EE 40 Final Exam

UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE40
Summer 09

SOLUTIONS

Frank Liao

FINAL EXAMINATION
Time allotted: 110 minutes

NAME: KEY _____ _____ _____
(print) Last First Student ID#

LAB SECTION: _____ / _____
Days / Time

I acknowledge that the UC rules on academic honesty apply. _____
Signature

INSTRUCTIONS:

1. **SHOW YOUR WORK. Partial credit will be given only if your methods are clear to the grader.**
2. **Clearly mark (BOX or UNDERLINE) your answers.**
3. **Specify the units on answers whenever appropriate. Points will be deducted for missing units.**
4. **Closed book, closed notes. You are allowed THREE 8.5" x 11" sheet of notes. Calculators are allowed.**

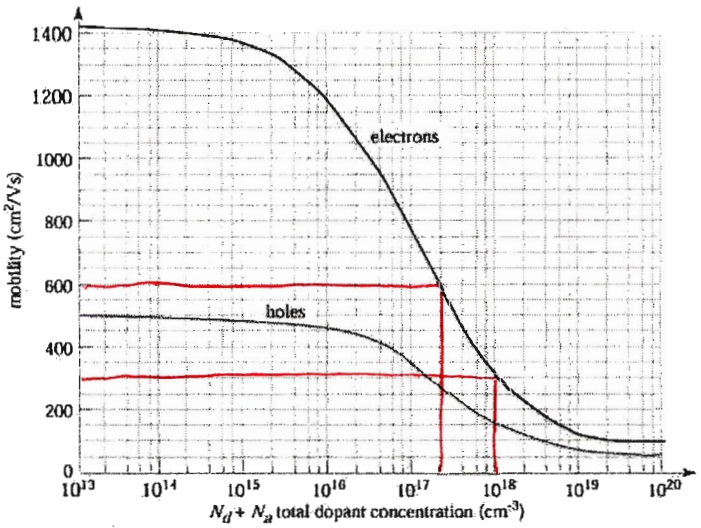
PHYSICAL CONSTANTS

Description	Symbol	Value
Electronic charge	q	1.6×10^{-19} C
Boltzmann's constant	k	8.62×10^{-5} eV/K
Thermal voltage at 300K	$V_T = kT/q$	0.026 V

PROPERTIES OF SILICON AT 300K

Description	Symbol	Value
Intrinsic carrier concentration	n_i	10^{10} cm^{-3}
Dielectric permittivity	ε_{Si}	1.0×10^{-12} F/cm

Electron and Hole Mobilities in Silicon at 300K



SCORE:	**1** _____	**/ 8**
	2 _____	**/ 28**
	3 _____	**/ 40**
	4 _____	**/ 34**
	5 _____	**/ 28**
	6 _____	**/ 12**

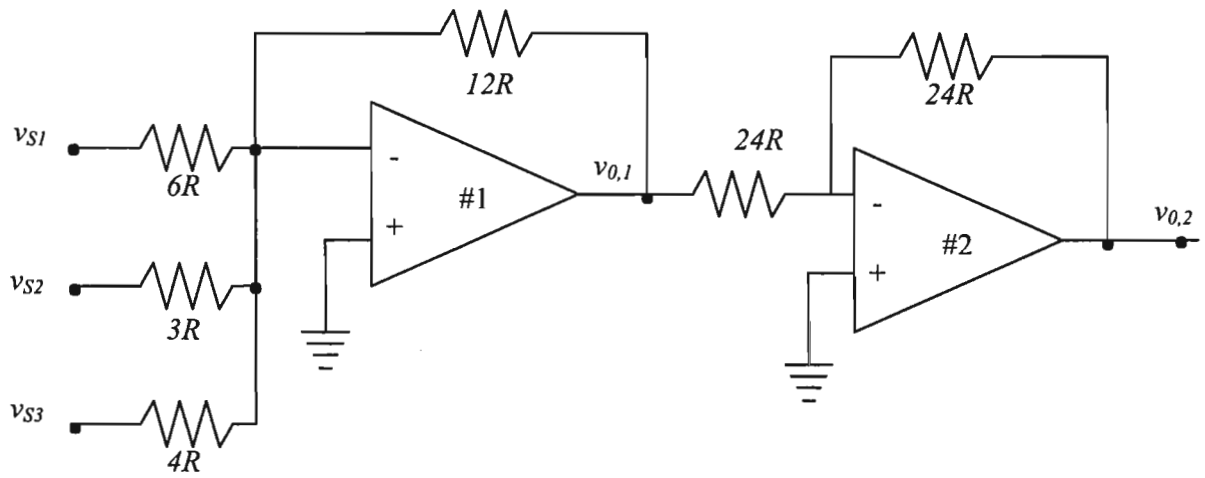
TOTAL: _____ **/ 150**

SOLUTIONS

EE 40 Final Exam

Problem 1 [8 points]: Multiple Choice Questions
Select only ONE choice. No credit will be given to multiple answers.

