

## Final Exam

Dec 15, 2009

Total Time Allotted: 3 hours

### **DO ALL WORK ON EXAM PAGES**

**(Exam paper has a total of 9 pages including cover page)**

1. Closed book exam. You are allowed to bring 3 sheets (8.5" x 11") of notes.
2. You can use a calculator. NO cell phone or computer.
3. If you put down the wrong answer, partial credits will be given only if you show the correct steps.
4. Points will be taken off for answers without units.

Last (Family) Name: **Solutions** \_\_\_\_\_

First Name: \_\_\_\_\_

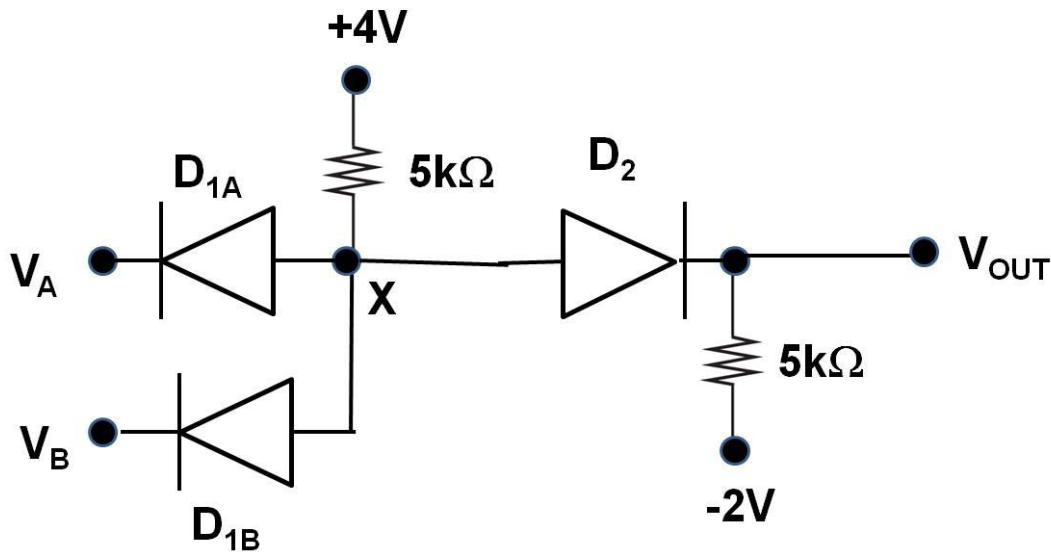
Student ID: \_\_\_\_\_ Discussion Session (# or TA): \_\_\_\_\_

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<b>Problem 1 (18 points)</b>	
<b>Problem 2 (25 Points)</b>	
<b>Problem 3 (22 Points)</b>	
<b>Problem 4 (25 Points)</b>	
<b>Problem 5 (25 points)</b>	
<b>Problem 6 (15 Points)</b>	
<b>Problem 7 (15 points)</b>	
<b>Problem 8 (25 Points)</b>	
<b>Problem 9 (30 points)</b>	
<b>TOTAL (200 points)</b>	

### Problem 1 Diode Circuits (18 points total)

All pn diodes shown below have a voltage drop of 0.6V when they are “ON”.



Indicate in the table whether the diodes  $D_{1A}$ ,  $D_{1B}$  and  $D_2$  are **ON/OFF** with the given  $V_A$  and  $V_B$  values ( 1point each) . Also give the corresponding  $V_{OUT}$  numerical values (3 points each) . NO PARTIAL CREDIT WILL BE GIVEN FOR THIS PROBLEM.

$V_A$	$V_B$	$D_{1A}$ (ON/OFF)	$D_{1B}$ (ON/OFF)	$D_2$ (ON/OFF)	$V_{OUT}$ (sign and numerical value)
0V	0V	ON	ON	ON	0V
0V	+4V	ON	OFF	ON	0V
+4V	+4V	OFF	OFF	ON	+0.7V

$V_A=0V$ ,  $V_B=0V$ ,  $D_{1A}$ is ON,  $D_{1B}$  is ON,  $V_X=+0,6V$ .  $D_2$  is ON.  $V_{OUT}=0V$

$V_A=0V$ ,  $V_B=+4V$ ,  $D_{1A}$  is ON,  $D_{1B}$  is OFF,  $V_X=+0,6V$ .  $D_2$  is ON.  $V_{OUT}=0V$

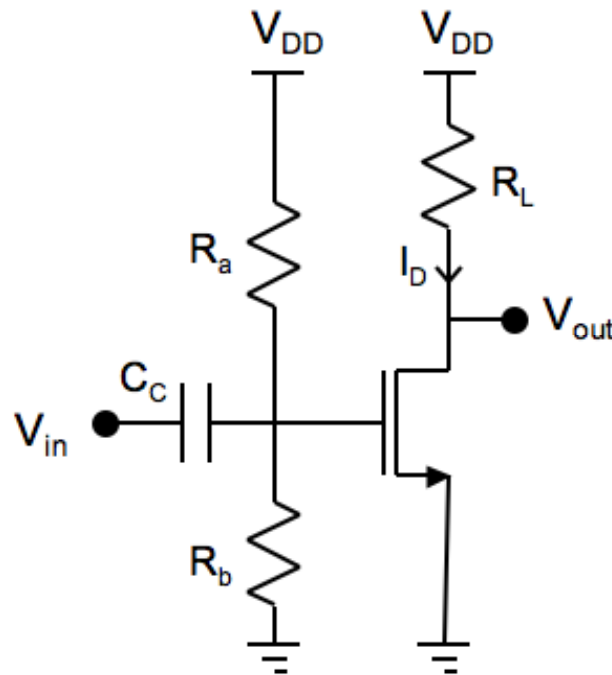
$V_A=+4V$ ,  $V_B=+4V$ ,  $D_{1A}$  is OFF,  $D_{1B}$  is OFF,  $D_2$  is ON ,

Current through  $D_2 = (4-(-2)-0.6)/10k\Omega = 5.4 \times 10^{-4}A$

$V_{OUT} = -2V + 5k\Omega \times 5.4 \times 10^{-4}A = -2V + 2.7V = +0.7V$

**Note:**  $V_X = 4V - 5k\Omega \times 5.4 \times 10^{-4}A = 4 - 2.7V = +1.3V$  (consistent with  $D_2$  ON)

Problem2: MOSFET Amplifier (25 points total)



Consider the N-channel MOSFET amplifier given above.  $i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$ ,  $V_{DD}=5V$ ,  $R_L=2k\Omega$ ,  $K=1mA/V^2$ ,  $V_T=1V$ . You can ignore  $r_d$  of the MOSFET.  $C_C$  is the input coupling capacitor. You can assume it is infinitely large.

