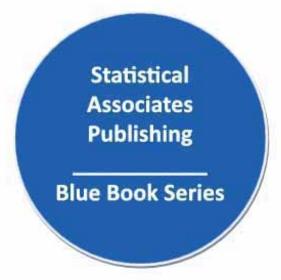
GAME THEORY

By G. David Garson

North Carolina State University School of Public And International Affairs



@c 2012 by G. David Garson and Statistical Associates Publishing. All rights reserved worldwide in all media. No permission is granted to any user to copy or post this work in any format or any media.

The author and publisher of this eBook and accompanying materials make no representation or warranties with respect to the accuracy, applicability, fitness, or completeness of the contents of this eBook or accompanying materials. The author and publisher disclaim any warranties (express or implied), merchantability, or fitness for any particular purpose. The author and publisher shall in no event be held liable to any party for any direct, indirect, punitive, special, incidental or other consequential damages arising directly or indirectly from any use of this material, which is provided "as is", and without warranties. Further, the author and publisher do not warrant the performance, effectiveness or applicability of any sites listed or linked to in this eBook or accompanying materials. All links are for information purposes only and are not warranted for content, accuracy or any other implied or explicit purpose. This eBook and accompanying materials is © copyrighted by G. David Garson and Statistical Associates Publishing. No part of this may be copied, or changed in any format, sold, or used in any way under any circumstances other than reading by the downloading individual.

Contact:

G. David Garson, PresidentStatistical Publishing Associates274 Glenn DriveAsheboro, NC 27205 USA

Email: gdavidgarson@gmail.com
Web: www.statisticalassociates.com

Table of Contents

Overview	4
Key Concepts and Terms	4
The Prisoner's Dilemma	4
Repeated games	4
Strategies	. 4
Pure strategies	5
Mixed strategies	5
Game matrix	5
Maximin, minimax, and saddle point	5
Zero-sum games	6
Constant-sum games	7
Saddle points	8
Dominance and reduced games	8
Deadlock	8
Odds in games without saddle points	9
Fair game value	10
Other types of games	10
Assumptions	10
Frequently Asked Questions	11
Is game theory purely a branch of logic?	11
What is an extensive form game?	11
Bibliography	11

Game Theory

Overview

Game theory is a branch of logic which deals with cooperation and conflict in the context of negotiations and payoffs. The theory of games can elucidate the incentive conditions required for cooperation, can aid understanding of strategic decisions of nations or actors in conflict, and can help in the development of models of bargaining and deterrence.

Key Concepts and Terms

The Prisoner's Dilemma

The "prisoner's dilemma" is the classic game in game theory literature. It centers on a game in which both actors would be better off cooperating, but both have an individual incentive to defect (not to cooperate) and as a result the likely outcome is one which is worse for both players than had they cooperated. This is illustrated in Table I below.

Repeated games

In real life, most games are repeated rather than single-shot. Repetition means each player has additional information based on past game decisions of the other player. This complicates calculation of choices and changes the equilibrium point. See Fink, Gates, and Humes, 1998: ch. 3. For instance, if the Prisoner's Dilemma is repeated a sufficient number of times, for instance, players may learn to take a strategic view and cooperate.

Strategies

A strategy is a plan of action that cannot be upset by an opponent or nature. In the prisoner's dilemma, the options are "confess" and "do not confess," and in one-round games, the strategy is the option taken. In multi-round games, strategies may be more complex. The purpose of strategies is to secure the most favorable game value in the long run. As an example, one strategy in multi-round games is *tit-for-tat*, in which the player responds to a given game move with a mirroring move.

Pure strategies

A pure strategy involves always pursuing the same strategy.

