University of California, Berkeley – College of Engineering

Department of Electrical Engineering and Computer Sciences

Summer 2010 Instructor: Josh Hug 2010-07-28

Last Name	Key
First Name	Answer
Student ID Number	7,216
Name of the person to your Left	Cooper
Name of the person to your Right	Tony
All the work is my own. I had no prior knowledge of the exam contents nor will I share the contents with others in EE40 who have not taken it yet. I have not cleverly hidden notes and/or computational devices that give me an unfair advantage. **(please sign)**	

Instructions (Read Me!)

- Once time is out at **~2:00 PM**, we'll announce that time is up, and you should stop writing immediately.
- There are 13 pages on the exam. All work should be on this exam. Don't hand in random sheets of paper.
- Please turn off anything that might make noise, unless it is necessary for you to stay alive. Remove all hats and headphones. Allow at least one empty seat between your neighbor and you
- There are no electronic devices allowed on this test, including calculators.
- You may use 2 pages of notes (8.5" x 11" or A4), front and back, handwritten.
- Partial credit will be given for incomplete answers, so please show your work. **The best possible thing you can do if you start running out of time is to write out things like the correct** node voltage equations for **problems you haven't completed**. Overly complex solutions will be receive a minor penalty.
- If you get stuck on an algebra problem, move on! The numerical part of a problem will be worth less than the conceptual part!
- **If you do something by inspection, very briefly explain your answer! Credit will not be given for answers without explanation. It's good enough to say something simple like "voltage divider".**
- If you prefer to solve problems with resistive variables as conductances, feel free to do so on this exam. If you don't know what I just said, don't worry about it.
- The occasional problem has a hint! If you're stuck, look to see if you missed a hint. If there isn't one, sorry, but at least you're all in the same boat.

	P1	P2	P3	P4	P5		Total
Earned							
Max	20	15	37	30	58		160

P1: Basics – More things you now know that you didn't know you knew (20 pts)

Credit will not be given for answers without explanation!

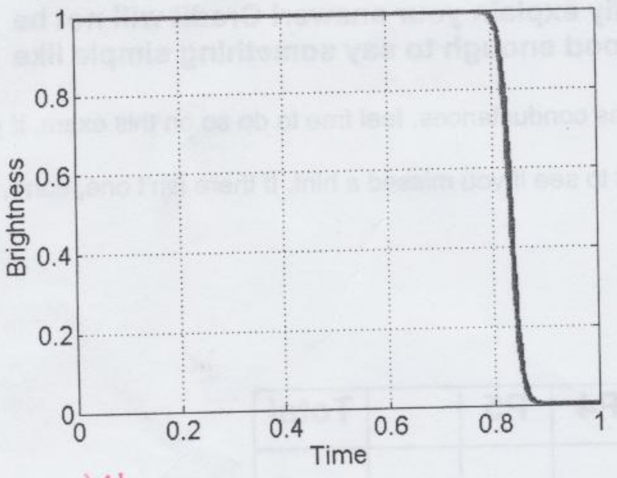
Credit for:
- Mention of current stops if removed ⇒ no energy
- resistance of wire ⇒ energy drops over time

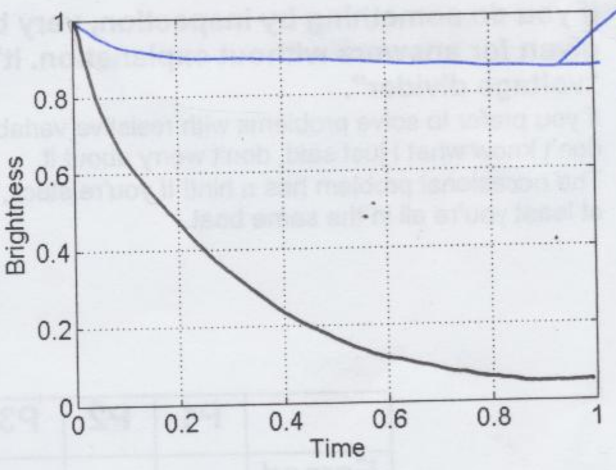
a) (5 pts) Both capacitors and inductors can be used to store energy for use later. Capacitors, though, are far more popular for energy storage than inductors, because they are easier to keep charged. In lab, for example, you can easily connect up a capacitor to a battery, disconnect it from the source, carry it around in your pocket to grab lunch, go back to lab, and then find that it's still holding a charge even after a long time. Explain why this is relatively difficult with inductors.

An inductor must maintain a current in order to hold energy. All real wires (including the inductor) have non-negligible resistance, so energy will be dissipated even with the inductor shorted.

