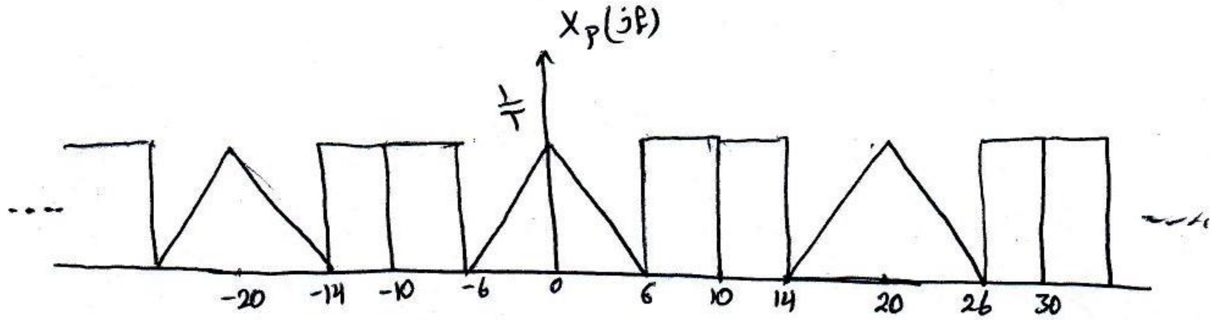
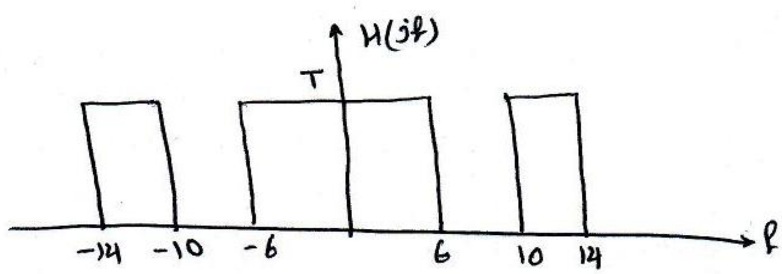
EE120 SIGNALS AND SYSTEMS, Spring 2011
Midterm #2 Solutions

Problem 1:

a) The minimum sampling rate is $f_s = 20^{khz}$. Sampling with this frequency, we obtain $X_p(jf) = \frac{1}{T}\sum_k X(j(f - kf_s))$, where $T = \frac{1}{20}^{ms}$. The following figure illustrates the spectrum of $x_p(t)$

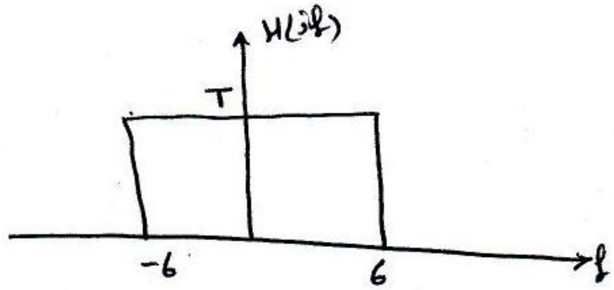


Therefore, combination of a low-pass and a band pass filter is sufficient to reconstruct the signal.

