EE 120: Signals and Systems
Department of EECS
University of California Berkeley

QUIZ 2
8 March 2012

LAST Name Ubed FIRST Name Seenk

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 4.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the four numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1	20	20
2(a)	15	15
2(b)	15	15
Total	**55**	55

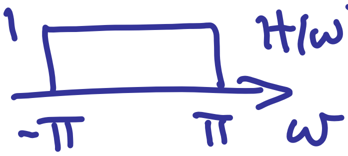
Q2.1 (20 Points) Evaluate the following integral:

$$\int_{-\infty}^{+\infty} \operatorname{sinc}^3 t \, dt$$

where $\operatorname{sinc} t = \dfrac{\sin(\pi t)}{\pi t}$.

Take a look at the bottom of the last page for potentially useful facts and formulas.

$h(t) = \operatorname{sinc} t \overset{\mathcal{F}}{\longleftrightarrow}$



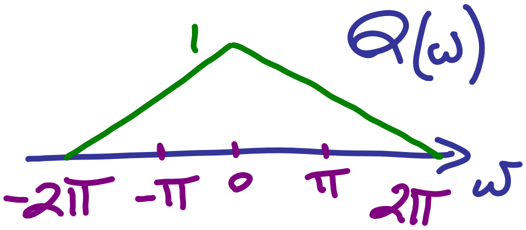
(let $A = \pi$ and $B = 1$ in the transform pair on p. 4)

$$\int_{-\infty}^{\infty} \operatorname{sinc}^3 t \, dt = \int_{-\infty}^{\infty} \operatorname{sinc}^2 t \operatorname{sinc} t \, dt = \int_{-\infty}^{\infty} \underbrace{h^2(t)}_{\text{let } q(t) = h^2(t)} h(t)\, dt$$

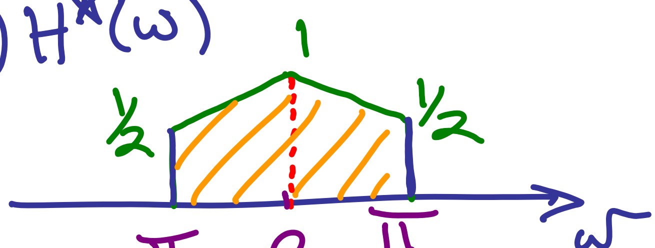
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} Q(\omega) H^*(\omega)\, d\omega$$

Plancherel-Parseval-Rayleigh Identity

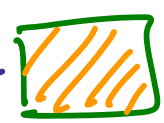
$$q(t) = h^2(t) \overset{\mathcal{F}}{\longleftrightarrow} Q(\omega) = \frac{1}{2\pi}(H * H)(\omega)$$



$Q(\omega) H^*(\omega)$



$$\int_{-\infty}^{\infty} Q(\omega) H^*(\omega)\, d\omega =$$



$$= \underbrace{\left(1 + \tfrac{1}{2}\right)\pi}_{\text{Twice the area of each trapezoid}} = \frac{3\pi}{2}$$

$$\int_{-\infty}^{\infty} \operatorname{sinc}^3 t \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q(\omega) H^*(\omega)\, d\omega = \frac{1}{2\cancel{\pi}} \cdot \frac{3\cancel{\pi}}{2} = \frac{3}{4} \Longrightarrow$$

$$\int_{-\infty}^{\infty} \operatorname{sinc}^3 t \, dt = \frac{3}{4}$$

Q2.2 (30 Points) Consider a continuous-time signal x described by

$$x(t) = \begin{cases} \cos(\omega_0 t) & |t| \leq T \\ 0 & |t| > T, \end{cases}$$

