Problem # 1 — Short Questions (30 pts.)

(5) Given the choices (voxels | B-rep mesh | CSG | sweep | instantiation), which is the prefered way to model :

A tapered, twiste airplane wing: **Brep**   A regular, modular spiral staircase: **Inst.**

A geometrical sculpture made of interpenetrating spheres and cylindrical holes: **CSG**

(3) A colored light has the RGB components (0.9 0.4 1.0). Circle the term that best describes its perceptual color:

pink   medium_grey   magenta   (light_purple)   bright_orange   light_brown   blue_cyan

(6) A cubic Bézier segment **with a cusp at t = 0.5** is subdivided into two parts at **parameter value   t= 0.25** (using deCasteljau). Circle **all** types of continuity that exist at the junction:

(G0   C0   G1)   C1   (G2)   C2   (G3)   C3   (G4)   C4

Circle **all** the types of continuity that hold for the larger of the two subdivided curve pieces:

(G0   C0)   G1   (C1)   G2   (C2)   G3   (C3)   G4   (C4)

(3) Given a perspective projection with the eye at z = 0, the image plane at z = −1, and a 3D scene object centered at z = −100, how does the perspective image change, if all world coordinates (x,y,z) are **doubled**, so that the image plane is now at z = −2 , and the scene object is centered at z = −200? The transformation experienced by the image is (circle all that apply):
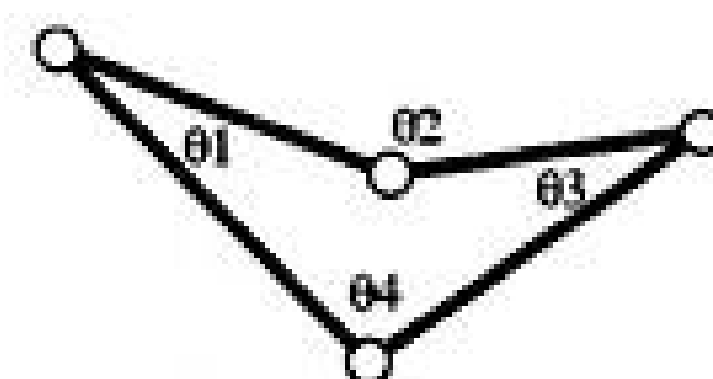
stays_same,   (unifom scaling,)   non-uniform scaling,   sheared,   none_of_the_previous

(3) How many DOF (degrees of freedom) are associated with all possible **lines** in **4D-space (x,y,z,t)** ? _____**4**_____
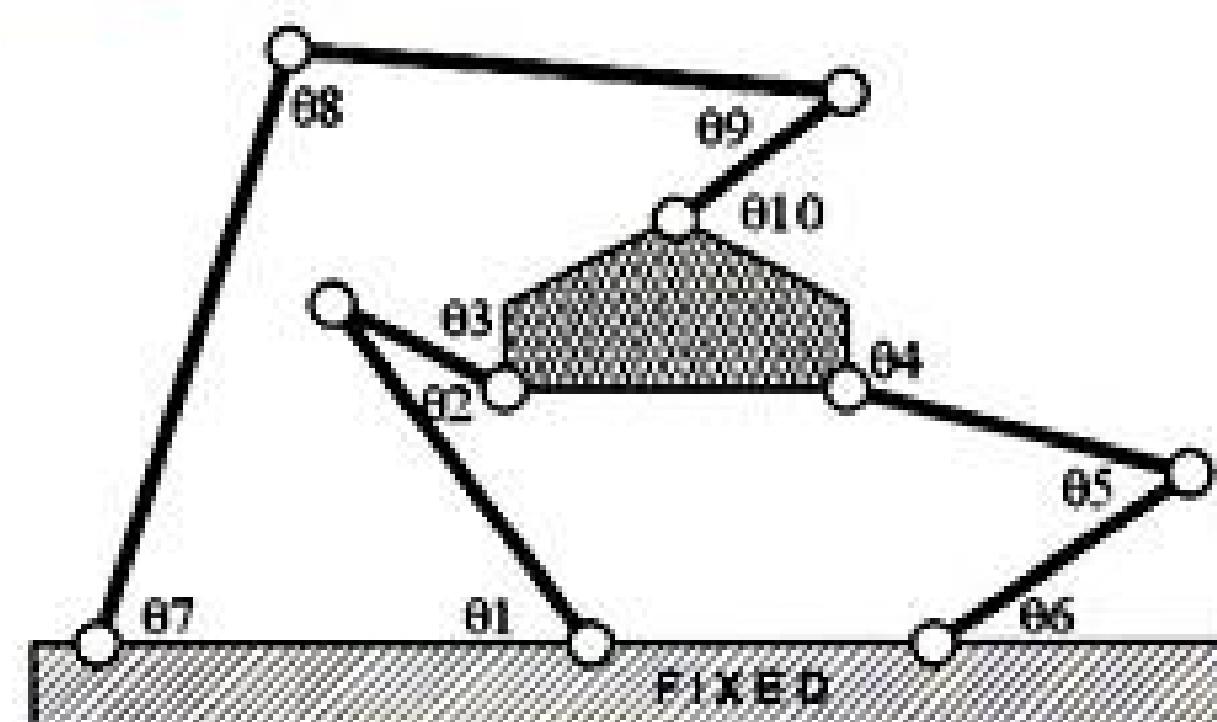
(3) How many DOF are associated with all possible configurations of a **4-bar loop** in **R3** ? The 4-bar loop is constructed from 4 rigid, **thin** line-segments connected by 4 ball-joints. _____**8**_____

(3) How many DOF (degrees of freedom) are associated with the **planar** mechanism in **R2** shown at right?

_____**4**_____

(4) Mr. **Phong** has introduced an illumination model that renders partially reflective surfaces. What is the **other** contribution that Mr. **Phong** has made to the field of computer graphics:

Phong shading : interpolating normal vectors

(4) What are the minimum and maximum number of vanishing points that can be obtained in a perspective projection of a scene containing two independently rotating cubes ?
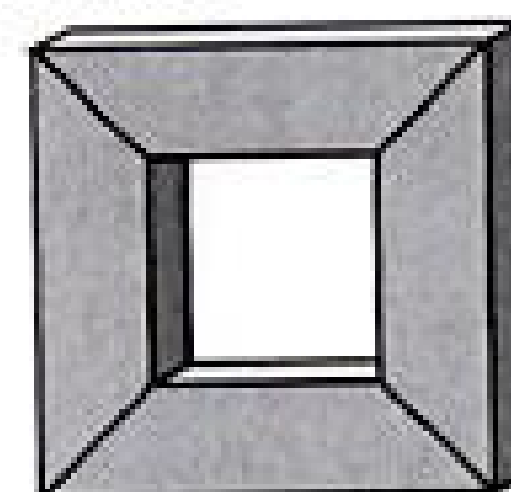
MIN: _____1_____   MAX: _____6_____
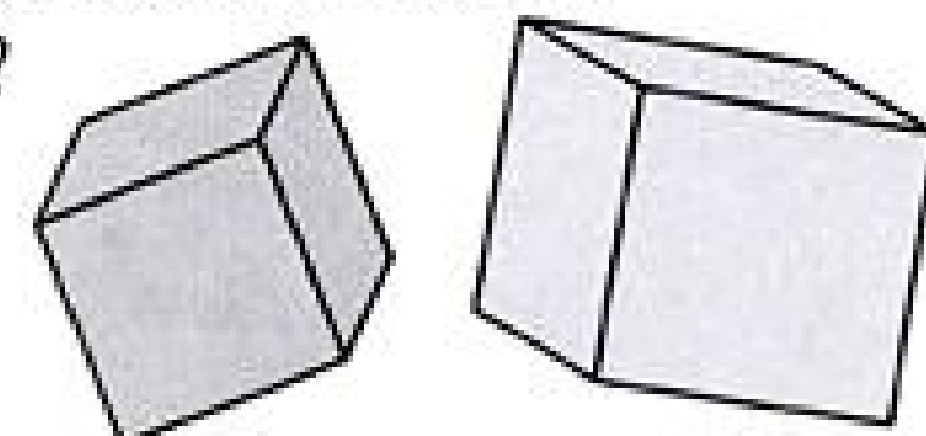
(4) What are the minimum and maximum number of vanishing points that can be obtained from a perspective projection of the picture frame made from 4 square, mitred dowels ? (All enhanced edges count, including the diagonals).

MIN: _____1_____   MAX: _____5_____

(4) Modify **one** of the four directional vector diagrams below to represent the perceived brightness observed in the directions opposite to the small arrows for an idealized **Phong** surface, illuminated with a directional light coming from the upper right (fat arrow); the relevant surface characteristics are kd=0.3, ks=0.7, Phong exponent=50.
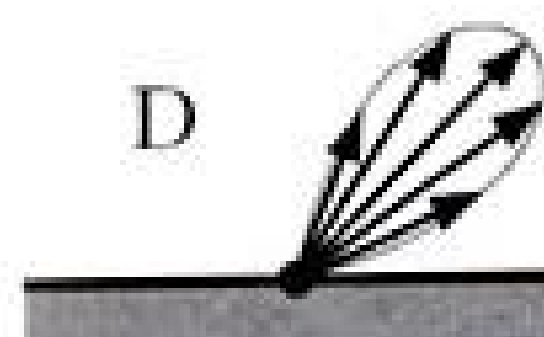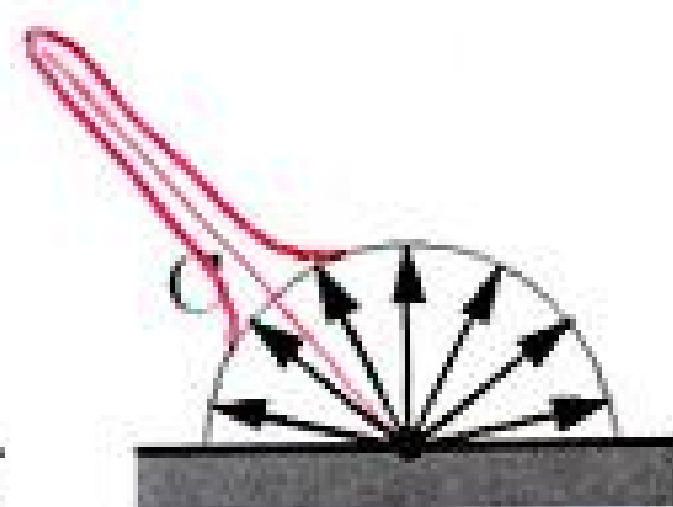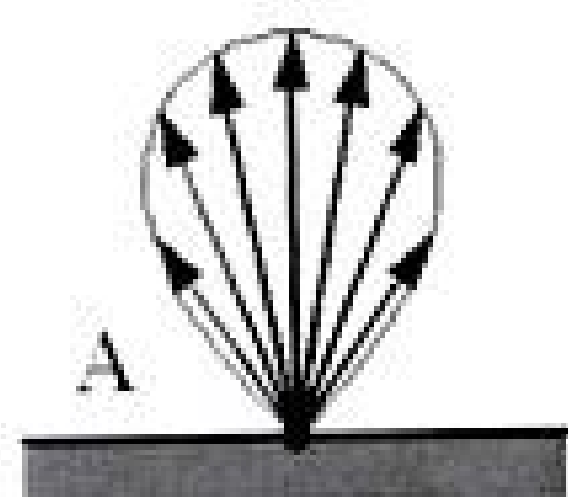
(4) Two of Grassman's Laws state that perceptual color space is 3-dimensional and continuous. Write down the **third law**:

metamers _add_ to form metamers

(4) Which of the following effects **can** be done with the type of ray-tracing renderer that you used in AS#5 and #6 ? (Check all that **can** be done).