- a) (3 points) Derive an expression for the transistor bias point  $V_{GSQ}$  as a function of  $V_{DD}$ ,  $R_a$  and  $R_b$ .

This is just a resistive divider,  $V_{GS} = V_{DD} \frac{R_b}{R_b + R_a}$

- b) (10 points) Determine the required ratio  $R_a/R_b$  such that the MOSFET  $g_m = 1mA/V$ .

Remember that  $g_m$  is defined as  $\frac{\partial i_{DS}}{\partial V_{GS}}$ .

First calculate  $V_{GS}$  from the equation for  $g_m$

$$g_m = \frac{\partial i_{DS}}{\partial V_{GS}} = K(V_{GS} - V_T) = 1mA/V$$

$$= 1mA/V^2 (V_{GS} - 1V) = 1mA/V$$

$$(V_{GS} - 1V) = 1V$$

$$V_{GS} = 2V$$

Next use the result from part a to determine the required ratio

$$V_{GS} = V_{DD} \frac{R_b}{R_b + R_a} = 5V \frac{R_b}{R_b + R_a} = 2V$$

$$\frac{R_b}{R_b + R_a} = \frac{2}{5}$$

$$R_a = 1.5R_b$$

- c) (6 points) What is the voltage bias point of the output  $V_{outQ}$ ? (If you cannot calculate a numerical answer leave the answer it in terms of the given circuit parameter values,  $R_a$  and  $R_b$ ). Use the results from part(b).

First solve for the quiescent current, then calculate the voltage:

$$I_{DS} = \frac{K}{2} (V_{GS} - V_T)^2 = \frac{1mA/V^2}{2} (2V - 1V)^2 = 0.5mA$$

$$V_{OUT} = V_{DD} - I_{DS} R_L = 5V - 0.5mA \cdot 2k\Omega = 4V$$

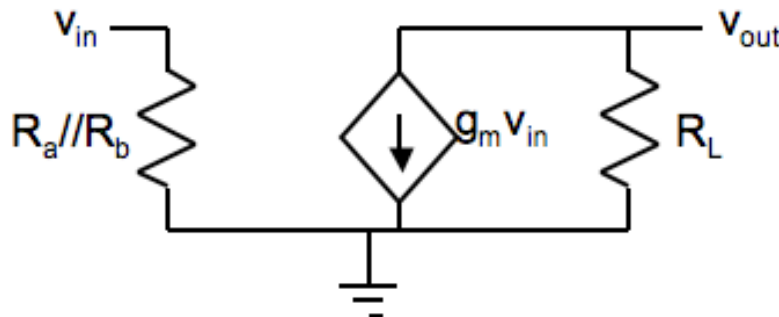
In terms of circuit parameters:

$$I_{DS} = \frac{K}{2} (V_{GS} - V_T)^2 = \frac{K}{2} \left( V_{DD} \frac{R_b}{R_b + R_a} - V_T \right)^2$$

$$V_{OUT} = V_{DD} - I_{DS} R_L = V_{DD} - \frac{K}{2} \left( V_{DD} \frac{R_b}{R_b + R_a} - V_T \right)^2 R_L$$

- d) (6 points) Draw the small-signal model for the amplifier and calculate the gain  $v_{out}/v_{in}$ . Clearly label the component values and small-signal voltages  $v_{in}$  and  $v_{out}$ . Use the results from part(b).

The small-signal model:

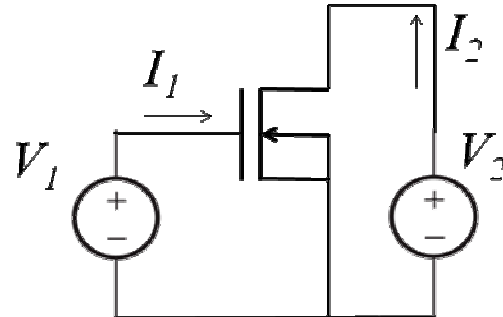


The gain is just  $g_m \cdot R_L = 2$

### Problem 3 MOSFET and diode: Qualitative behavior (22 points total)

a) You are given the following circuit, and the magical ability to change each of the following input parameters independently:  $V_1$ ,  $V_2$ ,  $K$ ,  $V_T$ . You also measure each of the following parameters:  $I_1$ ,  $I_2$ ,  $g_m$ ,  $r_d$ . Indicate in the table how an increase in each of the “input” parameters changes each of the “output” parameters. Please use these symbols:  $\uparrow$  = increase,  $\downarrow$  = decrease, -- = no change.

You may assume that the transistor is in saturation.



	$I_1$	$I_2$	$g_m$	$r_d$
$V_1$	--	$\uparrow$	$\uparrow$	$\downarrow$
$V_2$	--	$\uparrow$	$\uparrow$	$\downarrow$
$K$	--	$\uparrow$	$\uparrow$	$\downarrow$
$V_T$	--	$\downarrow$	$\downarrow$	$\uparrow$

$I_1$  is always zero, and is not affected by any of the input parameters.

- $I_2 = K(V_1 - V_T)^2(1 + \lambda V_2)$   $I_2$  increases with  $K$ ,  $V_1$  and  $V_2$ , and decreases with  $V_T$ .
- $g_m = 2\sqrt{KI_2}$   $g_m$  varies the same way as  $I_2$ .
- $r_d = \frac{1}{\lambda I_2}$   $r_d$  varies in the opposite way from  $I_2$ .

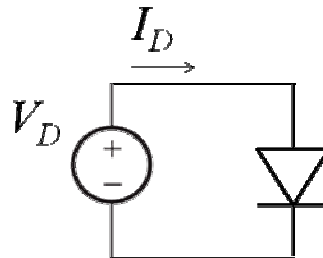
### Problem 3 continued

(b) The following diode circuit and the parameters are shown in the table. Fill in the table how an increase in each of the “input” parameters  $V_D, I_S, V_{Th}$  changes each of the “output” parameters. Please use these symbols:

↑ = increase, ↓ = decrease, -- = no change.