Open circuiting a capacitor, by contrast is effectively an ∞ resistance.

b) (5 pts) If you've ever had the batteries on a flash light run out, you know that as soon as the light starts dimming, your battery will very soon be dead. This is because a battery supplies its maximum voltage until right before it is depleted. On the axis on the left, I have plotted the brightness of a battery powered flashlight over the life of the battery. We can also power a flashlight using a capacitor. Assuming the brightness of a bulb is proportional to the voltage across it, on the right axis, **draw (qualitatively) the brightness vs. time for a capacitor powered flashlight.** Draw so that brightness is 1 at time=0, and has dropped to at least 0.1 by time=1. We just want the shape, not necessarily the exact values.





worth 3/5

All or nothing for Concave up, decaying curve similar to the one drawn.

Any downward sloping line that is concave up the whole way will work.

what an annoying flashlight!

(But if RC is big, no big deal)

c) (5 pts) A student in EE40 is not satisfied with the amount of energy stored in his one-capacitor circuit. He wants to store more energy, so puts 10 capacitors in series and charges them with a single voltage source. Is this a good idea? Why or why not?

+2 for capacitance dropping

+3 for explanation of why this affects energy

Bad plan! $E = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2 + \dots + \frac{1}{2}CV_{10}^2 = \frac{1}{2}C\left(\sum_{i=1}^{10} V_i^2\right)$

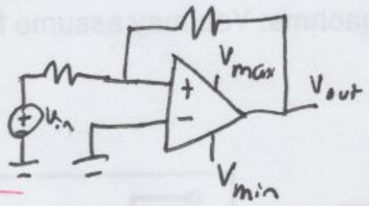
and $V_1 + V_2 + \dots + V_n = V_s \Rightarrow \sum_{i=1}^{n} V_i^2 \le V_s^2$ (If voltage is split up, total energy is less)

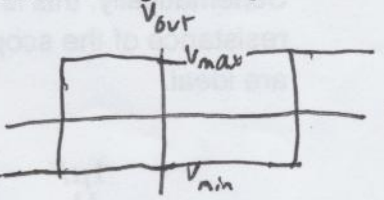
(not true if V_i are different)

Alternately: $C_{eq} = \frac{C}{10}$ so energy is roughly $\frac{1}{10}$th

d) (5 pts) In the lab, we have used op-amps with negative feedback (amplifier), with positive feedback (Schmitt trigger), and with no feedback (comparator). How does a Schmitt trigger differ from a comparator? In other words, how do their functions differ and/or when is it appropriate to use a Schmitt trigger vs. a comparator? [Hint: It may help to draw each configuration]

Schmitt Trigger

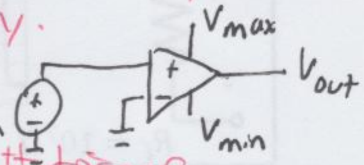


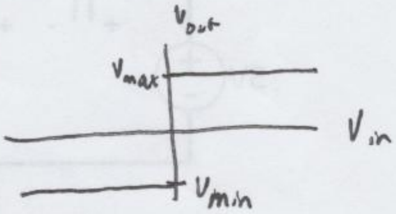


Credit for mentioning:

- Hysteresis vs. instantaneous change in output
- Memory vs. no memory.
- Some explanation of threshold voltages in Schmitt trigger
-

comparator:





Schmitt triggers have memory, holding their values until an input signal threshold is crossed.

In the example above, V_{out} will hold a value of V_{max} as long as $V_{in} > -\frac{R_1}{R_2}V_{max}$

Comparators, by contrast, have an output which is purely a function of the input signal. In the example above, V_{out} would be V_{max} if $V_{in} > 0$.

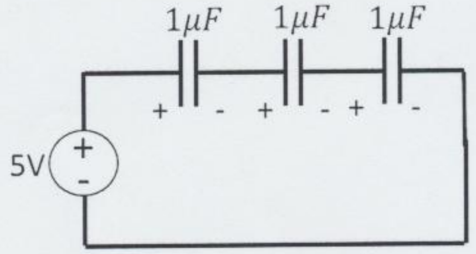
We use Schmitt Triggers when we want memory.

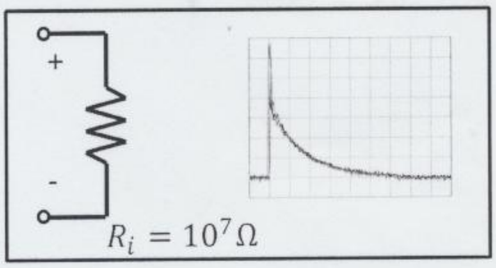
P2: Incapacitation **(15 pts)**

a) I went into lab to see what happens if you connect three $1\mu F$ capacitors in series with a 5V DC voltage source, because I was pretty sure that the charge would not be equal (due to the fact that real capacitors are not ideal).

My experimental procedure was as follows:
1. Set up three capacitors in series
2. Connect the plus side of the DC voltage source to the + terminal of the first capacitor
3. Connect the ground of the DC voltage source to the - terminal of the last capacitor
4. Connect up the oscilloscope probe with the + side on the + side of one of the capacitors, and the – side of the oscilloscope on the – side of the same capacitor. **I never disconnected the DC source, so leave it in for this entire problem!**

Schematically, this is shown below. The total resistance of the scope probe and internal resistance of the scope is 10 megaohms. You may assume for this problem that the capacitors are ideal.

