Problem #1 — Circle the correct answer (10 pts.; 2each, -3 each wrong)

| TRUE | FALSE | 2D translations can be represented by homogeneous orthonormal 3x3 matrices.

| TRUE | FALSE | In 3-space, any sequence of non-uniform scalings can be applied in arbitrary order without affecting the result.

| TRUE | FALSE | The Gouraud shading model will produce a uniform apparent brightness when applied to an isolated, irregular, planar Lambert polygon, illuminated with a single parallel light source, and viewed (perspectively) from a close-by eye-point.

| TRUE | FALSE | The Gouraud shading technique produces a planar {a*x + b*y + c} brightness distribution on triangular faces of a polyhedral object.

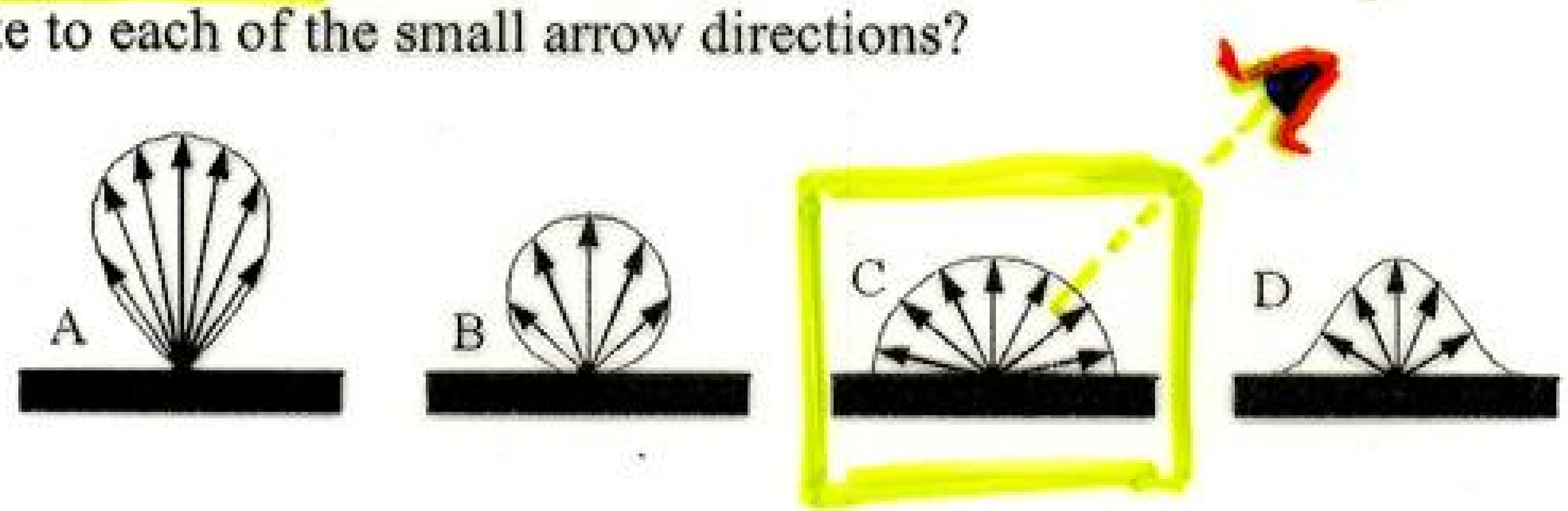
| TRUE | FALSE | The transpose of an orthonormal matrix is equal to its inverse.

Problem #2 — Circle the correct answers: (14 pts.)

(4) Circle all other operations with which a **rotation around the x-axis** does commute:

Mirroring in x; Translation in y; Uniform scaling; Non-uniform scaling; Rotation around z.

(4) Which of the four directional vector diagrams below describes most appropriately the perceived brightness observed on an ideal Lambert surface when viewing the surface from a direction opposite to each of the small arrow directions?



(6) A multi-segment cubic B-spline curve **with no cusps** is defined by six control points. Circle **all** the degrees of continuity that exist at its parametric midpoint (t=1.5):

G0 C0 G1 C1 G2 C2 G3 C3 G4 C4

C^{∞}
G^{∞}

Problem # 3 — Parametric Representation (12 pts.)

(6) Give a parametric representation of a ray that starts at the eye E, passes through pixel center P, and then goes off to infinity:

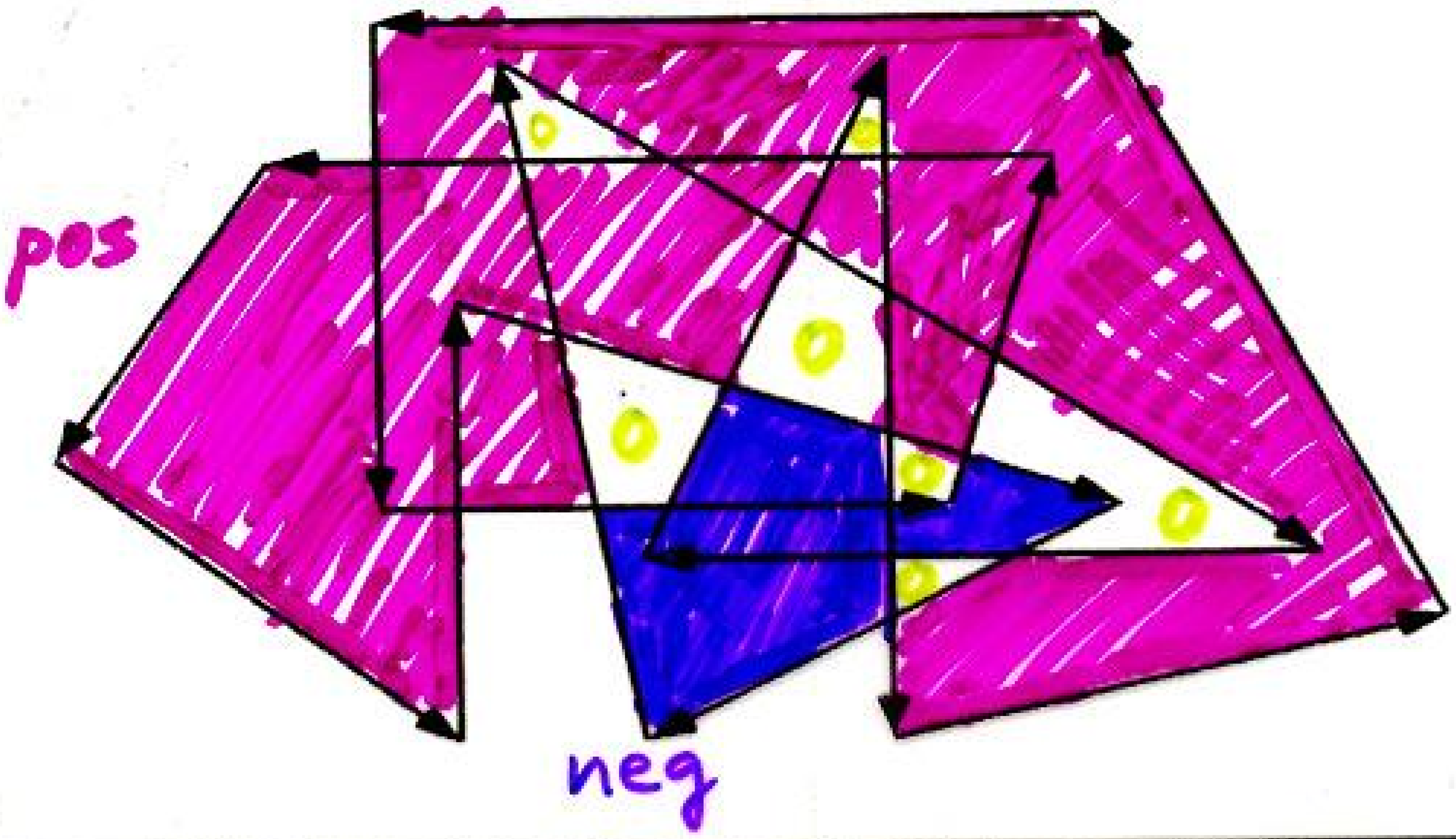
$P(t) = E + (P-E) \cdot t$ $\quad t = [0 \ldots \infty]$

(6) Give **two** reasons why a parametric curve representation: $\mathbf{x = F_x(t)}$, $\mathbf{y = F_y(t)}$, $\mathbf{z = F_z(t)}$ is preferable to the form: $\mathbf{y = f_y(x)}$, $\mathbf{z = f_z(x)}$.

- can handle multi-valued functions
- can make curve a function of time (re-param.)

Problem # 4 — Polygon-fill (8 pts.)

Paint the "inside" areas according to the NON-ZERO WINDING-NUMBER MODEL.



Problem # 5 — Short Questions (20 pts.)

(4) Given the choices (voxels | B-rep mesh | CSG | sweep | instantiation), which is the prefered way to model :

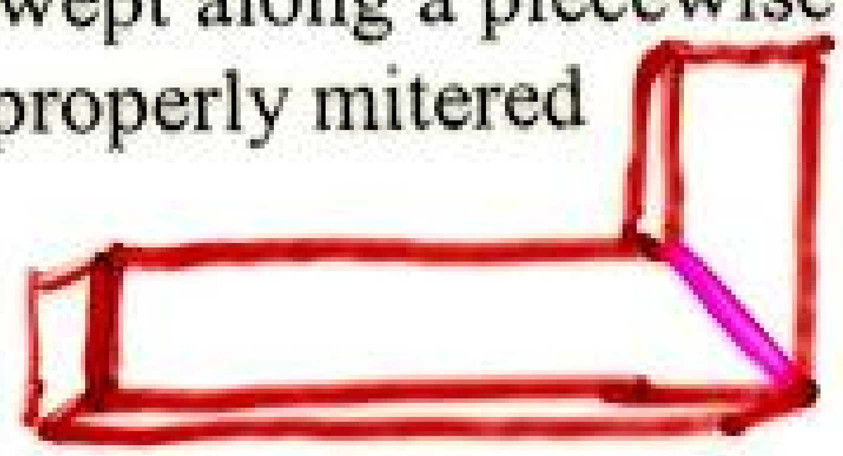
a) A perfect, rotationally symmetric ellipsoid ? CSG, Inst.

b) A piece of sponge (e.g., to wash your car) ? voxels

(4) A cubic B-spline in the x,y plane has the following control points:
A(0,0), B(0,1), C(0,2), D(0,3), E(0,4), F(0,5), G(0,6);

How long is the drawn curve? 4 units

(4) A **square** cross section of area 1 cm^2 is swept along a piecewise linear space path. What is area of the cross-sectional "rib" at a properly mitered joint that makes a 90 degree turn? $\sqrt{2}$



(4) A **rotation-minimizing** frame (RMF) is swept around a planar circular path. The RMF is initialized to the Frenet frame. How many degrees is it rotated relative to the Frenet frame after sweeping through a full revolution? 0°

no twisting!

