# EE40 Spring 2011 Midterm 1 Solutions

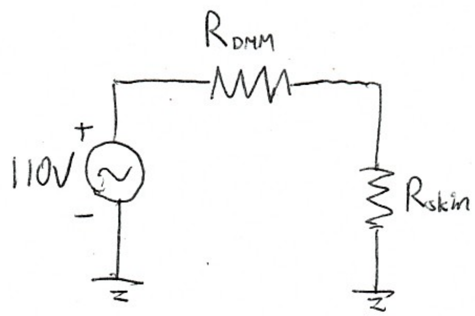
## Problem 1:

Identical to a problem from last year's midterm and a problem from HW6. The most common mistake was to assume that the voltmeter was a short, which is not true. Voltmeters have high resistance; ammeters have low resistance!

5. Suppose you stand barefoot on a wet floor with a hand-held digital voltmeter (DVM) in one hand. You insert one probe of the DVM into the hot output of a 110 V outlet and touch the other probe with your free hand.

Hypothetical experiment—don't try this!

a) Draw a circuit schematic of the situation. Use only circuit elements—sources and resistors—and their correct symbols. Do not draw pictures of elements. Assign reasonable values to all circuit components.




$R_{DMM} \sim 1M\Omega - 10M\Omega$

$R_{skin} \sim 10k\Omega - 100k\Omega$

We accepted values between 1k and $1M\Omega$ for $R_{skin}$. 10 pts. 3

b) Would you get hurt? Explain!

No. $R_{DMM}$ and $R_{skin}$ form a voltage divider. So, voltage across skin is relatively small.

$$V_{skin} \sim \frac{R_{skin}}{R_{skin} + R_{DMM}} \sim \frac{10k}{10k + 1M} \approx 1.1V, \text{ which is safe.}$$

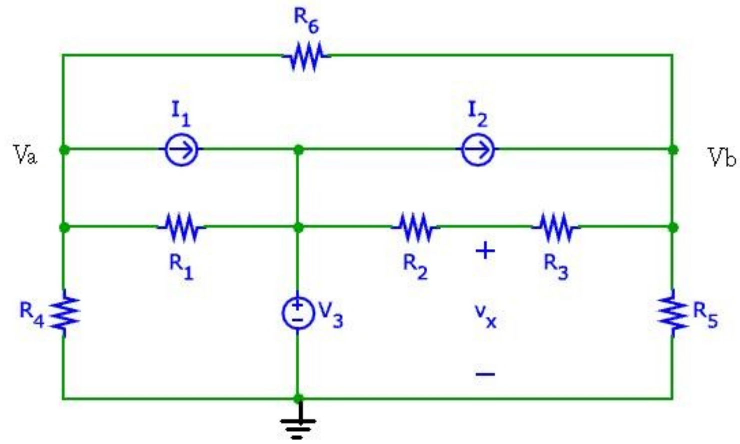
10 pts. 4

Or, since $R_{DMM}$ and $R_{skin}$ are in series,

$$I < \frac{V}{R_{DMM}} = \frac{110V}{1M\Omega} = 0.11mA, \text{ which won't hurt you.}$$

**Problem 2:**

Identical to a problem from HW6. The easiest way to do this problem is Node Voltage Analysis.




$$\frac{Va-Vb}{R6}+I1+\frac{Va-V3}{R1}+\frac{Va}{R4}=0$$
$$\frac{Va-Vb}{R6}+I2+\frac{V3-Vb}{R23}=\frac{Vb}{R5}$$

2 equations, 2 unknowns. Most of you did this part right. Some of you also wrote an equation for Vx, giving you 3 equations and 3 unknowns, which is ok too.

One mistake we saw pretty frequently was writing a KCL at $V_3$. This doesn't work because you can't easily express the current running through $V_3$.

Some of you tried other techniques such as mesh current analysis (5 equations, 5 unknowns) and superposition. In general, success rates using these techniques were pretty low.