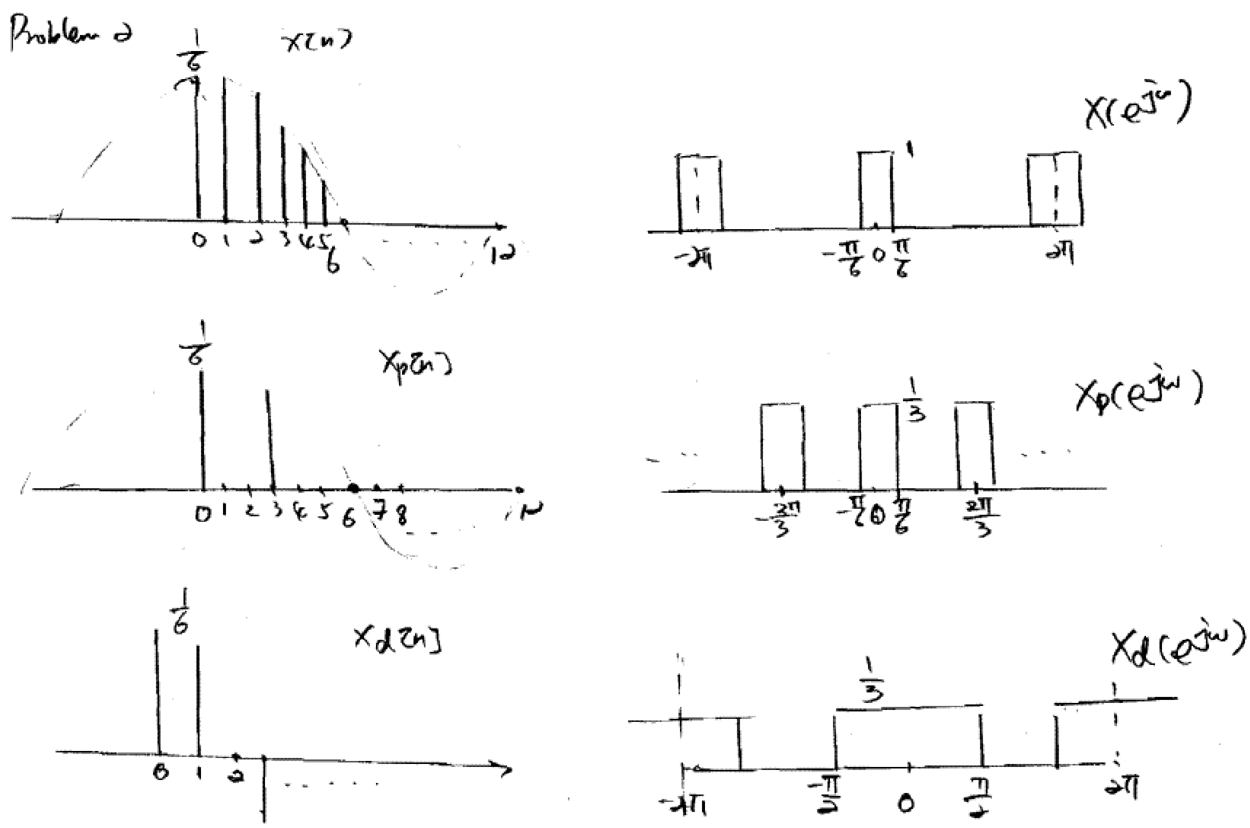


This example shows that Shannon sampling rate is not necessarily the lowest. If you use the Shannon rate ($f_s = 28^{khz}$) with a low pass filter reconstruction (with cut off frequency $f_c = 14^{khz}$) you get up to 5 points.

b) The answer is $f_s = 20^{khz}$ The spectrum of the sampled signal is the same as before. This time $H(jf)$is a low-pass filter with the cut off frequency $f_c = 6^{khz}$ and the gain T



Problem 2:



refer back to textbook. Fig 7.34 & Fig 7.35

Problem 3:

From fact 1, we know that the poles of X(s) will come in complex conjugate pairs and that X(s) will also be even (which means that poles will be symmetric through the origin). Additionally, from fact 2 we know that X(s) will have the form:

$$X(s) = \frac{A}{(s-a)(s-b)(s-c)(s-d)} = \frac{A}{(s-a)(s-a^*)(s+a)(s+a^*)}$$

From fact 3, we know that $a = -1 + j$ resulting in:

$$X(s) = \frac{A}{(s+1-j)(s+1+j)(s-1+j)(s-1-j)}$$

To calculate *A*, we use fact 4 because $\int_{-\infty}^{\infty} x(t)dt = \int_{-\infty}^{\infty} x(t)e^{0t}dt = X(0) = 1$

