EE 120: Signals and Systems FINAL S06
Department of EECS 15 May 2006
UNIVERSITY OF CALIFORNIA BERKELEY

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 120 minutes to complete. You will be given at least 120 minutes, up to a maximum of 170 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except four double-sided $8.5" \times 11"$ sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 22.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the twenty two numbered pages. If you find a defect in your copy, notify the staff immediately.
- **You will be given a separate document containing formulas and tables.**
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

You may use this page for scratch work only.
Without exception, subject matter on this page will *not* be graded.

F-S06.1 (30 Points) Each of the pole-zero diagrams (I)-(VI) on the next two pages belongs to a *causal* LTI system $\mathsf{H} : [\mathbb{R} \to \mathbb{R}] \to [\mathbb{R} \to \mathbb{R}]$ having a rational transfer function $\widehat{H}$ and an impulse response h. Each of the impulse-response plots (a)-(f) in the subsequent pages may correspond to *at most* one of the pole-zero diagrams (I)-(VI). Match each pole-zero diagram with an impulse response plot, or explain why no such match exists. Justify each of your choices succinctly, but clearly and convincingly. Show your work on this page.

(I)

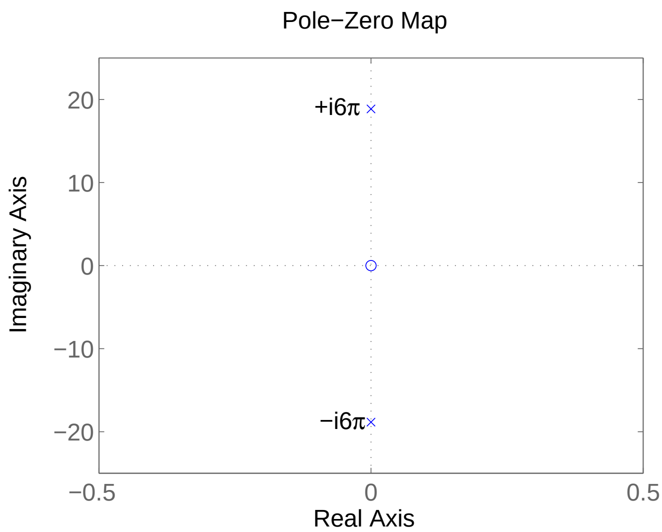
(II)

(III)

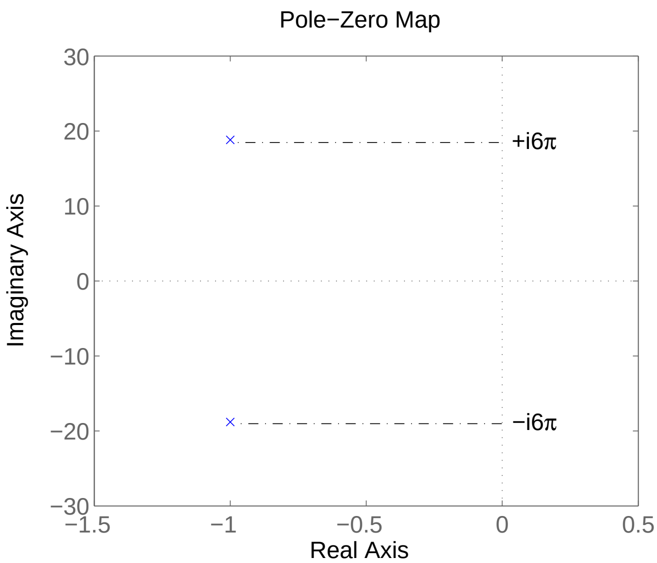
(IV)

(V)