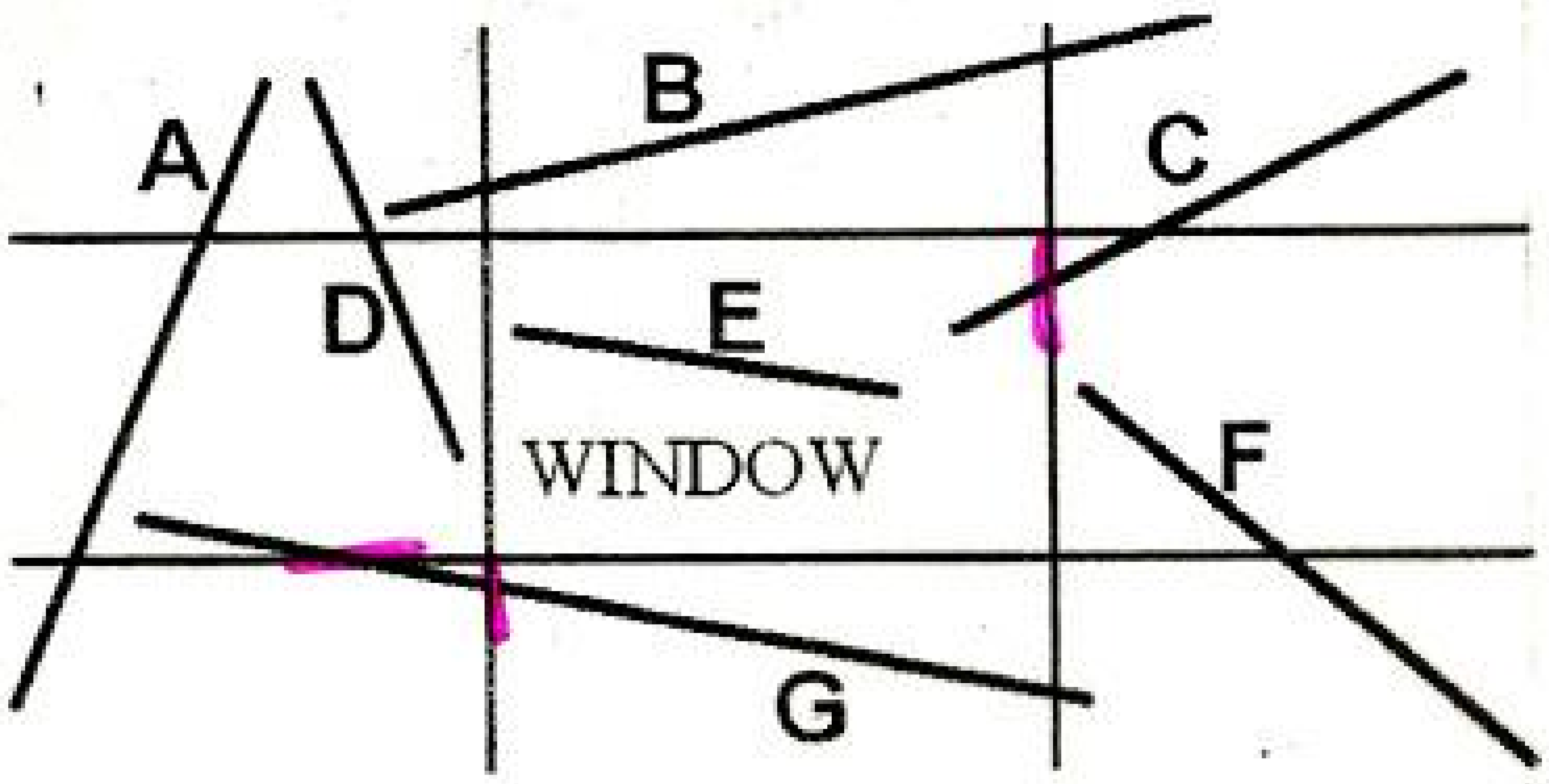
(4) Describe in one sentence the essence of a contribution that Mr. **Phong** has made to the field of computer graphics:

Phong
- Illumination : model specular highlights
- Shading : normal-vector interpolation

NOT: "realistic" surface modeling

Problem # 5 — Clipping (8 pts.)

For the figure below list all the line segments that, based on their "outcodes," can be trivially eliminated from being subjected to a more detailed line clipping algorithm.

