GB	14.8GB	7.9GB	4.5GB	2.7GB	14.9GB	8.52GB
Step time	2.99s	1.56s	0.79s	0.39s	0.19s	1.66s	0.76s
FLOPS util	47.9%	43.7%	43.5%	43.5%	43.8%	20.5%	41.2%

Table 8. Benchmarks for 3D U-Net with spatial partitioning. The first mesh dimension is used for data parallelism, and the second mesh dimension is used for spatial partitioning. Peak memory usage includes TPU-specific padding and does not scale linearly.

5.6 Image spatial partitioning

High-resolution images are often required in object detection and segmentation tasks, but they may not fit in the memory of a single device. This section discusses *spatial partitioning* of convolution neural networks (CNNs). The goal is to shard activation tensors (along non-batch dimension) instead of model weights. As usual, GSPMD can express it with the same sharding API. In fact, sharding annotations are *required only for the model inputs*; GSPMD can propagate the shardings on the spatial dimensions to all of the convolutional layers, because they share the same set of spatial dimensions. This technique has been used in MetNet-2 [14].

Performance experiments. We experimented with the 3D U-Net model [12], a popular dense 3D segmentation model which has been widely used in the medical image domain. With GSPMD, this model could run with the original resolution for CT images, which can be as large as 256x256x256. Spatial partitioning avoids downsampling which could affect accuracy. GSPMD makes this possible by simply annotating an input spatial dimension. We measure the performance of spatially partitioned 3D U-Net to evaluate GSPMD's convolution partitioning with halo exchange (Figure 5a).

Table 8 shows two sets of measurements. We first measure GSPMD's performance scaling with spatial partitioning only.

For image size 128x128x128, GSPMD achieves nearly linear scaling with a 15.7x step time reduction on 16 partitions.

We then measure the step time when combining spatial partitioning with data parallelism to keep the same number of total devices. This time we choose image size 256x256x256, which does not fit in a single device even with per-device batch size 1. We compare the results of 16-way and 32-way spatial partitioning, and found 32-way partitioning achieves 2x higher FLOPS utilization because it enables a higher per-device batch size.

6 Related Work

Because programming in a distributed heterogeneous environment is challenging, particularly for high-level practitioners, deep-learning frameworks do not force users to specify precisely how the distributed computation is done. For example, TensorFlow [5] has support for data parallelism, and basic model parallelism with graph partitioning by per-node device assignment. Mesh TensorFlow [33] helps the user to build large models with SPMD-style per-operator partitioning, by rewriting the computation in a Python library on top of TensorFlow; in comparison, GSPMD partitions the graph in the compiler based on lightweight annotations, without requiring the user to rewrite the model.

GSPMD is generalized from the back-end system used in GShard [23]. GSPMD expands the sharding representation with partial tiling and manual subgroups, and implements new techniques like priority sharding propagation, recursive partitioning and pipelining, making it more suitable for combined parallelism patterns and support much more use cases beyond MoE language models.

Tofu [39] is another parallelization system for large models, but it supports only limited partition strategies (e.g., "partition-n-reduce"), while GSPMD supports partitioning all dimensions of complex operators like `Convolution`.

Pipelining algorithms [15, 18, 25, 27, 37] focus on one type of parallelism, while GSPMD can be used either to express similar ideas with the help of vectorization (Section 3.3), or to work in combination of these implementations by additionally partitioning each pipeline stage. Some of these works also provide orthogonal pipeline scheduling techniques that could be used in GSPMD to reduce pipeline bubbles.

Zero [30] presents a set of optimizations to reduce memory redundancy in parallel training devices, by partitioning weights, activations, and optimizer state separately. GSPMD is more general: it does not distinguish these tensors and dimensions, and those specific partitioning techniques can be supported by annotating the corresponding tensor dimensions with a uniform API. Weight-update sharding [40] is another automatic parallelization transformation that achieves optimizer state sharding similar to Zero, and conceptually it can be viewed as a special case for GSPMD.

Combination of in-layer model parallelism and pipelining has also been studied in previous works [4, 28]. GSPMD provides a general implementation of many of their partitioning techniques under the same sharding annotation API. For example, the scatter/gather optimization across pipeline stages in [28] is automatically enabled for all of the configurations in Table 4, because the activations are fully sharded (scatter phase) and then combined on-demand (gather phase) in the next stage (`bSm` and `bSM` tensors in Figure 7).

FlexFlow [20] uses automated search to discover the partitioning strategy of operators in a graph for better performance. While FlexFlow focuses on determining the partitioning policy, GSPMD focuses on the mechanisms to transform an annotated graph. The two are complementary to each other: GSPMD can be used to define a search space and perform the transformation, and automated search combined with GSPMD could provide a fully automated system.