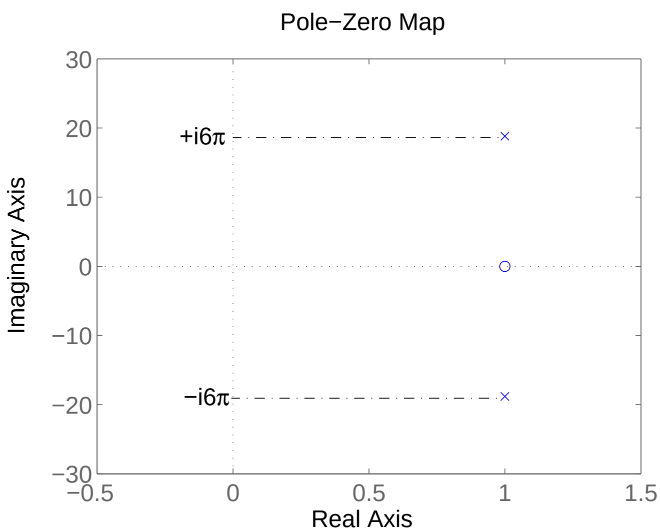
(VI)


Pole−Zero Map
+i6π
−i6π
Imaginary Axis
Real Axis
20
10
0
−10
−20
−0.5
0
0.5

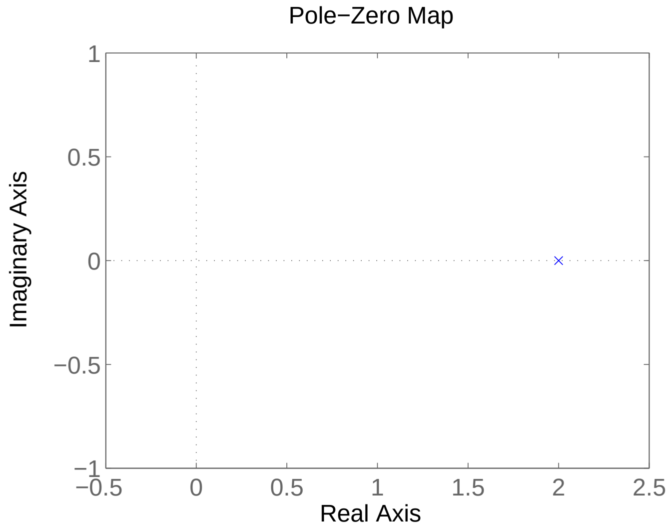
(I)


Pole−Zero Map
+i6π
−i6π
Imaginary Axis
Real Axis
30
20
10
0
−10
−20
−30
−1.5
−1
−0.5
0
0.5

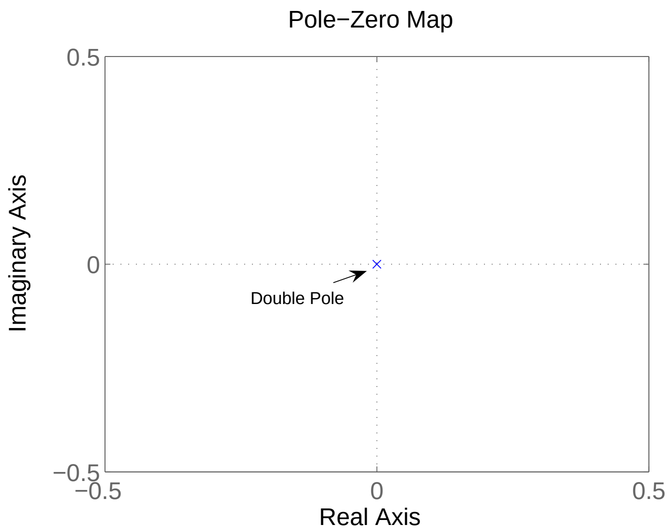
(II)


Pole−Zero Map
+i6π
−i6π
Imaginary Axis
Real Axis
30
20
10
0
−10
−20
−30
−0.5
0
0.5
1
1.5

