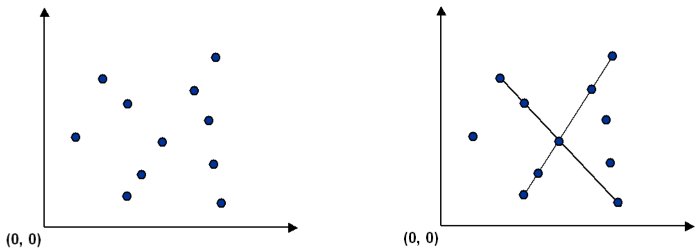
COLLINEAR POINTS

Spec FAQ Project Submit

Write a program to recognize line patterns in a given set of points.

Computer vision involves analyzing patterns in visual images and reconstructing the real-world objects that produced them. The process is often broken up into two phases: *feature detection* and *pattern recognition*. Feature detection involves selecting important features of the image; pattern recognition involves discovering patterns in the features. We will investigate a particularly clean pattern recognition problem involving points and line segments. This kind of pattern recognition arises in many other applications such as statistical data analysis.

The problem. Given a set of n distinct points in the plane, find every (maximal) line segment that connects a subset of 4 or more of the points.



Point data type. Create an immutable data type `Point` that represents a point in the plane by implementing the following API:

```
public class Point implements Comparable<Point> {
   public Point(int x, int y)                         // constructs the point (x, y)

   public   void draw()                               // draws this point
   public   void drawTo(Point that)                   // draws the line segment from this point to that point
   public String toString()                           // string representation

   public               int compareTo(Point that)     // compare two points by y-coordinates, breaking ties by x-coordinates
   public            double slopeTo(Point that)       // the slope between this point and that point
   public Comparator<Point> slopeOrder()              // compare two points by slopes they make with this point
}
```

To get started, use the data type [Point.java](), which implements the constructor and the `draw()`, `drawTo()`, and `toString()` methods. Your job is to add the following components.

- The `compareTo()` method should compare points by their y-coordinates, breaking ties by their x-coordinates. Formally, the invoking point (x_0, y_0) is *less than* the argument point (x_1, y_1) if and only if either $y_0 < y_1$ or if $y_0 = y_1$ and $x_0 < x_1$.

- The `slopeTo()` method should return the slope between the invoking point (x_0, y_0) and the argument point (x_1, y_1), which is given by the formula $(y_1 - y_0) / (x_1 - x_0)$. Treat the slope of a horizontal line segment as positive zero; treat the slope of a vertical line segment as positive infinity; treat the slope of a degenerate line segment (between a point and itself) as negative infinity.

- The `slopeOrder()` method should return a comparator that compares its two argument points by the slopes they make with the invoking point (x_0, y_0). Formally, the point (x_1, y_1) is *less than* the point (x_2, y_2) if and only if the slope $(y_1 - y_0) / (x_1 - x_0)$ is less than the slope $(y_2 - y_0) / (x_2 - x_0)$. Treat horizontal, vertical, and degenerate line segments as in the `slopeTo()` method.

- Do *not* override the `equals()` or `hashCode()` methods.

Corner cases. To avoid potential complications with integer overflow or floating-point precision, you may assume that the constructor arguments `x` and `y` are each between 0 and 32,767.

Line segment data type. To represent line segments in the plane, use the data type [LineSegment.java](), which has the following API:

```
public class LineSegment {
   public LineSegment(Point p, Point q)        // constructs the line segment between points p and q
   public   void draw()                        // draws this line segment
   public String toString()                    // string representation
}
```

Brute force. Write a program `BruteCollinearPoints.java` that examines 4 points at a time and checks whether they all lie on the same line segment, returning all such line segments. To check whether the 4 points p, q, r, and s are collinear, check whether the three slopes between p and q, between p and r, and between p and s are all equal.

```
public class BruteCollinearPoints {
   public BruteCollinearPoints(Point[] points)    // finds all line segments containing 4 points
   public           int numberOfSegments()        // the number of line segments
   public LineSegment[] segments()                // the line segments
}
```

The method `segments()` should include each line segment containing 4 points exactly once. If 4 points appear on a line segment in the order $p \rightarrow q \rightarrow r \rightarrow s$, then you should include either the line segment $p \rightarrow s$ or $s \rightarrow p$ (but not both) and you should not include *subsegments* such as $p \rightarrow r$ or $q \rightarrow r$. For simplicity, we will not supply any input to `BruteCollinearPoints` that has 5 or more collinear points.