7 Conclusion

GSPMD is a largely automated parallelization system for machine learning computations. It offers a simple yet powerful API which is general enough to combine different typical parallelism patterns. GSPMD offers an intuitive auto-completion feature that enables the user to annotate only a few tensors to partition the entire model efficiently. We have demonstrated that GSPMD is able to partition several image, speech and language models on up to thousands of Cloud TPUv3 cores, with good and predictable performance and memory scaling.

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A Appendix

A.1 XLA operators for dynamism

Although XLA requires static shapes, data access can have dynamic offsets based on run-time values. Table 9 summarizes a few operators in XLA that GSPMD uses to support dynamic behaviors across different partitions.

PartitionId () Returns the partition id (integer) of the current device running the program.
DynamicSlice (Tensor[T] operand, Tensor[int] start_indices, List[int] size_indices) Returns a slice (of shape size_indices) from the operand tensor where the offset is given as a tensor. Used to allow each partition to select different regions of a tensor, where the dynamic offset is often calculated from PartitionId.
DynamicUpdateSlice (Tensor[T] operand, Tensor[T] update, Tensor[int] start_indices) Returns a tensor that replaces a sub-region of the operand tensor with the given update tensor. start_indices specifies the offset of the region to be updated. Used similar to DynamicSlice, but to update different regions of a tensor by each partition.
Iota (List[int] dimensions, int64 iota_dimension) Creates an integer tensor of dimensions shape, with sequentially increasing values from 0 along the iota_dimension (similar to std::iota(), but multi-dimensional). Used to create a mask of a partitioned tensor on each partition by comparing it against the un-partitioned dimension size.
Select (Tensor[bool] pred, Tensor[T] on_true, Tensor[T] on_false) Selects each element between two tensors based on the predicate tensor. All three tensors have the same shape. Used to mask out a region of a tensor (e.g., padded region due to uneven partitions), where the mask is often computed using Iota.

Table 9. XLA operators used by GSPMD to handle non-uniform behavior between partitions.

A.2 Halo exchange details

We first introduce the window configurations for operators like `Convolution` that GSPMD has to consider. Each spatial dimension in the convolution has the following set of configurations.

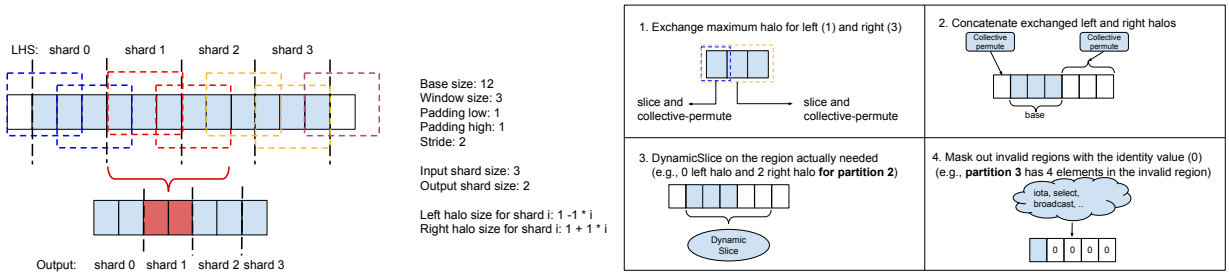
- **Stride** is the distance (in number of elements) that the window moves to produce the next output element.
- **Low/high padding** is the number of elements padded to the low/high end of the dimension in LHS (base).
- **Base dilation** is the dilation factor of the LHS, i.e., one plus the number of elements padded between every element (excluding low/high padding). No base dilation means the value is set to 1. Base dilation is applied before low/high padding.
- **Window dilation** is one plus the number of elements padded between every element in the RHS (window).

Non-constant halo size. We demonstrate that non-constant halo size is common using a simple example. Figure 9a shows a 4-way partitioned convolution, where the right halo sizes for the partitions are (1, 2, 3, 4) and can be expressed as a linear function of the partition ID: `partition_id + 1`.

Figure 9b describes the sequence of operations for a general halo exchange. First, we calculate the maximum sizes of left and right halos across partitions and perform the halo exchange of the maximum size (Steps 1 and 2). Since some partitions may have excessive halos than needed, we use `DynamicSlice` (based on the partition ID) to get the valid region for the current partition (Step 3). Finally, some partitions may include garbage values (e.g., halos from out-of-range input data), so we apply masking as described in Section 4.1.

Base dilation. Base dilation adds additional complexities to halo exchange, because the offset of each partition may be positioned at the dilation holes, which makes the edges have different behavior than the interior elements. We handle base dilation in 3 cases (Figure 10).

