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University of California, Berkeley – College of Engineering

Department of Electrical Engineering and Computer Sciences

Summer 2011 Instructor: Neel Shah 2011-07-08

Last Name	Key
First Name	Solution
Student ID Number	
Name of the person to your left	
Name of the person to your right	
All the work is my own. I had no prior knowledge of the exam contents nor will I share the contents with others in EE40 who have not taken it yet. I have not cleverly hidden notes and/or computational devices that give me an unfair advantage. (please sign)	

Instructions:

- You have 100 minutes to complete this exam, which we'll start at 12:15 PM. Once time is out at 1:55 PM, we'll announce that time is up, and you should stop writing immediately. The exact times will depend on when we can get everyone settled in.
- There are 10 pages on the exam. All work should be on this exam. Don't hand in random sheets of paper.
- Please turn off anything that might make noise, unless it is necessary for you to stay alive. Remove all hats and headphones. Allow at least one empty seat between your neighbor and you. All backpacks, laptops, and jackets should be up front, and nothing should be in the seats or on the floor between you and your neighbors.
- Calculators are allowed and recommended, but cell phone calculators are not.
- You may use 1 page of notes (8.5" x 11" or A4), front and back, handwritten or typed, no magnifying glasses!
- Partial credit will be given for incomplete answers, so please show your work. The best possible thing you can do if you start running out of time is to write out things like the correct node voltage equations for problems you haven't completed.
- If you get stuck on an algebra problem, move on! The numerical part of a problem will be worth less than the conceptual part!
- This is a hard test, but hopefully not too hard! If the average is too low, remember that we're going to nudge grades up if we went too hard on you and wrote problems that were too difficult.

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Total
Earned						
Max	25	25	50	50	50	200

Name________________________________ SID____________________________________

Problem 1. Short Answer

a) Why don't birds get electrocuted when they sit on high-voltage power lines? How could they get electrocuted?

Birds sit with both of their feet on the same power line. This places both of their feet at the same voltage. Thus no current flows through their body. If they place one foot on a different power line than another, then they risk electrocution, since their feet may be at different voltages and a lethal current may pass through their body.

+3 points for correct explanation of why birds don't get electrocuted

+3 points for correct explanation of how birds can get electrocuted

No partial credit

b) Suppose we create a Thévenin model of a circuit, and find that the Thévenin resistance is negative, and the Thévenin voltage is zero. Does this circuit supply power or consume power? Why?

A circuit with zero Thévenin voltage and a negative Thévenin resistance simply becomes a resistor with negative resistance. If $V_{TH} = 0$, then we just replace the Thévenin source with a short circuit. A negative resistance can be interpreted as a positive resistance that reverses current flow. So a voltage source hooked up to this thing would start receiving energy instead of delivering energy. Thus the circuit is a power supplier.

+2 points for choosing the circuit supplies power

+2 point for replacing the Thévenin source with a short circuit

+2 points for correctly interpreting the negative resistance [+1 if they get the +2 above but miss this part]

c) On your graphing calculator or other AA/AAA battery powered device, why do you have to plug in half of your batteries one way and half the other way?

AA/AAA batteries only operate at 1.5 V. In order to generate a large voltage we need to connect the batteries end to end. However it is difficult to do this in a space-efficient package. Flipping the batteries around allows easy series connection between batteries.

+3 points for recognizing the need for space-efficiency

+3 points for recognizing batteries need to be connected in series

No partial credit

d) Suppose you try to start your car one morning and find that you cannot start your car engine. However you can turn on the lights. Furthermore, you also notice that you can turn on the windshield wipers, but they move slowly. Give the most likely explanation for these findings.

This is a symptom of a battery with low charge remaining (i.e. almost dead). It has enough charge to generate a high voltage for a large impedance load (the lights) but not enough charge to maintain that voltage for a small impedance load (the wipers) and definitely not enough charge to provide the spark to start engine combustion.