a) What function does the following circuit implement? **[4 pts]**



a) $v_{o,2} = 12v_{s1} + 6v_{s2} + 8v_{s3}$

b) $v_{o,2} = 2v_{s1} + 4v_{s2} + 3v_{s3}$

c) $v_{o,2} = -\frac{1}{2}v_{s1} - \frac{1}{4}v_{s2} - \frac{1}{3}v_{s3}$

d) $v_{o,2} = \frac{1}{2}v_{s1} + \frac{1}{4}v_{s2} + \frac{1}{3}v_{s3}$

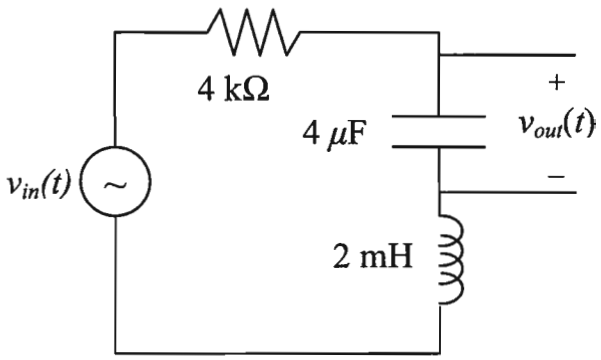
2) Consider the circuit below on the left:
a) What is the resonant frequency of this circuit? **[2 pts]**

a) 62.5 rad/s

b) 11.2×10^3 rad/s

c) 2×10^6 rad/s

d) 125×10^6 rad/s



b) What is the natural response of this circuit? **[2 pts]**

a) Underdamped

b) Critically Damped

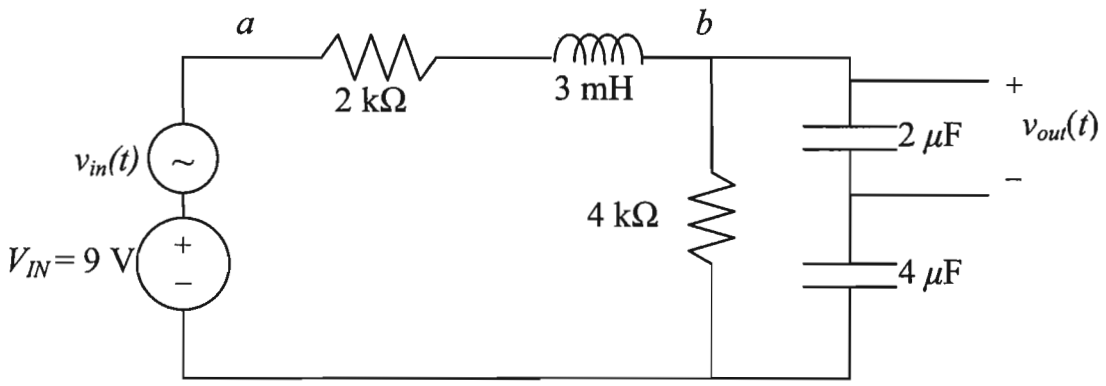
c) Overdamped

SOLUTIONS

EE 40 Final Exam

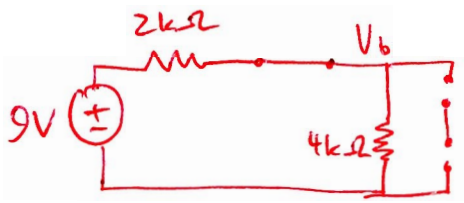
Problem 2 [28 points]: Phasor Analysis

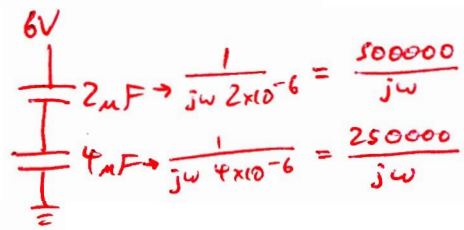
Consider the circuit below:



a.) Suppose $v_{in}(t) = 0$. What is v_{out} after a long time? **[10 pts]**

2 pts for realizing inductor is a short
2 pts for realizing capacitor is open





2 pts for capacitive divider

$$V_b = \frac{4k\Omega}{2k\Omega+4k\Omega} V_{in} = \frac{4}{6} V_{in} = \frac{2}{3}\cdot 9 = 6V$$