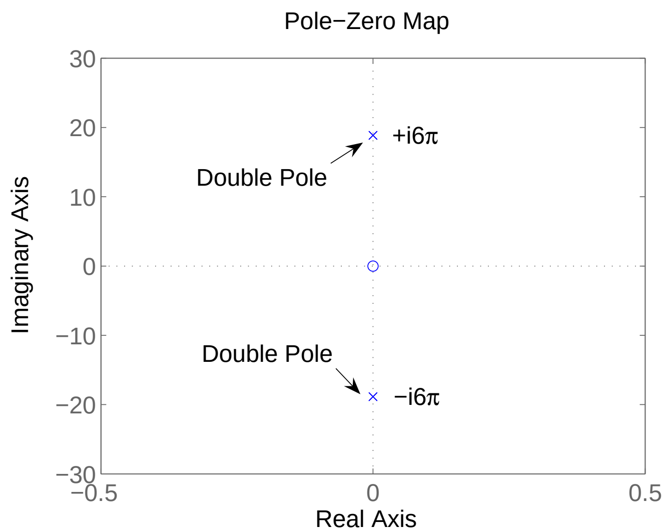
(III)


Pole−Zero Map
Imaginary Axis
Real Axis
1
0.5
0
−0.5
−1
−0.5
0
0.5
1
1.5
2
2.5

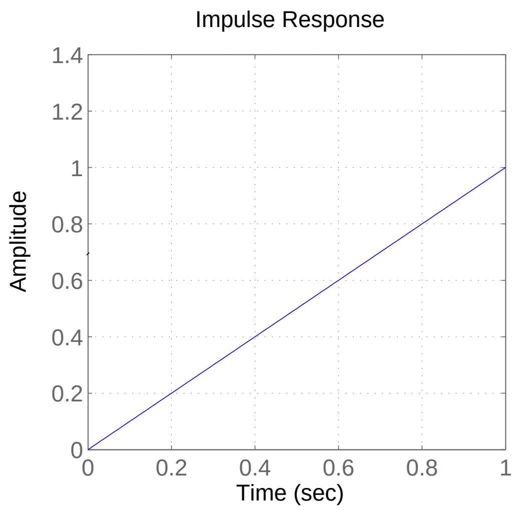
(IV)


Pole−Zero Map
Imaginary Axis
Real Axis
Double Pole
0.5
0
−0.5
−0.5
0
0.5

(V)

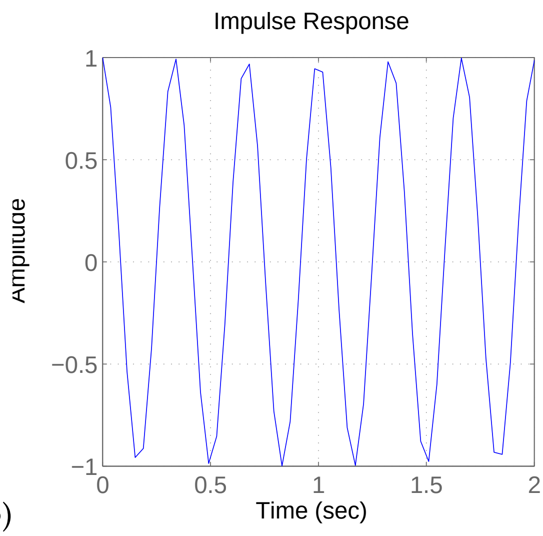

Pole−Zero Map
Imaginary Axis
Real Axis
Double Pole
+i6π
Double Pole
−i6π
30
20
10
0
−10
−20
−30
−0.5
0
0.5

(VI)



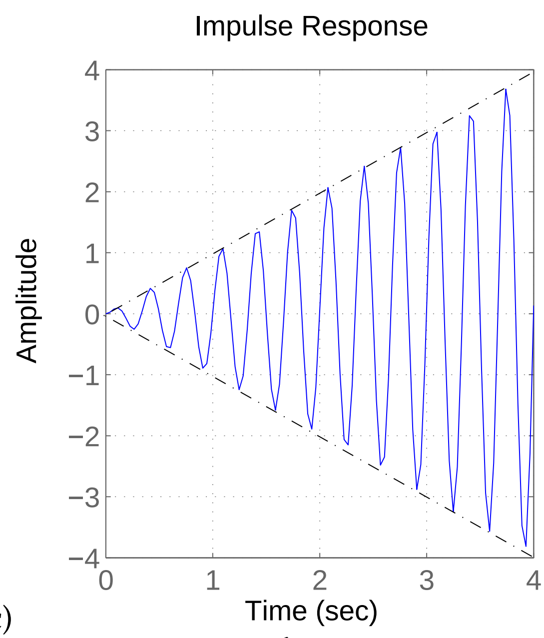
Impulse Response
Amplitude
Time (sec)