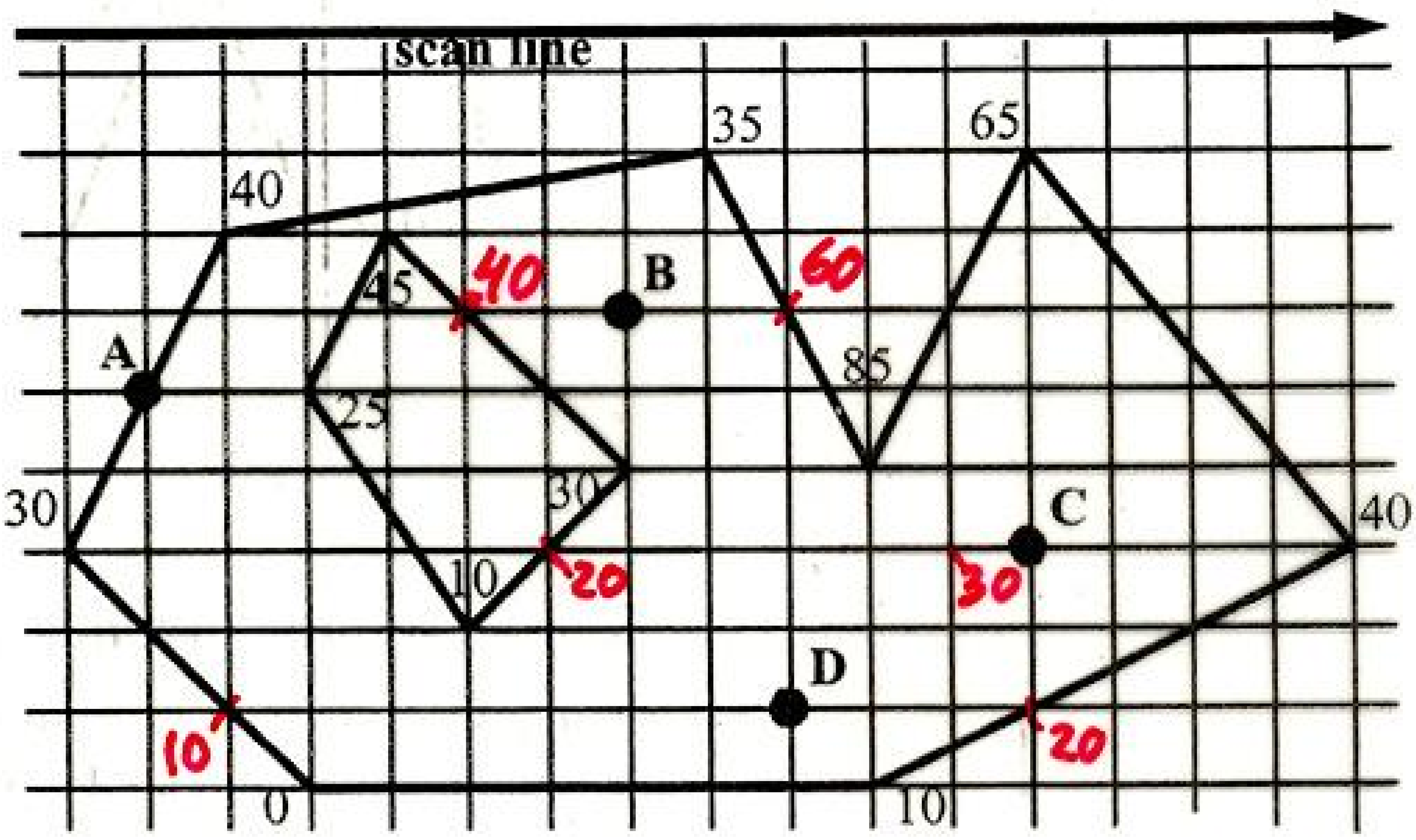


These lines can be trivially eliminated:

ABDF E

Problem # 7 — Gouraud Shading (12 pts.)

You are scan-line processing (in the usual way) the polygon below using **Gouraud** interpolation. The rendering intensities at the vertices are shown. Write out the intensities at the labeled points.