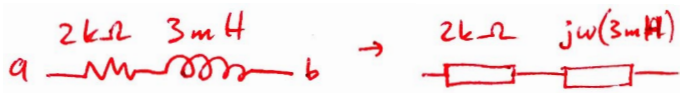
voltage divider:

$$\frac{V_{out}}{V_b} = \frac{\frac{500000}{j\omega}}{\frac{500000}{j\omega} + \frac{250000}{j\omega}} = \frac{50}{50+25} = \frac{2}{3}$$

2 pts for voltage divider

$$V_b \cdot \frac{V_{out}}{V_b} = 6\cdot\frac{2}{3} = 4V$$

b.) If the angular frequency, ω = 100 rad/s, what is the equivalent impedance, $\mathbf{Z_{eq}}$, between nodes *a* and *b*? You may express your answer as a complex number or phasor. **[2 pts]**



$j\omega L = j(100)(3\times10^{-3}) = j0.3$

$$Z_{eq} = 2000 + j0.3$$

$$\approx 2000 \angle 0.0086^\circ$$

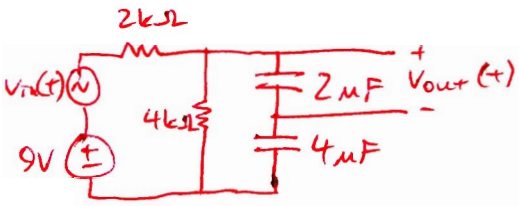
SOLUTIONS

EE 40 Final Exam

c.) *To simplify the math in this part, assume that* $\boldsymbol{Z_{eq}}$ *= 2 kΩ, (with no imaginary part). This is the* $\boldsymbol{Z_{eq}}$ *between nodes a and b (from part (b).*

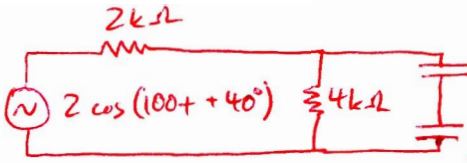
If $v_{in}(t) = 2\cos(100\,t + 40°)$ and $V_{IN} = 9$ V, what is $v_{out}(t)$ in cosine form? (Note: for partial credit, find $v_{out}(t)$ (in cosine form) if $V_{IN} = 0$ V (instead of 9 V). Please make it clear to the grader if you do this.) **[16 pts]**

4 pts for using superposition



Superposition:

solved in part A: $V_{out} = 4$V



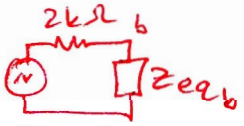
$\frac{1}{j\omega(2\mu F)} = \frac{-j}{100 \cdot 2\times10^{-6}} = \frac{-j10000}{2} = -j5000$

$\frac{1}{j\omega(4\mu F)} = \frac{-j}{100 \cdot 4\times10^{-6}} = \frac{-j10000}{4} = -j2500$

2 pts for getting right Z_{eq} of caps

6 pts for right KCL expression or 2 pts for using right Z_{eq} & 4 pts for voltage divider

$$Z_{eq_b} = \frac{4000 \cdot -j7500}{4000 - j7500}$$



$$\frac{V_b}{V_{in}} = \frac{Z_{eq_b}}{2000 + Z_{eq_b}} = \frac{\frac{-j30\times10^6}{4000 - j7500}}{\frac{8\times10^6 - j15\times10^6 - j30\times10^6}{4000 - j7500}}$$

$$2000 + Z_{eq_b} = \frac{(8\times10^6 - j15\times10^6) - j30\times10^6}{4000 - j7500} = \frac{-j30\times10^6}{8\times10^6 - j45\times10^6}$$

$$\frac{V_b}{V_{in}} = \frac{-j30}{8 - j45}$$

from part A: $\frac{V_{out}}{V_b} = \frac{2}{3}$

$$\rightarrow \frac{V_{out}}{V_{in}} = \frac{-j20}{8 - j45} = \frac{20\angle -90°}{45.7\angle -79.9°} = 0.438\angle -10.1°$$

+2 pts for capacitive divider

$$2\angle 40° \times 0.438\angle -10.1° = 0.876\angle 29.9° = 0.876\cos(100t + 29.9°)$$

perform superposition:

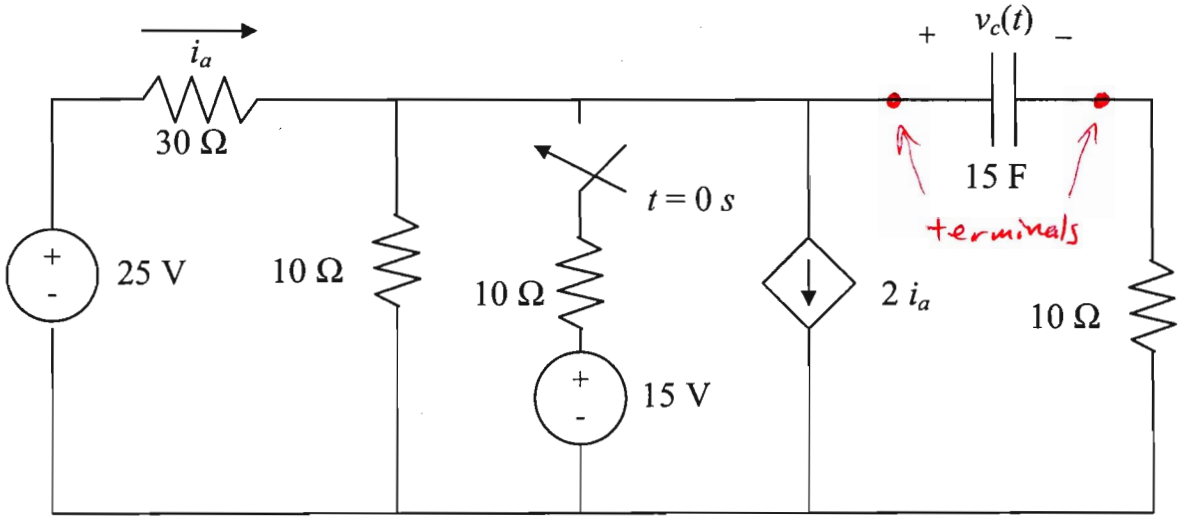
+2 pts for cosine form and correct phasor math

Total $\boxed{v_{out}(t) = 4 + 0.876\cos(100t + 29.9°)\ \text{V}}$

SOLUTIONS

EE 40 Final Exam

Problem 3 [40 points]: First-Order Transients
Consider the circuit below. The switch has been open for a long time and then it is closed at 0 s.

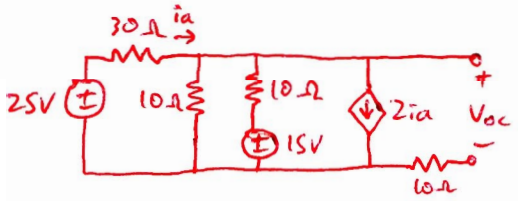


a.) What is the time constant, τ, of the circuit for $t \geq 0$ s? *(Note: because of the dependent source, you cannot use the equivalent resistance method of finding* R_{eq}*.)* There is more space for work on the following page. **[16 pts]**

This can be done by either finding V_{oc} and I_{sc} at the terminals, or zeroing all independent sources and applying a test voltage. This is the first option.

2 pts for deactivating sources
10 pts for right KCL
2 pts for treating dependent source correctly

for $t \geq 0$:



$$i_a = \frac{25 - V_{oc}}{30}$$

$$-i_a + \frac{V_{oc}}{10} + \frac{V_{oc} - 15}{10} + 2i_a = 0$$

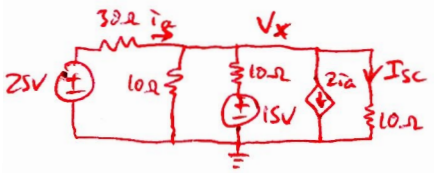
$$\frac{V_{oc} - 25}{30} + \frac{V_{oc}}{10} + \frac{V_{oc} - 15}{10} + \frac{25 - V_{oc}}{15} = 0$$

$$V_{oc} - 25 + 3V_{oc} + 3V_{oc} - 45 + 50 - 2V_{oc} = 0$$

$$5V_{oc} - 20 = 0$$

$$V_{oc} = \frac{20}{5} = 4\ V$$

10 pts for right KCL
2 pts for treating dependent source correctly
2 pts for right R_{eq}



$$i_a = \frac{25 - V_x}{30} \qquad I_{sc} = \frac{V_x}{10}$$

$$-i_a + \frac{V_x}{10} + \frac{V_x - 15}{10} + 2i_a + \frac{V_x}{10} = 0$$

$$\frac{V_x - 25}{30} + \frac{V_x}{10} + \frac{V_x - 15}{10} + \frac{50 - 2V_x}{30} + \frac{V_x}{10} = 0$$

$$V_x - 25 + 3V_x + 3V_x - 45 + 50 - 2V_x + 3V_x = 0$$

$$8V_x = 20$$

$$V_x = \frac{20}{8} = \frac{5}{2} \qquad I_{sc} = \frac{V_x}{10} = \frac{1}{4}$$