☑ Phong highlights on a glossy surface

☑ Specular reflections of other objects in the scene

☐ Caustic lines

☑ Image enlargement seen through a convex glass lens

☐ Color-bleeding from one diffuse surface onto another diffuse surface

☑ Total internal reflection on a glass/air interface

(4) What is the key difference between **ray-tracing** and **ray-casting** ? (Do **NOT** mention common features!).

→ handle reflection + refraction
↳ stops at first ray intersection

(6) Give a parametric vector description for all points in the triangle (**ABC**) where **A,B,C** are its corner locations in **R3**.

$$\vec{v} = \alpha \cdot \vec{A} + \beta \vec{B} + \gamma \vec{C} \text{ with } \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix} [0,1] \text{ and } \alpha + \beta + \gamma = 1$$

Problem #2 — CSG (10 pts.)

Given the 2-dimensional shape below and a 2-D computer graphics CSG system with only the primitives **unit-square** and **unit-circle**, draw a **simple CSG tree** that will model this shape. Use a **minimal number** of elements and of Boolean operations. Ignore transformations. Also show the **transformed, instantiated leaf objects** overlaid on the depicted shape.
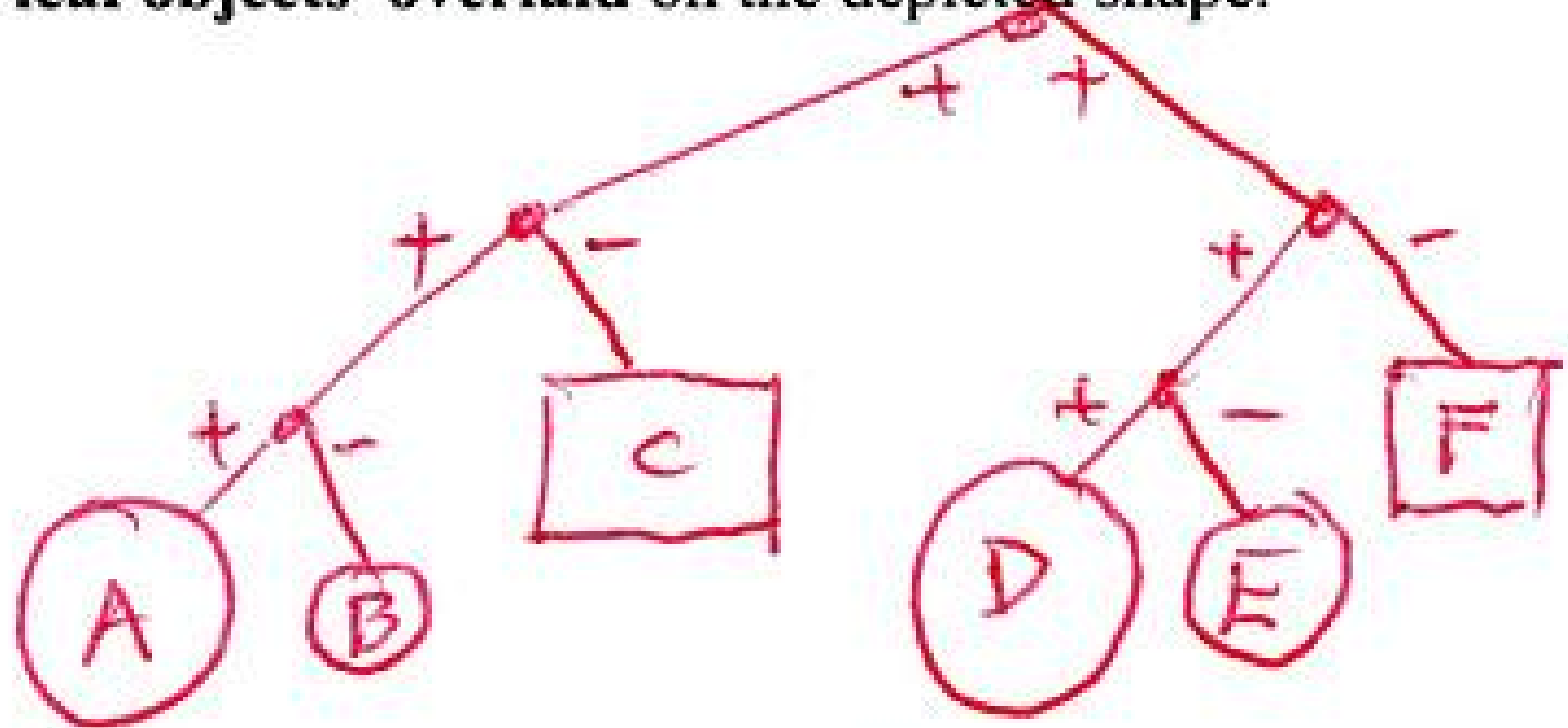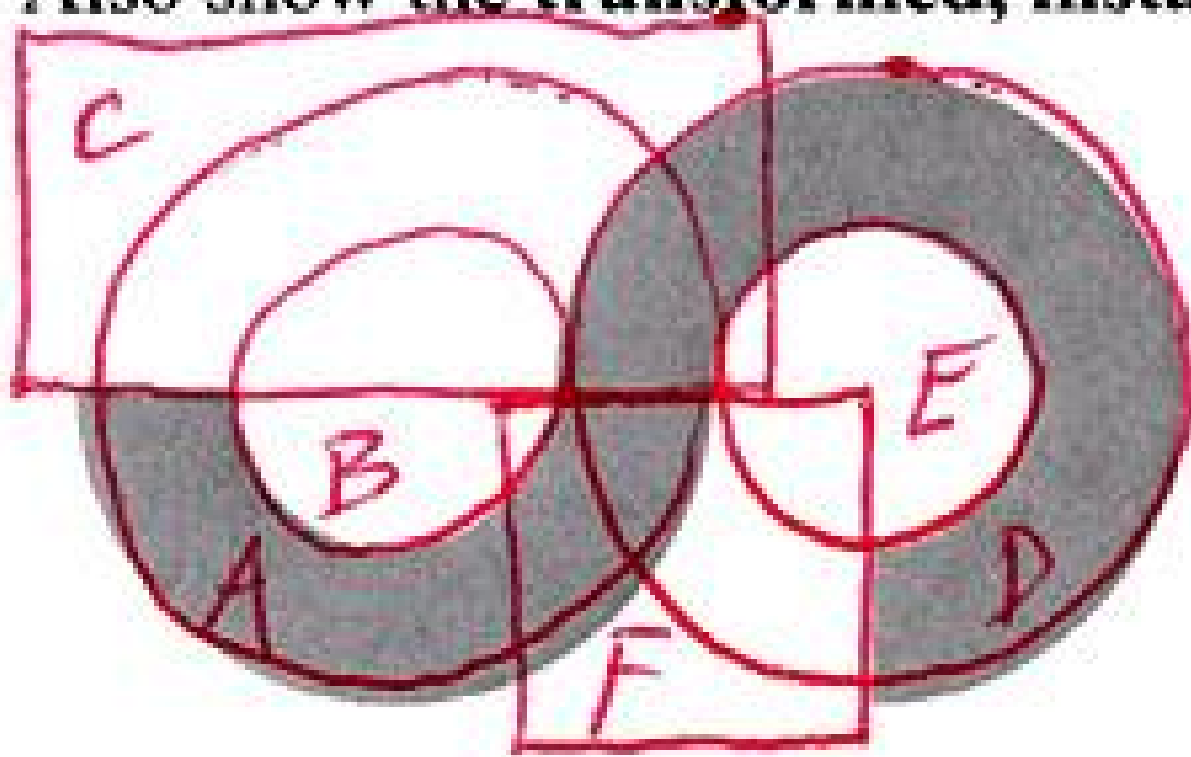
# Problem # 3 — Bézier Curve (10 pts.)

(A) Draw the cubic Bézier segments defined by the control polygon below; locate its **mid-point** and its tangent direction using the **deCasteljau method**.