(a)



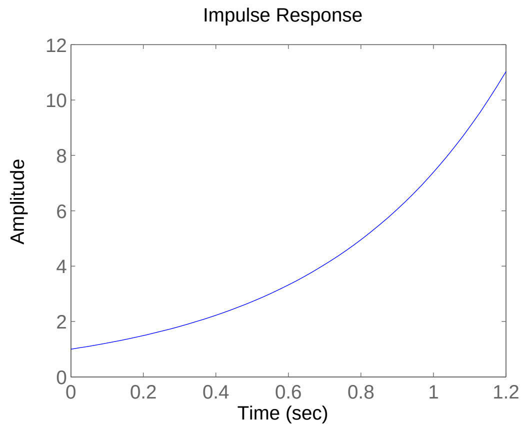
Impulse Response
Amplitude
Time (sec)

(b)



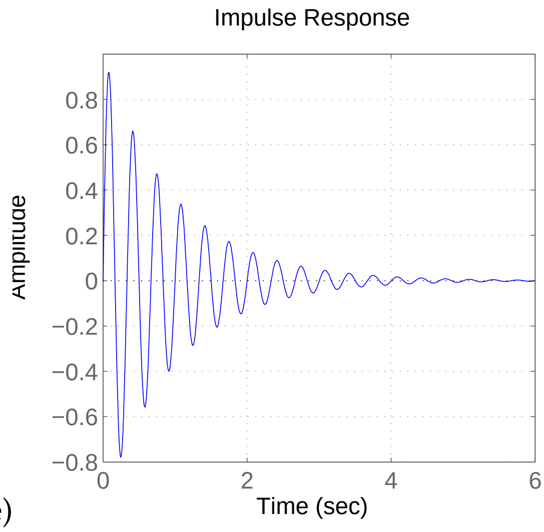
Impulse Response
Amplitude
Time (sec)

(c)



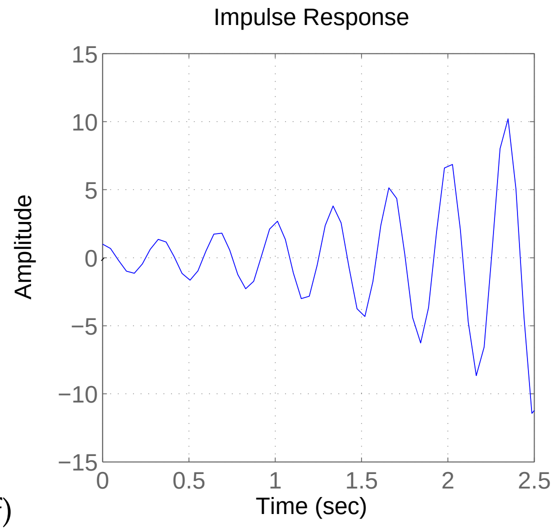
Impulse Response
Amplitude
Time (sec)

(d)



Impulse Response
Amplitude
Time (sec)

(e)



Impulse Response
Amplitude
Time (sec)

(f)

F-S06.2 (40 Points) A signal m phase-modulates a carrier of frequency f_c Hz to produce the signal

$$\forall t, \quad x(t) = \cos(2\pi f_c\, t + m(t)).$$

Suppose $|m(t)| \ll 1$, so this is narrow-band phase modulation (PM).

(a) Find a coherent demodulation scheme to recover the signal m from x. Give an algebraic or block-diagram description of your scheme. Explain why your scheme works.