2 pts for right R_{eq}

$$R_{eq} = \frac{V_{oc}}{I_{sc}} = 16\ \Omega$$

$$\tau = R_{eq} C = 16 \cdot 15 = \boxed{240\ sec}$$

SOLUTIONS

EE 40 Final Exam

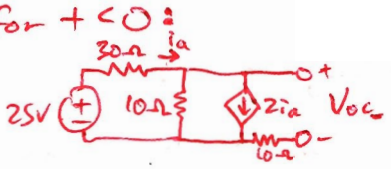
More space for part (a). Be sure to clearly indicate your final answer:

b.) Find the expression for $v_c(t)$ (piecewise expressions are acceptable) for all values of t. Write your final in the box on the next page. **[15 pts]**

for $t \geq 0$, $\tau = 240$ sec., final voltage $= V_{oc} = 4V$

$V(0^-) = V(0^+)$ due to presence of capacitor

for $t < 0$:



V_{oc-} = initial capacitor voltage

$i_a = \frac{25 - V_{oc-}}{30}$ $\quad -i_a + \frac{V_{oc-}}{10} + 2i_a = 0$

$\frac{V_{oc-}}{10} = -i_a$

$\frac{V_{oc-}}{10} = \frac{V_{oc-} - 25}{30}$

$3V_{oc-} = V_{oc-} - 25$

$V_{oc-} = \frac{-25}{2} = -12.5\,V = V(0^-) = V(0^+)$

$v(t \geq 0) = v(\infty) + (v(0) - v(\infty))\, e^{-t/\tau}$

$= 4 + \left(\frac{-25}{2} - 4\right) e^{-t/240}$

$v(t \geq 0) = 4 + -16.5\, e^{-t/240}$

$v(t < 0) = -12.5V$

40 pts for correct expression
(1 pt for $t<0$
3 pts for $t \geq 0$)

initial value:
3 pts for right KCL
1 pt for treating dependent source correctly
1 pt for correct value

for V_{oc}:
4 pts for right KCL
1 pt for treating dependent source correctly
1 pt for right V_{oc}
(note: the work may have been done in part (a))