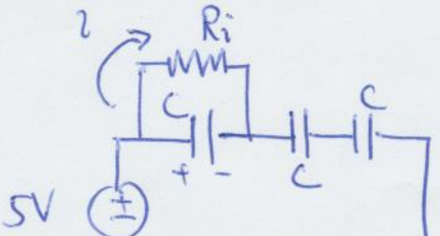




Oscilloscope

When I took my first measurement, I was surprised, because the scope showed me an exponential decay of charge as soon as I connected it to the oscilloscope for measurement (basically as soon as I performed step 4), even with the DC source still connected. I soon realized what was going on.

a. (5 pts) Why was the charge draining out of the measured capacitor?



The charge on the capacitor drains through the internal resistance of the scope.

+5, pretty much all or nothing.

+0 for totally wrong answer

+3, correct answer but incomplete explanation

+1, correct answer but incorrect explanation.

b. (5 pts) I took a measurement of how long it took this drainage to take, but in this problem, you will have to predict what I measured. Predict the approximate time that it took for the measured capacitor to drain to a very low voltage (was it milliseconds? seconds? minutes?).

+4 for correct answer, incomplete quantitative explanation

Takes about 3 time constants for capacitor to drain completely:

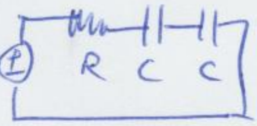
$\tau = RC = (10^7 \Omega)(1 \mu F) = 10 s.$

$3\tau = \underline{30 \text{ seconds}}$

+0 for no explanation.
+1 for correct answer, wrong explanation.
+2 for wrong answer, correct qualitative explanation.
+3 for ~~correct~~ no answer, correct quantitative analysis.
+3 for wrong answer, correct quantitative analysis.

c. (5 pts) One thing that I did not investigate was what happened to the voltages on the other capacitors as the charge was drained from the measured capacitor. However, you should hopefully be able to figure out what this measurement would give you, if you were to take it. If you measure one capacitor with the scope, does the voltage on the other capacitors decrease, stay the same, or increase?

Capacitor becomes open circuit, no longer plays a role:

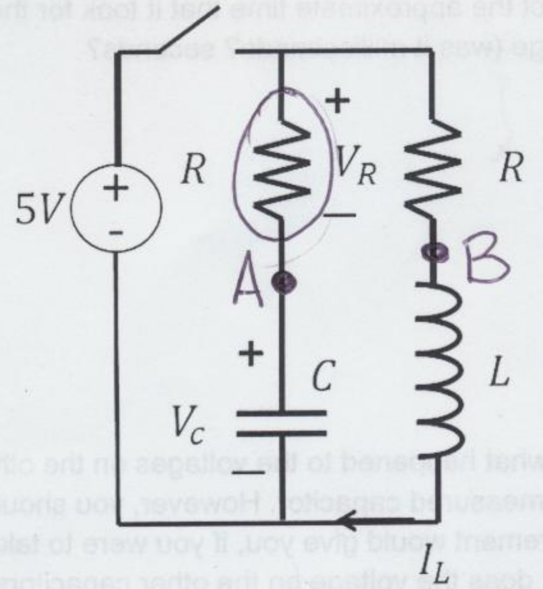


Since we now have fewer capacitors but the total voltage across them is the same, voltage across each individual one <u>increases</u>.

+5 for correct explanation.
+1 for wrong explanation.
+4 for incomplete explanation
+0 for no explanation/wrong answer

(By the way, in case you're curious, the voltage (and therefore charge) on the three capacitors was not evenly distributed! The reason has to do with uneven capacitor leakage, but we haven't discussed this in class, so I didn't ask you guys about this on the midterm. Students of mine from the future reading this exam, take note, you may be the ones who get to answer this harder problem that Summer 2010 didn't get to)

P3: RLRCRLC (37 pts)



For parts a through d: $R = 10\Omega$, $L = 1H$, $C = 1\mu F$
For part e: R, L, and C may be any value

a. (6 pts) In the circuit above, the switch has been open for all of history, and suddenly closes at t=0.What are V_c and I_l for extremely large t, if the switch now stays closed forever (i.e. what is the steady state with the switch closed)? (For this problem you don't need to use the label V_R, it is for a later part of the problem)

As t becomes large, capacitor looks like openckt. No current flows through resistor, hence $\boxed{V_c = V_s = 5V}$ +3pts