+3 points for identifying the battery as the cause of the problem

+3 points for identifying a low charge status as the problem with the battery

+2 point for distinguishing how the battery is affected by the various loads

Answers not following this key, but are technically valid (yet convoluted) receive 4 points (-1 for impracticality)

Name________________________________ SID____________________________________

Problem 2. Lightning Fast Cars

Suppose a bolt of lightning strikes the ground from 4000 meters up. From the brightness of the bolt you estimate it is a fairly tame lightning bolt, and the current traveling through the bolt is about $I_L = 55\ kA$.

a) Suppose you measured the lightning bolt with a photodetector, and found the lightning bolt lasted for $6\ ms$. How many coulombs of charge were transferred from the sky to the ground via the bolt? How many electrons is that?

$$charge = current \times time = 55\ kA \cdot 6\ ms = 330\ C$$
$$\#\ electrons = charge/1.6 \times 10^{-19} = 206 \times 10^{19} = 2.1 \times 10^{21}\ electrons$$

+4 points for getting the charge right (+3 points for equation setup, +1 point for correct arithmetic)

+4 points for getting the # electrons right (+3 point for equation setup, +1 point for correct arithmetic)

b) Based on the atmospheric conditions at the time of the lightning strike you estimate the electric field (in circuit terms, a spatial voltage gradient) of the lightning at $E_L = 140\ V/m$. What was the power delivery of the lightning bolt? How much energy did it contain?

$$power = voltage \times current = (electric\ field \times length) \times current = 140\ V/m\ \cdot\ 4000\ m\ \cdot 55\ kA = 30.8\ GW$$
$$energy = power \times time = 30.8\ GW \times 6\ ms = 184.8\ MJ$$

+3 points for getting the right voltage (560 kV) from the electric field (+2 points for equation, +1 point for arithmetic)

+3 points for getting the right power (+2 points for equation, +1 point for arithmetic)

+3 points for getting the right energy (no partial credit)

c) Suppose the lightning bolt struck a (parked and uninhabited) Tesla Roadster, and all the energy transferred into the Roadster's fully discharged battery pack. Given the Tesla Roadster battery pack operates at $V_T = 375\ V$ nominal and stores $53\ kWh$ of energy,[1] what percent of the battery did the lightning bolt re-energize?
(Note: $1\ kWh = 1000\ W \cdot 1\ hour = 1000\ J/sec\ \cdot 3600\ sec$)

First find the energy capacity of the battery pack: $53\ kWh = 191\ MJ$
Or first find the energy of the lightning bolt: $184.8\ MJ = 51.333\ kWh$
Then take the ratio: $184.8/191 \approx 96.8\%$
+5 points for getting the correct capacity in kWh of the bolt or in MJ of the battery
(+2 points if they set up the correct conversion but fail at the arithmetic)
+3 points for closing out and getting the correct ratio (+1 point if they set up right but fail at arithmetic)
Students may try by calculating the charge capacity of the battery (~500 kC). This obviously gives a small ratio.
+2 points for this method. (Doesn't work because charge is not conserved from a circuit perspective in this case)

d) How awesome is that? Why isn't this part of Tesla's marketing pitch? [Bonus +1 pt each]
+1 bonus point for something along the lines of "very awesome"
+1 bonus point for something along the lines of "because marketing is for fools" (it's okay, they don't have feelings)

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[1] Tesla Roadster Battery System White Paper, http://webarchive.teslamotors.com/display_data/TeslaRoadsterBatterySystem.pdf

Name__________________________________ SID_______________________________________

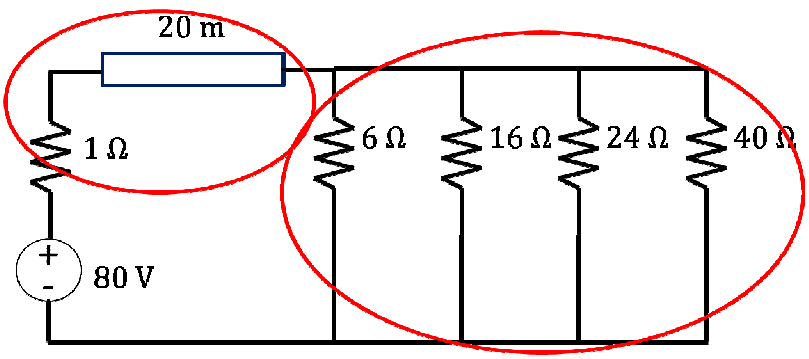
Problem 3. A Bigger Picture

Suppose you've outfitted your apartment with solar panels to power your appliances. The solar cell assembly outputs a DC voltage of $V_S = 80\ V$ and has a source resistance of $R_S = 1\ \Omega$. The cabling from the solar cell assembly has a length $l = 20\ m$ and attaches on one end to the solar cell assembly and on the other end to the power distribution assembly. This power distribution assembly then connects to all of your household appliances, which include a refrigerator $(16\ \Omega)$, a desktop computer $(40\ \Omega)$, an electric heater $(6\ \Omega)$, and a creepy robot vacuum cleaner $(24\ \Omega)$.

