CS 61B: Data Structures (Spring 2012)
Midterm II

If you want to relive taking the midterm, here it is in PostScript or PDF.

Solutions

Problem 1. (6 points) **A Miscellany.**

a. `AException` is a superclass of `BException`.

b. $\Theta(n^2)$.

c. We divide the proof into two separate cases: either $x \geq y$, or $x < y$.

In the first case, we have

$$\log (x + y) \leq \log (2x) = \log 2 + \log x \leq \log x + \log y$$

for all values $y \geq 2$ and $x \geq y$.

The second case, where $x < y$, is symmetric.

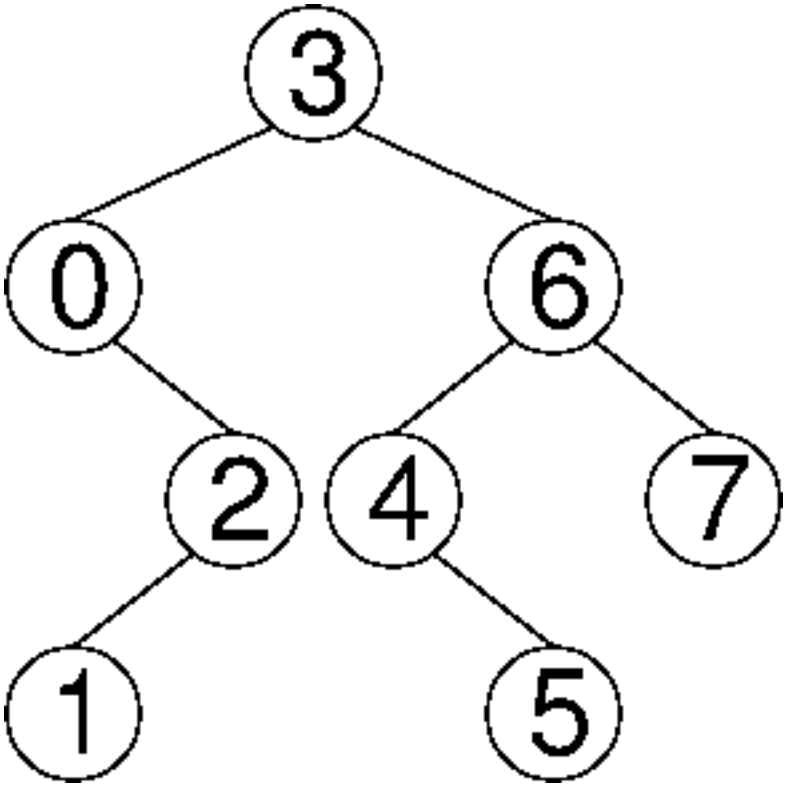
$$\log (x + y) < \log (2y) = \log 2 + \log y \leq \log x + \log y$$

for all values $x \geq 2$ and $y \geq x$.

Therefore, $\log (x + y) \leq \log x + \log y$ for all $x \geq 2$ and $y \geq 2$, so $\log (x + y) \in O(\log x + \log y)$.

Problem 2. (10 points) **Trees.**

a.

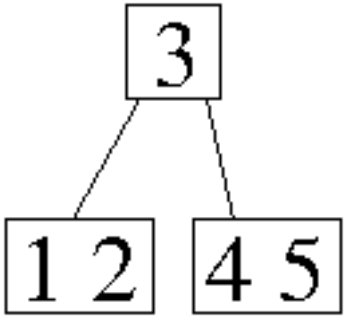


b.

X	6	3	6	4	8	5	1	2
X	6	3	6	2	8	5	1	4
X	6	3	1	2	8	5	6	4
X	6	2	1	3	8	5	6	4
X	1	2	6	3	8	5	6	4
X	1	2	5	3	8	6	6	4

c. A **binary heap** of height *h* contains at least 2^h keys. A **binary search tree** of height *h* contains at least *h* + 1 keys. A **2-3-4 tree** of height *h* contains at least $2^{h+1} - 1$ keys.

d.