(B) Now add a second cubic Bézier segment so that the composition of the two segments form a C1-continuous, closed, figure-8 curve. Also construct the mid-point and its tangent direction for the second Bézier segment using the **deCasteljau method**.
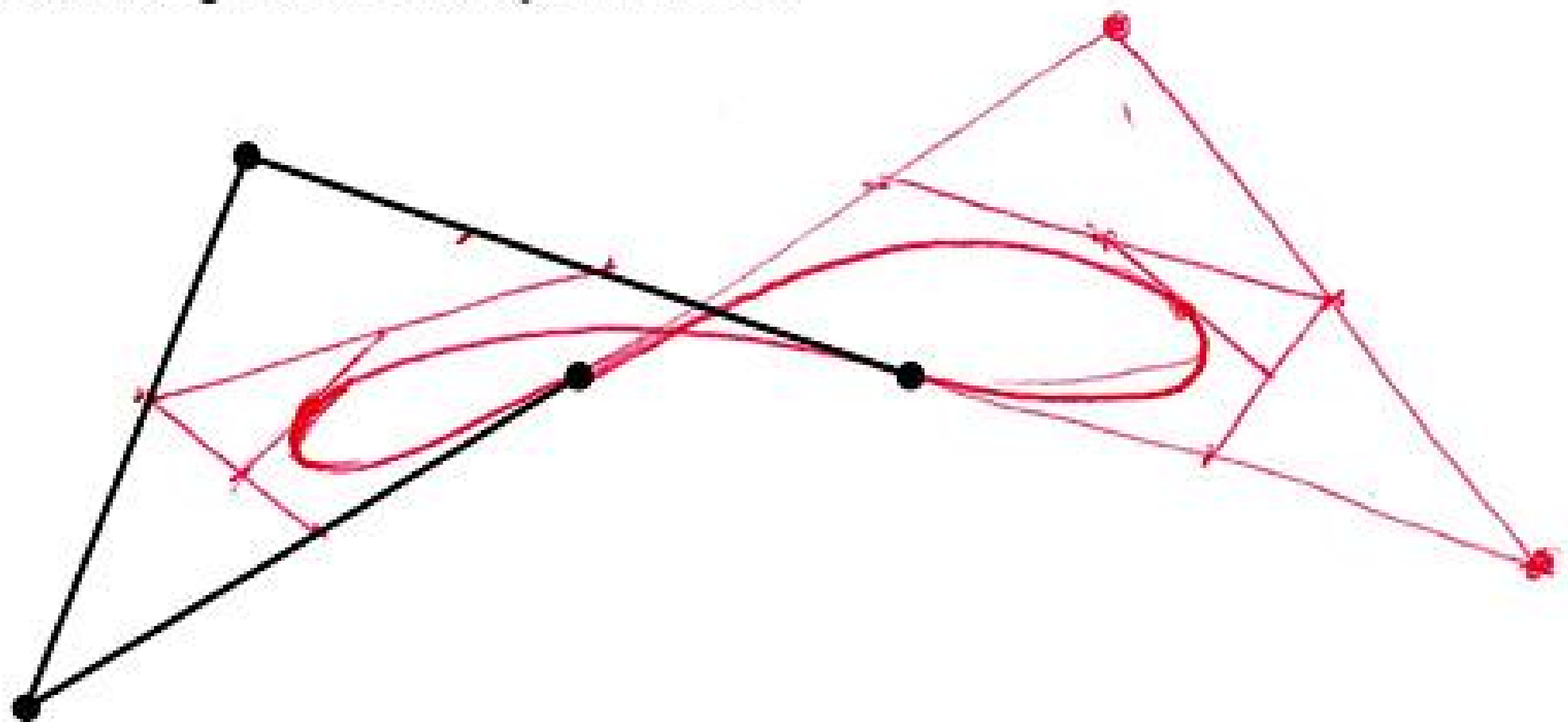

Problem # 4 — Polygon-fill (8 pts.)

Paint all "inside" areas according to the POSITIVE WINDING NUMBER model.
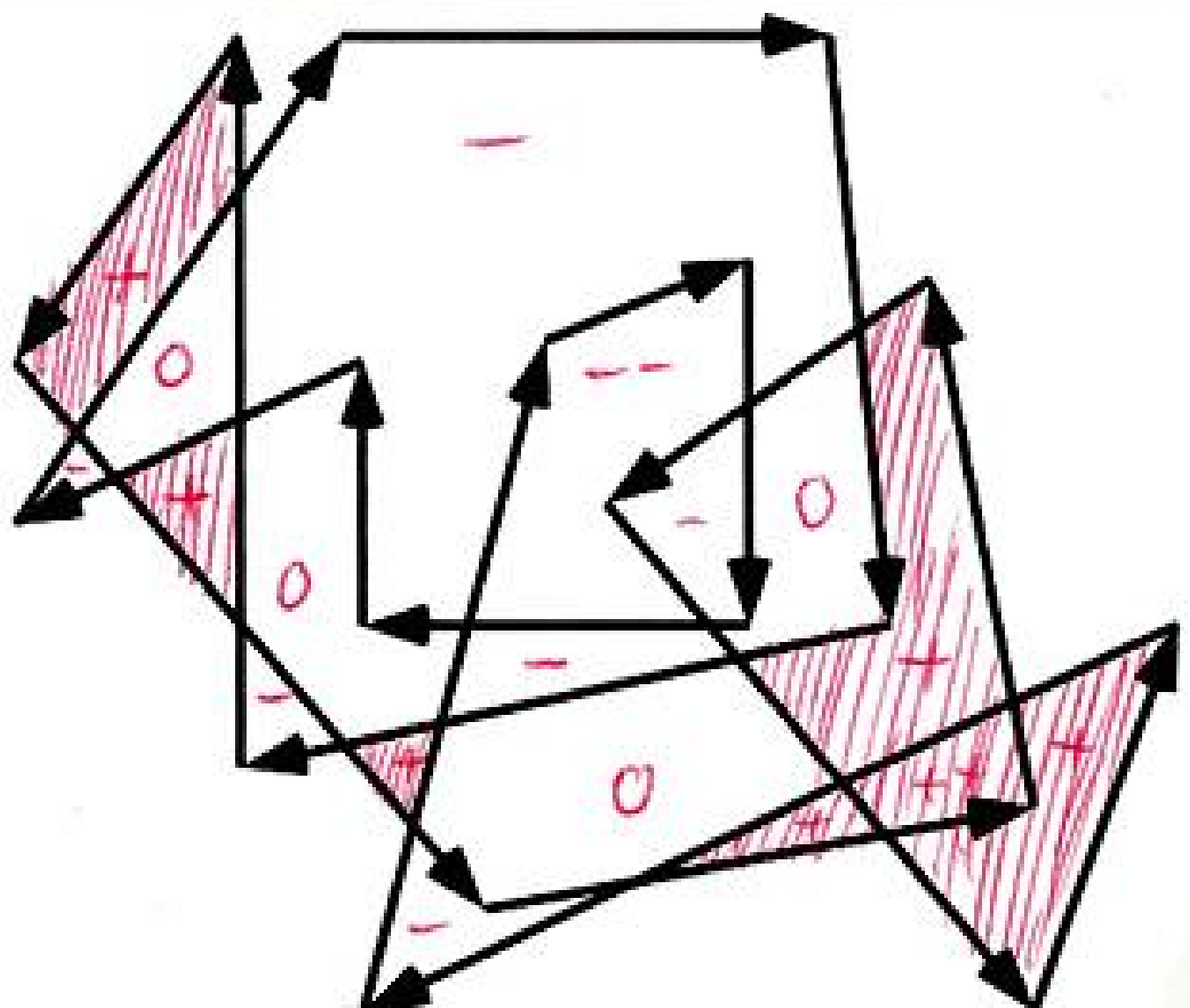
Problem # 5: — Polygon Clipping (8 points)

Show the polygon contour(s) including the spurious double segments on the Window frame that will be output from the Sutherland-Hodgman polygon clipping algorithm for the polygons shown below. Assume that the clipping sequence is: **c, d, a, b.** Do your draft on the left, and show the final result in the right figure by strongly tracing out all output line segments.
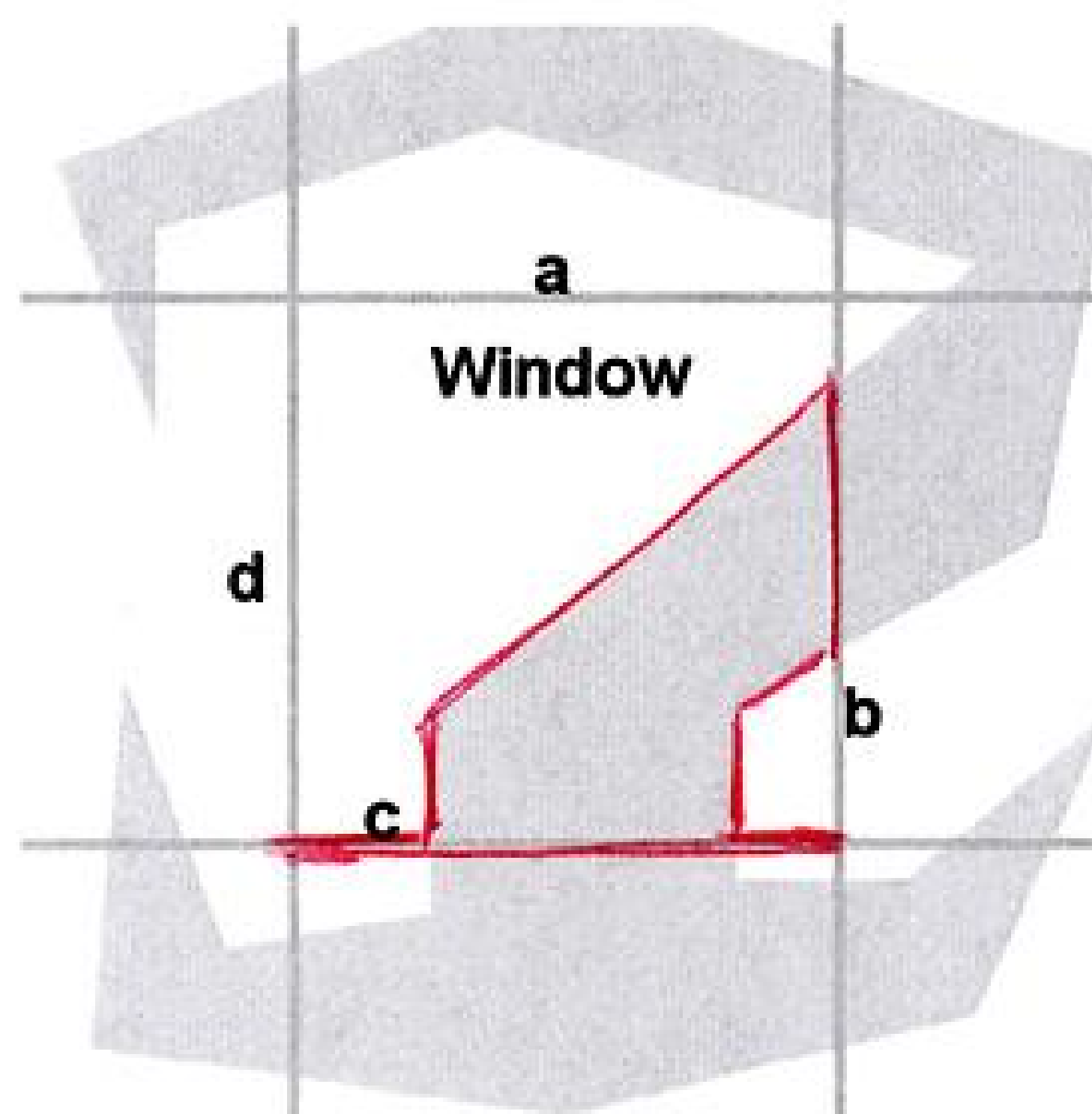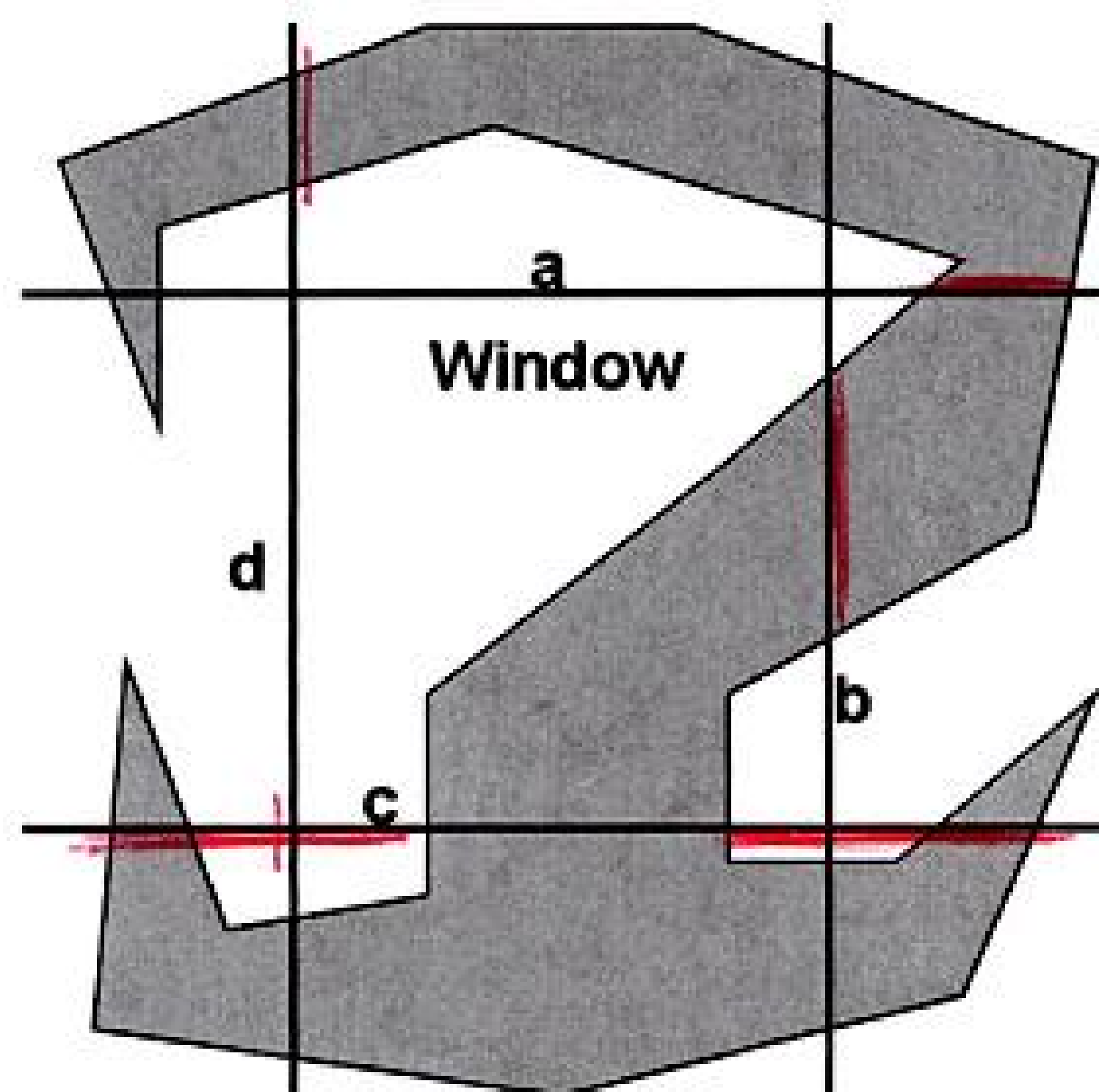
Problem # 6 — Phong Shading (12 pts.) .