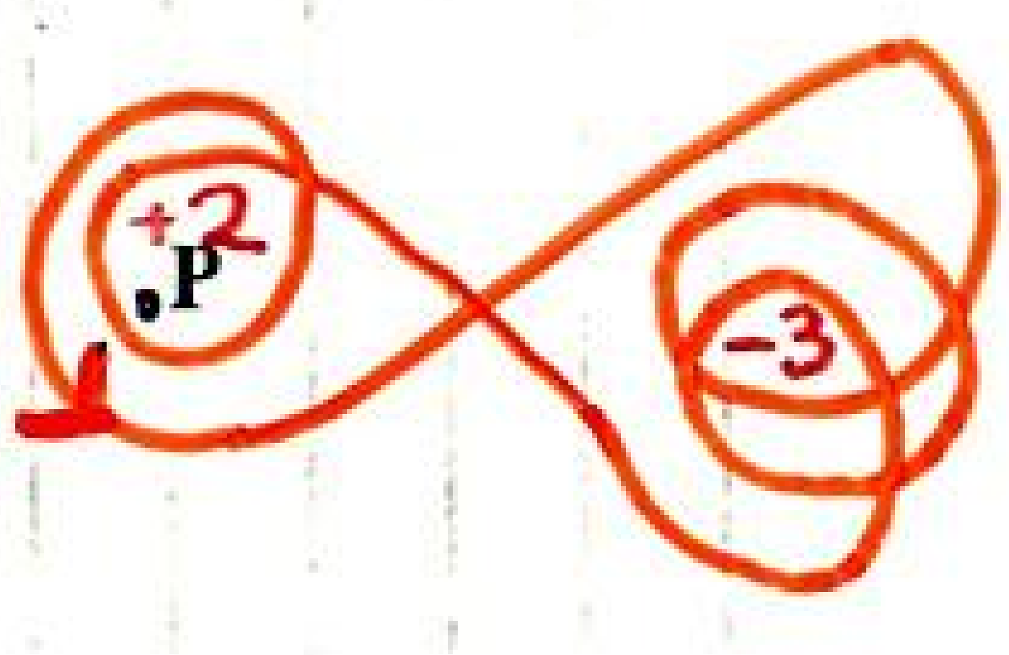
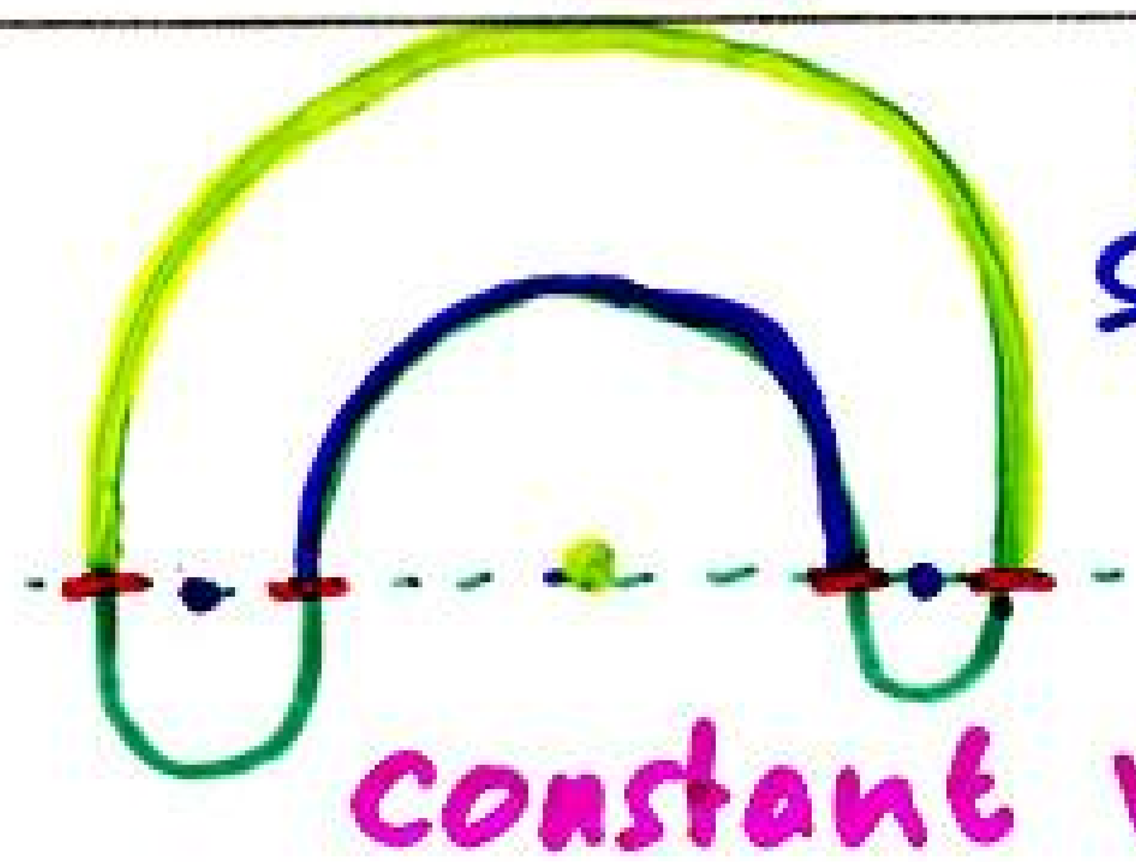
A = 35

B = 50

C = 32

D = 17

Problem # 8 — Polygon-fill (5 pts. each)

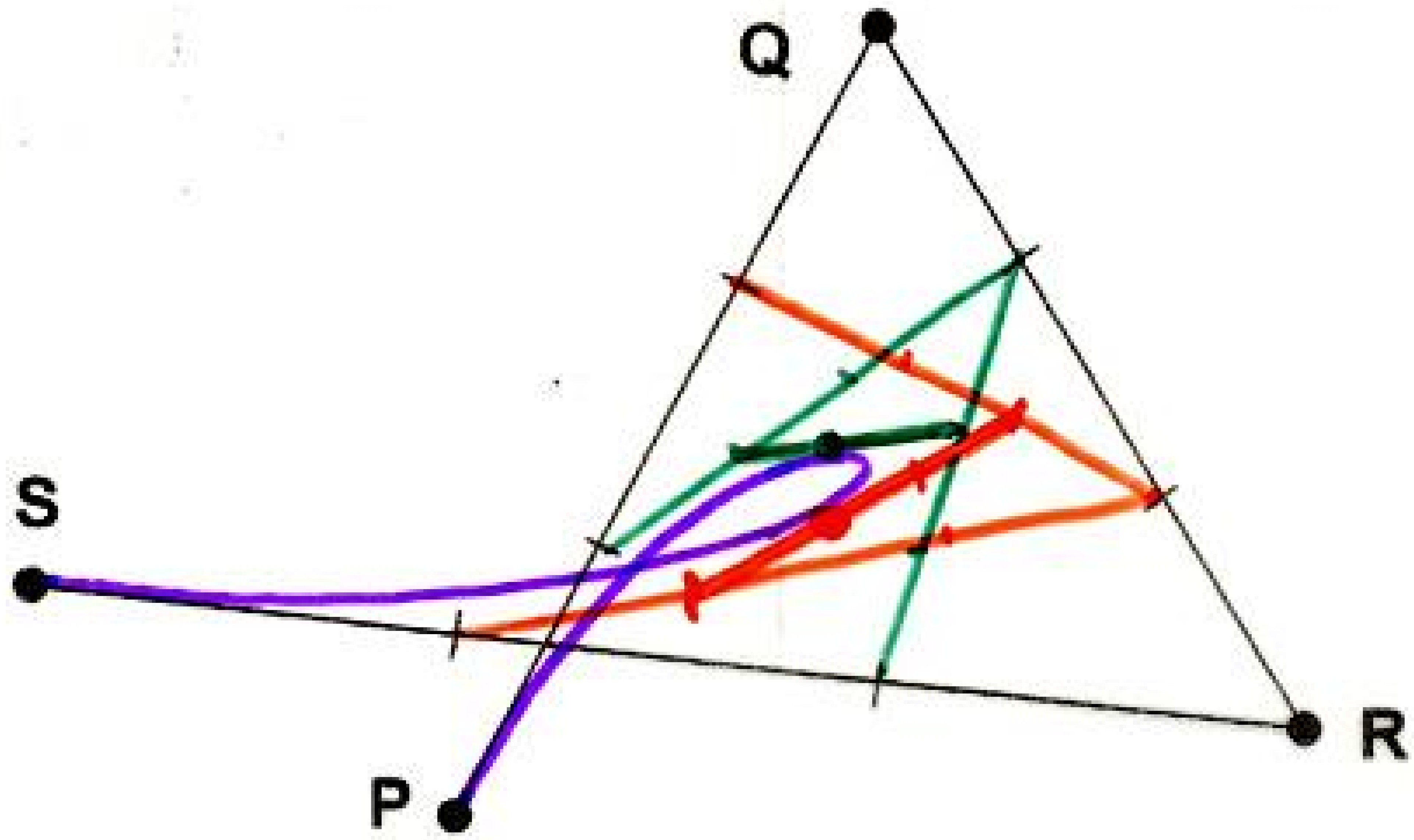
(A) Draw a curve with a turning number of -1 and a winding number of +2 around a point P.	(B) Draw a closed curve that has G1- and C1-contuinuity but not G2- or C2-continuity.
	