(b) Suppose the modulated signal suffers amplitude distortion so that the received signal is y instead of x,

$$\forall t, \quad y(t) = A(t)x(t) = A(t)\cos(2\pi f_c\, t + m(t)),$$

where $1 \leq A(t) \leq 2$ is the distortion. What signal does your demodulator generate, and how is it related to m?

(b) You may continue you solutions here:

(c) Modify the design of your demodulator so that the effect of the distortion A is eliminated. Remember, you don't know A.

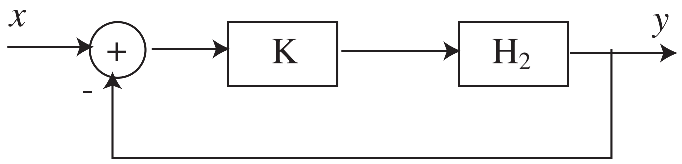
Hint: First, send y through a hard-limiter. A hard-limiter is a memoryless device g whose output is $\mathrm{sgn}(y(t))$ when its input is $y(t)$.

Explain why your scheme works.

F-S06.3 (40 Points) A plant with transfer function

$$H_2(s) = \frac{s+4}{s\,(s+3)}$$

is arranged in a feedback configuration with a proportional controller K, as shown in the figure below.



(a) What is the closed-loop transfer function?

(b) Plot the root locus for $K > 0$ and $K < 0$. Mark the location(s) of the closed-loop poles for $K = 0$ and $K \to \pm\infty$.

(b) You may continue you solutions here:

(c) For what values of K is the closed-loop system stable?

(d) Suppose $K = 1$ and $x(t) = u(t)$, the unit step. Determine $y(t), t \geq 0$. Express y in terms of its transient and steady-state components,

$$y(t) = y_{\mathsf{tr}}(t) + y_{\mathsf{ss}}(t).$$

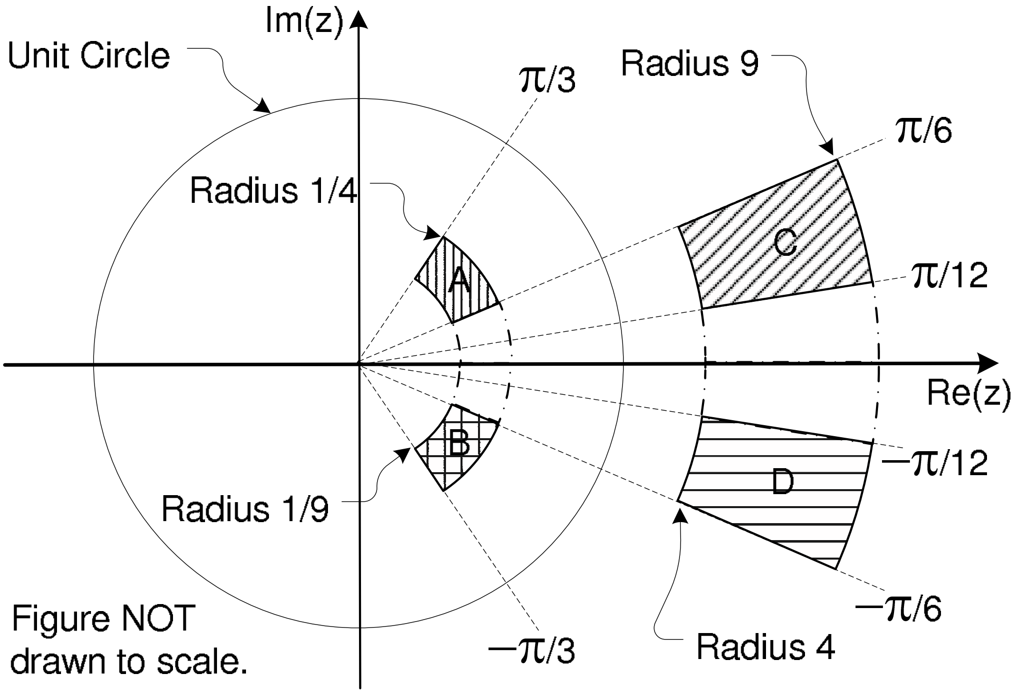
(e) What is the frequency response of the closed-loop system for $K = 1$?

