

Problem #1 — Circle the correct answer (22 pts.; 2 ea; -3 each wrong)

☒ TRUE ☐ FALSE | All rotations are always orthonormal matrices.

| TRUE ☒ FALSE | A perspective projection from 3D to 2D is a linear transformation.

☒ TRUE ☐ FALSE | In a perspective projection, the smaller the distance between the object and the center of projection, the larger the image of the object will be.

☒ TRUE ☐ FALSE | A sphere with a surface that acts as a Lambertian diffuse reflector will look to an observer like a uniformly lit flat circular or elliptical disk when illuminated only with an ambient light source and viewed with perspective projection.

☒ TRUE ☐ FALSE | In 3-space, any sequence of non-uniform scalings can be applied in arbitrary order without affecting the result.

| TRUE ☒ FALSE | In a perspective projection, changing the distance between camera and the object to be imaged will cause an affine change of the projected result.

☒ TRUE ☐ FALSE | A spherical Lambert emitter will look like a disk of uniform brightness from any direction.

☒ TRUE ☐ FALSE | The Gouraud shading technique produces a planar $\{a \cdot x + b \cdot y + c\}$ brightness distribution on triangular faces of a polyhedral object.

☒ TRUE ☐ FALSE | If a line segment AB is entirely behind the eye before the perspective projection, then the perspective projection of AB (without clipping) is identical to the line segment between the perspective projections of the two endpoints A and B.

☒ TRUE ☐ FALSE | The perspective warp transform expressed in 4D homogeneous coordinates is a linear operation.

| TRUE ☒ FALSE | In a perspective transformation, the midpoint of a straight line segment will be mapped to the middle of the projected line segment.

Problem #2 — Short Questions (pts.)

(6) Given the choices (voxels | B-rep mesh | CSG | sweep | instantiation), which is the preferred way to model:

a) A turbine wheel with 120 complicated fan blades ? instantiation

b) An Easter-egg approximated by 9000 triangles ?

B-rep

(4) Describe in one sentence the essence of the contribution that Mr. Gouraud has made to the field of computer graphics:

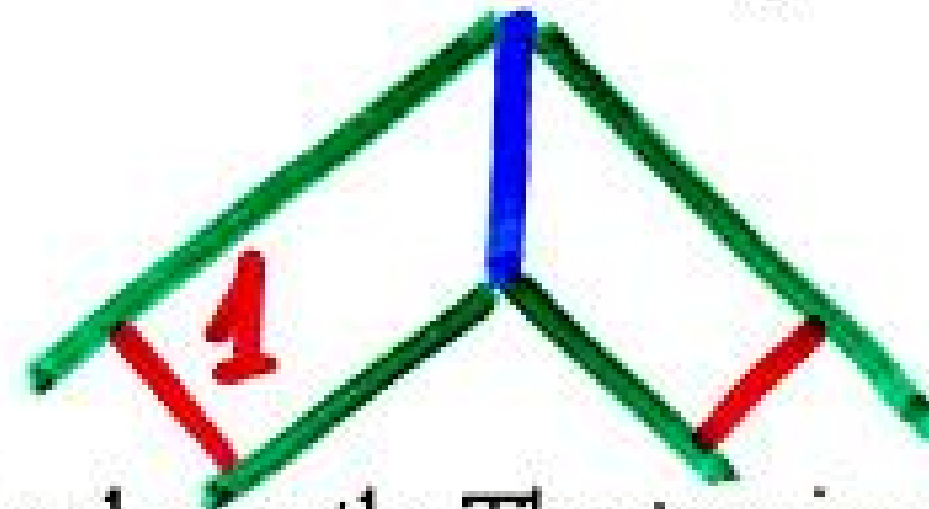
Gouraud shading: linear interpolation
of brightness

(6) The deCasteljau algorithm is used to subdivide a Bézier curve with no cusp into two parts at parameter value 0.333. Circle all the degrees of continuity that exist at that junction:

G0 C0 G1 C1 G2 C2 G3 C3 G4 C4

no C's !

(4) A unit-square cross section is swept along a piecewise linear space path. What is area of the cross section at a properly mitered joint that makes a 90 degree turn?



$\sqrt{2}$

(4) A torsion minimizing frame is swept around a circular path. The torsion minimizing frame is initialized to the Frenet frame, how many degrees is it rotated relative to the frenet frame after sweeping through a full revolution?

