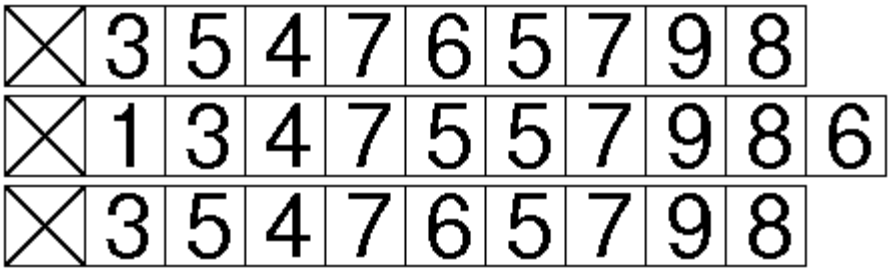
CS 61B: Data Structures (Autumn 2010)
Midterm II

Solutions

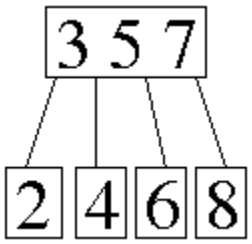
Problem 1. (10 points) **A Miscellany.**

a. Determining whether the graph contains a particular edge (x, y). Other acceptable answers include inserting an edge and deleting an edge. A particularly interesting solution, contributed by one student on his exam, is finding all the edges directed *into* a particular vertex. This takes $O(|V|)$ time with an adjacency matrix, and $O(|V| + |E|)$ time with an adjacency list.

b.



c.



d. `cdadcdad`.

e. Three acceptable answers (each worth full points):
If any one of the characters is zero, the hash code is zero.
Anagrams all have the same hash code.
A string's hash code doesn't change if you insert characters equal to one.

Problem 2. (8 points) **Algorithms.**

a, b. With `bottomUpHeap()`, create a heap containing the n numbers. This takes $\Theta(n)$ time. Then, call `removeMin()` j times, taking $\Theta(j \log n)$ worst-case time. The total worst-case running time is in $\Theta(n + j \log n)$.

An alternative solution uses a *max-heap*, which reverses the heap-order property, so that the greatest key is at the top of the heap. Starting from an empty max-heap, insert j keys (it doesn't matter which ones) into the heap. Then repeat the following operations for each remaining key: insert the key into the heap, and remove the maximum key from the heap. After you've processed all the keys, the heap contains the j smallest. At worst, this takes $\Theta(n \log j)$ time.

The optimal solution—which requires knowledge you won't learn until later in the semester—is to use a median-finding algorithm to find the jth-smallest key in $\Theta(n)$ expected time. Then, a linear scan through the array can find the other $j - 1$ smallest items, also in $\Theta(n)$ expected time.

c. Output-sensitive.

d. Starting from the root, walk down the tree. When you encounter a key less than k, go right. When you encounter a key greater than or equal to k, go left. Repeat until you reach a null pointer. (A useful intuitation is that this is equivalent to performing a `find` operation on the key $k - \varepsilon$, where ε is infinitesimal.)

As you go down the tree, keep track of the largest key you've encountered so far that's less than k. When you reach a null pointer, return that value. The tree might not contain any key less than k, in which case you can return `null` or some other value that means "no such key."

Problem 3. (7 points) **Sibling-based tree height.**

The main idea is that if `this` node has a next sibling, then `height()` should return the *maximum* height of `this` node and its siblings to the right. This information allows the parent to calculate its height. The other idea is to use a postorder traversal to calculate the heights.

```
public class SibTreeNode {
  public Object item;
  public SibTreeNode parent;
  public SibTreeNode firstChild;
  public SibTreeNode nextSibling;

  int height() {
    int h = 0;
    if (firstChild != null) {
      h = 1 + firstChild.height();
    }
    if (nextSibling != null) {
      int hSib = nextSibling.height();
      if (hSib > h) {
        return hSib;
      }
    }
    return h;
  }
}
```