$V_{Th}$  is the thermal voltage ( $kT/q$ ) and  $r_d$  is the small signal resistance

The diode I-V characteristics is :  $I_D = I_S (e^{V_D / V_{Th}} - 1) \cong I_S e^{V_D / V_{Th}}$



	$I_D$	$r_d$
$V_D$	↑	↓
$I_S$	↑	↓
$V_{Th}$	↓	↑

For the diode, we have the following equations:

$$I_D = I_S (e^{V_D / V_{Th}} - 1) \cong I_S e^{V_D / V_{Th}}$$

$$r_d = \frac{V_{Th}}{I_D}$$

(If you didn't have the equation for  $r_d$  on your formula sheet, you can easily derive it, or estimate from a sketch of  $I_D$ .) We see that:

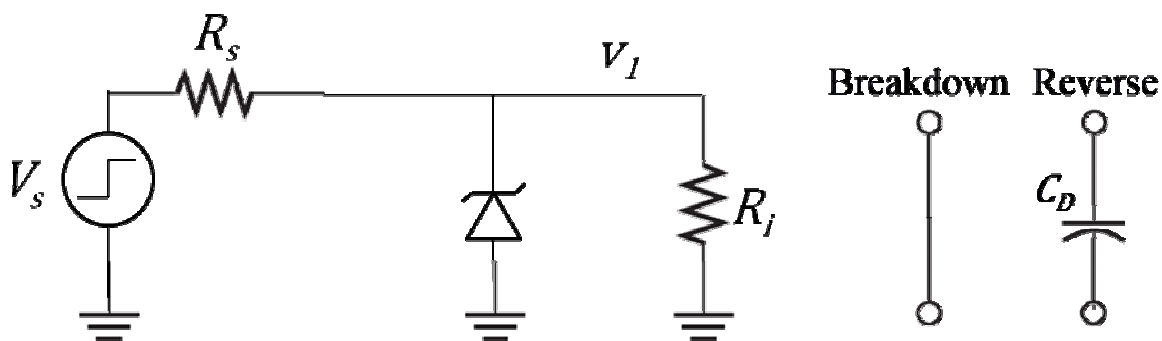
- $I_D$  increases with  $I_S$  and  $V_D$ , and decreases with  $V_{Th}$ .
- $r_d$  varies in the opposite way as  $I_D$ . For the dependence on  $V_{Th}$ , both the numerator and the denominator cause  $r_d$  to increase.

#### Problem 4 Diodes and transients (25 points total)

A circuit with input resistance  $R_i$  uses a Zener diode with breakdown voltage  $V_{BR}$  ( $=5V$ ) to protect itself from large input voltages.

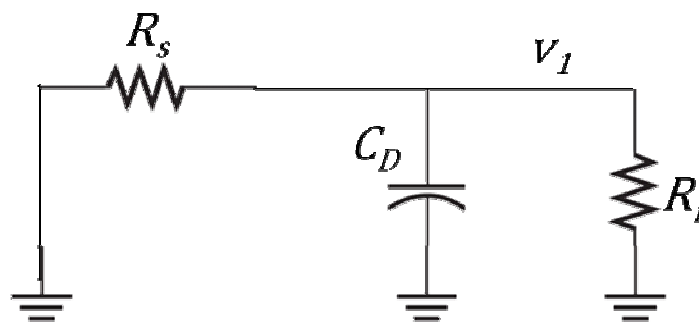
For this problem, we will augment the diode model used in class to include the diode's *capacitance* due to the p-n junction. The equivalent circuit of the Zener diode in the reverse-bias region is modeled as a capacitor  $C_D$  if there is no breakdown. The diode is modeled as a short plus  $V_{BR}$  if there is breakdown (see figures at right)

A large input voltage step with magnitude  $V_s$  is connected to the circuit through a resistance  $R_s$ . It is given that  $V_s = 14V$  and  $V_I(t=0) = 0$ . Let  $R_i = R_s$ .



- (a) Find an analytical expression for the time constant  $\tau$  in terms of  $R_s$ ,  $R_i$  and  $C_D$  when the diode is not conducting. (10 pts)

The natural response in the reverse-bias region can be found by setting the forcing function to zero and substituting a capacitor for the diode:

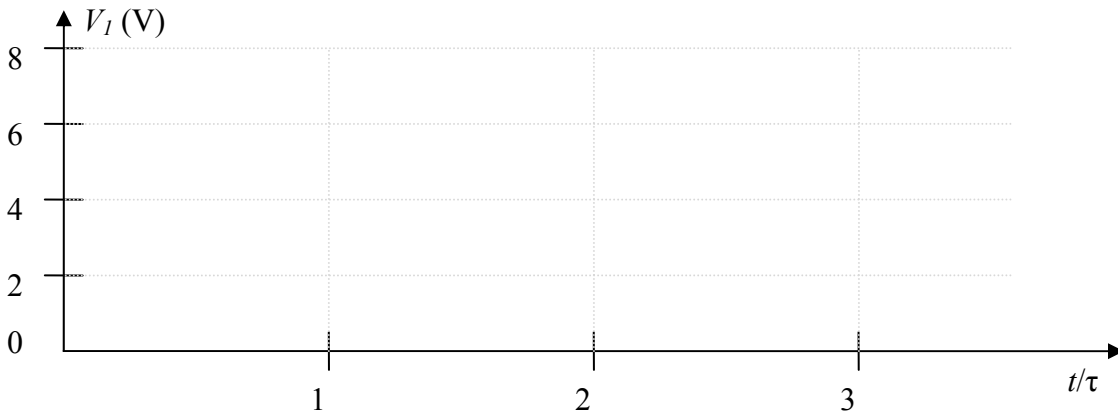


The resistors now appear in parallel. We can find the time constant in terms of the equivalent (parallel) resistance:

$$\begin{aligned}\tau &= R_{eq} C_D = (R_s || R_i) C_D = \frac{R_s R_i}{R_s + R_i} C_D \\ &= \frac{R_s}{2} C_D = \frac{R_i}{2} C_D\end{aligned}$$

- (b) Sketch the voltage  $V_I$ , versus time in units of  $\tau$ , on the axes given. Find the voltage at  $t = \tau, 2\tau, 3\tau$  and show the values on your plot. (15 pts) [Hint: Apply the method of assumed states for each time  $t$  and check consistency]

a)