0

(4) How many degrees of freedom are associated with all possible planar ellipses in R^3 ?

8

(4) How many degrees of freedom are associated with all possible infinitely long cylinders of some (variable) diameter D in R^3 ?

5

(6) What are the minimum and maximum number of vanishing points that can be obtained from a perspective projection of a regular five-sided prism ?

MIN: 1

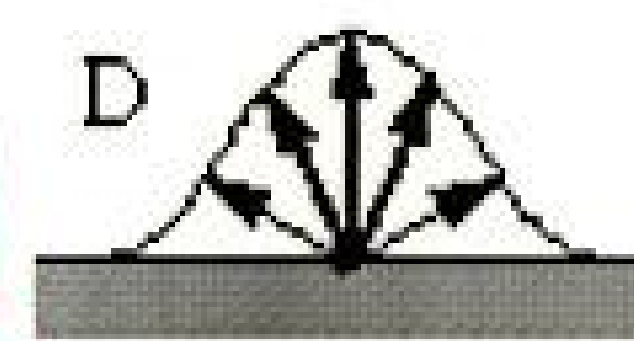
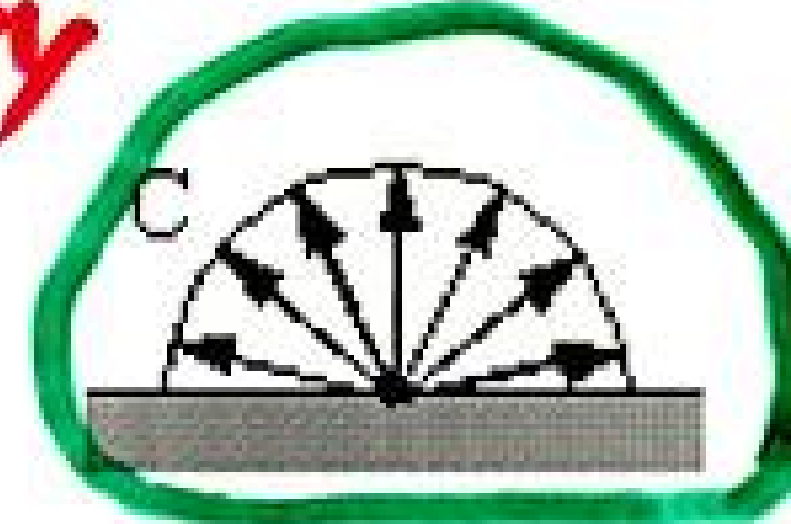
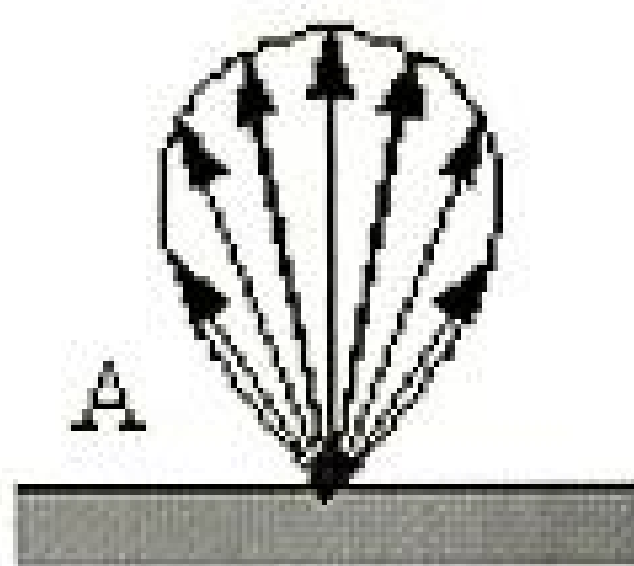
MAX: $6 = 1 + 5$