Solving these equations algebraically gets pretty messy so most of you decided to proceed numerically. The values for Va and Vb are:

| **Version** | **Va** | **Vb** |
|---|---|---|
| 1 | 1.92V | 11.38V |
| 2 | -2.19V | 13.92V |
| 3 | 0.75V | 10.35V |

From there, we can write:

$$V_x = V_3 + \frac{(V_b - V_3)\,R_2}{R_2 + R_3}$$

Quite a few of you just did a voltage divider without offsetting the result by $V_3$. The correct numerical answers are:

| Version | Vx |
|---|---|
| 1 | 3.84V |
| 2 | 6.78V |
| 3 | 8.67V |

**Problem 3:**

Surprise, surprise! Another problem straight from homework… the solution is reproduced below:

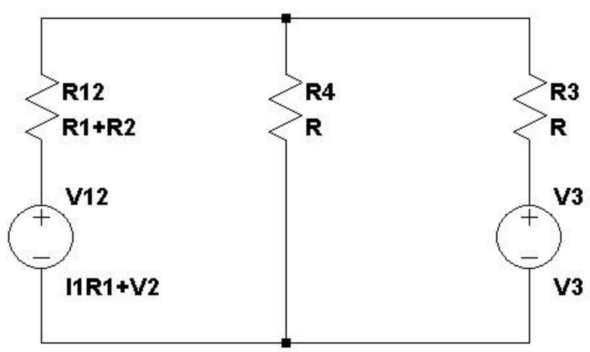
We note that the circuit can be greatly simplified through source transformations. Start off by transforming I1 into a voltage source.

$$V_1 = I_1 R_1$$

$V_1$ and $V_2$ as well as $R_1$ and $R_2$ can now be added in series.

$$V_{12} = I_1 R_1 + V_2$$

$$R_{12} = R_1 + R_2$$



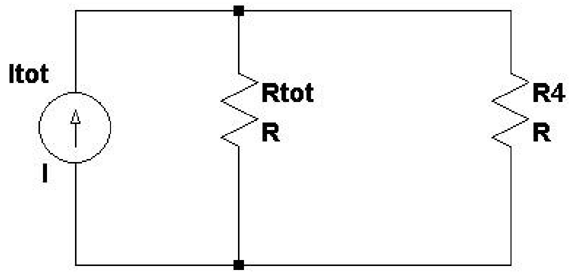

Next, we note that if we transform the two voltage sources into current sources, we can combine them in parallel.

$$I_{12} = \frac{V_{12}}{R_{12}} = \frac{I_1 R_1 + V_2}{R_1 + R_2}$$

$$I_3 = \frac{V_3}{R_3}$$

$$I_{tot} = I_{12} + I_3 = \frac{I_1 R_1 + V_2}{R_1 + R_2} + \frac{V_3}{R_3}$$

$$R_{tot} = R_{12} \parallel R_3 = \frac{(R_1 + R_2) R_3}{R_1 + R_2 + R_3}$$




Some of you might be tempted to combine R4 into Rtot, to simplify the circuit even further. However, we want to leave R4 intact because our final objective is to find the current flowing through R4. We lose that information if we lump R4 into everything else!

Finally, we note that all we're left with is a current divider. Thus,

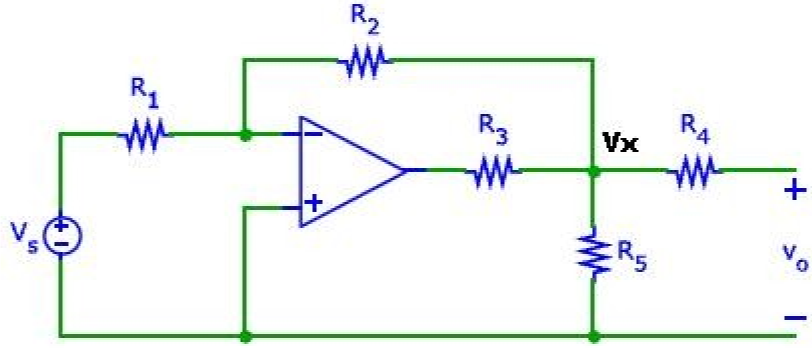
$$ix = I_{tot} \cdot \frac{R_{tot}}{R_{tot} + R_4}$$

The numerical answers for $I_{tot}$, $R_{tot}$, and $i_x$ are:

| **Version** | $\mathbf{I_{tot}}$ | $\mathbf{R_{tot}}$ | $\mathbf{i_x}$ |
|---|---|---|---|
| 1 | 5.25mA | 3.2kOhm | 1.50mA |
| 2 | 3.57mA | 3.94kOhm | 1.57mA |
| 3 | 10.4mA | 0.875kOhm | 1.16mA |

**Problem 4:**

The circuit in this problem is similar to one you analyzed for one of the practice assignments.