Observe that this is a first-order transient with initial value 0V. Ignoring the breakdown of the diode (i.e. assuming that the model valid at  $t = 0$  remains valid for all time), the final value is given by a voltage divider consisting of two equal resistors:

$$v_f = V_s/2 = 7 \text{ V}$$

This allows us to find the value at  $t = \tau$ :

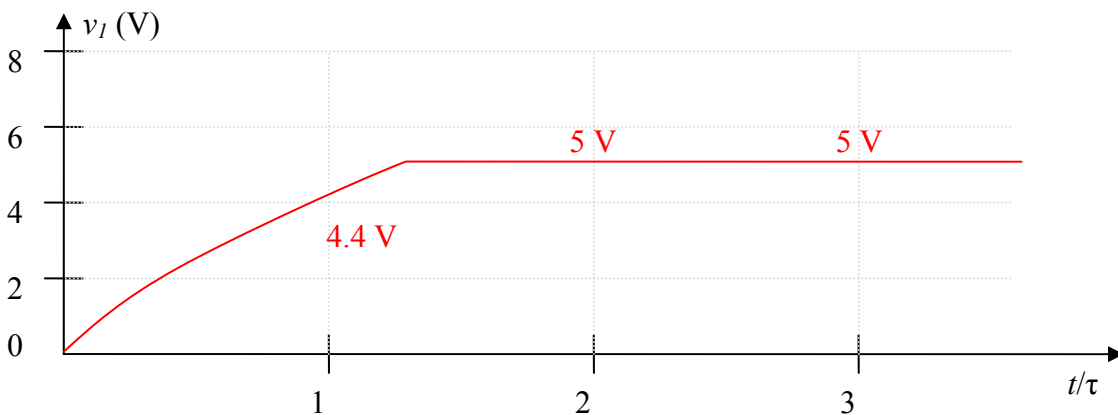
$$v_1(t = \tau) = v_f(1 - e^{-1}) = 4.42 \text{ V}$$

We can use the same approach to find the value at  $t = 2\tau$ :

$$v_f(1 - e^{-2}) = 6.05 \text{ V}$$

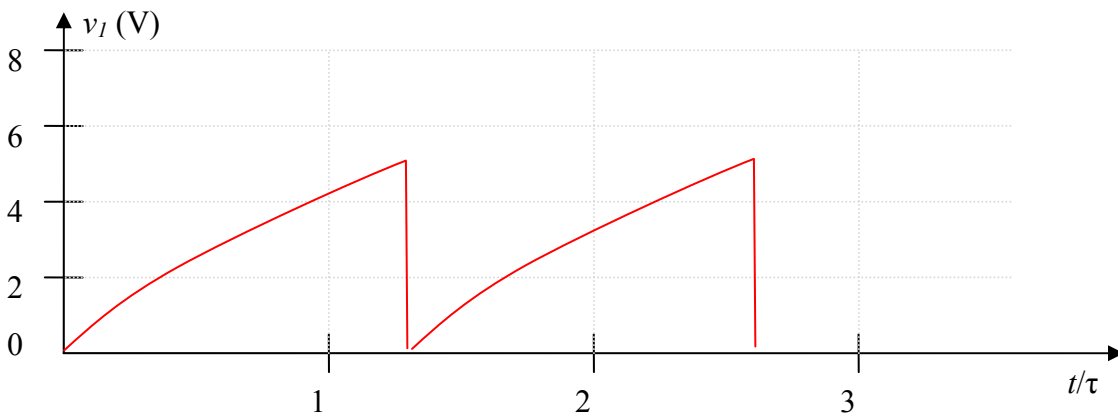
But this is greater than the diode's breakdown voltage. Therefore, somewhere between  $\tau$  and  $2\tau$ , the diode will start to conduct, and clip off any further increase in voltage. Therefore,

$$v_1 = V_{BR} = 5 \text{ V}$$





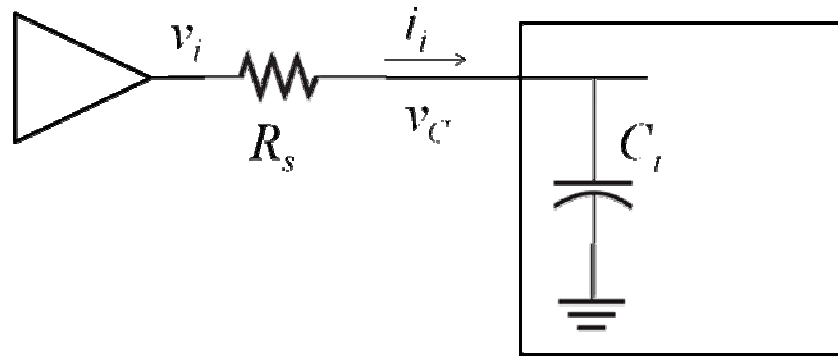
\*\*For those who ignore the  $V_{BR}$  in the Zener diode model after breakdown, the capacitance discharges immediately and completely at the onset of breakdown, returning the circuit to the state at  $t = 0^+$  and leading to a periodic waveform, like this:



You will also obtain full credit for this answer.

### Problem 5 Reactive Elements and Power (25 points total)

You are driving a sinusoidal signal  $v_i$  through a resistance  $R_s$ , into a device with a large parasitic input capacitance  $C_i$  ( $=100\text{pF}$ ). Besides the capacitor, no other current flows into the device.



- a) Does the current  $i_i$  lead or lag the applied voltage  $v_i$ ? (5 pts)

**The current leads the voltage**, because a capacitor is present, which differentiates voltage. Suppose the applied voltage is a sine function. At  $t = 0$ , the voltage is zero, but because the voltage is increasing, current is already flowing in the capacitor.