As t becomes large, inductor looks like short ckt Then $I_L = I_R = \frac{5}{10} = \boxed{0.5A}$ +3 pts -1.5 for no explanation on each

b. (8 pts) What are $V_c(t)$ and $I_l(t)$ for $t > 0$? Use the intuitive method. This works since if we write the first order ODEs for KCL (for $V_c(t)$, $I_L(t)$) at nodes A & B respectively, we see we get the DEs which we have previously solved with the intuitive method.

$$\boxed{\begin{aligned} V_c(t) &= V_f\left(1 - e^{-t/RC}\right) = 5\left(1 - e^{-t/10^{-5}}\right) \\ I_L(t) &= I_{L,f}\left(1 - e^{-t/(L/R)}\right) = 0.5\left(1 - e^{-t/0.1}\right) \end{aligned}}$$

For each: +2 for time constant, +1 for V_f/I_f, +1 for proper solution
-2 each for not saying at least "using intuitive method". or showing work.

If they use $V_c(t) = 3 + e^{-t/5}$: $\ln(0.16) = -t/5$

$-5\ln(0.16) = t$

c. (6 pts) How long (in seconds) does it take for $V_c(t)$ to reach 3.16 volts? (You may leave your answer in terms of any functions for which you would need a calculator to compute) If you did not get part b, assume

$V_c(t) = 3 + e^{-\frac{t}{5}}$ (Note, this is not the right answer for part b!)

Solve $V_c(t)$ from b) for t s.th. $V_c(t) = 3.16\,V$

$$3.16 = 5(1 - e^{-t/10^{-5}})$$

$$e^{-t/10^{-5}} = -\left(\frac{3.16}{5} - 1\right)$$

$$\frac{-t}{10^{-5}} = \ln\left(1 - \frac{3.16}{5}\right)$$

$$\boxed{t = -10^{-5}\ln\left(1 - \frac{3.16}{5}\right)}$$

+4 for correct idea

+2 for correct solution

d. (14 pts) Now assume the switch opens again at $t = 10^6$ seconds (yes this is long enough for the system to reach steady state). At $t = 10^6$ seconds, what is **the voltage across the resistor $V_R(10^6)$**? What is **the derivative of the current through the inductor $i_L'(10^6)$**?

+7 each

-2 each for no, or wrong units

-4 each for no work

-1 each for small errors

At $t = 10^6 - \varepsilon$, for small $\varepsilon > 0$, $V_R(t) = 0V$. When the switch opens, $-I_L(t)$ must flow through R since I_L cannot change instantaneously. Hence, $V_R(10^6 + \varepsilon) = -I_L R = -(0.5)(10) = \boxed{-5V}$

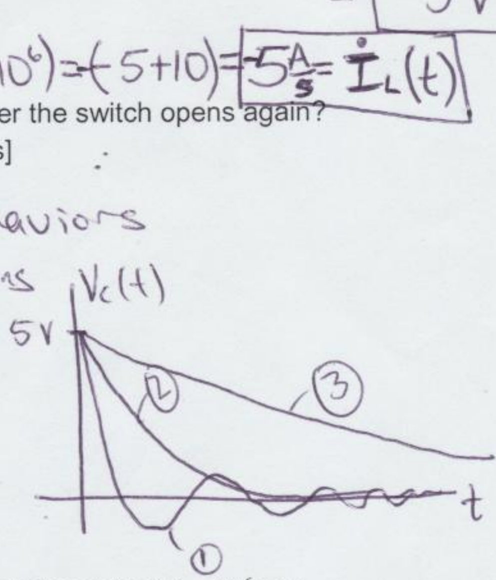
$V_L = L\dot{I}_L(10^6)$

By KVL $V_L(10^6) = +V_C(10^6) + V_R(10^6) + V_R(10^6) = -(5+10) = \boxed{-5\frac{A}{s} = \dot{I}_L(t)}$

e. (3 pts) What is the possible qualitative behavior of $V_c(t)$ for $t > 10^6$, after the switch opens again? [Reminder: for this part, we do not know the values for our components]

There are three possible behaviors

1. underdamped: exp. decay to 0V w/ oscillations
2. critically damped: exp. decay to 0V
3. overdamped: slow exp. decay to 0V



~~+1 for drawing or mentioning each case.~~

To be perfectly clear on part d: Switch is open from $t = -\infty$, closes at $t = 0$ sec, then opens again at $t = 10^6$ sec.

+1 for getting final value of 0

+2 for getting at least one of the shapes.

:) for listing all 3 behaviors.

P4: HPF Design**(30 pts)**

Telephone lines carry both data and voice signals on the same line by using low frequencies for voice, and much higher frequencies for data. As you know, the high and low frequency information can be separated.