Solar Cell Assembly	Cable	Electric Heater	Refrigerator	Vacuum	Desktop Computer	Battery Pack
1 Ω, 80 V	20 m	6 Ω	16 Ω	24 Ω	40 Ω	4 Ω, 45 V

a) Draw a circuit schematic of this described setup using the parts provided.

+2 points for grouping solar cell and cable in series
+2 points if cable splits solar cell from appliances



+3 points for grouping all appliances in parallel
+3 points if appliances connect to negative side of solar cell

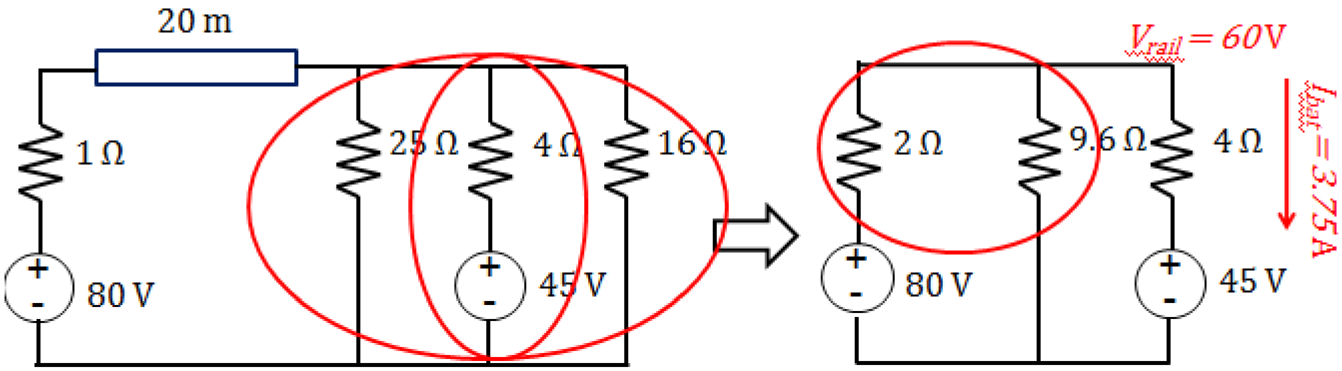
b) We want the cable resistance to be no more than $R_{cable} = 1\ \Omega$. Assuming a circular cross section and a cable resistivity of $\rho = 200\ \mu\Omega \cdot cm$, what should the diameter of the cable be?

$$R = \frac{\rho l}{A} = \frac{4\rho l}{\pi d^2} = \frac{4 \cdot (200\ \mu\Omega \cdot cm) \cdot (2000\ cm)}{\pi d^2} \rightarrow d = \left(\frac{1.6\ \Omega \cdot \mathrm{cm}^2}{\pi (1\ \Omega)}\right)^{1/2} = 0.713\ cm = 7.13\ mm$$

+4 points for the correct resistivity equation, +2 points if answer is "7.13 x $10^{\#}$", +3 points if units are all worked out right

c) Calculate the power the battery pack $(45\ V, 4\Omega)$ would absorb if connected between the power distribution assembly and the return line (the negative terminal of the solar cell voltage source or "ground") and only the refrigerator and desktop computer are connected.

Name__________________________________ SID_______________________________________



+2 points if battery pack is hooked up with the appliances
+2 points if all appropriate appliances are present/absent
(+1 point if some appliances are right)

+2 points for resistor simplification
+2 points for correct KCL setup
+2 points for correct rail voltage
+2 points for correct battery current
+1 point for correct power value

Calculating the rail voltage:

$$\frac{80 - v_{rail}}{2} = \frac{v_{rail}}{9.6} + \frac{v_{rail} - 45}{4} \; (+2\;points)$$

$$\frac{80}{2} + \frac{45}{4} = v_{rail}\left(\frac{1}{2} + \frac{1}{9.6} + \frac{1}{4}\right)$$

$$v_{rail} = \frac{51.25}{0.854} = 60\;V \; (+2\;points)$$

Calculating the battery input power:

$$power = current \times voltage = \left(\frac{v_{rail} - 45}{4}\right) \times 45\;V = 3.75\;A \; (+2\;points) \times 45\;V = 168.75\;W \; (+1\;point)$$

d) If the cable resistance is $R_{cable} = 1\;\Omega$ and the battery pack is disconnected, which three appliances can you run and maintain a line voltage of $V_{rail} \geq 61\;V$? If you then connect the battery pack, how much power will it absorb or deliver, and what is the new line voltage?