Mixed strategies

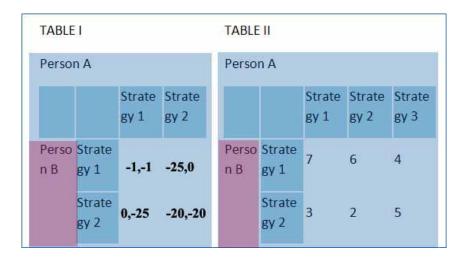
A mixed strategy involves randomly choosing among one's best strategies according to some proportions in order to maximize favorable game value when a pure strategy in repeated games would give the opponent an advantage of predictability. The part of a proportion (such as the 3 in the proportion 1:3) corresponding to a particular strategy is called the *oddment* of that strategy.

Game matrix

A game matrix is the table of all strategies of person A (as columns) versus all strategies of person B (as rows). The cell entries in a game matrix are *payoff* values, as illustrated below, with the first being the payoff to A and the second being the payoff to B. When the payoffs to strategies are quantified and placed in a matrix, the game is said to be in "strategic form."

Maximin, minimax, and saddle point

Consider the following tables:



In the prisoner's dilemma, let Strategy 1 b "Not confess" and Strategy 2 be "Confess". Assume each person considers only payoffs to him/herself, not payoffs to the other player. With full information and full trust, the solution would be for each person to choose Strategy 1, causing each person to serve only one year in jail. However, using conservative self-interested reasoning, Person B will select the strategy where the least to be gained is highest, or in this example, the most to be lost is lowest. The row containing the highest minimum (the maximum row minimum, or *maximin*) for Person B in Table I is the row for Strategy 2, where the maximin is minus 20. That is, the with Strategy 2 the worst Person B will get is 20 years in jail, whereas with Strategy 1 the worst is 25 years. Using conservative reasoning, Person A will also select Strategy 2.

Zero-sum games

Table II above-right shows a different formatting for a game matrix. In this format, only the payoffs to one player are shown, by convention the second player, B. Th game is "zero sum" because gain to one player, B, is seen as loss to the other player, A. The objective is assumed to be for Player B to maximize his/her payoff and for Player B to seek to minimize the payoff to A. Here the payoffs are positive, meaning higher values are desirable. By conservative strategy, Player B will select the maximin, which is Strategy 1, where the maximum row minimum is 4. Player A in Table II above-right will seek the column containing the lowest maximum (the

minimum column maximum, or minimax), which is Strategy 3, where the minimum column maximum is 5.

A zero-sum game could also be represented with dual notation as below, with payoffs adding to zero.

TABLE II						
Person A						
			Strate gy 2			
Perso n B	Strate gy 1	-7,7	-6,6	-4,4		
	Strate gy 2	-3,3	-2,2	-5,5		

A *non-zero-sum game* is, of course, any game in which the payoffs do not add to zero. That is, a gain for one player is not necessarily an equal loss for the other, or even a loss at all.

Constant-sum games

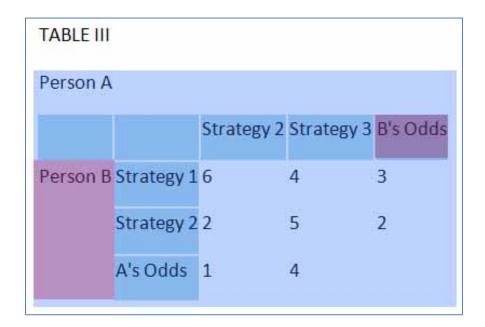
Constant-sum games are very similar to zero-sum games, but the payoffs add to a constant amount, not zero. This is typical when payoffs are expressed as percentages, which add to 100% in each cell. It is a type of zero-sum game since gain in percentage to one is loss in percentage to the other.

Saddle points

When the same cell is both the maximin and the minimax (not the case above), it is the *saddle point*. Any saddle point is also the *solution* to the game because it will be the payoff which results when the game is played by opponents using conservative rationality.

