UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE 105: Microelectronic Devices and Circuits — Fall 2011

MIDTERM EXAMINATION #1

Time allotted: 60 minutes

Solution

NAME: ______________________ ______________________

STUDENT ID#: __________________

INSTRUCTIONS:

1. **Unless otherwise stated, assume**
   a. **temperature is 300 K**
   b. **material is Si**
   c. **No Early effect**
2. **SHOW YOUR WORK. (Make your methods clear to the grader!)**
   Specially, while using chart, make sure that you indicate how you have got your numbers. For example, if reading off mobility, clearly write down what doping density that corresponds to.
3. **Clearly mark (underline or box) your answers.**
4. **Specify the units on answers whenever appropriate.**

SCORE: 1	________	**/ 20**
2	________	**/ 20**
3	________	**/ 20**
Total	________	**/ 60**

PHYSICAL CONSTANTS

Description	Symbol	Value
Electronic charge	q	1.6×10^{-19} C
Boltzmann's constant	k	8.62×10^{-5} eV/K
Thermal voltage at 300K	$V_T = kT/q$	0.026 V

PROPERTIES OF SILICON AT 300K

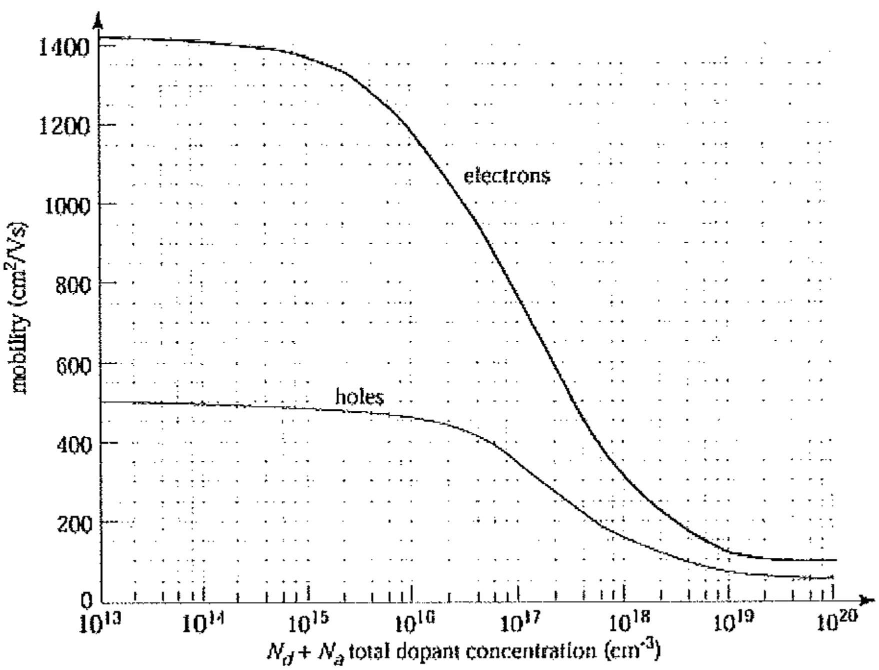
Description	Symbol	Value
Band gap energy	E_G	1.12 eV
Intrinsic carrier concentration	n_i	10^{10} cm^{-3}
Dielectric permittivity	ε_{Si}	1.0×10^{-12} F/cm

USEFUL NUMBERS

$V_T \ln(10) = \mathbf{0.060}$ **V** at T=300K

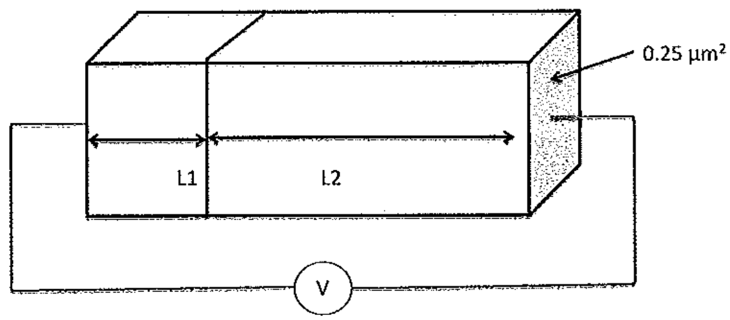
Depletion region Width: $W = \sqrt{\frac{2\varepsilon}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)\left(V_{bi}-V_{Applied}\right)}$