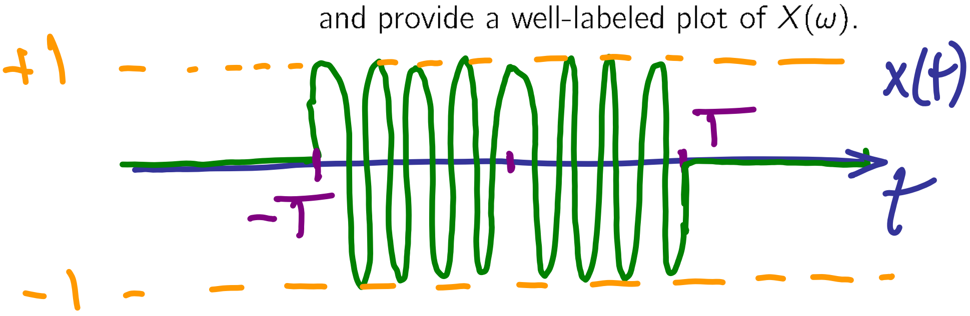
where you may safely assume that T is much greater than $2\pi/\omega_0$, the period of the cosine (i.e., $T \gg 2\pi/\omega_0$).

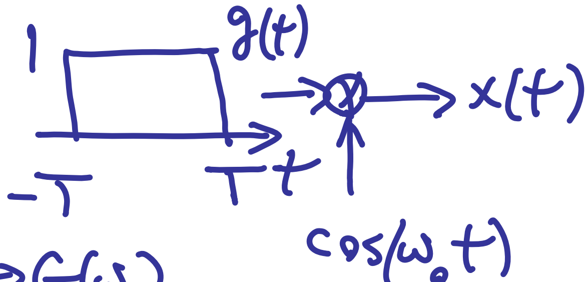
(a) (15 Points) Show that the spectrum of x is given by

$$X(\omega) = \frac{1}{\omega - \omega_0} \sin\Big((\omega - \omega_0)T\Big) + \frac{1}{\omega + \omega_0} \sin\Big((\omega + \omega_0)T\Big) \quad \text{for all } \omega,$$

and provide a well-labeled plot of $X(\omega)$.

We can think of $x(t)$ as follows

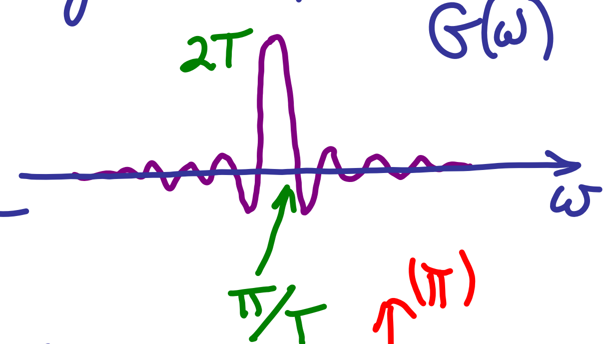




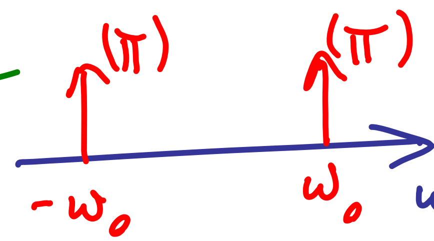
From the transform pair $g(t) \longleftrightarrow G(\omega)$ on p. 4, we know (letting $B=1$):