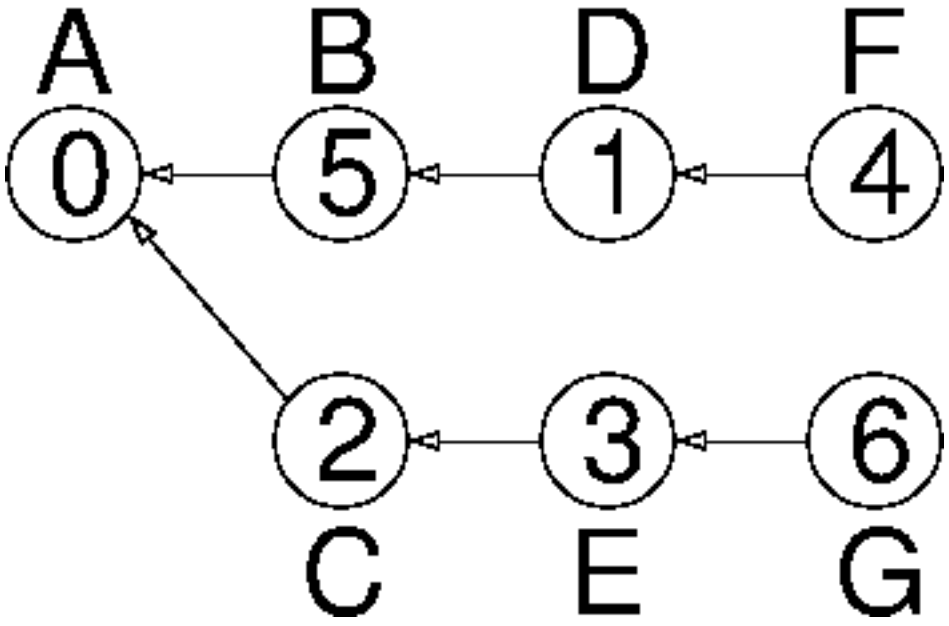


e. Write a loop that iterates through `k`, keeps track of the minimum key so far, and counts the number of keys in `k` (except `k[0]`) that are smaller than every previous key.

```
int min = k[0];
int depth = 0;
for (int i = 1; i < k.length; i++) {
  if (k[i] < min) {
    min = k[i];
    depth++;
  }
}
```

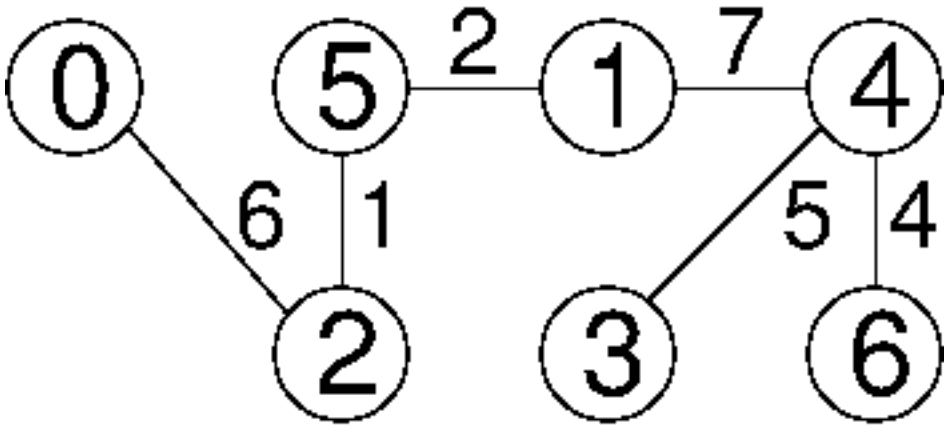
Problem 3. (9 points) **Graphs.**

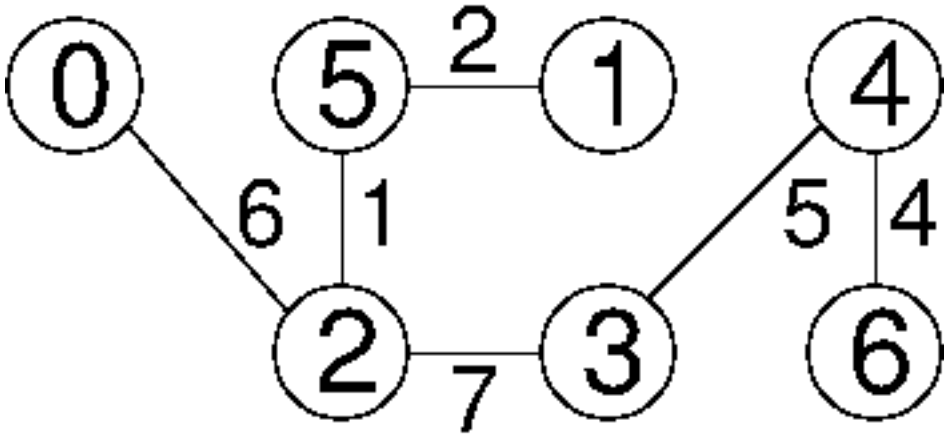
a.



b. (6, 0)

c.





d. Do you think that *T* **must** include the edge of *G* with the least weight? Yes. What about the edge with the second-least weight? Yes. What about the edge with the third-least weight? No.

Kruskal's algorithm must choose the first two edges, because it is impossible that the endpoints of those edges are already connected by a path. However, the third edge is rejected if the first three edges form a triangle.

e. $\Theta(n^2 + e \log e) = \Theta(n^2 + e \log n)$; either expression is correct.

f. Asymptotic notation is meaningful only as the number of edges *e* approaches infinity. In a 100-vertex undirected graph, the number of edges cannot exceed 5,050.

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