If the student is clever s/he will recognize the wording gives away that the combination is the one with the highest net resistance, which also have to be the ones with highest individual resistances. So the combo necessarily excludes the heater. The equivalent resistance is $40\;\Omega||24\;\Omega||16\;\Omega = 7.74\;\Omega$ and the rail voltage is $80V \cdot 7.74\Omega/9.74\Omega = 63.5\;V$.

When the battery is attached, the new line voltage can be found by the same method:

$$\frac{80 - v_{rail}}{2} = \frac{v_{rail}}{7.74} + \frac{v_{rail} - 45}{4} \therefore v_{rail} = 58.3\;V$$

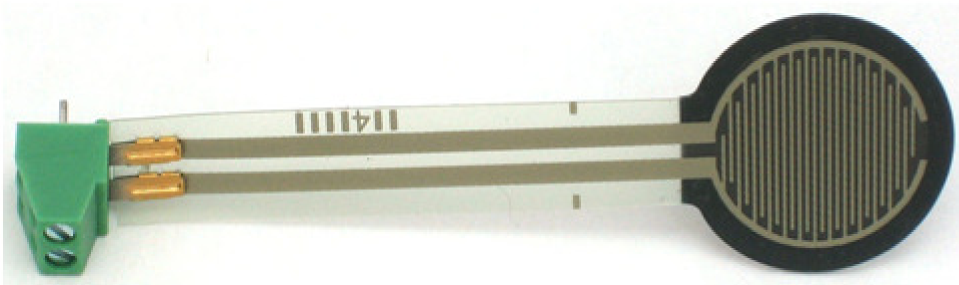
The new power absorbed by the battery pack is $\frac{58.3-45}{4} \times 45 = 149.5\;W$

+3 points for the correctly identified appliance combination
+3 points for the correctly calculated equivalent resistance [appropriate appliance combo or not]
+2 points for the correctly calculated rail voltage [appropriate appliance combo or not]
+3 points for the new line voltage with battery (+1 for method [independent of right appliances], +1 for right answer)
+2 points for the power into/out of the battery (+1 for method [independent of appliances], +1 for right answer)

Name________________________________ SID____________________________________

Problem 4. A Resistive Force Sensor Amplifier

The photograph below is of a force sensing resistor (FSR). The resistance of this device changes inverse-proportionally to the applied force. The characteristic of this particular FSR is $R_{FSR} = 10000/F_{FSR}$, where F_{FSR} is the applied force in Newtons and R_{FSR} is the resistance in Ohms. (So for $F_{FSR} = 10\ N$, $R_{FSR} = 1\ k\Omega$).



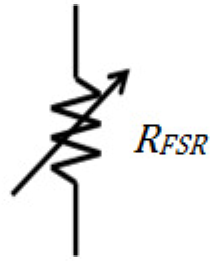


Figure 1: (left) Photograph of Interlink force sensing resistor. (right) Schematic representation of FSR.