All are semicircles

constant velocity

Problem # 9 — Bézier Curve (10 pts.)

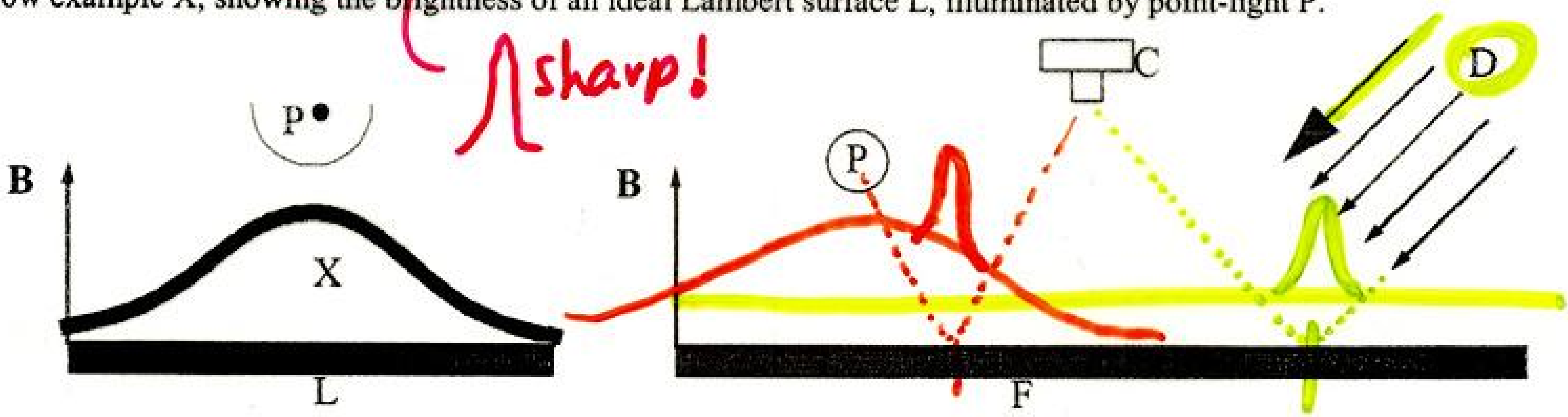
For the given cubic Bezier segment (P,Q,R,S), find the points at **t = 1/3** and **t = 2/3** and their tangent direction using the deCasteljau method. Then sketch the resulting curve.



Problem # 10 — Illumination (10 pts.)

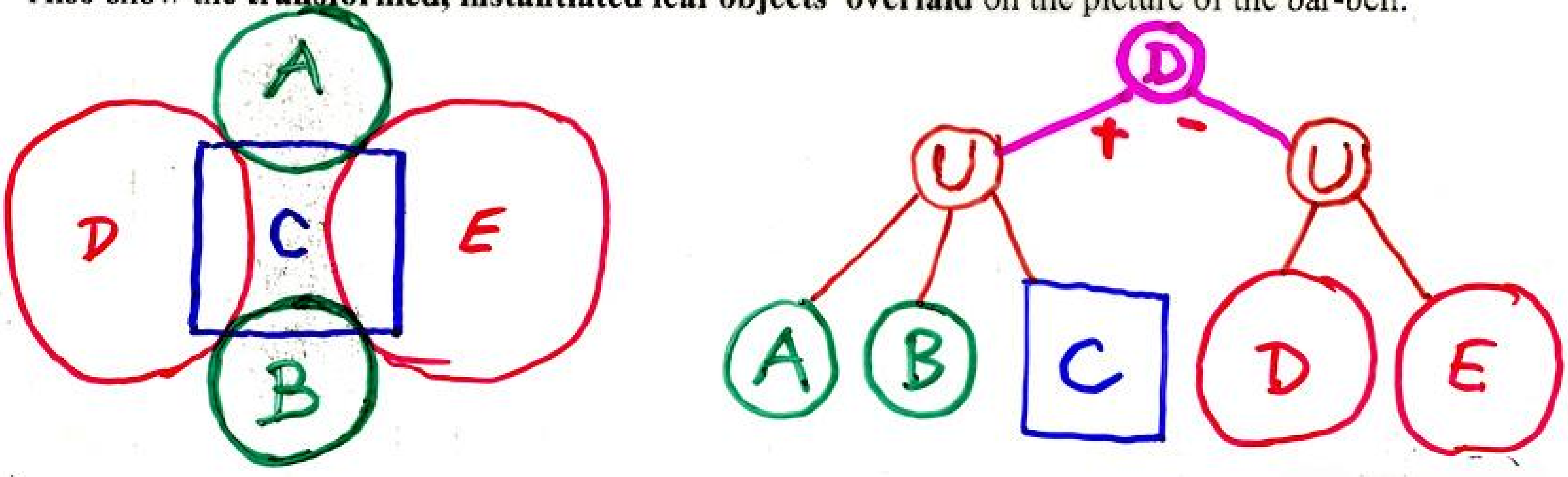
Sketch apparent brightness B, as seen from camera C, along real face F (Phong model, $K_{amb}=K_{diff}=K_{spec}=0.5$, $N_{phong}=50$), illuminated by point-light P and directional light D. Follow example X, showing the brightness of an ideal Lambert surface L, illuminated by point-light P.