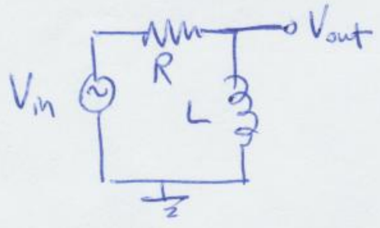
Imagine that Terrible Phone Company Incorporated has given you a faulty DSL modem which is missing the internal high pass filter it's supposed to have. Any time anybody uses the phone, your internet connection goes down because the voice signals interfere with the modem signals.

a. (15 pts) Design a simple high pass filter using a resistor and either a capacitor OR an inductor. Your filter should have a break frequency[1] of $\omega = 10000\ radians/sec.$

Draw your filter below showing:

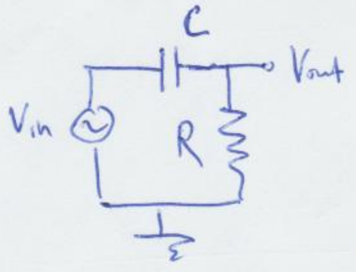
1. A cosine source V_{in} (representing the signal from the phone company)
2. A resistor
3. An inductor OR a capacitor
4. A label V_{out} that shows which component will be connected to the two input terminals of your DSL modem [i.e. you will attach the modem in parallel with this component]

Pick values for your resistor and capacitor/inductor which will give the desired break frequency.



KCL @ V_o: $\frac{V_o - V_{in}}{R} + \frac{V_o}{j\omega L} = 0$ 8

$\frac{V_o}{V_{in}} = \frac{j\omega L/R}{1 + j\omega L/R}$ $R/L = 10000\ rad/sec$



KCL @ V_o: $j\omega C(V_o - V_{in}) + \frac{V_o}{R} = 0$

$\frac{V_o}{V_{in}} = \frac{j\omega RC}{1 + j\omega RC}$ $\frac{1}{RC} = 10000\ rad/sec$

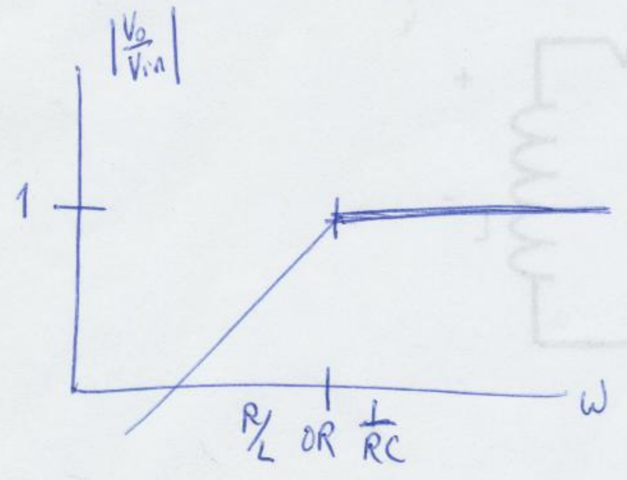
+10 for correct design
+5 for correct R/L/C values
+2 for incorrect attempt
(if got neither design nor values)

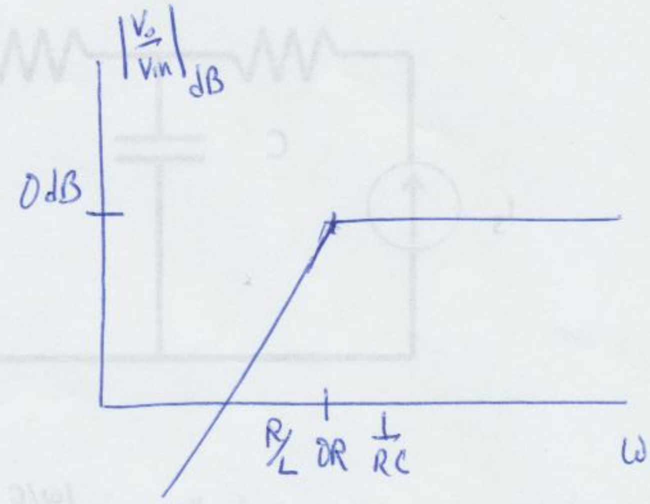
* Higher-order filters were accepted as long as they were correct; otherwise, subject to the same grading here.

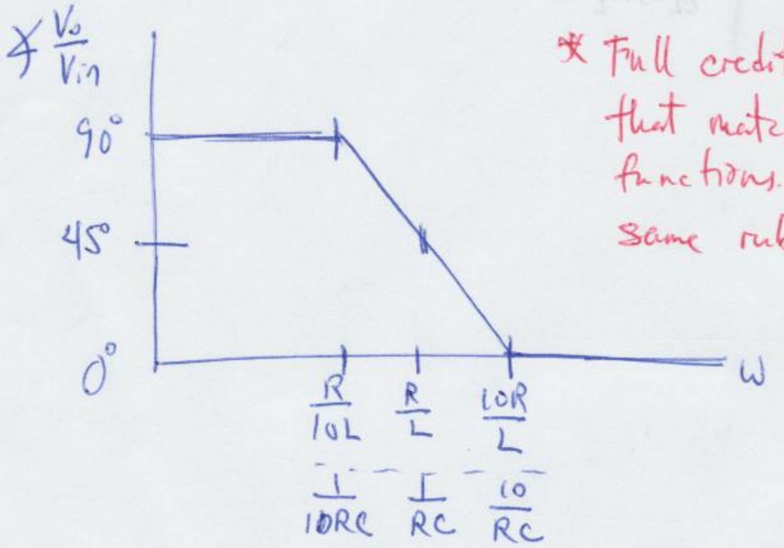
[1]: Note, in case you've forgotten what a break frequency is, you can alternately design your high pass filter so that the magnitude of the transfer function is 1/10 when $\omega = 1{,}000\ rad/sec$