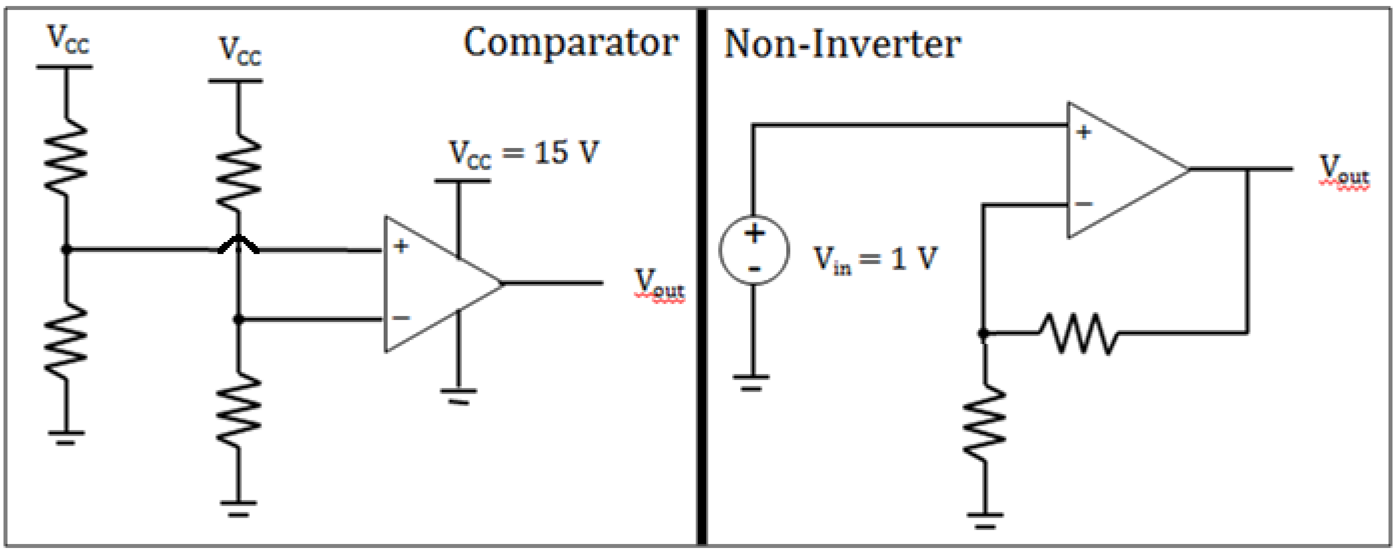


Figure 2: (left) Comparator circuit template. (right) Non-inverting amplifier circuit template. Both op-amps are ideal.

a) Using the open-loop comparator circuit template below on the left, choose resistor values that force $v_{out} = 15\ V$ when the applied force is above $F_t \geq 50\ N$ and $v_{out} = 0\ V$ when $F_t < 50\ N$. Assume an ideal comparator, i.e. $A \rightarrow \infty$. One of the four resistor slots need to be the FSR! Clearly state which one you chose and explain your decision.

We need to place the FSR either on the top of the + voltage divider or on the bottom of the – voltage divider.
As the force rises the resistance of the FSR drops; placing it at the top of the v+ divider will bring the v+ input higher, and placing it at the bottom of the v- divider will bring the v- input lower. Either case will trip a switch to $v_{out} = 15\ V$.
When $F = 50\ N$, $R_{FSR} = 200\ \Omega$. So the other resistor chosen with the FSR needs to yield a v+ value higher than v-.
+8 points for identifying either correct location of FSR, +7 points for a good explanation for the FSR placement
+6 points for appropriately choosing other resistors (+2 for each resistor)

Name________________________________ SID____________________________________

b) The comparator is sensitive to noise or fluctuations in the force sensing resistance. Small variations in resistance around the threshold point can cause the comparator to swing wildly between its rails. What circuit can we use instead to eliminate this problem? Don't worry about how to implement the circuit, just mention the name and qualitatively describe how it works.

Schmitt Trigger (+4 points)

Works based on positive feedback, latching condition, hysteresis (+1 points for mentioning each of these = +3 total)

c) Using the non-inverting amplifier circuit template below, choose i) which of the two resistor slots the FSR occupies and ii) what the other resistor value should be such that the circuit converts a force range of $F_{FSR} \in (0,100)N$ to $V_{out} = (1,15)V$ in a positive linear relationship (i.e. dV_{out}/dF_{FSR} is positive and constant).

For a linear relationship, the FSR needs to occupy the slot between v- and ground. We setup the V_{out} vs. F_{FSR} relation:

$$V_{out} = V_{in}\left(1 + \frac{R}{R_{FSR}}\right) = 1V \cdot \left(1 + \frac{F_{FSR} \cdot R}{10000}\right) = 1 + \frac{RF_{FSR}}{10000}$$

We automatically get $V_{out} = 1\ V$ when $F = 0\ N$. We solve for R by setting $V_{out} = 15\ V$ and $F_{FSR} = 100\ N$:

$$R = \frac{10000 \cdot (V_{out} - 1)}{F_{FSR}} = \frac{10000 \cdot (15 - 1)}{100} = 1400\ \Omega$$

+9 points for correctly placing the FSR, +9 points for determining the other resistor

+4 points for using the ideal non-inverting relation or deriving it from VCVS model

Name________________________________ SID____________________________________

Problem 5. Characterizing a Better Audio Amplifier

In class we looked at the suitability of a single-stage amplifier for driving low resistance loads, such as loudspeakers. We found the circuit provided good voltage gain but had high output resistance, as shown in the Thévenin model below. This unfortunate property prevented this amplifier from being effective at driving small loads such as a speaker.