You are processing the **Phong surface** shown in profile below with **Phong** interpolation. The computed dot-products between the averaged vertex normals and the directional light of strength 1.0 are as indicated. At the indicated points A, B, C, and D, compute the resulting brightness values for an observer looking form the light direction, assuming **ka = kd = 0.3; ks = 0.5; ksp = 50.**
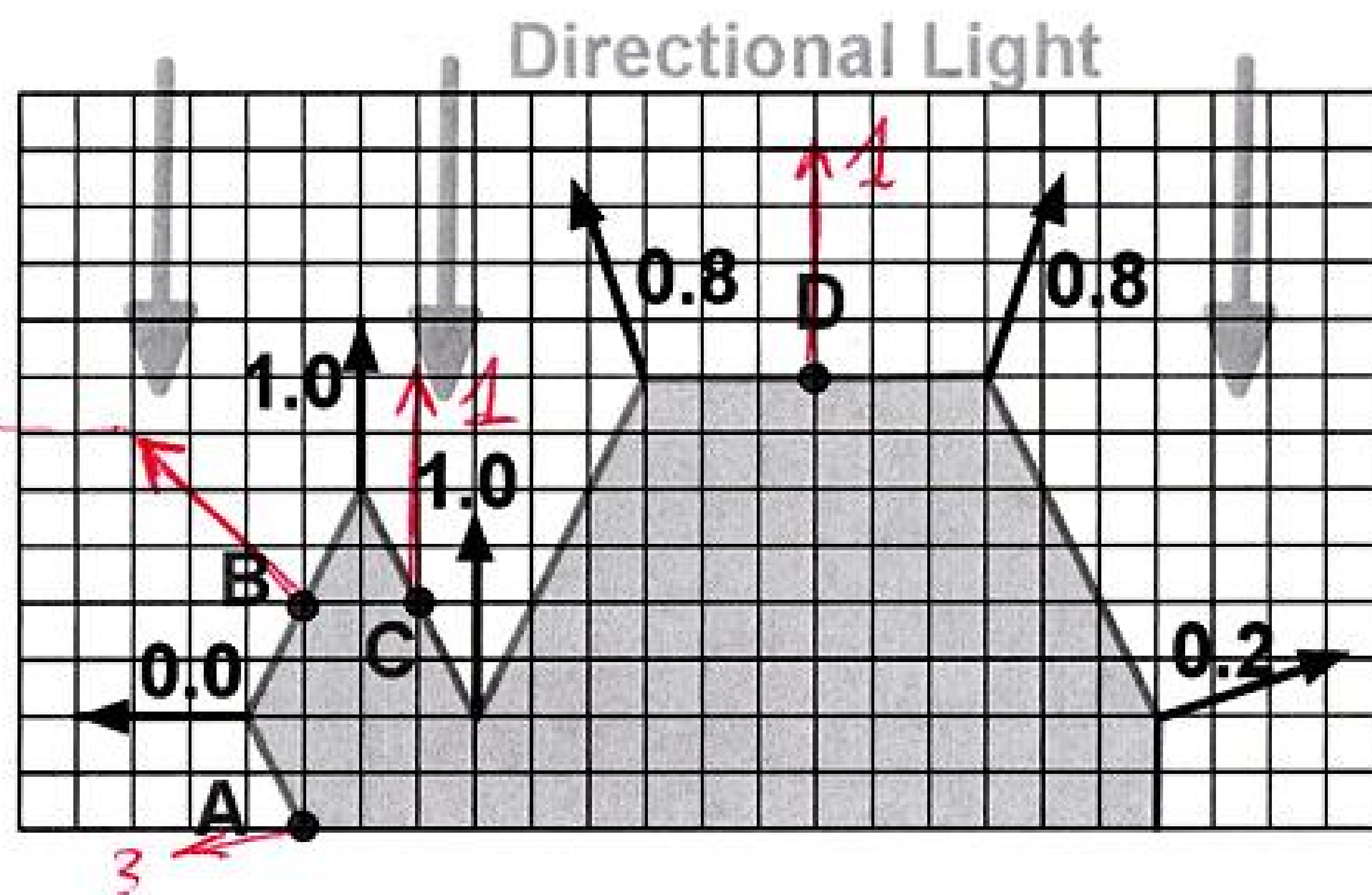
Brightness @ A = 0

Brightness @ B = $0.707 \times 0.3 + 0$
$= 0.23$

Brightness @ C = $0.3 + 0.5 = 0.8$

Brightness @ D = $0.3 + 0.5 = 0.8$

Problem # 7 — Circle the correct answer (10 pts.; –2 each wrong)

(**TRUE**) FALSE | Translations in R3 are commutative.

(**TRUE**) FALSE | Any R3 combination of translations and rotations is an affine transformation.

(**TRUE**) FALSE | In R3, any number of **non**-uniform scalings and a single mirroring operation on a main coordinate plane can be applied in arbitrary order to get same the result.

(**TRUE**) FALSE | In R3, asymmetrical 4x4 matrices can be used to describe translations of points represented by homogeneous coordinates.

| TRUE (**FALSE**) In a **perspective projection**, the midpoint of a straight line segment will be mapped to the middle of the projected line segment.

(**TRUE**) FALSE | The **oblique parallel projection** of a line segment AB (without clipping) is identical to the line segment between the parallel projections of the endpoints A and B.

| TRUE (**FALSE**) A piece of surface will have the same perceptual color regardless of which of **two metamers** is used to illuminate it.

(**TRUE**) FALSE | A spherical **Lambert emitter** will look like a disk of uniform brightness when looked at from any direction.

| TRUE (**FALSE**) The **Gouraud shading** technique produces a planar {a*x + b*y + c} brightness distribution on planar faces of a convex polyhedral object.