+3/5 for light algebra mistakes

b. (15 pts) Draw the Bode Magnitude and Phase plots for your filter. Make sure to label the break frequency, the values at the break frequency, and all axes. [Full credit for loglog for magnitude and semilogx for phase. Max 10 pts if you plot on a linear scale]







* Full credit given to Bode plots that matched incorrect transfer functions. Otherwise, subject to same rubric below.

Magnitude plots

+3 for correct shape
+1 for labeling ω axis
+1 for labeling $\left|\frac{V_o}{V_{in}}\right|$ axis
+1 for labeling break frequency
+1 for labeling 1 or 0dB.

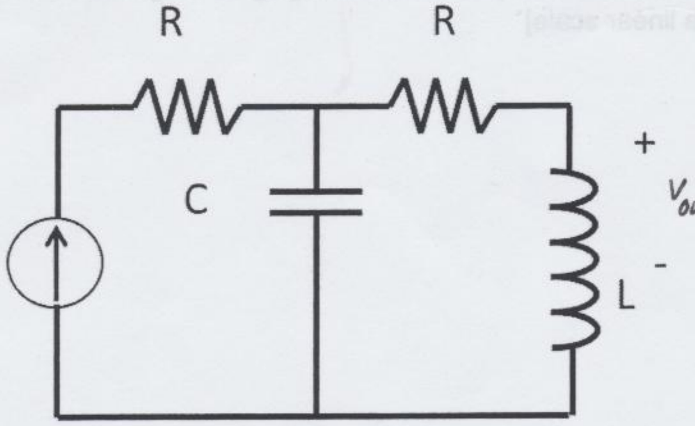
Phase plot

+4 for correct shape ⟹ +2/4 for single arctan ~~arctan~~ error
+ other points, same breakdown as magnitude plot.

P5: Filter Analysis **(58 pts)**

+5 for using impedances
+10 for correct setup

+5 for correct algebra
↓
w/some partial credit if close
No partial credit for algebra ~~that~~ is ~~clearly~~ clearly deliberately wrong in order to reach target

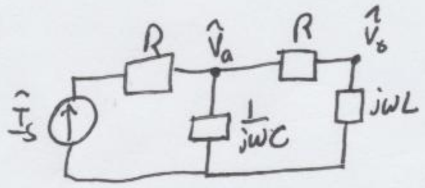


I_s is sinusoidal

a. (20 pts) Show that $H(jw) = \frac{V_{out}}{I_s}$ is $\frac{j\omega/C}{\frac{1}{CL}+j\omega\frac{R}{L}-\omega^2}$

There are many ways to do this problem, once you have swapped out components for impedances. Let's start there:

+ 1/20 - trying to find Z_{eq}



I think the best plan from here is to use node voltage and find $\hat{V}_a$, then use voltage divider to find $\hat{V}_0$.

Node $\hat{v}_a$ $\qquad -\hat{I}_s + \hat{V}_a \cdot jwC + \frac{\hat{V}_a}{R+jwL} = 0$

$$\hat{V}_a\left(jwC + \frac{1}{R+jwL}\right) = \hat{I}_s$$

$$\hat{V}_a = \frac{\hat{I}_s}{jwC + \frac{1}{R+jwL}}$$

And Voltage divider gives us:

$$\hat{V}_0 = \frac{jwL}{R+jwL} \cdot \hat{V}_a$$

$$\hat{V}_0 = \cancel{\frac{jwL}{(jwC + \frac{1}{R+jwL})}} = \frac{jwL\hat{I}_s}{(jwC + \frac{1}{R+jwL})\cdot(R+jwL)} = \frac{jwL\hat{I}_s}{(jwC(R+jwL)+1)}$$

$$= \frac{jwL\hat{I}_s}{jwRC - w^2LC + 1}$$

divide top and bottom by LC and get:

$$\frac{\hat{V}_{out}}{\hat{I}_s} = \boxed{\frac{jw/C}{jw\frac{R}{L} - w^2 + \frac{1}{LC}}}$$

+7/7 if right

b. (7 pts) Is this a band pass, low pass, high pass, or band stop filter? ~~+4~~ +1/7 w/ no explanation

Band Pass.