- b) Suppose  $v_C(t)$  is known to be  $4 \cos(\omega t)$  V. Calculate the current  $i_i(t)$  at 22 MHz. (5 pts)

First, find the current  $i_i$ :

$$v_C = V_C e^{j\omega t} = V_C \cos(\omega t)$$

$$\Rightarrow i_i = \frac{v_C}{Z_C} = \frac{V_C e^{j\omega t}}{Z_C}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\pi f C} = \frac{72.3}{j} \Omega$$

$$\Rightarrow i_i = 55.3 \text{ mA} \sin(\omega t)$$

- c) Calculate the peak power dissipated in the capacitor at 22 MHz? (10 pts)

Now we can find the power after knowing current, using the derivation described on Hambley p. 226:

$$p_C(t) = i_i v_C = V_C I_i \cos(\omega t) \sin(\omega t)$$

$$= V_C I_i \frac{1}{2} \sin(2\omega t)$$

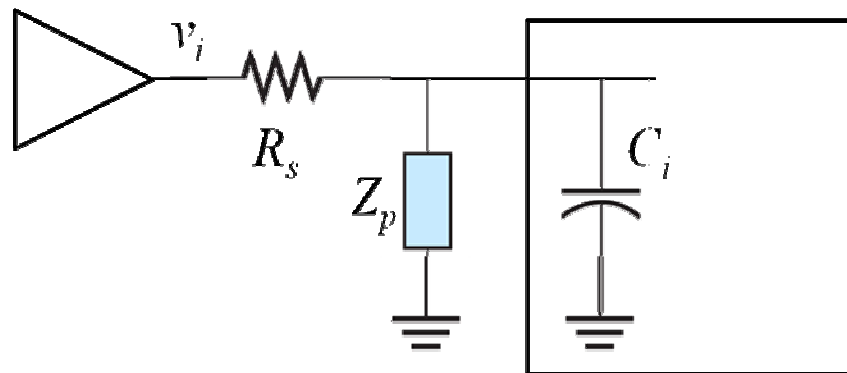
The peak power is just the amplitude of this waveform:

$$\Rightarrow P_C = |p_C| = \frac{V_C I_i}{2}$$

$$\Rightarrow P_C = \frac{4 \text{ V} \cdot 55.3 \text{ mA}}{2} = \mathbf{0.111 \text{ W}}$$

Note: The time-average power in a reactive element always averages to **zero** (as we can see in this particular case from the fact that it is a sinusoid).

d) You want to reduce the peak power consumption by adding an extra parallel element  $Z_p$  as shown. Indicate whether you will choose a **resistor**, **inductor**, or **capacitor**, and justify your choice in 10 words or less (further words will not be graded). (5 pts)

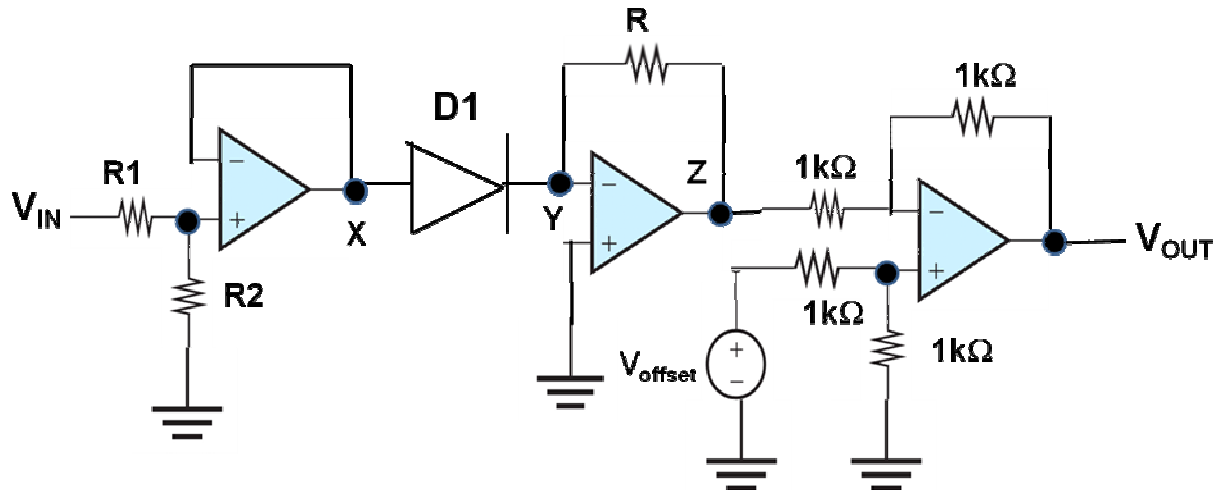


In order to reduce the power, we must reduce the current  $i_1$ . Only an inductor can achieve this. Possible justifications include:

- "Resonance" give infinite impedance of LC.
- "Increase impedance after resistor"
- "Positive reactance cancels negative reactance"

### Problem 6 Cascade Op Amp Circuits (15 points total)

The goal is to design an exponential amplifier such that  $V_{OUT} = \exp(V_{IN})$  with the following op amp circuit. Ideal op amp operation is assumed. Voltages are in units of volts.



The Diode D1 has I-V characteristics:  $I = 1\text{mA} \cdot [\exp(V_D/V_{Th}) - 1]$ , where the thermal voltage  $V_{Th}=0.026\text{V}$  at room temperature.

(a) (4 points) Choose the ratio  $R1/R2$  such that  $V_X/V_{IN} = 0.026$ . Show your work.

First op amp circuit is a voltage follower

$$V_X = V_{IN} \cdot R2/(R1+R2) \text{ or } 1 + R1/R2 = 1/0.026 = 38.46$$

Therefore  $R1/R2 = 37.46$

(b) (5 points) What numerical value of  $R$  (in  $\Omega$ ) will you choose? Show your work.

$$V_Z = -R \cdot I = -R \cdot 1\text{mA} \cdot [\exp(V_X/V_{Th}) - 1] = -R \cdot 1\text{mA} \cdot [\exp(V_{IN}) - 1]$$

We will choose  $R = 1\text{k}\Omega$  to make the pre-exponential factor = 1

$$V_Z = -[\exp(V_{IN}) - 1]$$

(c) (6 points) What numerical value of  $V_{offset}$  (in volts) will you choose? Show your work.