<u>**Electron and Hole Mobilities in Silicon at 300K**</u>



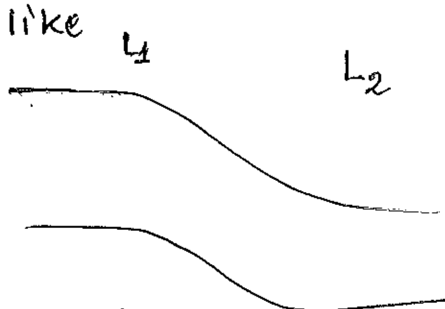
Prob 1. [20]

(a) [10 pts] Consider a Si slab as shown below. The slab has been doped with two different concentrations of acceptor ions 10^{18} /cm^3 and 10^{16} /cm^3 respectively in region L1 and L2 as shown in the figure. L1=10 μm and L2=30 μm. If a voltage of 5V is applied across the sample as shown in the figure, approximately calculate the current flowing in the sample. The cross sectional area of the sample is 0.25 μm^2.



Two regions have two different hole density. Therefore diffusion will happen in equilibrium and a junction will form.

L_1 region loses a lot more holes; so energy band diagram looks like



$$\therefore \boxed{I \simeq I_s e^{\frac{V_D}{V_T}} = 4.68 \times 10^{-19} e^{5/0.026} \text{ A}}$$

$$I_s = A q n_i^2 \left[\frac{D_p}{L_p N_{L_1}} + \frac{D_p}{L_p N_{L_2}} \right]$$

$$\simeq A q n_i^2 \frac{D_p}{L_p} \frac{1}{N_{L_2}}$$

Full credit if you are here

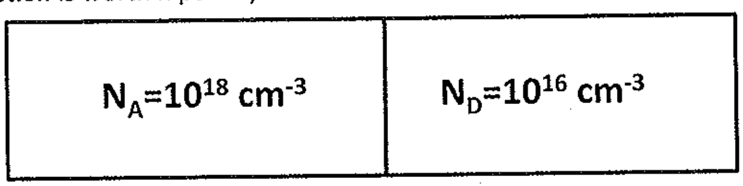
$$D_p = \frac{KT}{q} \mu_p = 0.026 \times \underbrace{450}_{} = 11.7 \text{ cm}^2/\text{sec}$$

from plot using $N_A + N_D \approx 10^{16}$

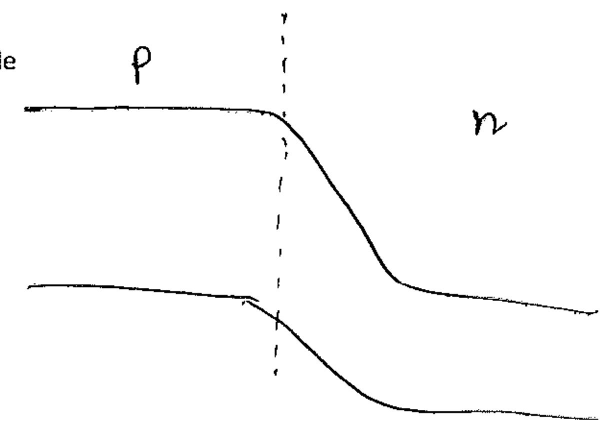
Assume $L_p \ll L_1$ & L_2; Take $L_p \sim 1$ μm (see hw2, prob 3)

$$\therefore I_s = 0.25 \times 10^{-8} \times 1.6 \times 10^{-19} \times 10^{20} \times \frac{11.7}{10^{-4}} \cdot \frac{1}{10^{16}} = 4.68 \times 10^{-19} \text{ A}$$