(f) Suppose $K = 1$ and $x(t) = \sin(2t)\, u(t)$. What is the steady-state response $y_{\mathsf{ss}}(t)$?

F-S06.4 (40 Points) A *causal* LTI system $\mathsf{H} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ has impulse response h and rational transfer function $\widehat{H}$.

The figure below shows shaded regions where the poles and zeroes of $\widehat{H}$ reside on the complex z-plane. Regions A and B indicate the general locations of the poles, whereas regions C and D indicate the general locations of the zeroes of the system.

The figure is not drawn to scale, but the relative sizes and placements of the angles and radii is correct (e.g., the radius $1/9$ is drawn to be smaller than the radius $1/4$). Each of the radii $1/9$ and $1/4$ indicates the boundary defined by at least one pole (e.g., there is at least one pole at radius $1/4$ and at least one pole at radius $1/9$). The same is true of the radii defining the zero regions.



In this problem, additional LTI systems are introduced whose impulse responses are related to h. You are asked to draw inferences about various properties of those systems, such as the placement of their poles and zeroes, etc.

<u>NOTE</u>: If a pole or zero region of one of the additional systems corresponds to a particular region A, B, C, or D of the system H, you must indicate the correspondence by using the same letter. For example, if a pole cluster of one of the additional systems is due to a mapping of the poles of region A, then you must make that correspondence clear by labeling that cluster with the letter A.

(a) Determine $\mathsf{RoC}(h)$. Is the system H stable?

(b) An LTI system H_{TR} is characterized by the impulse response h_{TR}, where

$$h_{\mathsf{TR}}(n) = h(-n), \quad \forall n.$$

Provide a well-labeled sketch of the pole and zero regions of the transfer function $\widehat{H}_{\mathsf{TR}}$; determine $\mathsf{RoC}(h_{\mathsf{TR}})$; and explain whether the system H_{TR} is stable.

(c) A system $\mathsf{H}_{\mathsf{INV}}$ is characterized by the transfer function

$$\widehat{H}_{\mathsf{INV}}(z) = \frac{1}{\widehat{H}(z)} \,.$$

Provide a well-labeled sketch of the pole and zero regions of the transfer function $\widehat{H}_{\mathsf{INV}}$. What is $\mathsf{RoC}(h_{\mathsf{INV}})$, if $\mathsf{H}_{\mathsf{INV}}$ is known to be stable?

(d) Consider an LTI system G characterized by the impulse response g, where

$$g(n) = (-1)^n\, h(n), \quad \forall n.$$

Select the strongest assertion from the choices below. Explain your choice.

(1) The system G must be stable.
(2) The system G could be stable.
(3) The system G cannot be stable.

Select the strongest assertion from the choices below. Explain your choice.

(I) The system G must be causal.
(II) The system G could be causal.
(III) The system G cannot be causal.

Provide a well-labeled sketch of the pole and zero regions of the transfer function $\widehat{G}$ of G.

(e) Consider an LTI system F characterized by the impulse response f, where

$$f(n) = \begin{cases} h\left(\frac{n}{2}\right) & \text{if } n \text{ is even} \\ 0 & \text{otherwise.} \end{cases}$$

Select the strongest assertion from the choices below. Explain your choice.

(1) The system F must be stable.
(2) The system F could be stable.
(3) The system F cannot be stable.

Select the strongest assertion from the choices below. Explain your choice.

(I) The system F must be causal.
(II) The system F could be causal.
(III) The system F cannot be causal.

Provide a well-labeled sketch of the pole and zero regions of the transfer function $\widehat{F}$ of F.

F-S06.5 (40 Points) Consider a function $h : \mathbb{Z} \to \mathbb{R}$ having Fourier transform H and Z transform $\widehat{H}$. The Z transform $\widehat{H}$ can be written as

$$\widehat{H}(z) = \sum_{n=0}^{N} h(n)\, z^{-n}\,,$$

where N is a positive integer (finite, of course), $h(0) \neq 0$, and $h(N) \neq 0$.