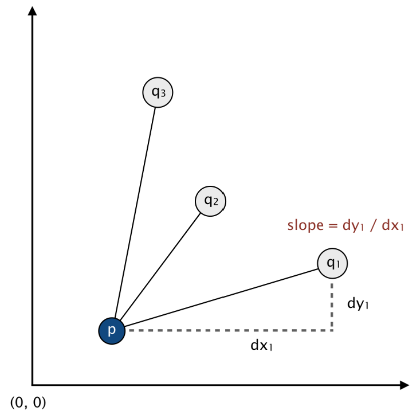
Corner cases. Throw an `IllegalArgumentException` if the argument to the constructor is `null`, if any point in the array is `null`, or if the argument to the constructor contains a repeated point.

Performance requirement. The order of growth of the running time of your program should be n^4 in the worst case and it should use space proportional to n plus the number of line segments returned.

A faster, sorting-based solution. Remarkably, it is possible to solve the problem much faster than the brute-force solution described above. Given a point p, the following method determines whether p participates in a set of 4 or more collinear points.

- Think of p as the origin.
- For each other point q, determine the slope it makes with p.
- Sort the points according to the slopes they makes with p.
- Check if any 3 (or more) adjacent points in the sorted order have equal slopes with respect to p. If so, these points, together with p, are collinear.

Applying this method for each of the n points in turn yields an efficient algorithm to the problem. The algorithm solves the problem because points that have equal slopes with respect to p are collinear, and sorting brings such points together. The algorithm is fast because the bottleneck operation is sorting.



Write a program `FastCollinearPoints.java` that implements this algorithm.

```
public class FastCollinearPoints {
   public FastCollinearPoints(Point[] points)     // finds all line segments containing 4 or more points
   public           int numberOfSegments()        // the number of line segments
   public LineSegment[] segments()                // the line segments
}
```

The method `segments()` should include each *maximal* line segment containing 4 (or more) points exactly once. For example, if 5 points appear on a line segment in the order $p \rightarrow q \rightarrow r \rightarrow s \rightarrow t$, then do not include the subsegments $p \rightarrow s$ or $q \rightarrow t$.

Corner cases. Throw an `IllegalArgumentException` if the argument to the constructor is `null`, if any point in the array is `null`, or if the argument to the constructor contains a repeated point.

Performance requirement. The order of growth of the running time of your program should be $n^2 \log n$ in the worst case and it should use space proportional to n plus the number of line segments returned. `FastCollinearPoints` should work properly even if the input has 5 or more collinear points.

Sample client. This client program takes the name of an input file as a command-line argument; read the input file (in the format specified below); prints to standard output the line segments that your program discovers, one per line; and draws to standard draw the line segments.

```
public static void main(String[] args) {

    // read the n points from a file
    In in = new In(args[0]);
    int n = in.readInt();
    Point[] points = new Point[n];
    for (int i = 0; i < n; i++) {
        int x = in.readInt();
        int y = in.readInt();
        points[i] = new Point(x, y);
    }

    // draw the points
    StdDraw.enableDoubleBuffering();
    StdDraw.setXscale(0, 32768);
    StdDraw.setYscale(0, 32768);
    for (Point p : points) {
        p.draw();
    }
    StdDraw.show();

    // print and draw the line segments
    FastCollinearPoints collinear = new FastCollinearPoints(points);
    for (LineSegment segment : collinear.segments()) {
        StdOut.println(segment);
        segment.draw();
    }
    StdDraw.show();
}
```

Input format. We supply several sample input files (suitable for use with the test client above) in the following format: An integer *n*, followed by *n* pairs of integers (*x*, *y*), each between 0 and 32,767. Below are two examples.

```
% cat input6.txt        % cat input8.txt
6                       8
19000  10000             10000      0
18000  10000                 0  10000
32000  10000              3000   7000
21000  10000              7000   3000
 1234   5678             20000  21000
14000  10000              3000   4000
                         14000  15000
                          6000   7000
```

```
% java-algs4 BruteCollinearPoints input8.txt
(10000, 0) -> (0, 10000)
(3000, 4000) -> (20000, 21000)

% java-algs4 FastCollinearPoints input8.txt
(3000, 4000) -> (20000, 21000)
(0, 10000) -> (10000, 0)

% java-algs4 FastCollinearPoints input6.txt
(14000, 10000) -> (32000, 10000)
```

Web submission. Submit a .zip file containing only `BruteCollinearPoints.java`, `FastCollinearPoints.java`, and `Point.java`. We will supply `LineSegment.java` and `algs4.jar`. You may not call any library functions except those in `java.lang`, `java.util`, and `algs4.jar`. You may use library functions in `java.util` only if they have already been introduced in the course. For example, you may use `Arrays.sort()`, but not `java.util.HashSet`.

This assignment was developed by Bob Sedgewick and Kevin Wayne.