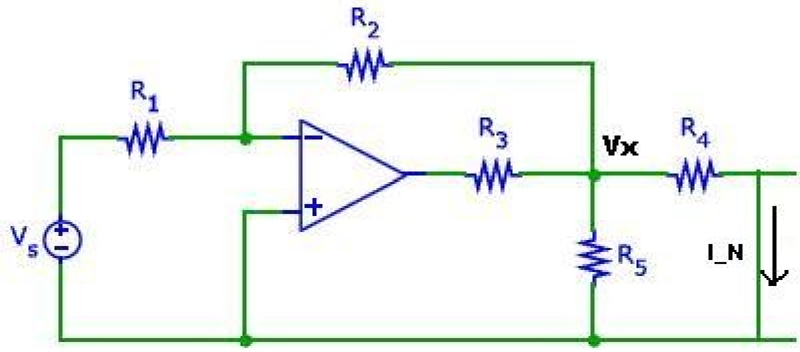

In particular, we note that since the op-amp can output any amount of current, the voltage Vx is simply:

$$V_X = V_S \cdot \frac{R_2}{R_1}$$

If you don't believe this, write the KCL equation at the negative terminal of the op-amp.

In addition, $R_5$ has no effect on the voltage at Vx.

With these observations, this problem becomes trivial. We want to model the circuit as a current source in parallel with a resistor. To find $I_N$, we short the output at find the current.




$$I_N = \frac{V_X}{R_4} = V_S \cdot \frac{R_2}{R_1 R_4}$$

To find $R_N$, we find the open circuit voltage.

$$V_{OC} = V_X = V_S \cdot \frac{R_2}{R_1}$$

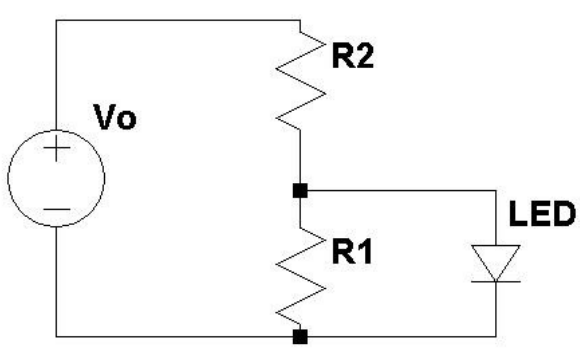
$$R_N = \frac{V_{OC}}{I_N} = R_4$$

An alternative method of finding $R_N$ is to turn off $V_S$ and note that the resistance seen at the output is simply $R_4$.

| **Version** | $\mathbf{I_N}$ | $\mathbf{R_N}$ |
|---|---|---|
| 1 | 0.544mA | 9.8kOhm |
| 2 | 3.36mA | 1.1kOhm |
| 3 | 3.42mA | 9.5kOhm |

**Problem 5:**

This is the approach we were expecting:



We need at least 2V to get any current through the LED. Thus, there is no way the LED can be on at $V_o$=1V so we don't have to worry about that condition. As for the other condition, we match the range of "bright on" to the $V_o$ range. In other words,

$I_D$ = 1.5mA when $V_o$ = 3V
$I_D$ = 4.5mA when $V_o$ = 5V

From the graph, we see that 2.5V gives 1.5mA of current and 3.5V gives 4.5mA of current. From this, we can write two equations (is this starting to look like a homework problem?)