$$G(\omega) = \frac{2}{\omega}\sin(\omega T)$$

$$G(0) = \lim_{\omega \to 0} \frac{2T\cos(\omega T)}{1} = 2T$$

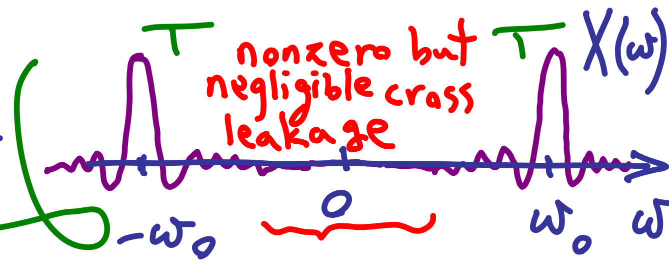


The spectrum of the cosine is

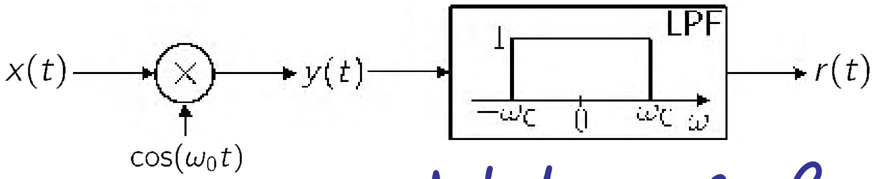


$$X(\omega) = \frac{1}{2}G(\omega - \omega_0) + \frac{1}{2}G(\omega + \omega_0)$$

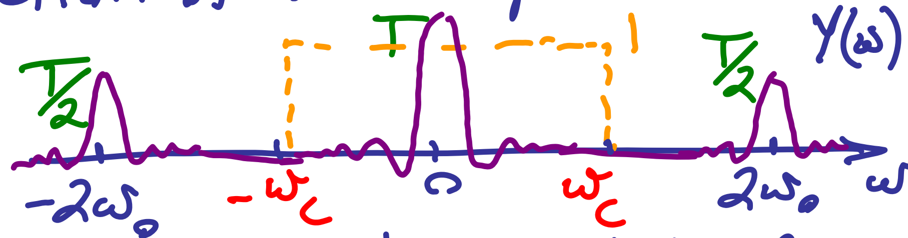
$$X(\omega) = \frac{\sin((\omega - \omega_0)T)}{\omega - \omega_0} + \frac{\sin((\omega + \omega_0)T)}{\omega + \omega_0}$$



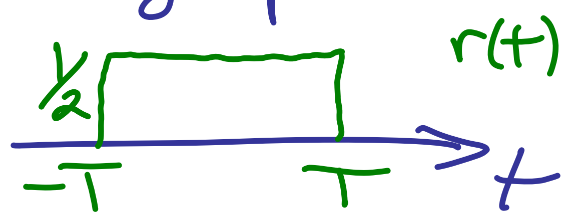
(b) (15 Points) We process the signal x according to the diagram below. Determine, and provide a well-labeled plot of, the signal r. You may assume that ω_c, the cutoff frequency of the low-pass filter, is much larger than π/T and less than, or equal to, ω_0 (i.e., $\pi/T \ll \omega_c \le \omega_0$).



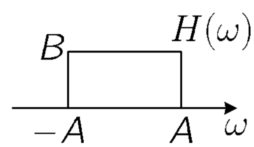
This is simply a demodulation of the square pulse g used in the construction of x. The spectra for $Y(\omega)$ is shown below:



The signal r is a good approximation, but is not exactly equal, to $g/2$. The reason is that the sinc replicas do have nonzero overlaps (g is not bandlimited).



Potentially useful facts:

- Parseval-Plancherel-Rayleigh Identity
$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, Y^*(\omega)\, d\omega\,.$$
-
$$h(t) = \frac{B}{\pi t}\sin(At) \quad \overset{\mathcal{F}}{\longleftrightarrow}$$


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$$g(t) = \begin{cases} B & |t| \le T \\ 0 & |t| > T \end{cases} \quad \overset{\mathcal{F}}{\longleftrightarrow} \quad G(\omega) = \frac{2B}{\omega}\sin(\omega T).$$
- $$\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos(2\alpha).$$

Method 2:

$$y(t) = g(t)\cos^2(\omega_0 t) = \frac{g(t)}{2} + \frac{g(t)}{2}\cos(2\omega_0 t)$$

$\downarrow$

LPF

$\downarrow$

$$r(t) \approx \frac{g(t)}{2}$$

approximate because there is spectral leakage from the sinc centered at $\pm 2\omega_0$, but the leakage is very small given the inequality $\frac{\pi}{T} \ll \omega_c \le \omega_0$