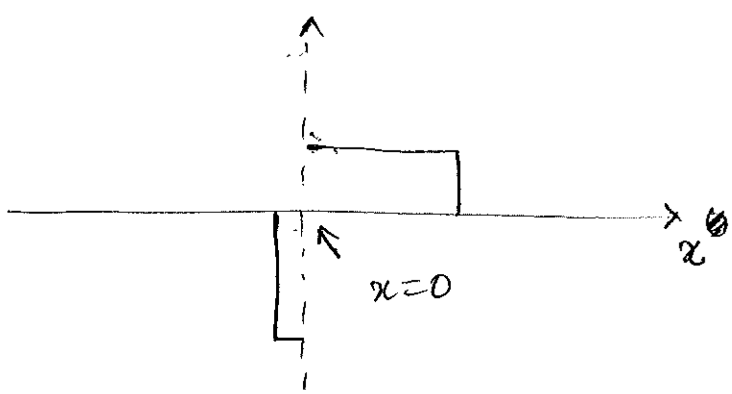
(b) [10 pts] (each question is worth 2 points) Consider a diode shown below:



(i) Draw the energy band profile



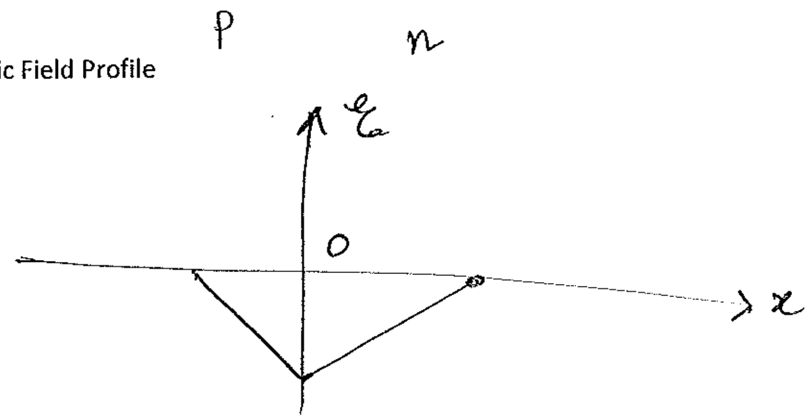
(ii) Draw the profile of the depletion region. You can make an abrupt depletion region approximation.



$$W \propto \sqrt{\frac{1}{N}}$$

so higher doped region has smaller depletion width.

(iii) Draw the Electric Field Profile



(iV) Calculate the built in voltage.

$$V_{bi} = \frac{KT}{q} \ln \frac{N_A N_D}{n_i^2}$$

$$= \frac{KT}{q} \ln \frac{10^{34}}{10^{20}} = 14 \times 60 \text{ mV}$$

$$= 840 \text{ mV}$$

(V) Calculate the depletion width

$$W = \sqrt{\frac{2\varepsilon_{si}}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right) V_{bi}}$$

$$\cong \sqrt{\frac{2 \times 10^{-12}}{1.6 \times 10^{-19}} \frac{1}{N_D} \times 0.84}$$

$$\cong \sqrt{\frac{2 \times 10^{-12}}{1.6 \times 10^{-19}} \times \frac{1}{10^{16}} \times 0.84}$$

$$\cong \sqrt{0.84 \times 10^{-9}} \text{ cm}$$

$$\boxed{W = 2.89 \times 10^{-5} \text{ cm}}$$

Prob 2. [20 pts]

(a) [7 pts] Consider two diodes:

D1: $N_A=10^{18}$ and $N_D = 10^{16}$ /cm^3 and

D2: $N_D=10^{18}$ and $N_A = 10^{16}$ /cm^3

If both diodes are forward biased with the same amount of voltage, do you expect a difference in the current for the two diodes? Why or Why not? Clearly justify your answer.