Case 1 ($V_o$=3V):

$$\frac{3V-2.5V}{R_2}=\frac{2.5V}{R_1}+1.5mA$$

Case 2 ($V_o$=5V):

$$\frac{5V-3.5V}{R_2}=\frac{3.5V}{R_1}+4.5mA$$

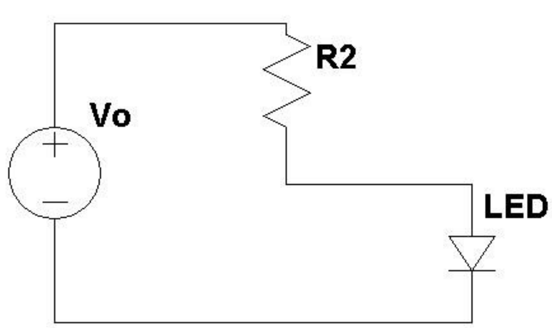
Isolating for $R_2$ for both equations the equating the two:

$$\frac{0.5V}{\left(\frac{2.5V}{R_1}+1.5mA\right)}=\frac{1.5V}{\left(\frac{3.5V}{R_1}+4.5mA\right)}$$

$$\frac{1.75}{R_1}+0.00225=\frac{3.75}{R_1}+0.00225$$

$$\frac{1.75}{R_1}=\frac{3.75}{R_1}\ \text{(what?!)}$$

Some of you might have gotten to this point and given up on this approach. Think! How could the above expression possibly hold? …What if $R_1$ is infinity? Remember that an infinite resistance is just an open circuit. Thus, we can just remove $R_1$ entirely.



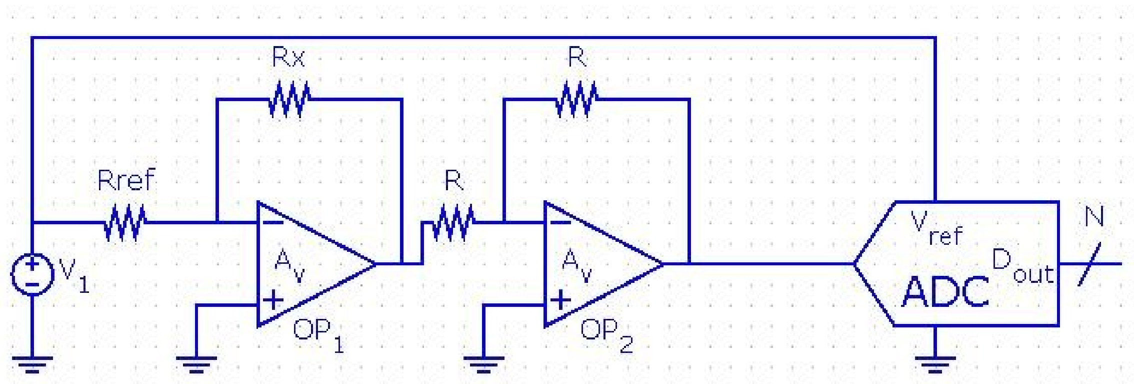

Plugging back into one of the original equations:

$$R_2 = \frac{0.5V}{\left(\frac{2.5V}{R_1} + 1.5mA\right)} = 333\Omega$$

Yeah, the solution is just a 333-Ohm series resistor.

**Problem 6:**

We might have done you a great disservice by drawing $R_X$ outside the box. This is the solution we were expecting:




The output of the first stage is:

$-V_1 \cdot \frac{R_X}{R_{ref}}$, which we note increases linearly with $R_X$

The second stage inverts the signal, since we want the input to be positive. Thus, the input to the ADC is:

$$V_{in} = V_1 \cdot \frac{R_X}{R_{ref}}$$

$D_0 = 1000 \cdot \frac{V_{in}}{V_{ref}} = 1000 \cdot \frac{R_X}{R_{ref}}$, which is what we want

We saw some pretty creative solutions. A good fraction of you reversed the positions of $R_X$ and $R_{ref}$ but connected $V_1$ to $V_{in}$ and the output of the opamp to $V_{ref}$. This works great on paper and we gave full credit for this solution. In practice, however, we generally want $V_{ref}$ to be constant.

Some of you tried to use $V_1$ as a dual supply. Unfortunately, this approach doesn't work because it results in a floating source.

## Moral of this midterm: Do the homework!