- $stride \times per\_shard\_window\_count$ is divisible by $dilation$, where $per\_shard\_window\_count$ is the number of windows to be processed by each partition (i.e., the number of output elements for each partition). This condition guarantees that all partitions start with the same number of (interior or low) padding elements before the first data element in the LHS, so that we can use the same low padding configuration. Halo exchange occurs on the non-dilated and non-padded base region, and the limit index of required data for Partition i can be represented as $a \times i + b$, where a and b are both integer constants. This limit determines the right halo size.
- $stride = 1$ but $per\_shard\_window\_count$ is not divisible by $dilation$. In this case, the low padding sizes vary on different partitions, but it is a static configuratio. Using `Pad` and `DynamicSlice` on the operand also would not work, because those operators would be applied before dilation, so everything would be multiplied by the dilation factor. Fortunately, with $stride = 1$, all positions on the padded and dilated base region are valid window starts, and we can use the maximum low padding on all partitions to ensure that each partition calculates all required windows, then perform a `DynamicSlice` on the output of the partitioned operator to remove unnecessary data. The limit index of required data on the non-padded base region for Partition i can be represented as $(a \times i + b)/c$, where a, b and c are all integer constants and "/" is integer division.



(a) Convolution halo size depends on shard offset. Input data required by each partition are indicated by dotted windows of a unique color.

(b) Sequence of operations for a general halo exchange.

Figure 9. Non-constant halo size in a partitioned convolution and the solution with padding and slicing.

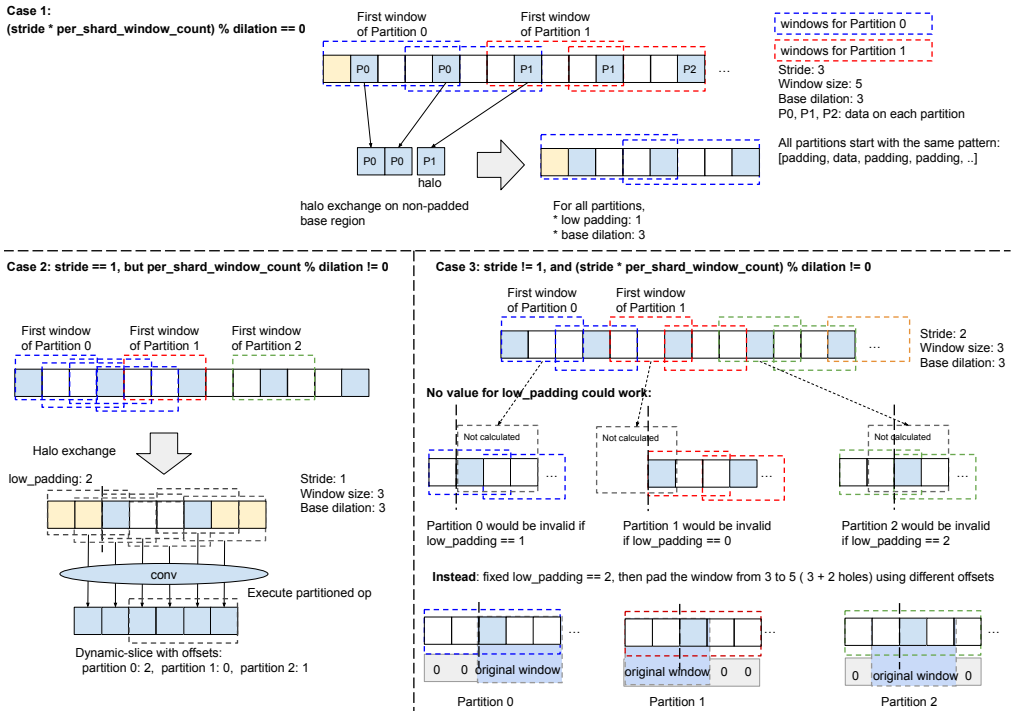


Figure 10. `Convolution` partitioning with base dilation.

• $stride \neq 1$ and $stride \times per\_shard\_window\_count$ is not divisible by $dilation$. If neither of the above conditions are true, different partitions could start with different number of padding elements, and not all offsets are valid window starts. Consider the last example in Figure 10. Whatever low padding we choose, some partition will be invalid, because valid windows could be skipped since $stride \neq 1$. A solution to this problem is to pad the window in addition to padding the base area. We can use the maximum low padding required by the partitions on the base area, and increase the window size by that low padding amount. The positions of the additional window padding vary on different partitions, which can be implemented as a `Pad` followed by a `DynamicSlice`. The window padding is used to mask off the unaligned elements in the base area, so that the start of the non-padding window element will be aligned with the desired start in the base area.

Window dilation. If the RHS is replicated, window dilation only affects the effective window size when the operator is partitioned based on its LHS. If the dilated RHS is also partitioned, which typically occurs in the gradient computation of strided convolutions, handling window dilation is still simpler than handling base dilation, because there is no low/high padding on the RHS. We skip the implementation details.