(**TRUE**) FALSE | Using the **Frenet frame** to define the orientation of the cross section swept along an arbitrary, smooth, closed space curve with no points of zero curvature anywhere, will guarantee that the cross sections will smoothly match up where the two curve ends join to close the loop.
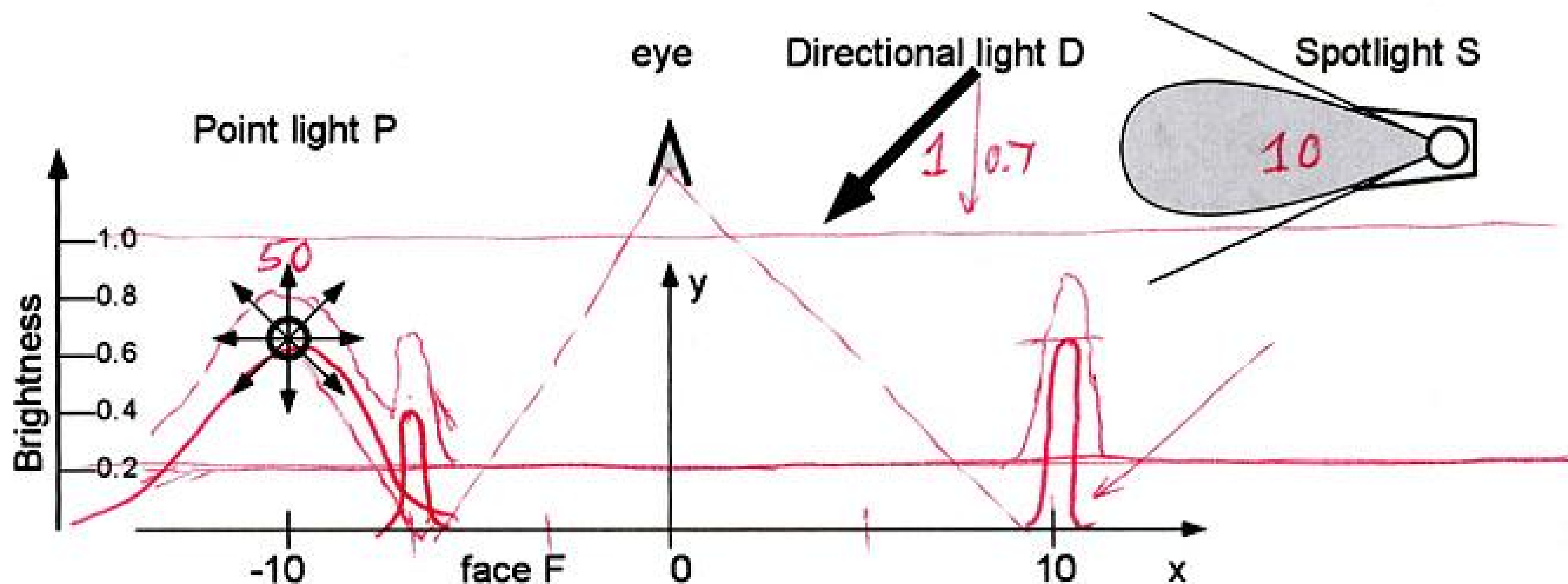
Problem # 8: Polygon Rasterization (10 pts)

Using the rasterization paradigm discussed in class (lower left pixel-corner sampling), mark with a strong dot all the **sample points** that fall onto the **polyon border** and that "**belong**" to the polygon, and thus would turn on the corresponding pixel. (Apparent coincidences are meant to be exact coincidences).
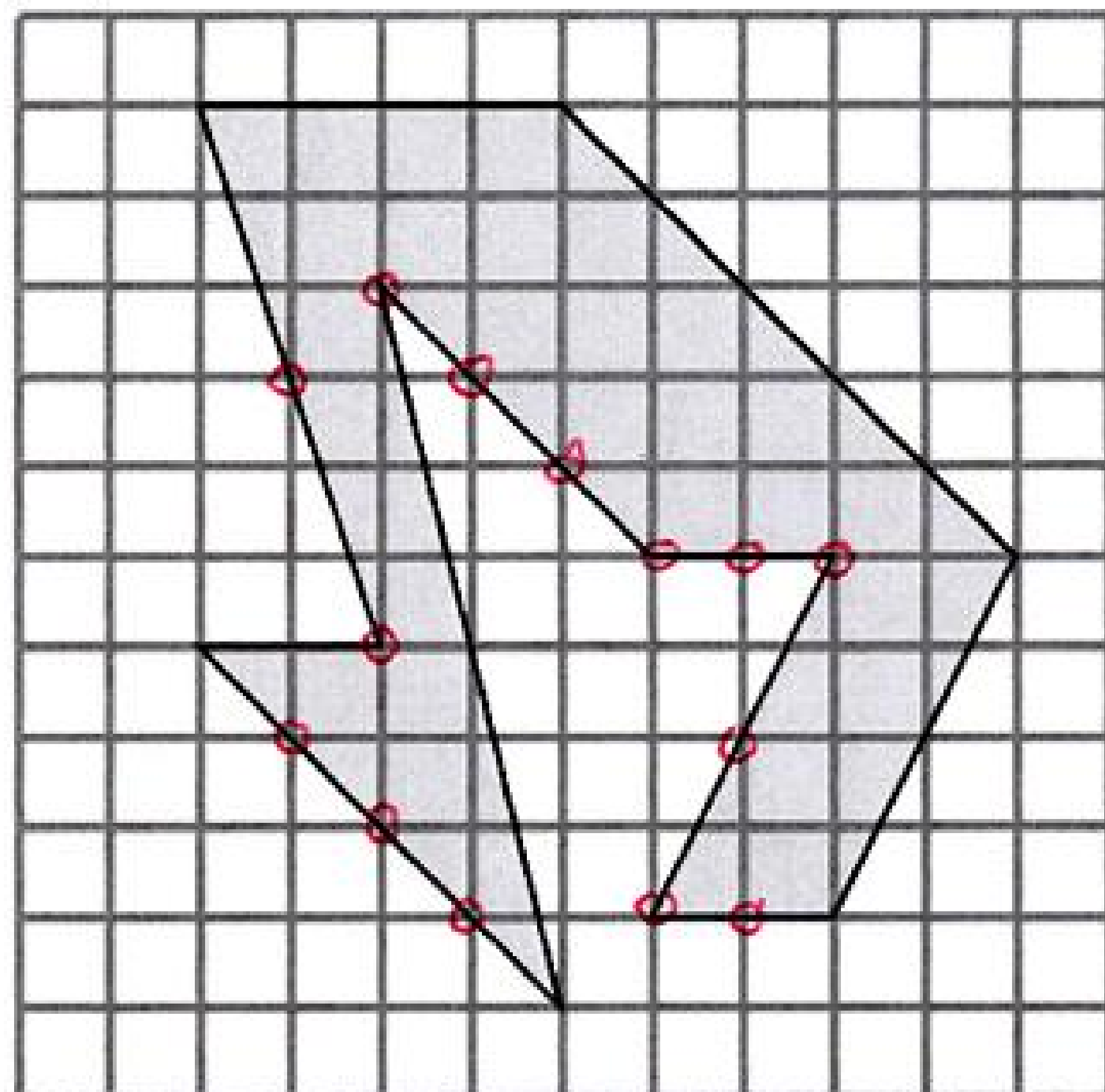
14 dots

Problem # 9 — Illumination (18 pts.)

Draw observed brightness B (to indicated scale), as seen from camera eye at (0, 10, 0), along real face F (Phong model, $K_{amb} = K_{diff} = 0.3$; $K_{spec} = 0.6$, $N_{phong} = 50$), illuminated by lights P, D & S. Point light P is of intensity 50, located at (-10, 5, 0). Directional light D is of intensity 1 and shines **from** direction (1000, 1000, 0). Spotlight S is of intensity 10 and shines from point (20, 10, 0) in direction (-1, 0, 0); its angular falloff is 4.

eye   Directional light D   Spotlight S

Point light P

50   1 0.7   10

Brightness

—1.0
—0.8
—0.6
—0.4
—0.2

-10   face F   0   10   x

At what x-values does "eye" observe a peak in Lambert reflection ?   x = -10

At what x-values does "eye" observe a peak in Phong reflection ?   x = -6.6, $+10$

Problem # 10 — Refraction and Reflection (12 pts.)

Two rays enter a square glass prism (refractive index n = 1.5) with a square evacuated cut-out as shown.

Ray-trace each of the two rays through the interactions with the **first 3** glass surfaces encountered (but not further), and show the directions of the emerging rays after that.