Transfer function goes to zero for small ω, and goes to 0 for large ω, and in the middle it is some non-zero number

+5/7 - right plot wrong answer

+3/7 - right algebra for asymptotes, wrong answer.

c. (15 pts) If $I_s = 5\cos(10t)$, $R = 10\Omega$, $L = 1H$, and $C = 5mF = \frac{1}{200}F$, what is $V_{out}(t)$ after a long time (in steady state)? Your answer may include arctan if necessary. Your answer must be a function of time.

+5 finding $H(j10)$

$$\hat{I} = 5\angle 0$$

$\cancel{Z_R = 10}$

$\cancel{Z_L = j10}$

$\cancel{Z_C =}$

+5 converting to cosine form

just plug in R, C, L, ω:

$$H(j\cdot 10) = \frac{j\cdot 10\cdot 200}{200 + 10j\cdot 10 - 10^2} = \frac{2000j}{100 + 100j} = \frac{20j}{1+1j}$$

+5 right algebra

3/5 for almost right algebra.

$$|H(j\cdot 10)| = \frac{20}{\sqrt{1^2+1^2}} = \frac{20}{\sqrt{2}}$$

$$\angle H(j\cdot 10) = \angle 20j - \angle(1+1j) = \pi/2 - \pi/4 = \pi/4$$

$\cancel{V_{out}(t) = I}$

$$\hat{V}_{out} = \hat{I}_s \cdot H(j\omega)$$
$$= |\hat{I}_s|\cdot|H(j\omega)| \angle (\angle\hat{I}_s + \angle H(j\omega))$$
$$= 5\cdot\frac{20}{\sqrt{2}}\angle(0 + \pi/4)$$

$$\text{so } V_{out}(t) = \frac{100}{\sqrt{2}}\cos(10t + \pi/4)$$

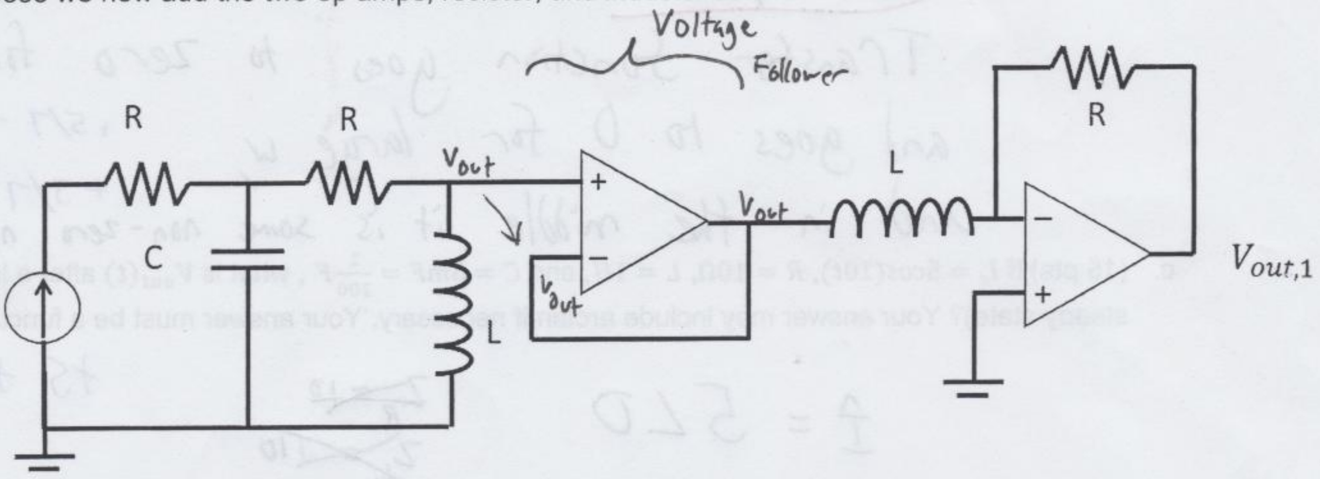
special case: Right answer ~~w~~ without plugging in R, L, C. : 12/15

Leaving everything like $\frac{10/5\times10^{-3}}{\left(\sqrt{\frac{1}{5\times10^{-3}} - 100^2 + 100\cdot 10}\right)}$ etc. : 13/15

Forgetting to multiply by I_s : 14/15

d. The infamous hard problem (do this one last)

Suppose we now add the two op amps, resistor, and inductor as shown below.



i. (3 pts) What is the low frequency asymptote of $V_{out,1}$ [the answer is not zero]? (Reminder of what the low frequency asymptote is: The LFA of the original transfer function (way back in part a), would be $j\omega L$)

$$\hat{V}_{out,1} = -\frac{\hat{V}_{out}}{j\omega L}\cdot R \qquad \text{Thus} \quad H_1(j\omega) = -\frac{H(j\omega)\cdot R}{j\omega L}$$

~~1/3~~ for ~~giving this.~~

$$H_1(j\omega) = -\frac{Rj\omega/C}{\left(\frac{1}{CL} + j\omega\frac{R}{L} - \omega^2\right)\cdot j\omega L}$$

largest order term

Smallest order term

Smallest order term in denominator will dominate for small ω.

$$LFA\left(H_1(j\omega)\right) = -\frac{Rj\omega/c}{j\omega/c} = \boxed{-R} \leftarrow \text{also ok}$$

$$LFA \text{ Of magnitude} = |-R| = \boxed{R}$$

3/3 for R or −R.