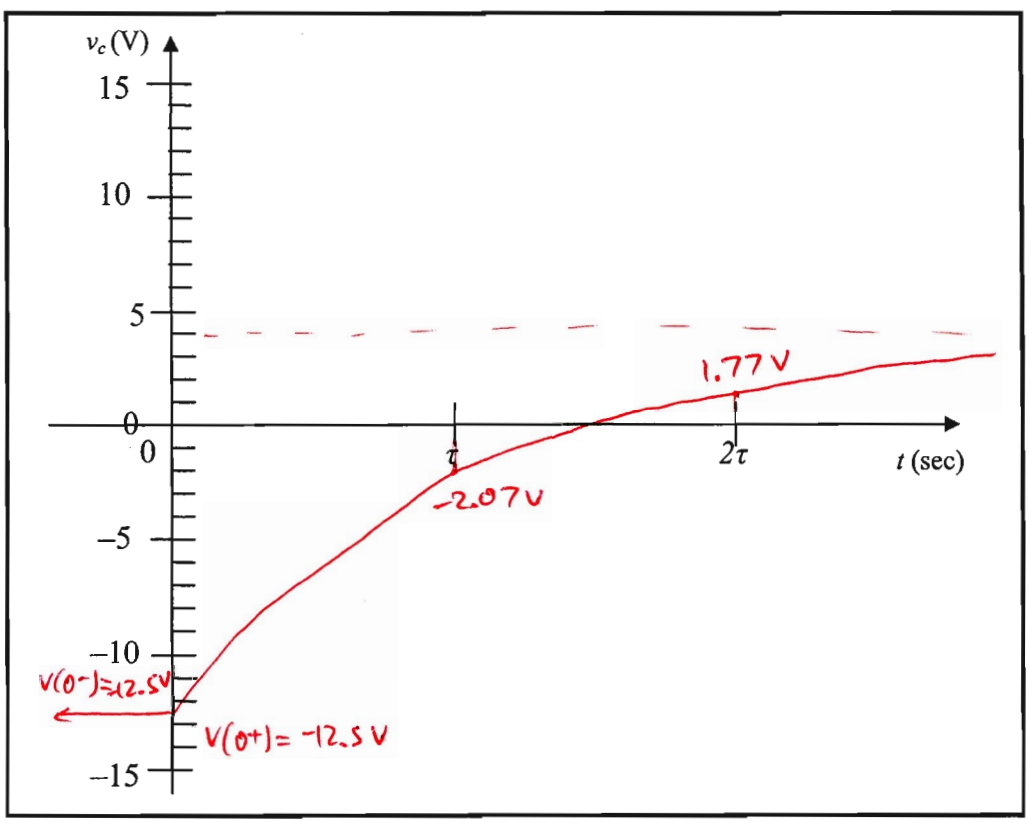
SOLUTIONS

EE 40 Final Exam

c.) Plot $v_c(t)$ for $t \geq 0$ s. Label the values of $v_c(0^-)$, $v_c(0^+)$, $v_c(t = \tau)$ and $v_c(t = 2\tau)$; be consistent with the given axes. **[9 pts]**

Answer:

Expression for $v_c(t)$: $v(t) = \begin{cases} -12.5\,V & t < 0 \\ 4 - 16.5\,e^{-t/240}\ V & t \geq 0 \end{cases}$



2 pts for right shape

2 pts for $v_c(0^-)$

3 pts for equating $v_c(0^-)$ to $v_c(0^+)$

2 pts for $v_c(\tau)$

SOLUTIONS

EE 40 Final Exam

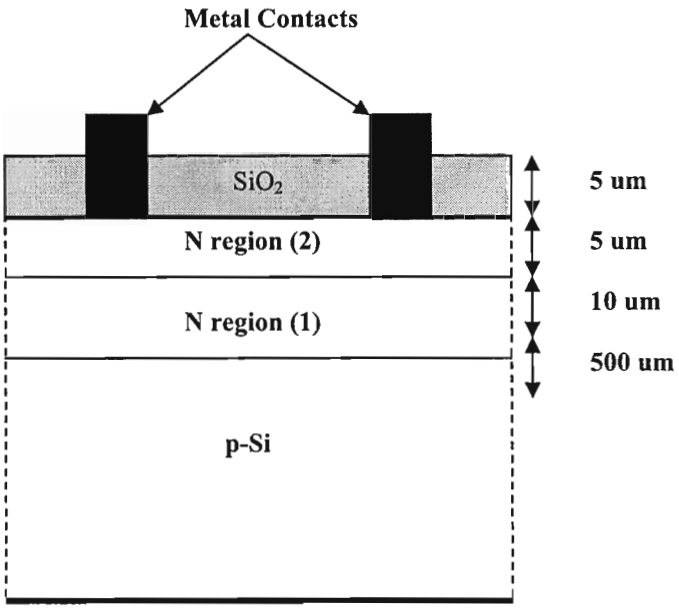
Problem 4 [36 points]: Doping and Resistivity (You too can play with semiconductors!)

Suppose you decide to have fun with some fabrication equipment you purchased on eBay and set up in your garage.

a) You purchase some p-type wafers with a background concentration of 10^{15} cm^{-3}. On top of the p background, you use an ion gun to dope a region with a donor concentration of $2\text{x}10^{17}$ cm^{-3} and a depth of 15 μm. Using the ion gun is so fun, that you dope again on top of the new n region with a donor concentration of $8\text{x}10^{17}$ cm^{-3} and a depth of 5 μm. What is this doping step usually called in a typical fabrication process? (It's two words.) **[2 pts]**

ION IMPLANTATION

Next, silicon oxide is deposited on top of the substrate and then selectively etched to create openings for the metal contacts. After that, metal contacts are then deposited. Your final cross section is shown here:



(Drawing not to scale)

You would like to use this structure as a resistor.

b) Find the majority and minority carrier concentration for all three regions. **[8 pts]**

Write your answers here:

p-type region

majority carrier concentration: 10^{15} cm^{-3} ← 1 pt

minority carrier concentration: 10^5 cm^{-3} ← 2 pt.

10^{15} cm$^{-3} = N_a$ = holes ← majority carrier concentration

$n = \frac{n_i^2}{N_a} = \frac{10^{20}}{10^{15}}$ ← electrons ← minority carrier concentration

n-type region (1)

majority carrier concentration: $2\text{x}10^{17}$ cm^{-3} ← 1 pt

minority carrier concentration: 500 cm^{-3} ← 1 pt

$N_d = 2\text{x}10^{17}$ ← electrons ← majority carrier concentration

$p = \frac{n_i^2}{2\text{x}10^{17}} = \frac{10^{20}}{2\text{x}10^{17}}$ ← holes ← minority carrier concentration

n-type region (2)

majority carrier concentration: 10^{18} cm^{-3} ← 2 pt.

minority carrier concentration: 10^2 cm^{-3} ← 1 pt.