For D1: current is dominated by holes

D2: current is dominated by electrons

since for the same density

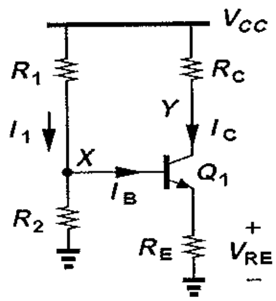
$\mu_n > \mu_p$

$\boxed{I_{D_2} > I_{D_1}}$

(b)[3 pts] Please use one of the three options (i) increases (ii) decreases or (iii) no change to fill out the following table. When considering one parameter change, assume everything else is kept fixed.

Parameter	Early Voltage	β
Increase width of the base	decreases	decreases
Increase emitter Doping	no change	increases
Increase collector doping	decreases	no change

(c) [10 pts]



For the circuit shown above, it is known that V_{cc} =5V, β=100, I_s =10^{-14} A and the desired I_c= 1 mA. Choose the values of R_c, R_E, R_1 and R_2 that makes sure that the transistor is in forward active region. Clearly write down all the approximations that you are making.

$I_c \cong I_s e^{V_{BE}/V_T}$

$\therefore V_{BE} = V_T \ln \frac{I_c}{I_s}$

$= \frac{kT}{q} \ln \frac{10^{-3}}{10^{-14}}$

$= 60\,mV \times 11$

$= 660\,mV$

$\boxed{\text{Assume } R_E = R_c = 1K}$

$\therefore V_{RE} \cong 1V$

$V_{CE} = 5V - 1V - 1V$

$= 3V$

$\therefore \boxed{V_{CE} > V_{BE} ;\ \text{forward active}}$

$V_X = V_{BE} + V_{RE}$

$= 1{\cdot}66\,V$

$\boxed{\text{Assume } I_1 \cong 100 I_B}$

$\therefore I_1 = 100 \times \frac{I_C}{\beta} = 10^{-3} A$

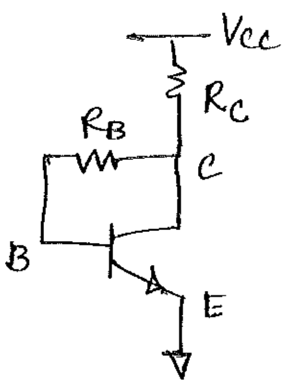
$\therefore \boxed{R_2 = \frac{1{\cdot}67V}{10^{-3}A} = 1{\cdot}67\,K\Omega}$

$\boxed{R_1 = \frac{(5 - 1{\cdot}67)V}{10^{-3}\,A} = 3{\cdot}33\,K\Omega}$

Prob 3 [20 pts]

(a) [10 pts]

(i) [3 pts] Explain how a self-biased circuit helps improve the tolerance to variation in β.