1/3 for correct transfer function.

ii. (3 pts) What is the high frequency asymptote of the $V_{out,1}$ [the answer is not zero]?

HFA $(|H_1(j\omega)|)$ – largest order term dominates

$$= -\frac{Rj\omega/c}{-j\omega^3 L} = \frac{R/C}{\omega^2 L} = \boxed{\left|\frac{R}{CL\omega^3}\right|}$$

– any of these are ok, giving 3/3

Again, 1/3 for transfer function being right

– Yes, this means the right transfer function gets 1 point on i. & ii.

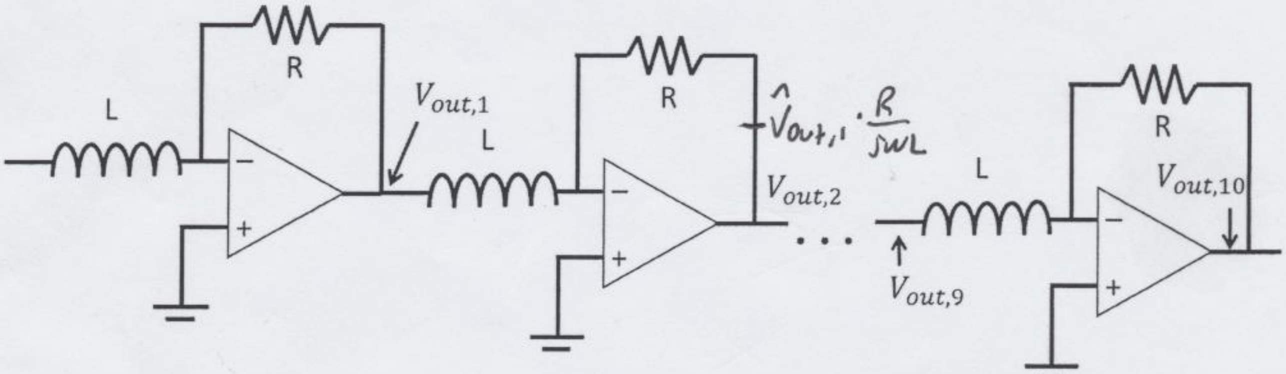
+1 for each if consistent w/answers from i. and ii.

iii. (2 pts) What happens to very low frequency signals? What happens to very high frequency signals?

Low frequency current signal is multiplied by a factor of R (and inverted)

High frequency are attenuated, with attenuation increasing w/ square of frequency.

e. Now suppose that we add 9 more op-amp stages with the same configuration as shown below. The original circuit is still connected to the left, it's just omitted for space reasons.



i. (3 pts) What is the low frequency asymptote of $V_{out,10}$ [the answer is not zero]?

1/3 right transfer function

Each stage multiplies output phasor by a factor of $-\frac{R}{jwL}$.

Therefore after 10 stages, we have:

$$\frac{jw/c}{\left(\frac{1}{CL} + jw\frac{R}{L} - w^2\right)} \cdot \frac{(-1)^{10} \cdot R^{10}}{j^{10} \cdot w^{10} \cdot L^{10}} = \frac{jw/c}{\left(\frac{1}{CL} + jw\frac{R}{L} - w^2\right)} \frac{R^{10}}{-w^{10} L^{10}}$$

3/3 right asymptote

only smallest order term matters

$$LFA: \left| \frac{jw/c \cdot R^{10}}{-\frac{w^{10} L^{10}}{CL}} \right| = \left| \frac{jw \cdot R^{10}}{-w^{10} \cdot L^{9}} \right|$$

$$\boxed{= \frac{R^{10}}{w^{9} \cdot L^{9}}}$$

ii. (3 pts) What is the high frequency asymptote of the $V_{out,10}$ [the answer is not zero]?

$$HFA: \left| \frac{jw/c \cdot R^{10}}{-w^{12} L^{10}} \right| = \boxed{\frac{R^{10}}{C L^{10} w^{11}}}$$

Same as i. (1/3 - Transfer function, 3/3 right asymptote)

iii. (2 pts) What happens to very low frequency signals? What happens to very high frequency signals?

+1 each if consistent with i. and ii.

Very low frequency signals are amplified w/ amplification increasing w/ 9th power of w. (as w decreases)

High frequency signals are attenuated, w/ attenuation decreasing w/ 11th power of w.