$n = N_{d_1} + N_{d_2} = 10^{18}$ ← electrons ← majority carrier concentration

$p = \frac{n_i^2}{n} = \frac{10^{20}}{10^{18}}$ ← holes ← minority carrier concentration

SOLUTIONS μ is from chart where $N_a+N_d = 2\times10^{17}\ cm^{-3}$

c) Find the sheet resistance for n region (1). **[5 pts]**

$$\rho = \frac{1}{(1.6\times10^{-19}\ C)(2\times10^{17}\ cm^{-3})(600\ cm^2/V\cdot s)} = 0.052\ \Omega\text{-cm}$$

$$R_s = \frac{\rho}{t} = \frac{0.052\ \Omega\text{-cm}}{1\times10^{-4}\ cm} = 52\ \Omega/\square$$

2 pts for resistivity
1 pt for right mobility
2 pts for sheet resistance

d) Find the sheet resistance of n region (2). **[4 pts]**

From MOBILITY CHART w/ $N_a+N_d = 1\times10^{18}$

$$\rho = \frac{1}{q n \mu_n} = \frac{1}{(1.6\times10^{-19}\ C)(1\times10^{18}\ cm^{-3})(385\ cm^2/V\cdot s)} = 0.029\ \Omega\text{-cm}$$

(300)

$$R_s = \frac{\rho}{t} = \frac{0.021\ \Omega\text{-cm}}{5\times10^{-4}\ cm} = 38\ \Omega/\square$$

1 pt for resistivity
2 pts for mobility
1 pt for sheet resistance

e) What is the total sheet resistance? **[2 pts]**

THE REGIONS ARE IN PARALLEL!

$$R_s = R_{s1} // R_{s2} = 23\ \Omega/\square$$

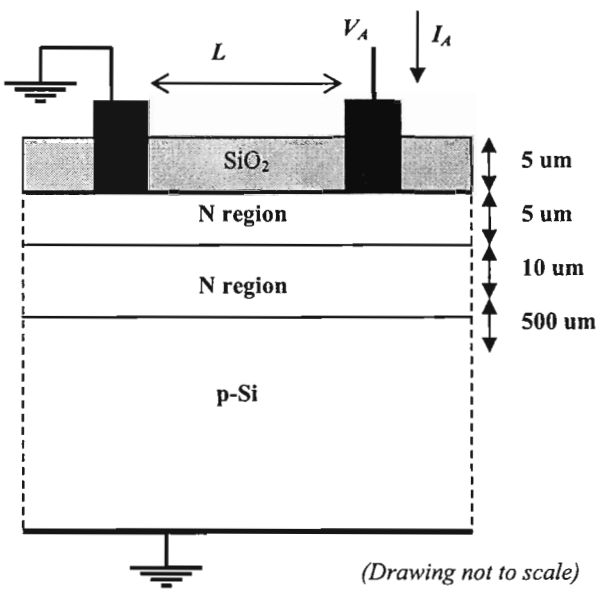
For the rest of the problem, assume your total sheet resistance from part (e), R_s, is equal to 5 Ω/square.

To test your resistor, you apply a bias, V_A, between the two metal contacts and according to common practice, you also connect the p-type substrate to ground. See the figure below. (Note: $T = 300$ K.)

f) If the length, L, between the contacts is 50 μm and the width, W, (going into and out of the paper) of this entire region is 10 μm, what is the current I_A for the following values of V_AV? (Follow the sign convention given in the figure.) **[3 pts]**

$$R = R_s\left(\frac{L}{W}\right) = 5\ \Omega/\square \cdot \left(\frac{50\ \mu m}{10\ \mu m}\right) = 25\ \Omega$$

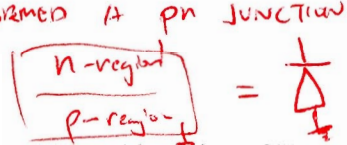
V_A	I_A
0.1 V	4 mA
0.5 V	20 mA
–0.5 V	–20 mA



(Drawing not to scale)

j) You notice at a negative bias of –1 V, you measure large amounts of negative current, I_A. Please explain what is happening. Be as specific as possible. **[6 pts]**

WE HAVE FORMED A pn JUNCTION.