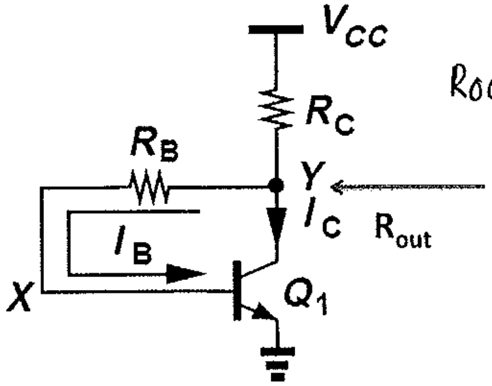
$$V_{BE} = V_{CC} - I_C R_C - I_B R_B$$

$$= V_{CC} - I_C R_C - \frac{I_C}{\beta} R_B$$

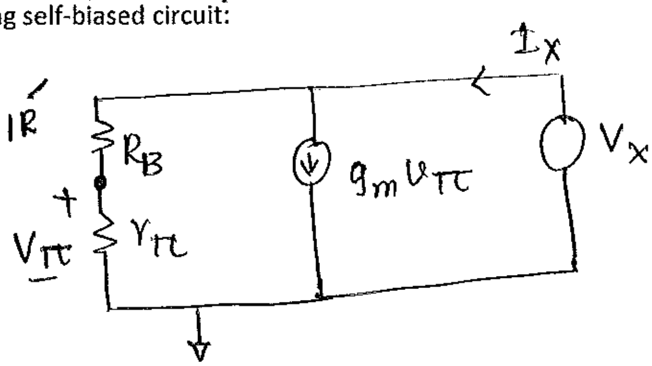
$$\boxed{\beta \uparrow \quad I_C \uparrow \quad V_{BE} \downarrow \quad I_C \downarrow}$$

Therefore, if I_C changes due to change in β, V_{BE} changes to change I_C back. Thus effect of β variation is minimized.

(ii) [7 pts] Find out the output resistance of the following self-biased circuit:



$R_{out} = R_C \| R'$



$$R' = \frac{V_X}{I_X} \quad - ①$$

$$I_X = g_m V_\pi + \frac{V_\pi}{r_\pi} = \frac{V_\pi}{r_\pi}(1+\beta) \quad - ②$$

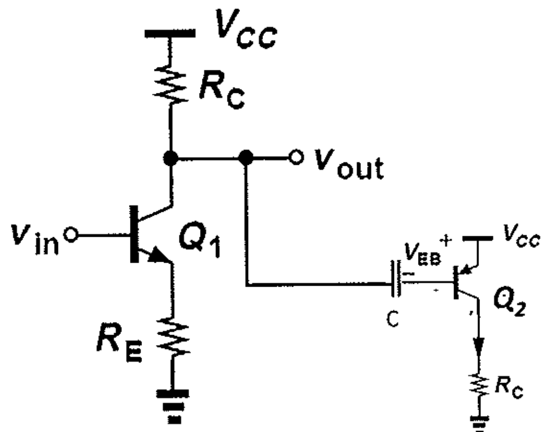
$$V_\pi = \frac{r_\pi}{r_\pi + R_B} V_X \quad - ③$$

From ② & ③, $I_X = \frac{\beta+1}{r_\pi} \cdot \frac{r_\pi}{r_\pi + R_B} V_X$

$$\therefore V_X / I_X = R' = (r_\pi + R_B)/(\beta+1)$$

$$\therefore \boxed{R_{out} = \left(R_C \,\|\, \frac{r_\pi + R_B}{\beta+1}\right)}$$

(b) [10 pts] For the following amplifier circuit, find out the expressions for input resistance, output resistance and voltage gain.



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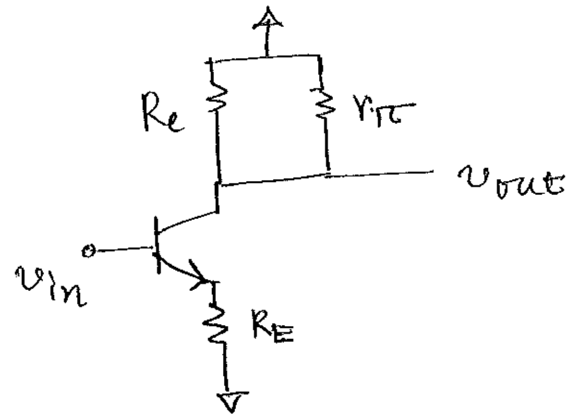

$$R_{in} = r_{\pi_1} + (\beta_1 + 1) R_E$$

$$R_{out} = R_c \parallel r_{\pi_2}$$

$$A_v \cong -\frac{R_c \parallel r_{\pi_2}}{1/g_{m_1} + R_E}$$