sharp!

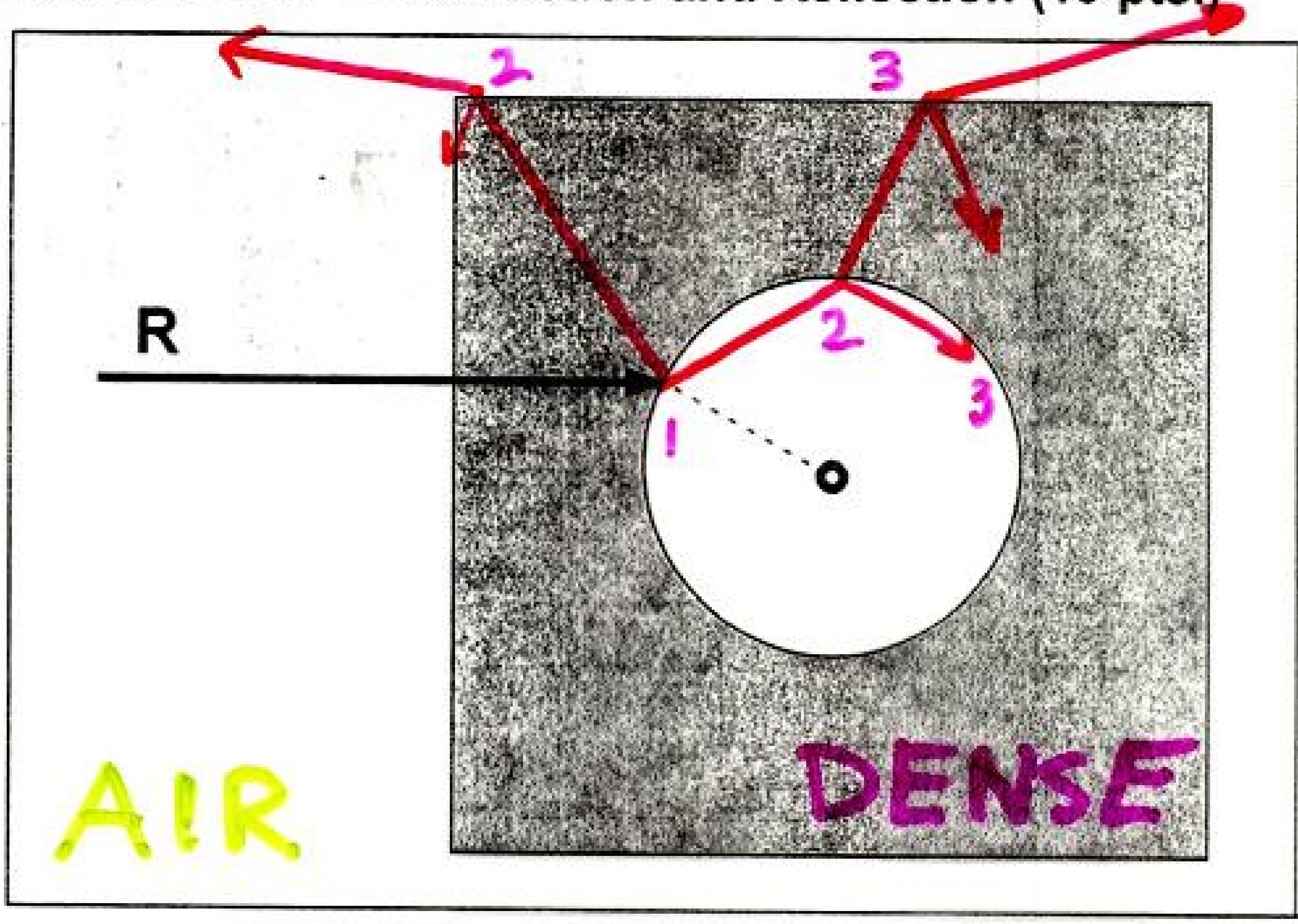


Problem # 11 — CSG (10 pts.)

Given the 2-dimensional bar-bell below and a 2-D computer graphics CSG system with only the primitives **unit-square** and **unit-circle**, draw a **simple CSG tree** that will model the bar-bell. Use a **minimal number** of elements and of Boolean operations (transformations do not count). Also show the **transformed, instantiated leaf objects overlaid** on the picture of the bar-bell.