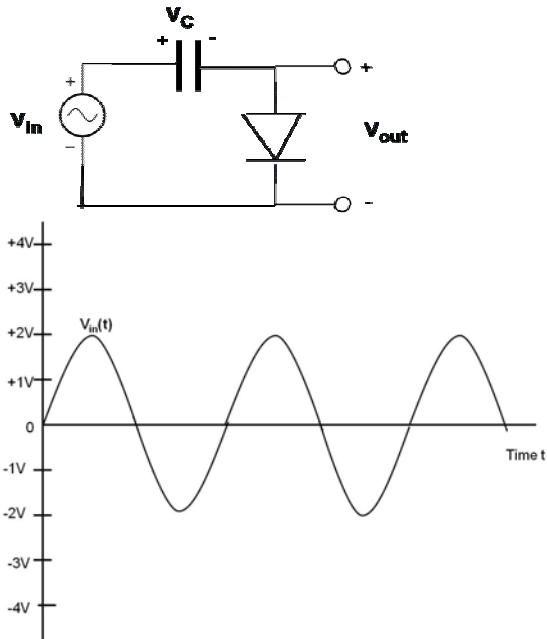
The Third op amp circuit is a differential inverting amplifier with gain = 1:

$$V_{OUT} = -(V_Z - V_{offset}), \text{ If we choose } V_{offset} = +1\text{V},$$

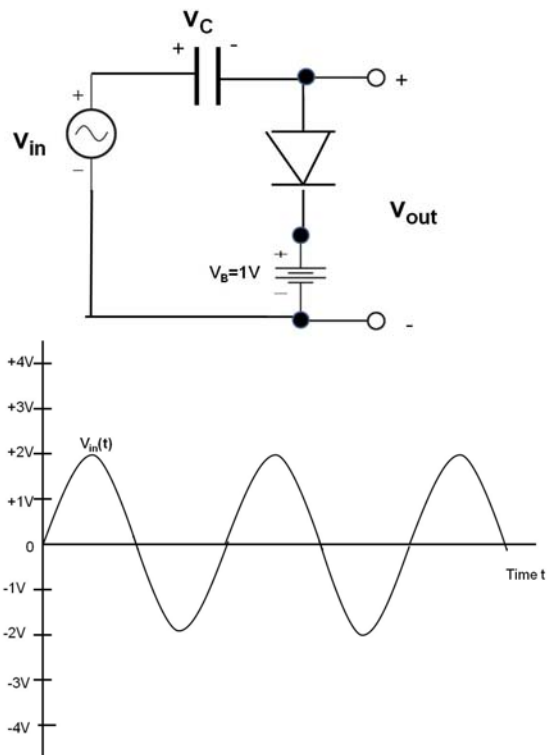
$$V_{OUT} = \exp(V_{IN})$$

### Problem 7 Wave Shaping Circuits (15 points total)

(A) (10 points) The following diode-capacitor circuit has  $v_{in}(t) = V_m \sin(\omega t)$  shown in the graph. The diode has perfect rectifier behavior.  $v_C$  is initially equal to 0 at  $t=0$ . Sketch  $v_C(t)$  using a dash line and sketch  $v_{out}(t)$  as a dotted line in the same graph for all times  $t$ .

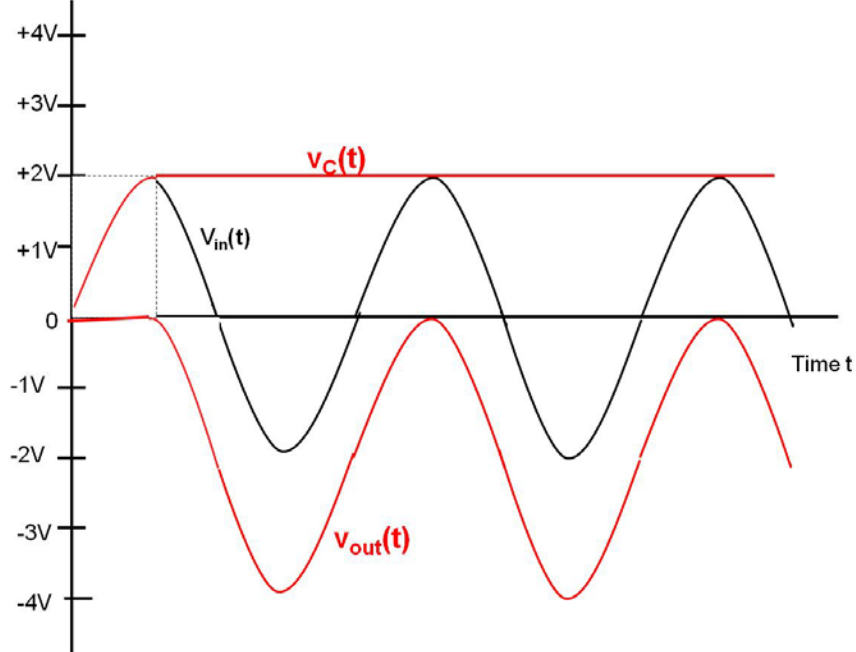


(B) (5 points) A DC voltage source  $V_B = +1V$  is added to the diode-capacitor circuit. Sketch only the **steady-state**  $v_{out}(t)$  in the same graph (i.e. after one cycle of  $V_{in}$ ).

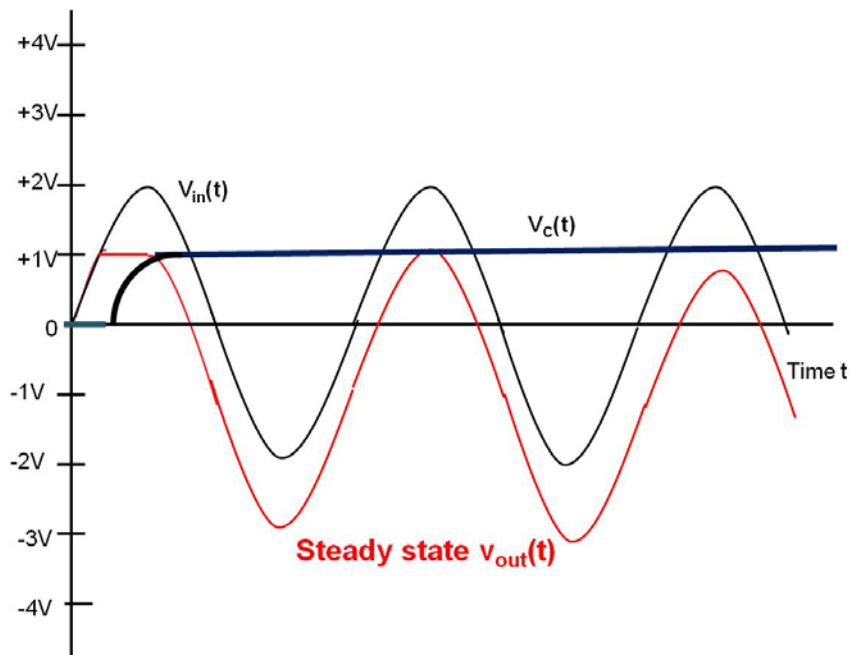


### Problem 7 answers

(A)  $V_{\text{out}}(t) = V_{\text{in}}(t) - V_{\text{c}}(t)$



(B) You only have to show the steady state waveform to obtain full credit.