WHEN $V_A < -0.7$ V the pn junction turns on, causing large amounts of current in forward bias. (this corresponds to negative current in our picture.)

+3 pts for mentioning pn junction
+3 for forward bias

k) You also notice for large positive values of V_A, you measure large amounts of positive current, I_A. Please explain what is happening. Be as specific as possible. **[4 pts]**

FOR LARGE POSITIVE VALUES OF V_A; THE PN JUNCTION IS IN REVERSE BIAS, THE LARGE CURRENT COMES FROM REVERSE BREAKDOWN, MOST LIKELY AVALANCHE.

+1 pt for mentioning pn junction
+1 for reverse bias
+2 for breakdown

SOLUTIONS

EE 40 Final Exam

Problem 4 [28 points]: MOSFET Behavior

In two tables below, label the region of operation and the find the drain current based on the information given. Ignore channel-length modulation. Definitions: (1) If $V_{GS}=V_T$, then the MOSFET is off. (2) If $V_{DS}=V_{DSsat}$, then the FET is in velocity saturation.

For NMOS Transistor:

$\mu_n C_{ox}$ = 100 μA/V^2; C_{ox}=0.345 μF/cm^2, v_{sat} = 8 x10^6 cm/s, $\lambda = 0$

V_T (V)	V_{DSsat} (V) (velocity saturation)	W/L (μm/μm)	V_S (V)	V_D (V)	V_G (V)	Region of Operation	I_D (A)
0.5	1	20/0.25	0	1	1	Velocity Sat.	2.76 mA
0.5	1	20/0.25	0.3	1	1	Saturation	160 μA
0.3	0.5	10/0.1	0.3	0.4	1	linear	350 μA
0.3	0.5	10/0.1	0.2	1.5	1.5	Velocity Sat.	2.76 mA

since $V_{GS} = 1V = V_{DSsat}$

$V_{DS} = 1.3\ V > V_{DSsat}$

Space for work: (Recommended for partial credit) (More space available on the next page.)

VELOCITY SAT.: $I_D = W C_{ox} (V_{GS} - V_T) v_{sat}$

SATURATION: $I_D = \frac{W}{2L} \mu_n C_{ox} (V_{GS} - V_T)^2$

LINEAR: $I_D = \frac{W}{L} \mu_n C_{ox} \left(V_{GS} - V_T - \frac{V_{DS}}{2}\right) V_{DS}$

For PMOS Transistor:

$\mu_p C_{ox}$ = 50 μA/V^2; C_{ox}=0.345 μF/cm^2, v_{sat} = 6 x10^6 cm/s, $\lambda = 0$

$V_{GS} = 2V$! (NOT -2 V)

V_T (V)	V_{DSsat} (V) (velocity saturation)	W/L (μm/μm)	V_S (V)	V_D (V)	V_G (V)	Region of Operation	I_D (A)
–0.75	–1	20/0.25	0	1	2	OFF	0 A
–0.75	–1	20/0.25	1	0.5	0	SATURATION	–125 μA
–0.3	–0.5	10/0.1	1.25	1	0.5	LINEAR	–406 μA
–0.3	–0.5	10/0.1	2	1	0	VELOCITY SATURATION	–3.52 mA

Space for work: (Recommended for partial credit) (More space available on the next page.)

VELOCITY SATURATION: $I_D = W C_{ox} (V_{GS} - V_T) v_{sat}$

SATURATION: $I_D = -\frac{W}{2L} \mu_n C_{ox} (|V_{GS} - V_T|)^2$

LINEAR: $I_D = -\frac{W}{L} \mu_n C_{ox} \left(|V_{GS} - V_T - \frac{V_{DS}}{2}|\right)(|V_{DS}|)$

SOLUTIONS

Additional work space: (Recommended for partial credit)
Please make your methods clear to the grader if you use this space.

Problem 5 [12 points]: EE Technology

T	F	Moore's Law describes the phenomenon where the number of transistors on a computer chip doubles every 1.5 to 2 years.
T	F	Any linear circuit can be reduced to an equivalent circuit comprising a current source with a parallel resistance.
T	F	A good voltage source should have a small series resistance.
T	F	A good current source should have a small parallel resistance.
T	F	SPICE is capable of solving voltages and currents in an electric circuit, but it cannot generate Bode plots.
T	F	Decreasing the equivalent resistance, R_{eq}, of a transistor in a logic gate will decrease the time delay, τ, of that logic gate, but will increase its dynamic power consumption. (Assume the clock frequency, f, doesn't change.)

SOLUTIONS

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(If you use this page for extra work and want partial credit, please make that very clear to the grader.)