(a) Assume that $\widehat{H}$ does *not* have all its zeros on the unit circle, and that the impulse response h satisfies the following property:

$$\forall n, \quad h(n) = h(N-n)\,.$$

(i) Prove that if z_0 is a zero of $\widehat{H}$, then so is $1/z_0^*$; that is,

$$\widehat{H}(z_0) = 0 \quad \Longrightarrow \quad \widehat{H}(1/z_0^*) = 0\,.$$

This statement means that $\widehat{H}$ has at least one conjugate reciprocal pair of zeroes not residing on the unit circle.

(a) Continued:

(ii) Assume N is an odd positive integer. Show that the frequency response H can be expressed as follows:

$$\forall \omega, \quad H(\omega) = A(\omega) e^{i\theta(\omega)},$$

where $A(\omega) \in \mathbb{R}$ and $\theta(\omega) = \alpha\omega$, for some constant $\alpha \in \mathbb{R}$. Determine $A(\omega)$ and α (and hence $\theta(\omega)$) in terms of N and the impulse response values $h(n)$.

(iii) Suppose h represents the impulse response of an FIR filter $\mathsf{H} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$. Is the following statement true or false? Explain your reasoning succinctly, but clearly and convincingly.

The filter H has a causal and stable inverse $\mathsf{G} : [\mathbb{Z} \to \mathbb{R}] \to [\mathbb{Z} \to \mathbb{R}]$ having transfer function

$$\widehat{G}(z) = \frac{1}{\sum_{n=0}^{N} h(n)\, z^{-n}} \; .$$

(b) For this part, suppose $\widehat{H}$ has *no* conjugate reciprocal pair of zeroes; that is,

$$\widehat{H}(z_0) = 0 \quad \Longrightarrow \quad \widehat{H}(1/z_0^*) \neq 0.$$

Assume, also, that $\widehat{H}$ has no zero on the unit circle.

Consider a finite-length signal $f : \mathbb{Z} \to \mathbb{R}$ that shares these properties with h; that is, the Z transform $\widehat{F}$ of F has no zero on the unit circle and no conjugate reciprocal pair of zeroes (anywhere). Furthermore, F, the DTFT of f, has the same phase as H; that is, if $H(\omega) = |H(\omega)|e^{i\theta_h(\omega)}$ and $F(\omega) = |F(\omega)|e^{i\theta_f(\omega)}$, then

$$\forall \omega, \quad \theta_h(\omega) = \theta_f(\omega).$$

(i) Let $q(n) = (h * f_{\mathsf{TR}})(n)$, where $f_{\mathsf{TR}}(n) = f(-n)$, the time-reversed counterpart of f. Prove that q is an even signal by showing that $Q(\omega) \in \mathbb{R}, \forall \omega$. Determine $\theta_q(\omega) \stackrel{\triangle}{=} \angle Q(\omega)$.

(ii) Express each of $\widehat{Q}(z)$ and $\widehat{Q}(1/z)$ in terms of the Z transforms $\widehat{H}$ and $\widehat{F}$. Then, use the symmetry of q, as well as what you know about h and f, to prove that

$$\widehat{H}(z_0) = 0 \quad \Longleftrightarrow \quad \widehat{F}(z_0) = 0.$$

That is, prove that if z_0 is a zero of $\widehat{H}$, then it is also a zero of $\widehat{F}$, and vice versa.

(b) Continued:

(iii) Use the result of Part (b)(ii) to show that

$$f(n) = \beta\, h(n), \quad \exists \beta \in \mathbb{R}.$$

(iv) Argue that β in Part (b)(iii) is actually a *positive* real number.

The lesson from this problem is that, under certain circumstances, a signal h can be recovered (to within a positive scale factor β) from *only* the phase of its Fourier transform!!!

LAST Name ______________________ FIRST Name ______________________

Lab Time ______________________

Problem	Points	Your Score
Name	10	
1	30	
2	40	
3	40	
4	40	
5	40	
Total	**200**	