### Problem 8 Miscellaneous Questions (25 points total)

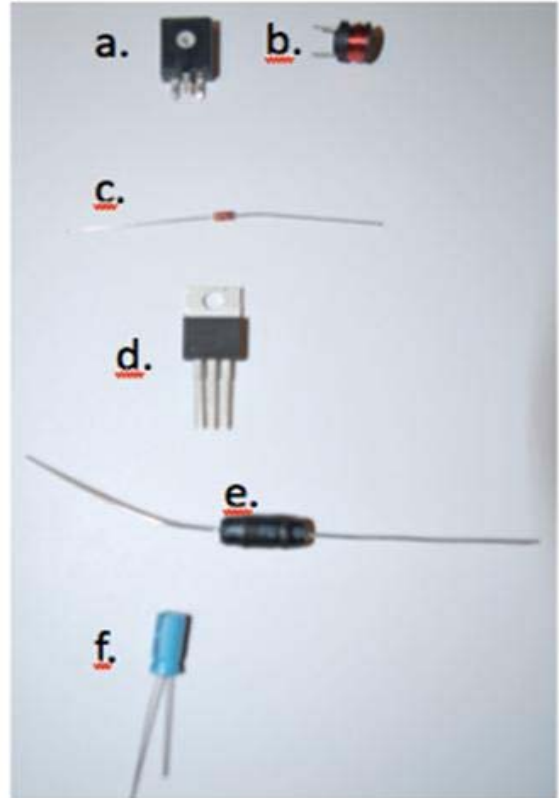
Please circle either True (T) or False(F) for the following statements:

- |   |   |   |
|---|---|---|
| T | F | Miniaturization gives faster integrated circuit performance                                   |
| T | F | A state-of-the-art solid-state transistor has critical dimensions on the order of 45 nm.      |
| T | F | The maximum power that a solar cell can deliver in practice equals $I_{sc} \bullet V_{oc}$ .  |
| T | F | KVL and KCL can be applied to circuits with nonlinear circuit elements                        |
| T | F | Superposition Principle can be applied to circuits with nonlinear circuit elements            |
| T | F | CMOS logic circuits dissipated power only during switching                                    |
| T | F | Ideal capacitors and inductors have zero instantaneously power at all times.                  |
| T | F | An ideal OP Amp has zero gain for common-mode input signal                                    |
| T | F | A real OP Amp voltage follower circuit has very large input resistance                        |
| T | F | For MOSFETs with fixed $V_{GS}$ , triode region has less current than saturation region.      |
| T | F | All combinatorial logic functions can be synthesized by NOR gates alone.                      |
| T | F | Mesh analysis has fewer independent equations than Nodal Analysis in general                  |
| T | F | The Transresistance Amplifier Model uses a current-controlled voltage source.                 |
| T | F | Amplifier will have no waveform distortion if phase shift is linear proportional to frequency |
| T | F | We can use SOP Method to simplify sequential logic design.                                    |
| T | F | Bias and offset current sources are desirable for Differential Amplifiers.                    |
| T | F | Harmonic distortion of an amplifier will become bigger with larger signal amplitudes.         |
| T | F | Overshooting of amplifier with impulse input is related to the amplifier's bandwidth          |
| T | F | A pure resistive load has maximum reactive power.   |
| T | F | A full wave rectifier circuit requires at least 4 diodes                                      |
| T | F | Magnitude Bode Plot of any RLC circuit cannot have a slope larger than 20 dB/decade           |
| T | F | A Ripple Counters requires both the input signal and a separate clock signal.                 |
| T | F | MOSFET with the drain tied to the gate will operate in the saturation mode when it is ON.     |
| T | F | A pn diode can be used as a photodetector.  |
| T | F | "Don't care" as output in truth tables will give simpler Boolean algebraic expressions.       |

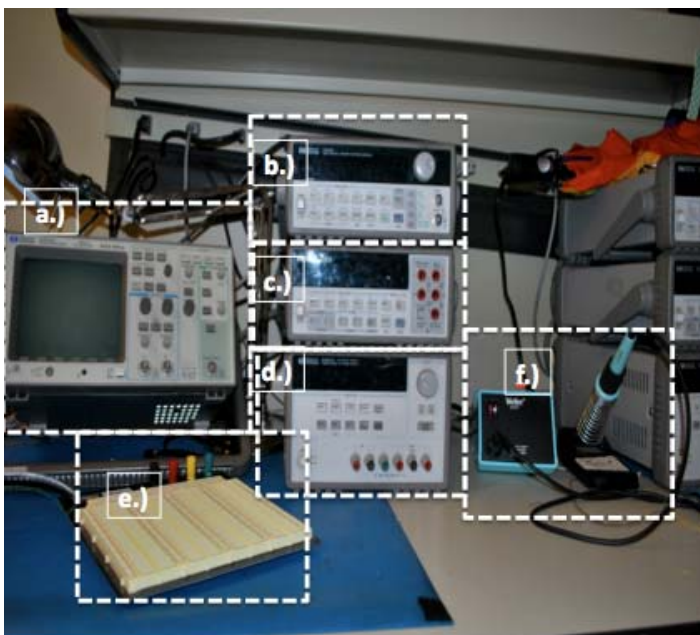
### Problem 9 LAB Questions (30 points total)

(A ) Write the names of circuit components shown below.

- a.) Potentiometer (1pt)
- b.) Inductor (1pt)
- c.) Diode (1pt)
- d.) Transistor (1pt)
- e.) Inductor (1pt)
- f.) Capacitor (1pt)



(B)- Write the name of the instruments' labeled below.



