EE 120: Signals and Systems
Department of EECS
University of California Berkeley

QUIZ 1
9 February 2012

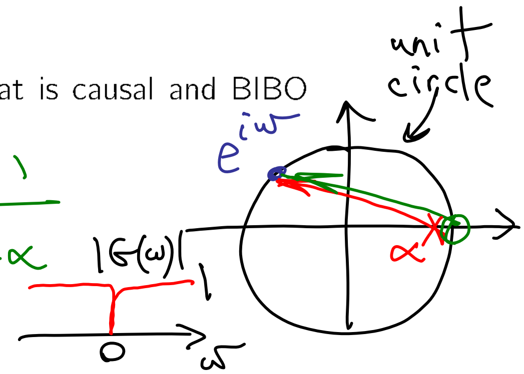
LAST Name Ter Nautsch FIRST Name Phil

- **(5 Points)** Print your name and lab time in legible, block lettering above.
- This quiz should take up to 20 minutes to complete. You will be given at least 20 minutes, up to a maximum of 30 minutes, to work on the quiz.
- **This quiz is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the quiz. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct.
- We will provide you with scratch paper. Do not use your own.
- **The quiz printout consists of pages numbered 1 through 3.** When you are prompted by the teaching staff to begin work, verify that your copy of the quiz is free of printing anomalies and contains all of the three numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your quiz. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this quiz.

Problem	Points	Your Score
Name	5	5
1(a)	15	15
1(b)(i)	10	10
1(b)(ii)	10	~~10~~ 8
1(b)(iii)	15	15
Total	**55**	~~55~~ 53

upon Regrade! Even the instructor is not perfect!

Q1.1 (50 Points) Consider a discrete-time LTI system G that is causal and BIBO stable, and whose frequency response is given by



DC Notch filter

$$G(\omega) = \frac{1 - e^{-i\omega}}{1 - \alpha\, e^{-i\omega}}, = \frac{e^{i\omega} - 1}{e^{i\omega} - \alpha}$$

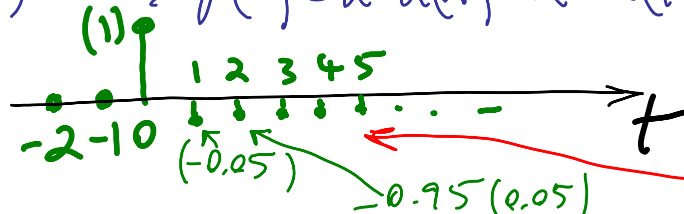
where $\alpha = 0.95$.

$G(\omega) \approx 1$ for ω not near $\omega = 0$
$= 0$ for $\omega = 0$

(a) (15 Points) Determine a reasonably simple expression for, and provide a well-labeled plot of, the system's impulse response g.

Consider a causal, BIBO stable system F whose frequency response is $F(\omega) = \frac{1}{1-\alpha e^{-i\omega}}$. The corresponding impulse response is $f(n) = \alpha^n u(n)$. Rewriting $G(\omega) = \frac{1}{1-\alpha e^{-i\omega}} - \frac{e^{-i\omega}}{1-\alpha e^{-i\omega}}$, we note that $g(n) = f(n) - f(n-1) \Longrightarrow g(n) = \alpha^n u(n) - \alpha^{n-1} u(n-1) \Longrightarrow$

$$g(n) = \alpha^{n-1}[\alpha u(n) - u(n-1)]$$



(b) (35 Points) Define another discrete-time LTI system H whose impulse response h is related to g according to $h(n) = \cos(\omega_0 n)\, g(n)$ for all integer n. Assume ω_0 is positive and *not* in a small neighborhood of the zero frequency.

(i) (10 Points) Show that the frequency response of the system H is given by

$$\forall \omega, \quad H(\omega) = \frac{G(\omega + \omega_0) + G(\omega - \omega_0)}{2}.$$

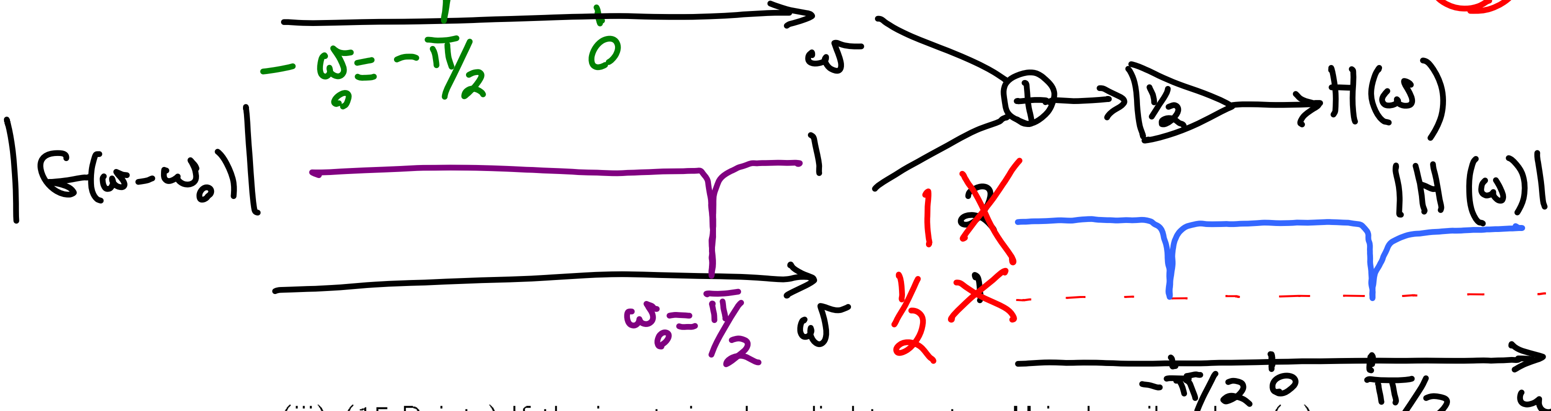
$$h(n) = \cos(\omega_0 n)\, g(n) = \frac{1}{2} e^{i\omega_0 n} g(n) + \frac{1}{2} e^{-i\omega_0 n} g(n) \Longrightarrow$$