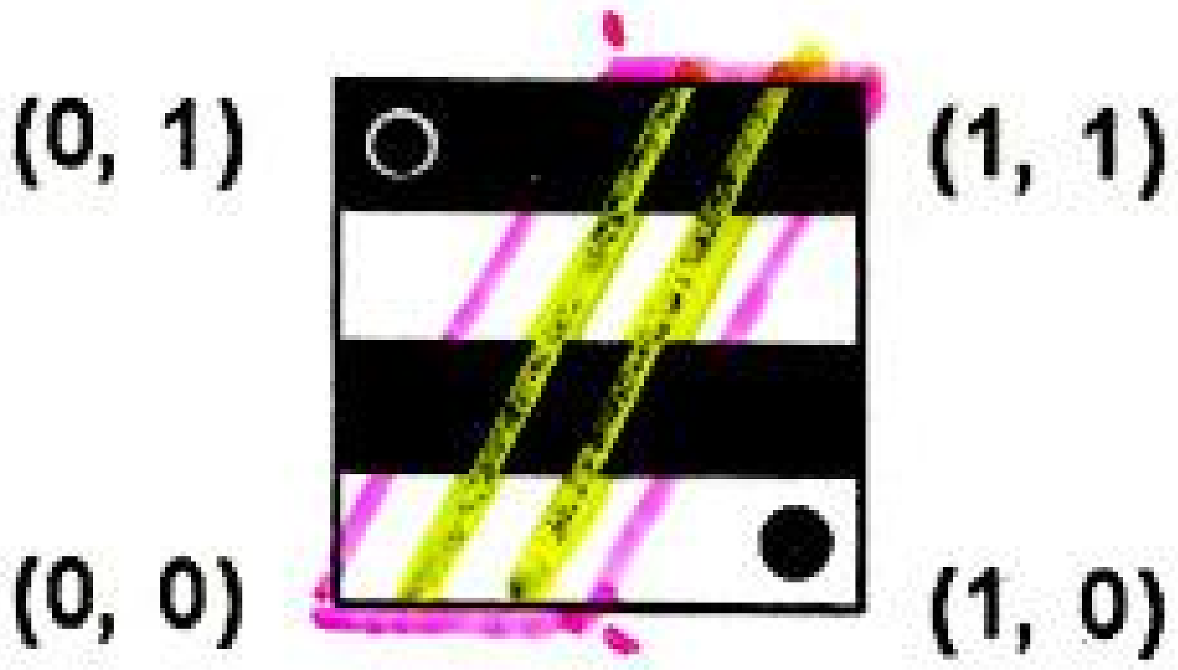
Problem # 12 — Refraction and Reflection (10 pts.)

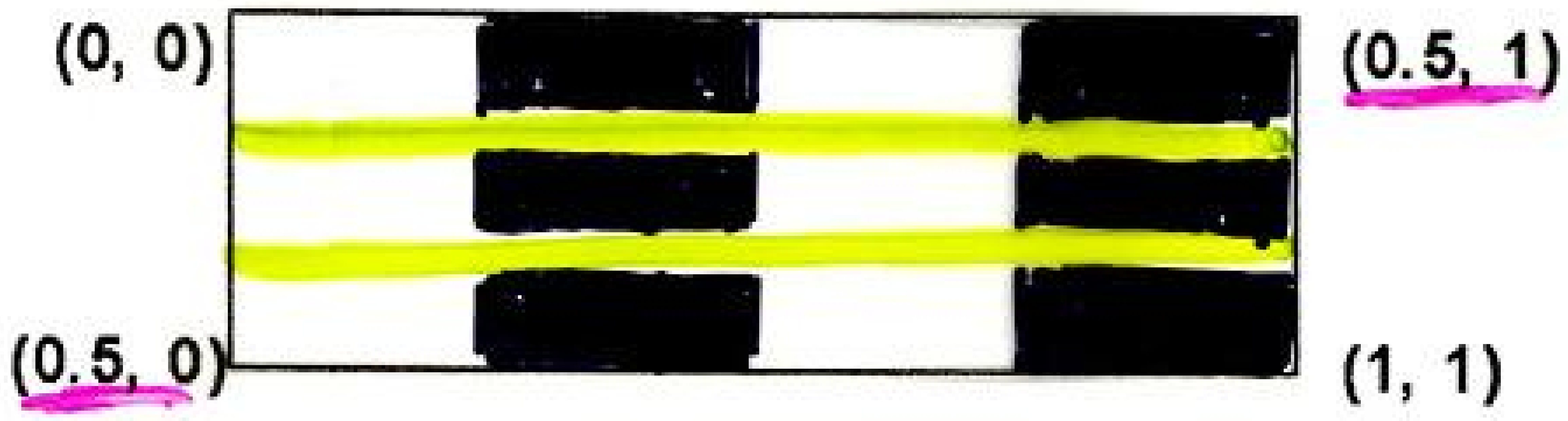


Ray R has entered a glass cube (refractive index **n=1.5**) with a spherical evacuated hollow as shown. Ray-trace this ray through all interactions with **2** subsequent glass surfaces encountered, and show the directions of the emerging rays after that.

Problem # 13 — Texture Mapping (8 pts.)

Use the texture map below and apply it to the rectangular surface on the right, carefully observing the given texture coordinates (u,v).





Problem # 14 — Surface "Decoration" (6 pts.)

You should understanding the fundamental principles behind the following "decoration" techniques": Texture-mapping (T), Bump-mapping (B), Displacement-mapping (D), and Environment-mapping (E). Indicate with the proper labels (*) which of these four techniques do the following:

(a): **Affect** the surface normal used for the lighting calculation: (B) (D)

(b): **Use** the surface normal as an entry into a look-up table: (E)