Dominance and reduced games

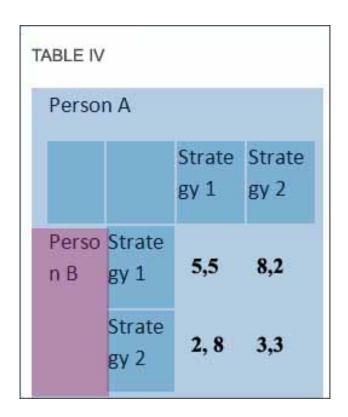
In Table II above-right, Considering only strategies 1 and 2, Person A will always prefer strategy 2 over strategy 1. If Person B chooses strategy 1, Person A will see B gain only 6 points rather than the 7 were A to instead choose strategy 1. Likewise A will see B gain only 2 rather than 3 points if A selects strategy 2. That is, for Person A, strategy 2 dominates strategy 1. This means column 1 can be eliminated from the game, resulting in the matrix in Table III below. After dropping a column, the matrix is called a reduced game.



Deadlock

Deadlock occurs when both players have a dominant strategy which results in a suboptimal outcome for both. In Table IV below, let strategy 1 be "Agree" and Strategy 2 by "Disagree." Player A will always agree because by selecting Strategy 1, Player A will get a higher payoff whatever strategy Player B selects. Likewise,

Player B will always agree also because Strategy 1 results in a higher payoff whatever strategy Player B selects. Barring mistakes by the other player, neither playerwill ever get the optimal payoff of 8.



Odds in games without saddle points

In the reduced game in Table III, there was no saddle point. The maximin was 4 but the minimax was 5. Person B will prefer strategy 1 because 4 is the least to be won (and possibly 6), whereas with strategy 2 the least to be won is 2 (or possibly 5). Person A will prefer strategy 3 because the most to be lost is 5, whereas under strategy 2 the most to be lost is 6. However, person A may want to play strategy 2 once in a while if Person A is consistently playing his strategy 1 because then Person B will lose only 2, Person A can't do this consistently, however, because then Person B will move to strategy 2 and Person A will lost 5. Person A wants to usually play strategy 3, but "sneak in" a strategy 2 once in a while. The proportion for the "once in a while" is determined by the odds. To compute the odds for

Table III, subtract the cells within each column or row, ignore minus signs, and place the answer in the *opposite* column. Thus, for instance, 6 - 1 = 4 and 4 - 5 = -1, giving 1 and 4 for A's odds, which means A should randomly use strategy 3 once for every four times strategy 3 is used.

Fair game value

The value of a game equals either person's odds played against any single strategy of the opponent. Thus, for Table III above, [(3x6)+(2x2)]/(3+2) = 4.4. A fair game is one with a value of zero.

Side payments. In some games the payoffs are loaded in favor of one player of the other. A side payment is the amount the favored player must pay the less favored player in order for the game to be fair. In the foregoing example, B should make a side payment of 4.4 units to A before each game if the game is to be fair.

Other types of games

Note that there are many other types of games than zero-sum games played under conservative rationality. Assumptions about rationality may be varied, for instance, and games may be *cooperative* rather than competitively zero-sum. Also, payoffs may be ordinal rather than interval and information may not be full.

Assumptions

Game theory usually assumes players respond rationally based on payoffs in the game. The most common assumption is one of *conservative rationality,* in which players choose strategies that assume maximum average gains or minimum average losses.

Most applications of game theory assume conditions of full information, a condition rarely met in the real world.

Frequently Asked Questions

Is game theory purely a branch of logic?

No, political scientists have long been interested in a behavioral, approach to game theory, testing out the implications of formal game theory using small group experiments.

What is an extensive form game?

A game in extensive form is represented as a horizontal tree diagram. The first set of branches are alternative strategies for player 1; the second set of branches from the end nodes of the first set, are the responsive strategies for player 2; the third set of branches from the end notes of the second set are the next strategies for player 1; and so on, until the final end nodes, which are the payoff amounts for each path through the tree.

Bibliography

- Binmore, K. (1992). Fun and games: A text on game theory. Lexington, MA: D. C. Heath.
- Dresher, M. (1961). *Games of strategy: Theory and applications*. Englewood Cliffs, NJ: Prentice Hall, 1961.
- Fink, Evelyn C., Scott Gates, and Brian D. Humes (1998). *Game theory topics: Incomplete information, repeated games, and n-player games*. Quantitative Applications in the Social Sciences Series No. 122. Thousand Oaks, CA: Sage Publications.
- Gates, S. and B. D. Humes (1997). *Games, information, and politics: Applying game theoretic models to political science*. Ann Arbor, MI: University of Michigan Press.
- Hargreaves Heap, S. and Y. Varoufakis (1995). *Game theory: A critical introduction*. London: Routledge.