R

S

— either of.
these 2 solutions
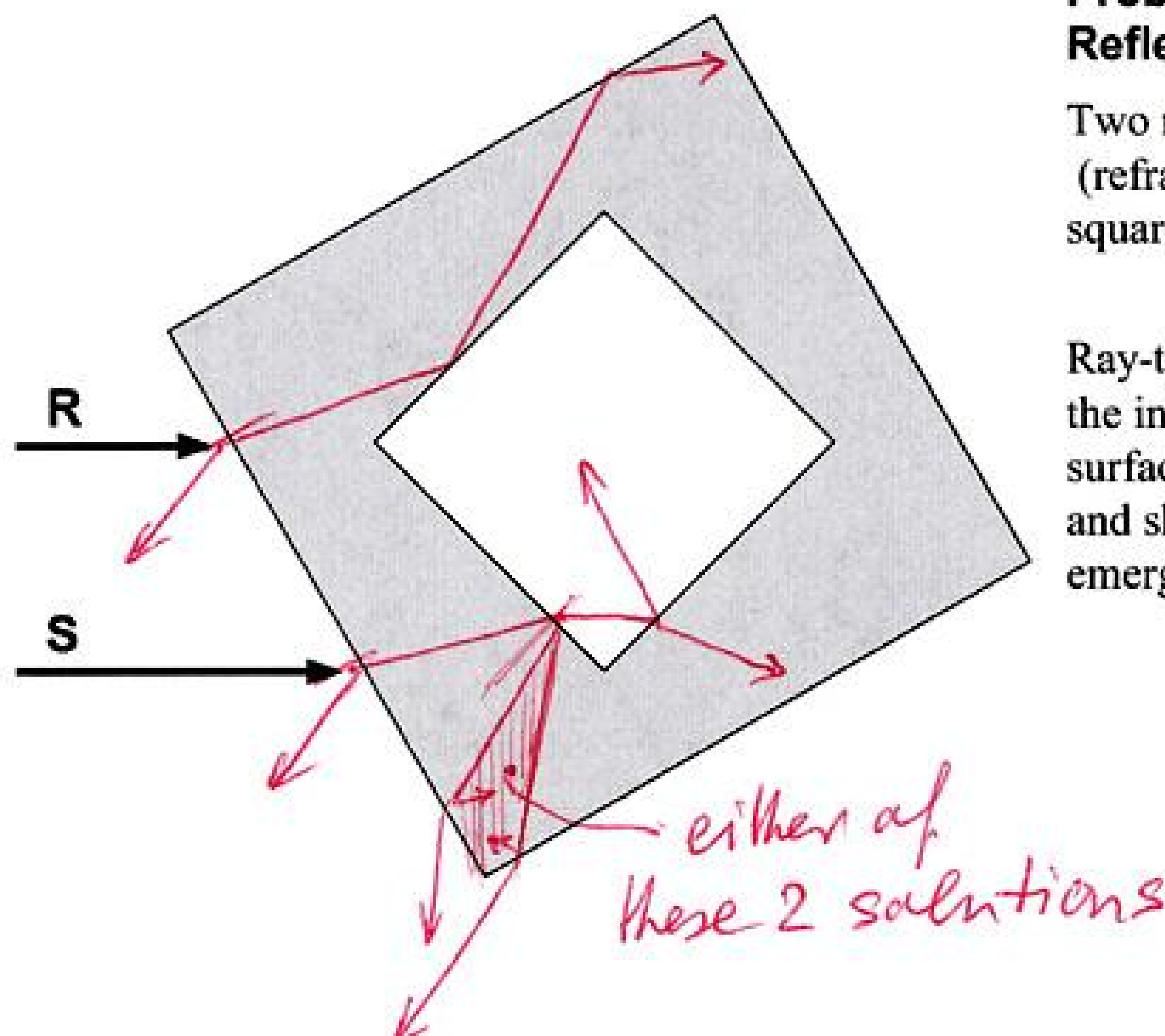
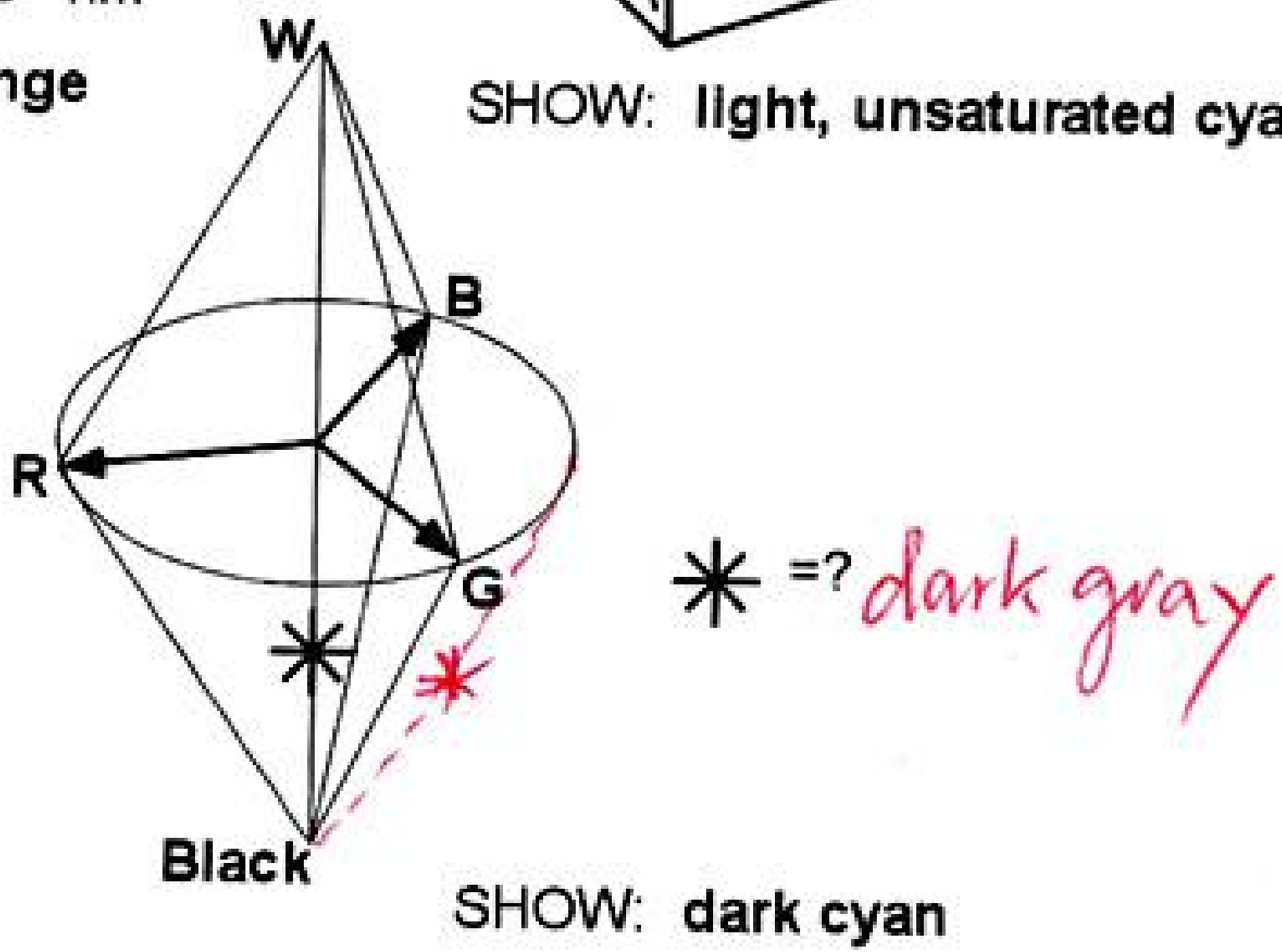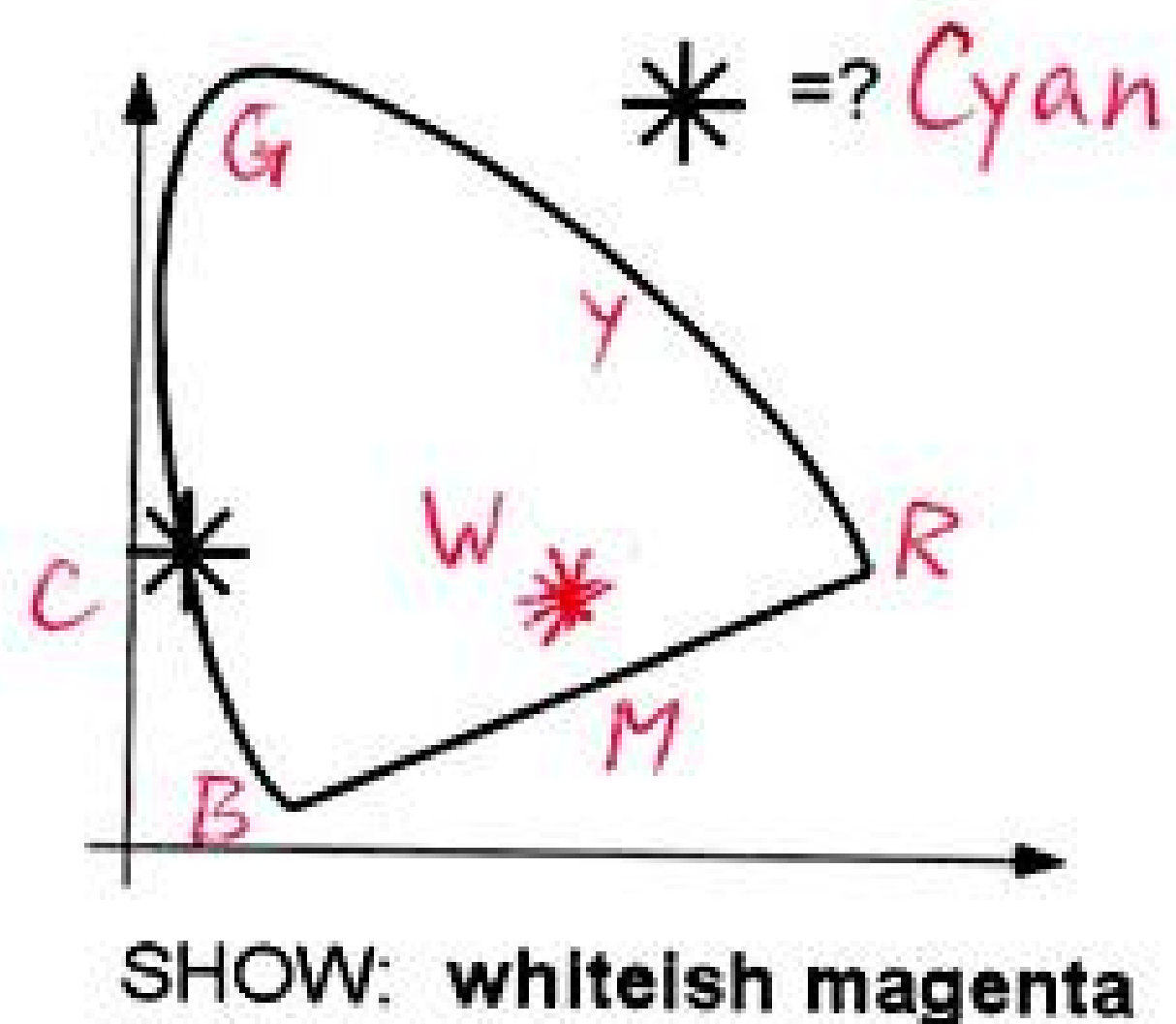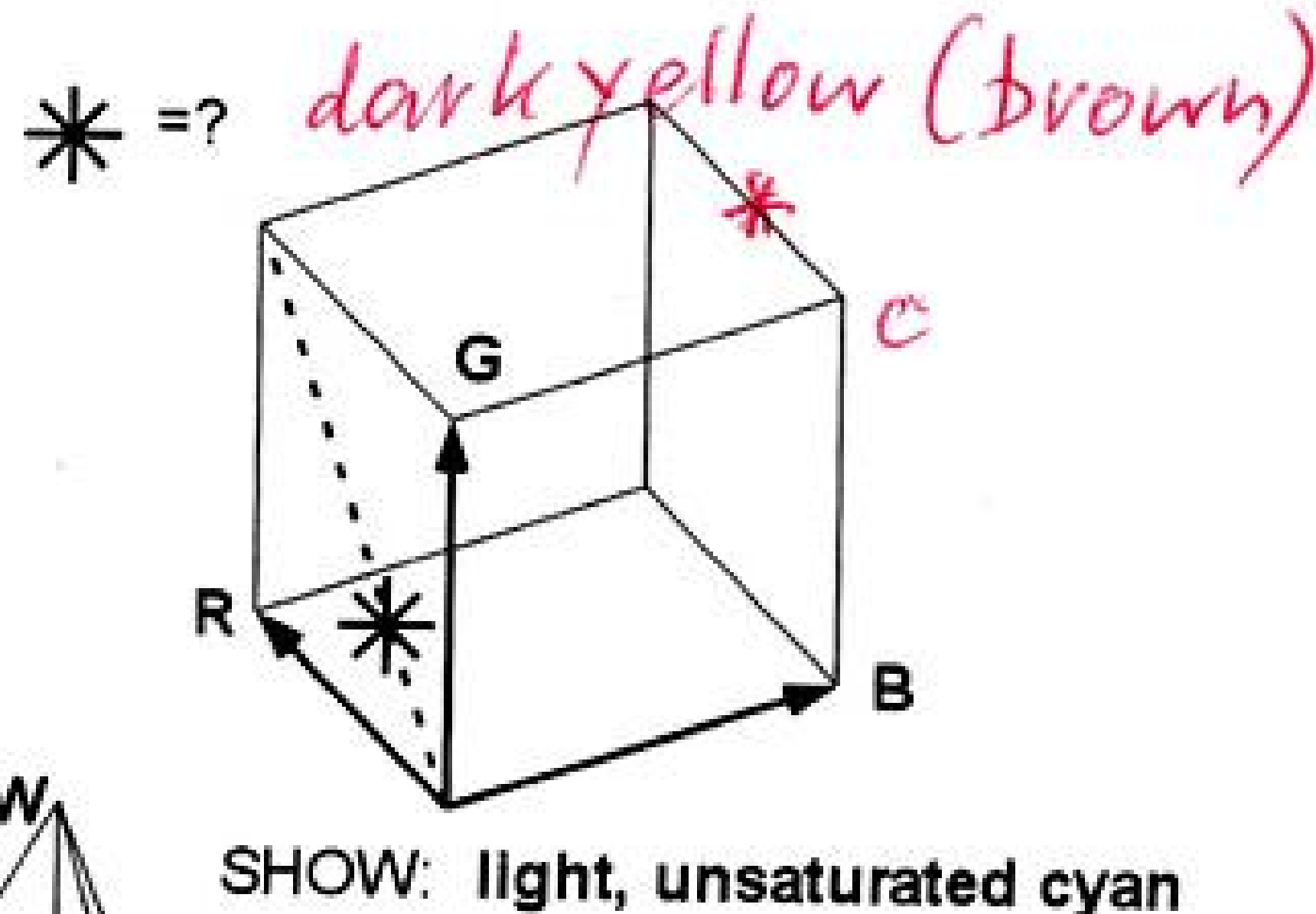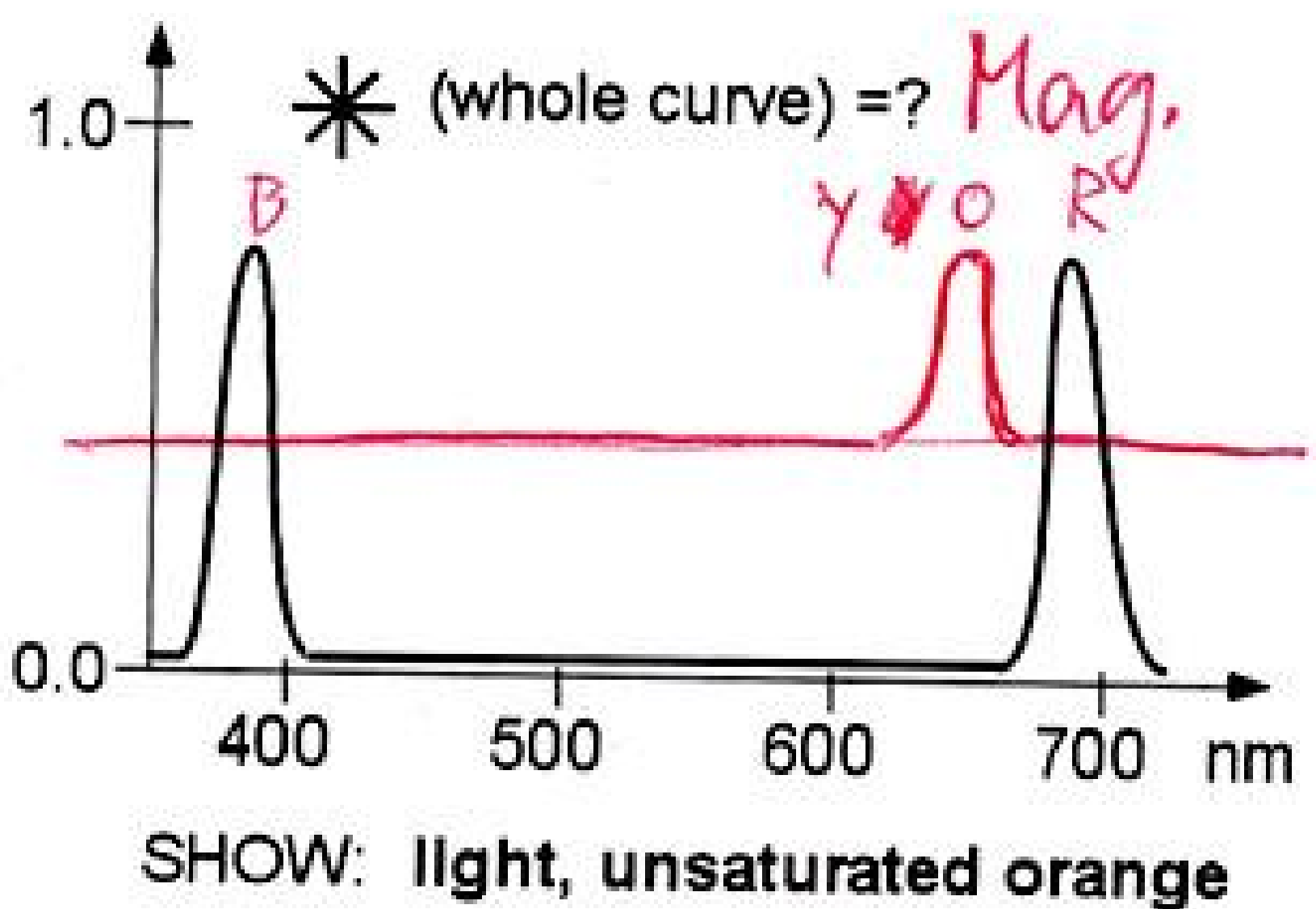# Problem # 11 — Reflection & Filtering (10 pts)