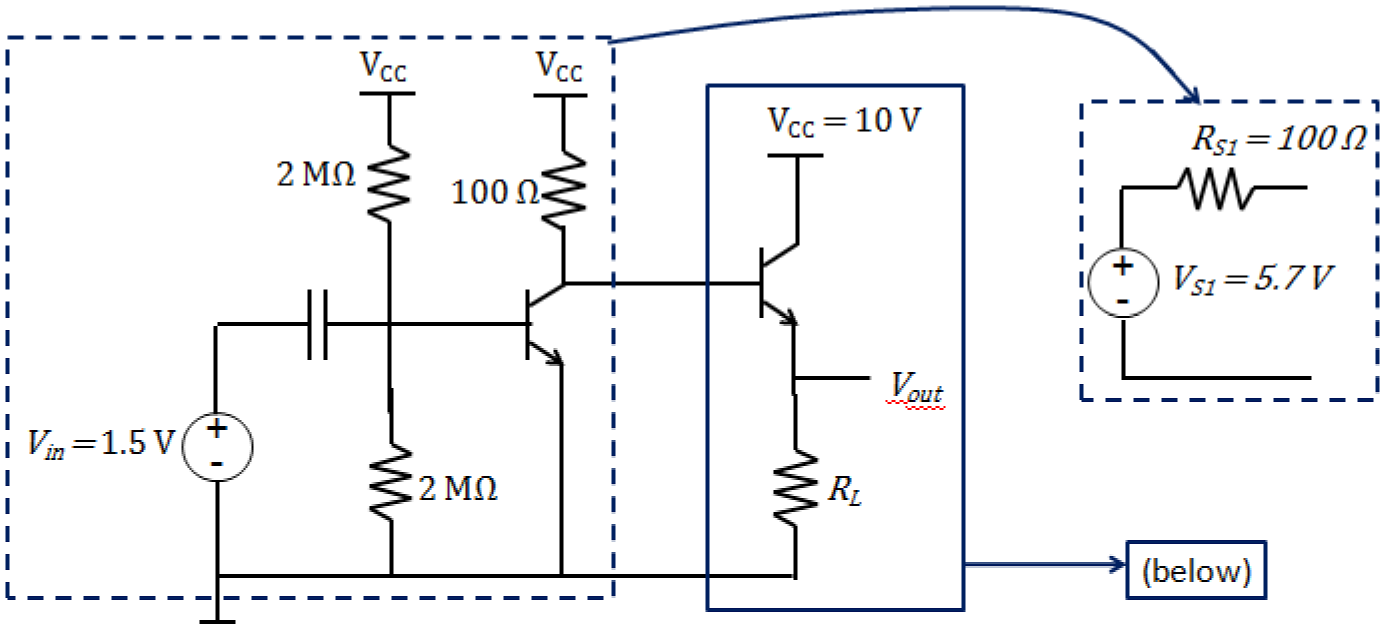
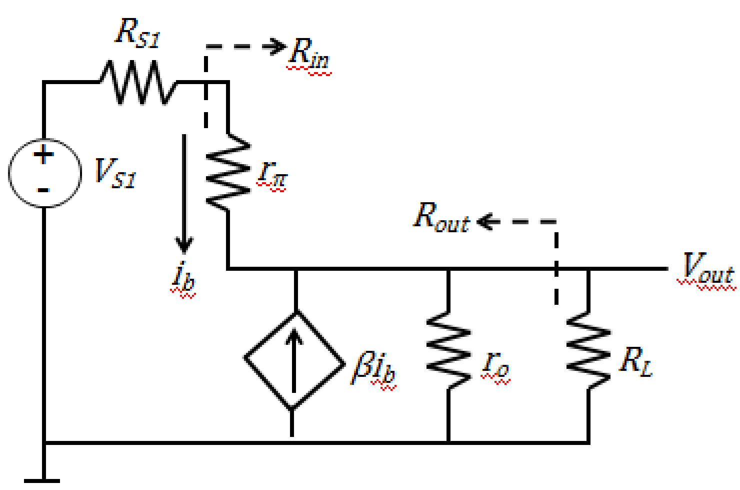


Figure 3: Two-stage amplifier circuit with Thévenin model of first stage to right.