$$H(\omega) = \frac{1}{2} \underbrace{\sum_n g(n) e^{-i(\omega-\omega_0)n}}_{G(\omega-\omega_0)} + \frac{1}{2} \underbrace{\sum_n g(n) e^{-i(\omega+\omega_0)n}}_{G(\omega+\omega_0)} \Longrightarrow$$

$$H(\omega) = \frac{G(\omega+\omega_0) + G(\omega-\omega_0)}{2}$$

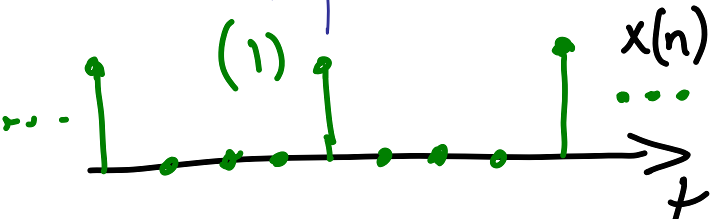
(ii) (10 Points) Provide a well-labeled, approximate sketch of the magnitude response plot $|H(\omega)|$. Assume that $\omega_0 = \pi/2$.

8 Pts ☺



(iii) (15 Points) If the input signal applied to system H is described by $x(n) = \sum_{\ell=-\infty}^{+\infty} \delta(n - 4\ell)$, determine a reasonably simple expression for the corresponding output y. Again, assume that $\omega_0 = \pi/2$.

x is 4-periodic $\Rightarrow p = 4 \Rightarrow \omega_0 = \frac{2\pi}{4} = \pi/2$

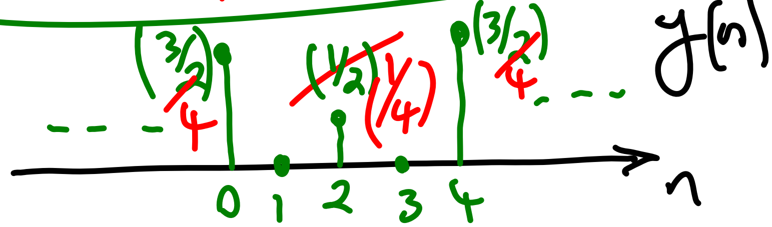


$\Rightarrow x(n) = \sum_{k \in \langle 4 \rangle} X_k e^{ik\frac{\pi}{2}n}$, where

$X_k = \frac{1}{p} = \frac{1}{4} \Rightarrow x(n) = \frac{1}{4}\sum_{k=-1}^{2} e^{ik\pi/2\, n} \Rightarrow$

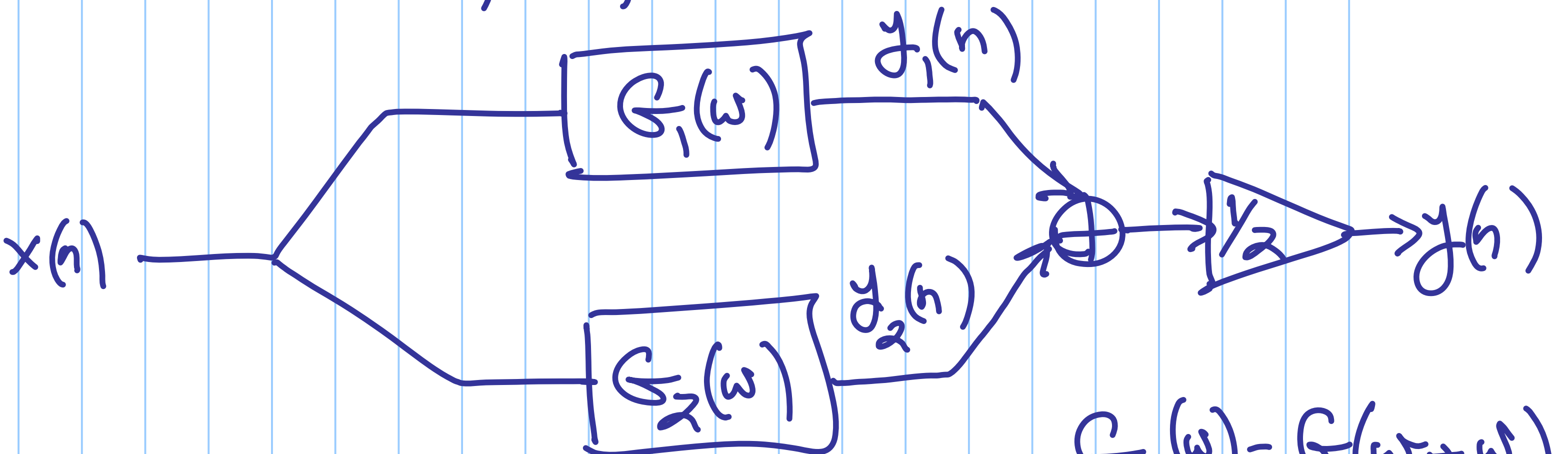
$y(n) = \frac{1}{4}\sum_{k=-1}^{2} H(k\frac{\pi}{2}) e^{ik\pi/2\, n} \approx \frac{1}{\cancel{4}\,8} e^{-i\frac{\pi}{2}n} + \frac{1}{\cancel{2}\,4} + \frac{1}{\cancel{4}\,8} e^{i\frac{\pi}{2}n} + \frac{1}{\cancel{2}\,4} e^{i\pi n}$