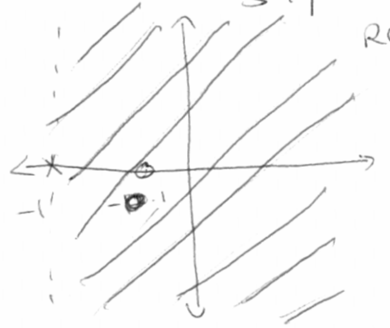
$$X(0) = \frac{A}{(1-j)(1+j)(1+j)(1-j)} = 1 \rightarrow A = 4$$

Since $x(t)$ is real and even, we know that the signal is two sided or finite, which means that the ROC will be a strip in the s-plane. In this case, the strip will be between the poles with real parts at ±1, or $-1 < Re\{s\} < 1$.

$$X(s) = \frac{4}{(s+1-j)(s+1+j)(s-1+j)(s-1-j)}, with\ ROC\ \ -1 < Re\{s\} < 1$$

Problem 4:

$H_1(s) = \frac{s+0.1}{s+1}$



Pole: $s = -1$

Zero: $s = -0.1$

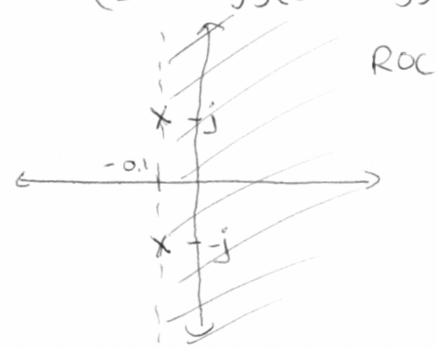
ROC: $Re\{s\} > -1$

stable

Freq. resp: bottom right fig.

$H_2(s) = \frac{1}{s^2 + 0.2s + 1.01}$

$= \frac{1}{(s+0.1+j)(s+0.1-j)}$



Pole: $s = -0.1 \pm j$

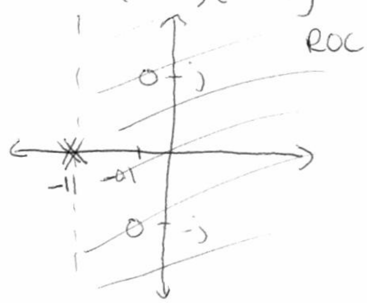
Zero:

ROC. $Re\{s\} > -0.1$

stable

Freq Resp: upper right fig.

$$H_3(s) = \frac{s^2+0.2s+1.01}{s^2+2s+1}$$
$$= \frac{(s+0.1+j)(s+0.1-j)}{(s+1)(s+1)}$$



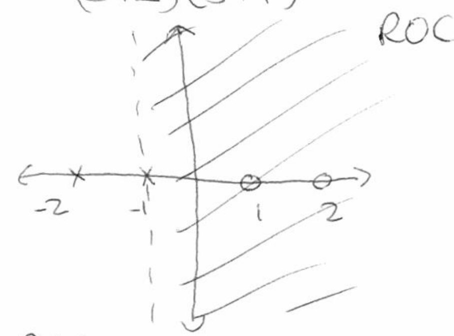
Pole: $s = -1, s = -1$

Zero: $s = -0.1 \pm j$

ROC: $Re\{s\} > -1$ <u>stable</u>

Freq Resp: upper left

$$H_4(s) = \frac{s^2-3s+2}{s^2+3s+2}$$
$$= \frac{(s-2)(s-1)}{(s+2)(s+1)}$$



Poles: $s=-1, s=-2$

Zero: $s=1, s=2$

ROC: $Re\{s\} > -1$ <u>stable</u>

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Problem 5:

$$X(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-z^{-1}\right)} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1-z^{-1}},$$

Solving the equation $A\left(1-z^{-1}\right)+B\left(1-\frac{1}{2}z^{-1}\right)=1$, we obtain $A=-1$ and $B=2$. (8 points)

a) Assuming ROC is $|z|>1$

$$x[n] = -\left(\frac{1}{2}\right)^n u[n] + 2u[n].$$ (6 points)

b) Assuming ROC is $\frac{1}{2}<|z|<1$

$$x[n] = -\left(\frac{1}{2}\right)^n u[n] - 2u[-n-1].$$ (6 points)