However if we hook up this amplifier to a second stage (typically called a "follower" for this reason) we find that we can deliver voltage much better to low loads. A simple follower circuit consists of one transistor and when attached to the first stage, looks like the solid-outlined part of the transistor circuit above. For this problem you will only deal with the equivalent circuit model of the follower coupled to the Thévenin model of the first stage amplifier, which looks like:

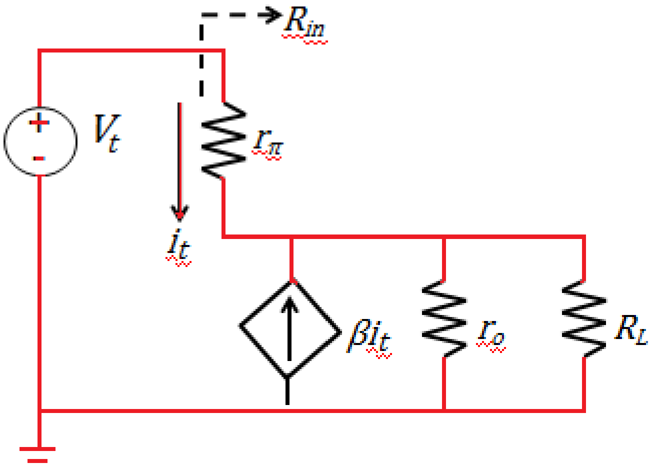


$r_\pi = 950\ \Omega$
$r_o = 125\ k\Omega$
$\beta = 150$
$R_L = 8\ \Omega$
$V_{S1} = 5.7\ V$
$R_{S1} = 100\ \Omega$

Figure 4: Equivalent circuit model of BJT emitter follower circuit.

Name__________________________________ SID______________________________________

a) Calculate the input resistance seen by the driving stage $S1$. Hint: Apply a test voltage v_{in} at the point marked R_{in}, and find the test current i_{in}. Remember to include the load resistance!



First define the test current i_t:

$$i_t = \frac{V_t - V_{out}}{r_\pi}$$

KCL at V_{out} to get it in terms of V_t:

$$i_t + \beta i_t = V_{out}/(r_o || R_L)$$

$$\frac{(\beta+1)(V_t - V_{out})}{r_\pi} = V_{out}\left(\frac{1}{r_o} + \frac{1}{R_L}\right)$$

$$\frac{\beta+1}{r_\pi}V_t = V_{out}\left(\frac{1}{r_o} + \frac{1}{R_L} + \frac{\beta+1}{r_\pi}\right)$$

$$V_{out} = V_t\left(\frac{1}{r_o} + \frac{1}{R_L} + \frac{\beta+1}{r_\pi}\right)^{-1}\left(\frac{\beta+1}{r_\pi}\right) = AV_t$$

$$A = \left(\frac{\beta+1}{r_\pi}\right)/\left(\frac{1}{r_o} + \frac{1}{R_L} + \frac{\beta+1}{r_\pi}\right) = \left(\frac{151}{950}\right)/\left(.000008 + .125 + \frac{151}{950}\right) = 0.56$$

Now combine to get $R_{in} = v_t/i_t$:

$$i_t = \frac{(1-A)}{r_\pi}V_t \rightarrow R_{in} = \frac{r_\pi}{1-A} = \frac{950}{0.44} = 2158\ \Omega$$