For the set-up illustrated below, in which two lights (A & B) shine on painted canvas C, which is observed through Filter (that **passes** color F), -- specify what OBSERVER sees (Fill in the table !):

Light A
Light B
Filter F
Canvas C
OBSERVER
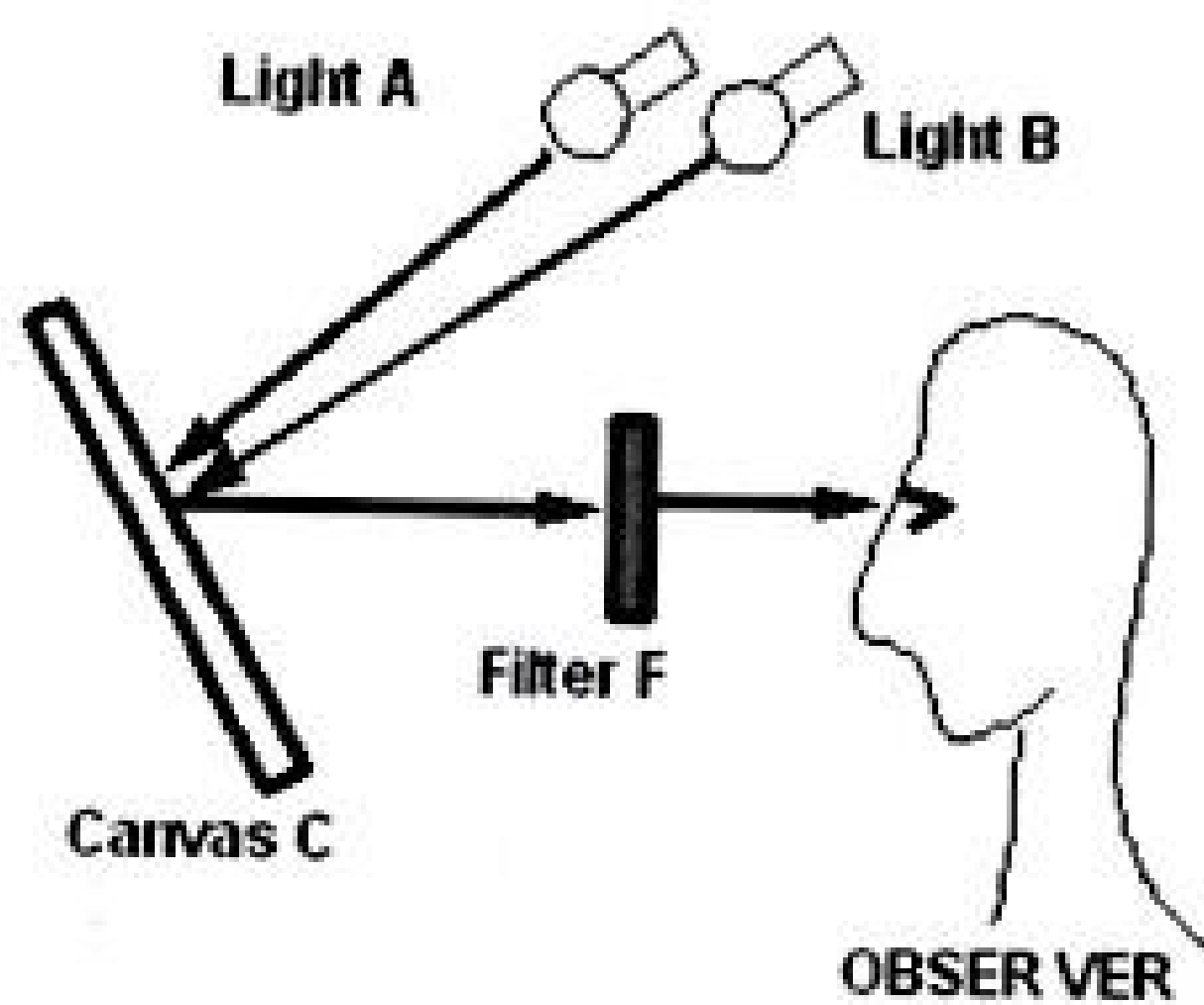
Light A	Light B	Painting	Filter F	OBSERVED
none	Magenta	Cyan	Blue	? *blue*
Blue	Green	Yellow	Blue	? *black*
Cyan	Red	Magenta	Yellow	? *red*

Problem # 12 — Color Spaces (12 pts.)

For each of the diagrams of a color space: (A) Name the perceptual color indicated by: * =?

(B) Represent the requested color in the diagram (SHOW: "color") with " * ".