- Luce, R. D. and H. Raiffa (1957). *Games and decisions: Introduction and critical survey*. NY: Wiley and Sons.
- Maynard Smith, J. (1982). *Evolution and the theory of games*. Cambridge, UK: Cambridge University Press.
- Morrow, J. (1994). *Game theory for political scientists*. Princeton, NJ: Princeton University Press.
- Myerson, R. (1991). *Game theory: Analysis of conflict*. Cambridge, MA: Harvard University Press.
- Ordeshook, P. C. (1986). *Game theory and political theory*. Cambridge, UK: Cambridge University Press.
- Rapoport, A. (1966). *Two-person game theory*. Ann Arbor, MI: University of Michigan Press.
- Rasmusen, E. (1989). Games and information. Cambridge, UK: Basil Blackwell.
- Riker, William (1967). Bargaining in a three-person game. *American Political Science Review*, 61(3): 642-656.
- Rosenthal, Edward C. (2011) *The complete idiot's guide to game theory*. London: Alpha Books/Penguin.
- Smoker, P. (1973). International relations simulations: A summary. In H. Alker, K. Deutsch, and A. Stoetzel, eds. (1973). *Mathematical approaches to politics*. San Francisco, CA: Jossey Bass: 417-464.
- Spaniel, William (2011). *Game Theory 101: The basics*. Kindle Direct Publishing, 2012.
- Von Neumann, J. and O. Morgenstern (1944). *Theory of games and economic behavior*. Princeton, NJ: Princeton University Press. The seminal work in game theory.
- Williams, Kenneth C. (2012). *Introduction to game theory: A Behavioral approach. NY: Oxford University Press.*

Zagare, F. (1984). *Game theory: Concepts and applications*. Quantitative Applications in the Social Sciences Series No. 41. Thousand Oaks, CA: Sage Publications.

Copyright 1998, 2008, 2012 by G. David Garson and Statistical Associates Publishers. Worldwide rights reserved in all languages and on all media. Do not copy or post in any format or on any medium. Last update: 10/12/2012.

Statistical Associates Publishing Blue Book Series

Association, Measures of

Assumptions, Testing of

Canonical Correlation

Case Studies

Cluster Analysis

Content Analysis

Correlation

Correlation, Partial

Correspondence Analysis

Cox Regression

Crosstabulation

Curve Estimation

Data Distributions and Random Numbers

Data Imputation/Missing Values

Data Levels

Delphi Method

Discriminant Function Analysis

Ethnographic Research

Evaluation Research

Event History Analysis

Factor Analysis

Focus Groups

Game Theory

Generalized Linear Models/Generalized Estimating Equations

GLM (Multivariate), MANOVA, and MANCOVA

GLM (Univariate), ANOVA, and ANCOVA

GLM Repeated Measures

Grounded Theory

Hierarchical Linear Modeling/Multilevel Analysis/Linear Mixed Models

Kaplan-Meier Survival Analysis

Latent Class Analysis

Life Tables

Logistic Regression

Log-linear Models,

Longitudinal Analysis

Multidimensional Scaling

Multiple Regression

Narrative Analysis

Network Analysis

Nonlinear Regression

Ordinal Regression

Partial Least Squares Regression

Participant Observation

Path Analysis

Power Analysis

Probability

Probit and Logit Response Models

Reliability Analysis

Resampling

Research Designs

Sampling

Scales and Standard Measures

Significance Testing

Structural Equation Modeling

Survey Research

Time Series Analysis

Two-Stage Least Squares Regression

Validity

Weighted Least Squares Regression

Statistical Associates Publishing http://www.statisticalassociates.com sa.publishers@gmail.com