$\Rightarrow y(n) \approx \frac{1}{\cancel{2}\,4}\cos(\frac{\pi}{2}n) + \frac{1}{\cancel{2}\,4} + \frac{1}{\cancel{2}\,4}(-1)^n \Rightarrow y(n) = \begin{cases} 0 & n \text{ odd} \\ 3/\cancel{2}\,4 & n \bmod 4 = 0 \\ 1/\cancel{2}\,4 & n \bmod 4 = 2 \end{cases}$



No penalty here b/c answer is consistent w/ previous error. ☺

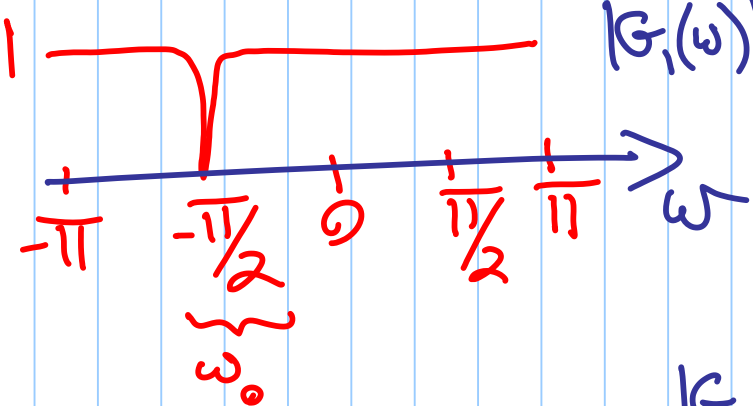
You can think of H as a parallel interconnection, as follows:



$G_1(\omega) = G(\omega + \omega_0)$

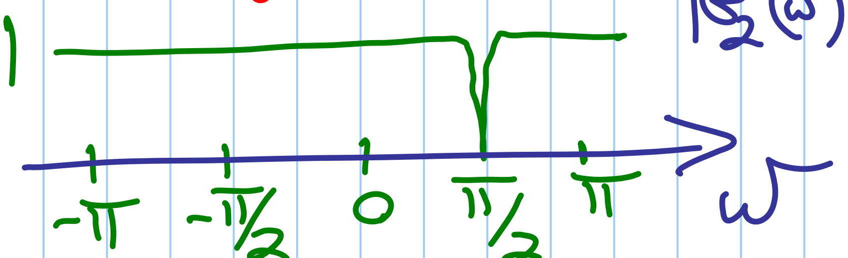
$G_2(\omega) = G(\omega - \omega_0)$

$|G_1(\omega)| = |G(\omega + \omega_0)|$



$G_1(\omega) \approx 1$ for ω not near $-\omega_0 = -\pi/2$.

$|G_2(\omega)| = |G(\omega - \omega_0)|$



$G_2(\omega) \approx 1$ for ω not near $\omega_0 = \pi/2$.

$$y_1(n) \approx \frac{1}{4} + \frac{1}{4} e^{i\frac{\pi}{2}n} + \frac{1}{4} e^{i\pi n} \quad \leftarrow e^{-i\frac{\pi}{2}n} \text{ zapped}$$

$$y_2(n) \approx \frac{1}{4} e^{-i\frac{\pi}{2}n} + \frac{1}{4} + \frac{1}{4} e^{i\pi n} \quad \leftarrow e^{i\frac{\pi}{2}n} \text{ zapped}$$

$$y(n) = \frac{y_1(n) + y_2(n)}{2} \approx \frac{1}{4} + \frac{1}{4}\cos\left(\frac{\pi}{2}n\right) + \frac{1}{4}(-1)^n = \begin{cases} 0 & n \text{ odd} \\ 3/4 & n \bmod 4 = 0 \\ 1/4 & n \bmod 4 = 2 \end{cases}$$