✳ (whole curve) =? *Mag,*
Y O R

1.0
B
0.0
400   500   600   700   nm

SHOW: light, unsaturated orange

✳ =? *dark yellow (brown)*

G
C
R
B

SHOW: light, unsaturated cyan

✳ =? *Cyan*

G
Y
C
W
R
M
B

SHOW: whiteish magenta

W
B
R
G
Black

✳ =? *dark gray*

SHOW: dark cyan

Problem # 13 — Texture Mapping (8 pts.)

Use the texture map below and apply it to the rectangular surface on the right, carefully observing the given texture coordinates (u,v).

(0, 1) ... (1, 1)

(0, 0) ... (1, 0)

(0, 0.25) ... (0.5, 1)

(0.5, 0) ... (1, 0.75)
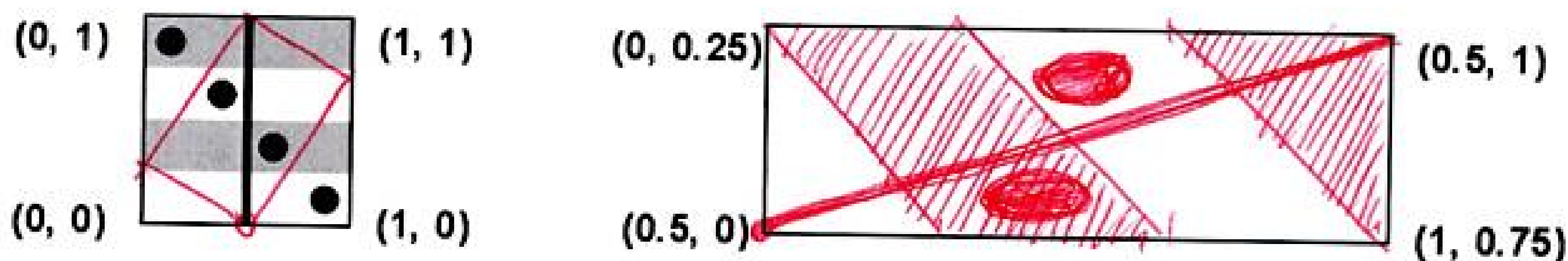
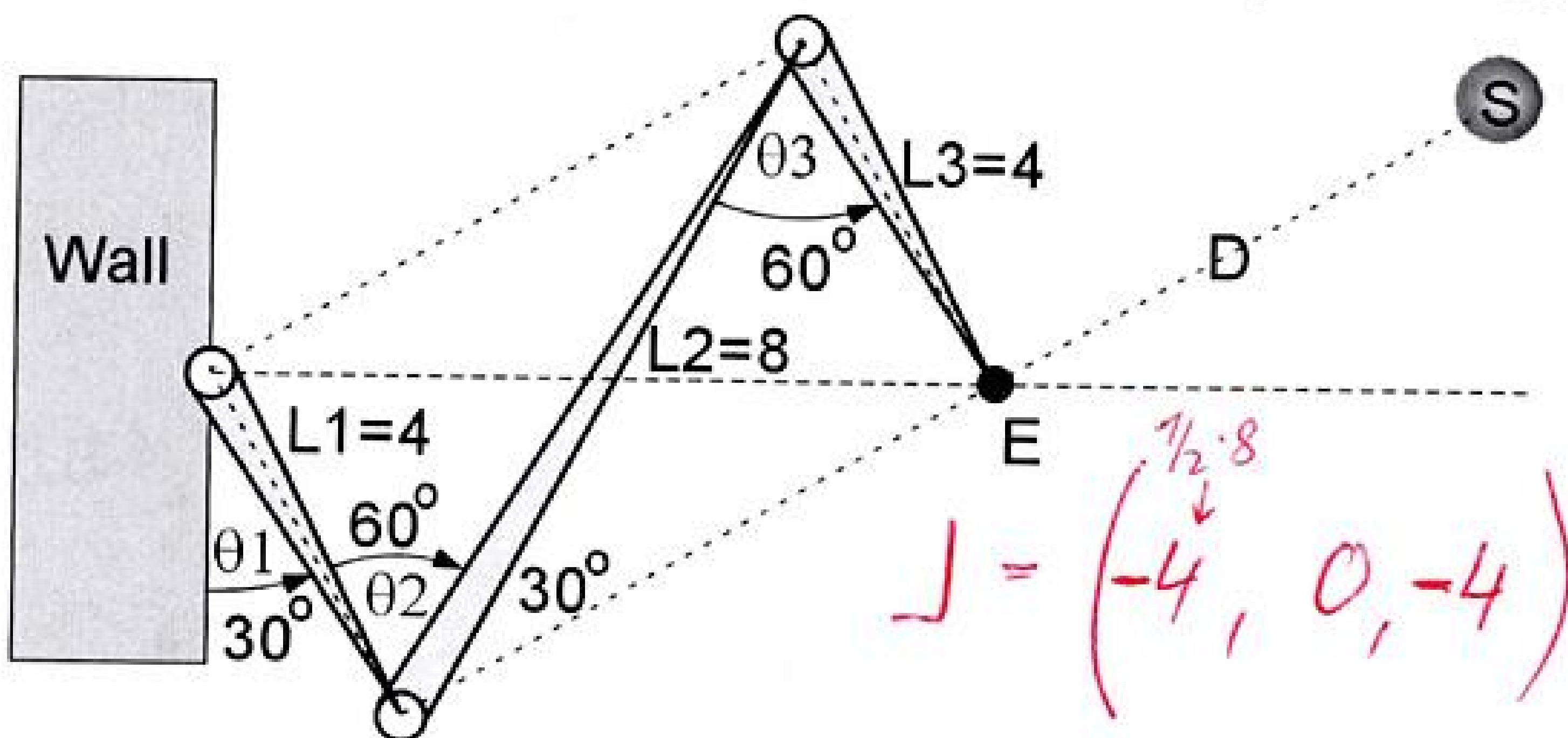## Problem # 14 — Perspective Warp (10 pts.)

A bundel of lines intersects at location (x, y, f) in the VRCS in 3-space. Where do they intersect after the {Shirley} perspective transform? (Here is the relevant homogeneous matrix):

$$M_p = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} x \\ y \\ f \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ nf+f^2-fn \\ f \end{pmatrix} \quad \text{to 3D} \quad \begin{pmatrix} nx/f \\ ny/f \\ f \end{pmatrix}$$

$$x = \underline{nx/f} \qquad y = \underline{ny/f} \qquad z = \underline{f}$$

Problem # 15 — Inverse Kinematics (10 pts.)

Write out the (**numerical entries** in the) Jacobian for the 3-link articulated arm with respect to the distance D of its end-effector E from the center of the sphere S, based on the three actuator angles $\theta 1$, $\theta 2$, $\theta 3$, (measured in radians). { sin 30° = cos 60° = 0.5; sin 60° = cos 30° = 0.866 }

Wall

L3=4

$\theta 3$

60°

L2=8

S

D

E

L1=4

$\theta 1$

60°

$\theta 2$

30°

30°

$$J = \begin{pmatrix} -4, & 0, & -4 \end{pmatrix}$$

½·8

Problem # 16 — Fill in the blanks with an appropriate answer (10 pts.)

The ___*inverse*___ of an orthonormal matrix is equal to its transpose.

The BRDF of a general, natural surface has __5__ argument(s);
         the BRDF of an ideal Lambert surface has __1__ argument(s) (variables).

The assignment of color to individual faces starts at *the bottom / leaves* of the scene hierarchy.

The Cohen-Sutherland line clipping algorithm relies on the *outcodes* of the end-points.

Problem # 17 — Ambient Occlusion (8 pts.)

In the silhouette below mark the locations with maximum ("* max") and minimum ("* min") ambient occlusion. Assume a simple hemispherical sky model with no distance penalty.
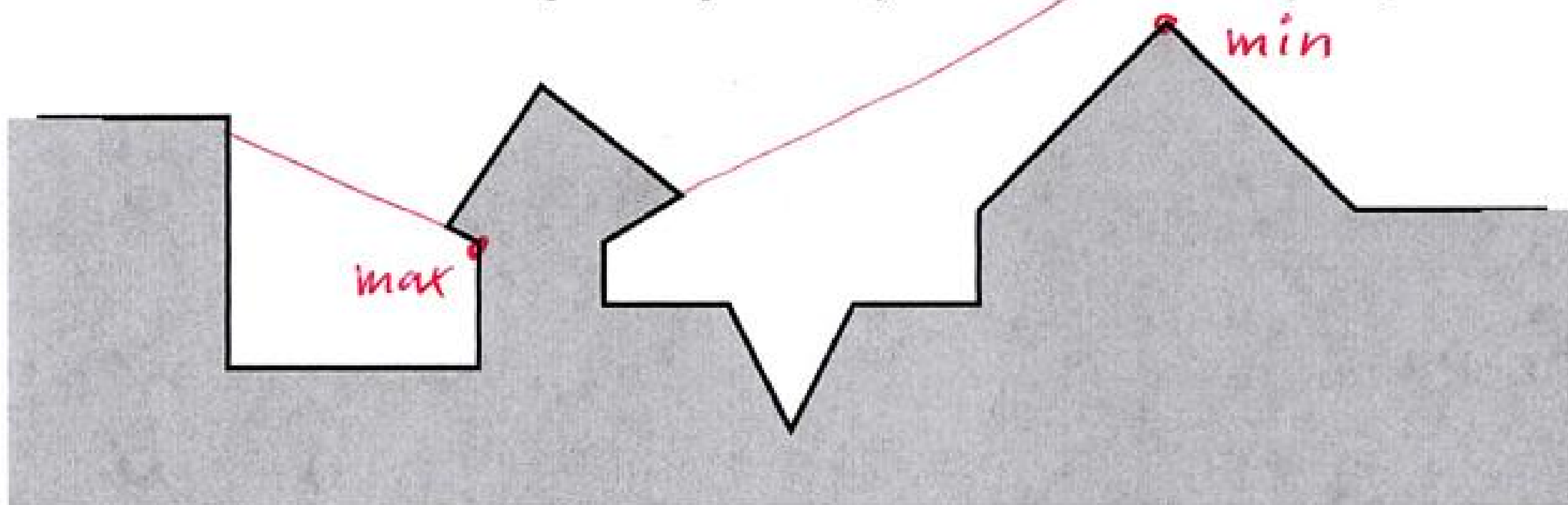
Problem # 18 — Catmull-Clark Subdivision (8 pts.)

Draw the result of applying one iteration of Catmull-Clark subdivision to the mesh below. Then **circle strongly** all vertices (both original and new ones created) that are **extraordinary**.