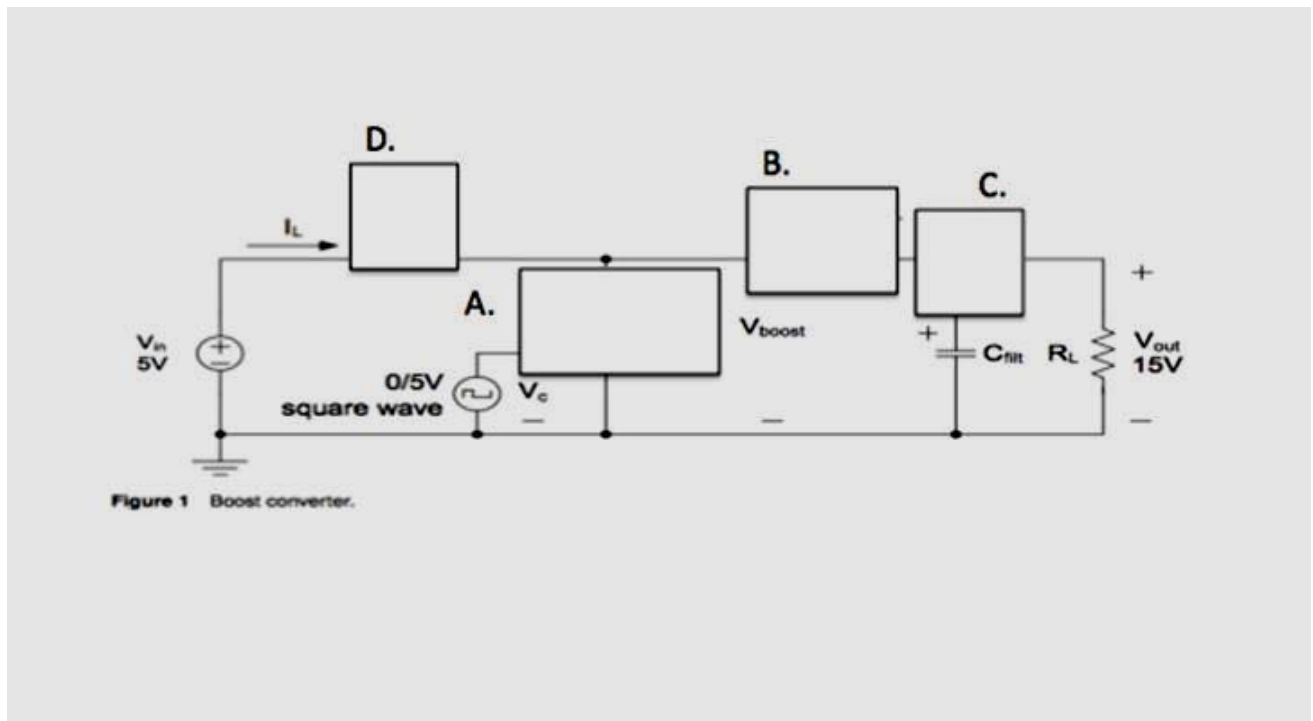
- a.) Oscilloscope (1pt)
- b.) Function Generator (1pt)
- c.) The Digital Multimeter (1pt)
- d.) Power supply (1pt)
- e.) Breadboard (1pt)
- f.) Soldering Iron/Solder (1pt)



(C) Which instrument do we use to measure current and voltage? (2pt)

- Multimeter
- Avometer

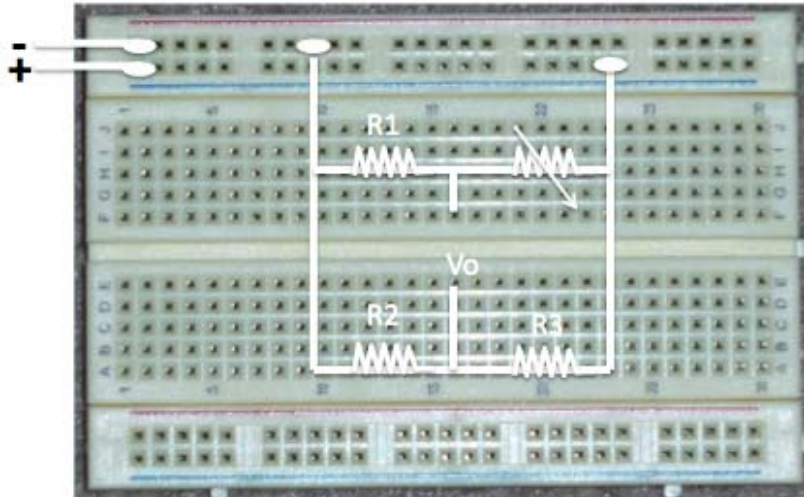
(D) Draw the missing components of the boost converter circuit. (6pt)



(E) Write the two basic components of a microcontroller. (2pt)

- CPU
- RAM
- In/Out Ports
- Timer

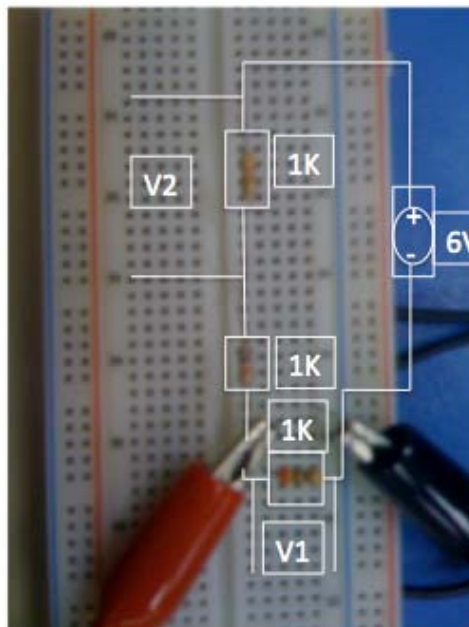
(F) Draw a full bridge (i.e. Wheatstoen Bridege) circuit diagram on breadboard with using symbols given below. (4pt)



(G) Please briefly explains, how does a *strain gauge work*? (2pt)

A **strain gauge** is a device used to measure the strain of an object. When an electrical conductor is stretched, it will become narrower and longer. Thus its electrical resistance increases end-to-end. Conversely, when a conductor is compressed, it will broaden and shorten, hence its electrical resistance decreases end-to-end.

(H) Write down voltage values of V1 and V2 in the circuit shown below? (2pt)



V1.) \_\_\_\_\_ **0V** (1pt)

V2.) \_\_\_\_\_ **3V** (1pt)