+2 points for defining the test current appropriately

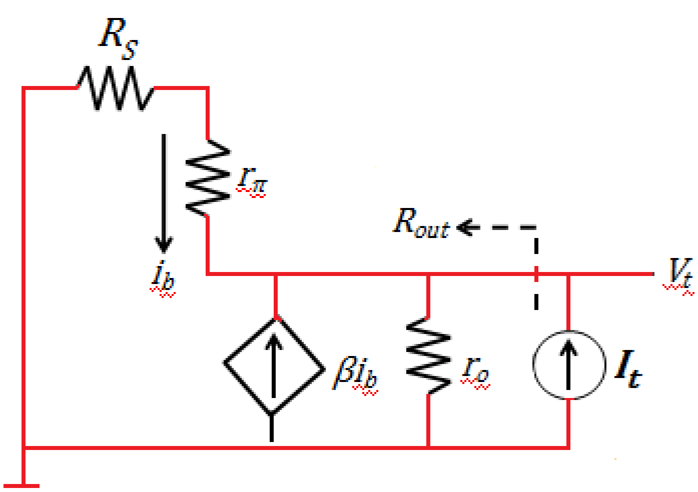
+5 points for recognizing KCL needs to be set up at the output

+10 points for setting up KCL correctly

+5 points for getting the right expression for R_{in}

+2 points for executing the arithmetic correctly

b) Calculate the output resistance and Thévenin voltage seen by the speaker R_L. Hint: Apply a test current source i_{out} at the point marked R_{out}, and find the test voltage v_{out}. Remember to include the source resistance!



All we need is KCL at the output node:

$$I_t + \beta i_b = \frac{V_t}{r_o} + \frac{V_t}{r_\pi + R_S}$$

$$I_t - \frac{\beta V_t}{r_\pi + R_S} = V_t\left(\frac{1}{r_o} + \frac{1}{r_\pi + R_S}\right)$$

$$I_t = V_t\left(\frac{1}{r_o} + \frac{\beta+1}{r_\pi + R_S}\right)$$

$$R_{out} = \frac{V_t}{I_t} = \left(\frac{1}{r_o} + \frac{\beta+1}{r_\pi + R_S}\right)^{-1}$$

Plugging in numbers:

$$R_{out} = \left(\frac{1}{125000} + \frac{151}{1050}\right)^{-1} = 6.95\ \Omega \approx 7\ \Omega$$

+5 points for recognizing KCL is needed at the output node, +10 points for setting up KCL appropriately

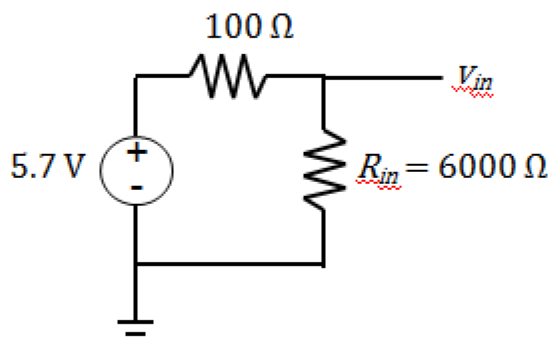
+3 points if they get the sign conventions right on the βi_b term

+4 points if they work through and get the right expression for R_{out}, +2 points for executing the arithmetic correctly

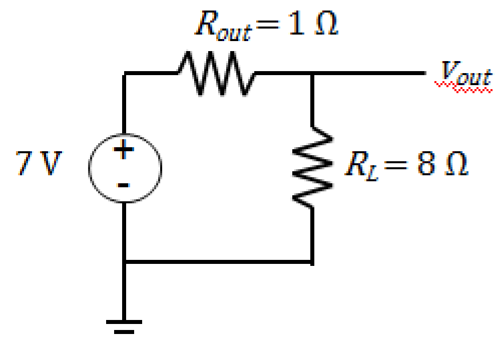
Name_______________________________ SID___________________________________

c) Assuming $V_{TH} = 7\ V, R_{out} = 1\ \Omega, R_{in} = 6\ k\Omega$: find i) the input voltage to the follower and ii) the output voltage with the speaker connected. Draw the input and output equivalent circuits.
Note: The given values are not the answers for parts a) or b), so don't try anything silly with them!

Input



Output



Input voltage to the follower:

$$v_{in} = 5.7V \cdot \left(\frac{6000}{6000 + 100}\right) = 5.6\ V$$

Output voltage to the speaker:

$$v_{out} = 7V \cdot \left(\frac{8}{8 + 5}\right) = 4.3\ V$$

+3 points for setting up the input circuit correctly
+3 points for setting up the output circuit correctly
+3 points for getting the input voltage correct (+2 if correct voltage divider setup, +1 for arithmetic)
+3 points for getting the output voltage correct (+2 if correct voltage divider setup, +1 for arithmetic)