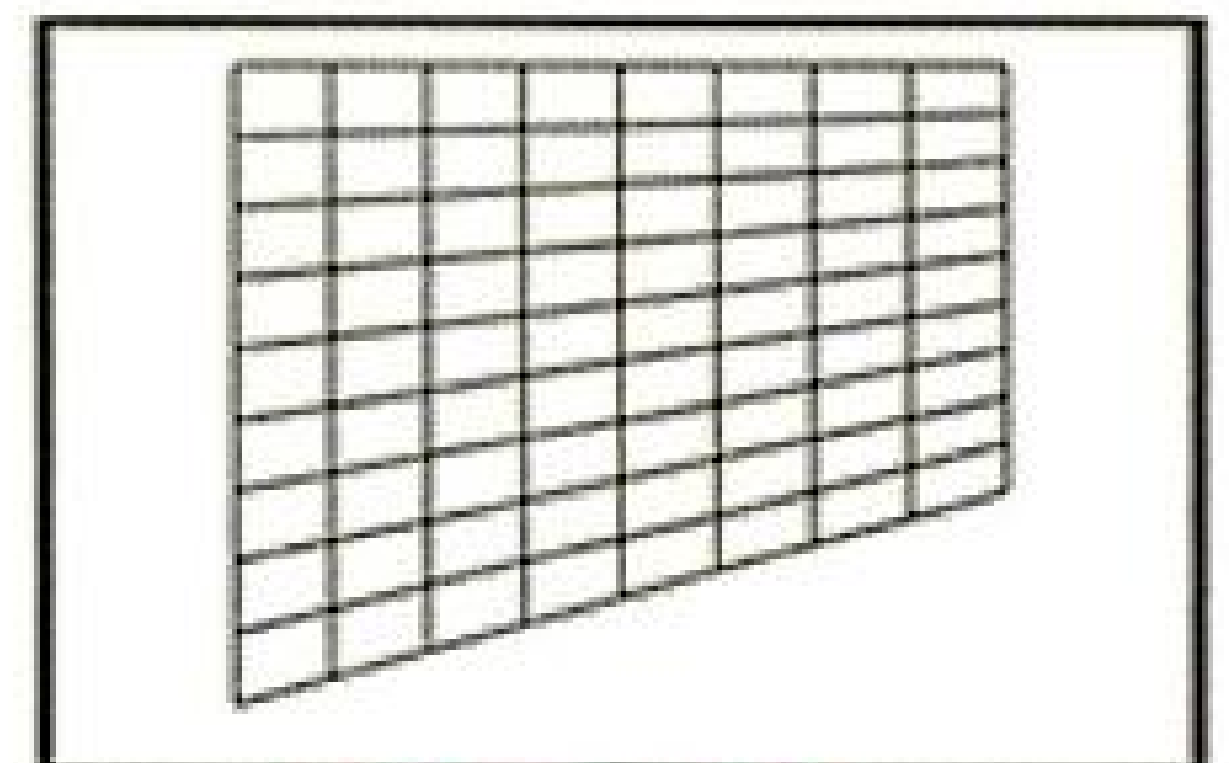
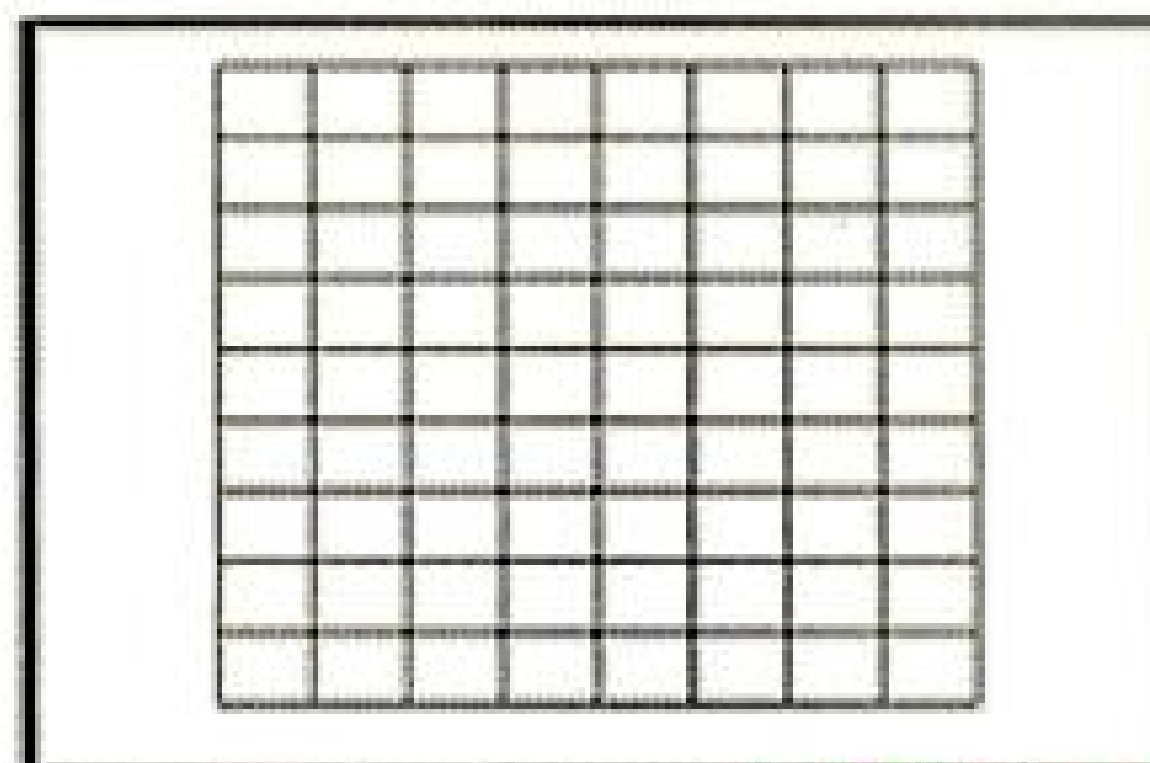
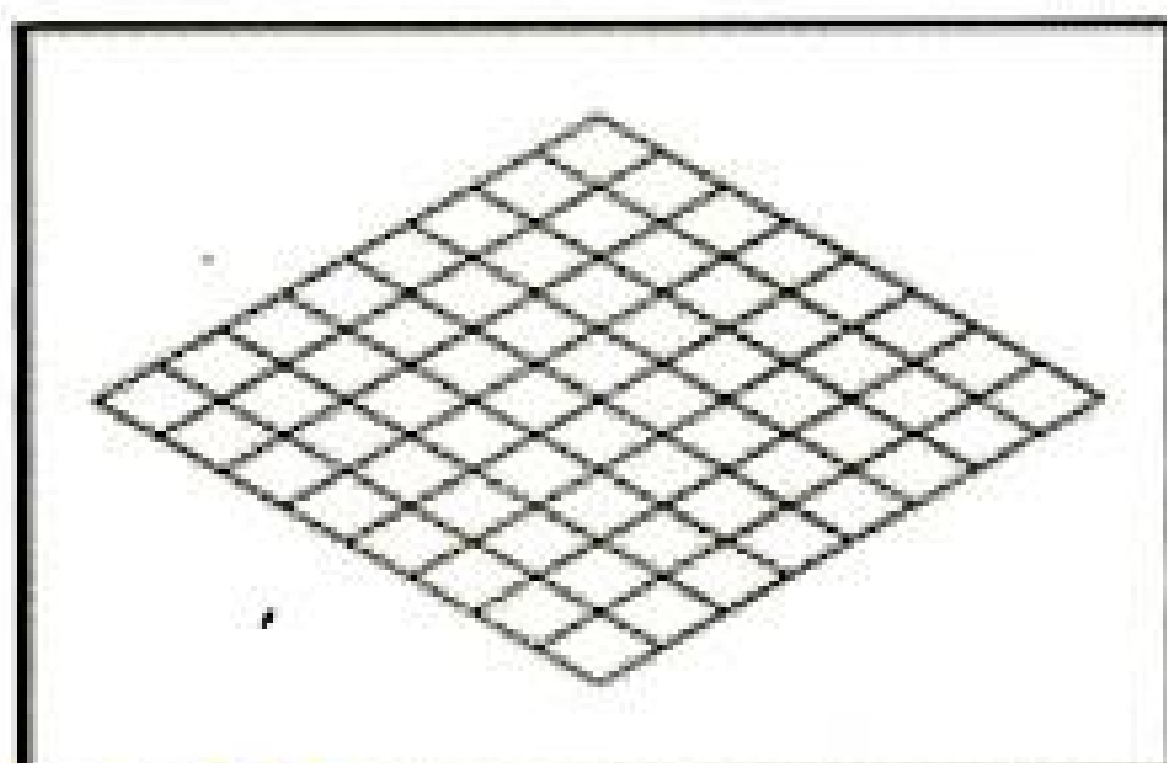
(4) Which of the four directional vector diagrams below describes most appropriately the perceived brightness observed on an ideal Lambert surface when viewing the surface from a direction opposite to each of the small arrow directions?

escape probability

dir. independent !



(6) The following images are all snapshots of an orthogonal planar grid, but taken with different cameras from different locations in 3-space. Determine the type of projection used in each case; circle the proper answer below each image.



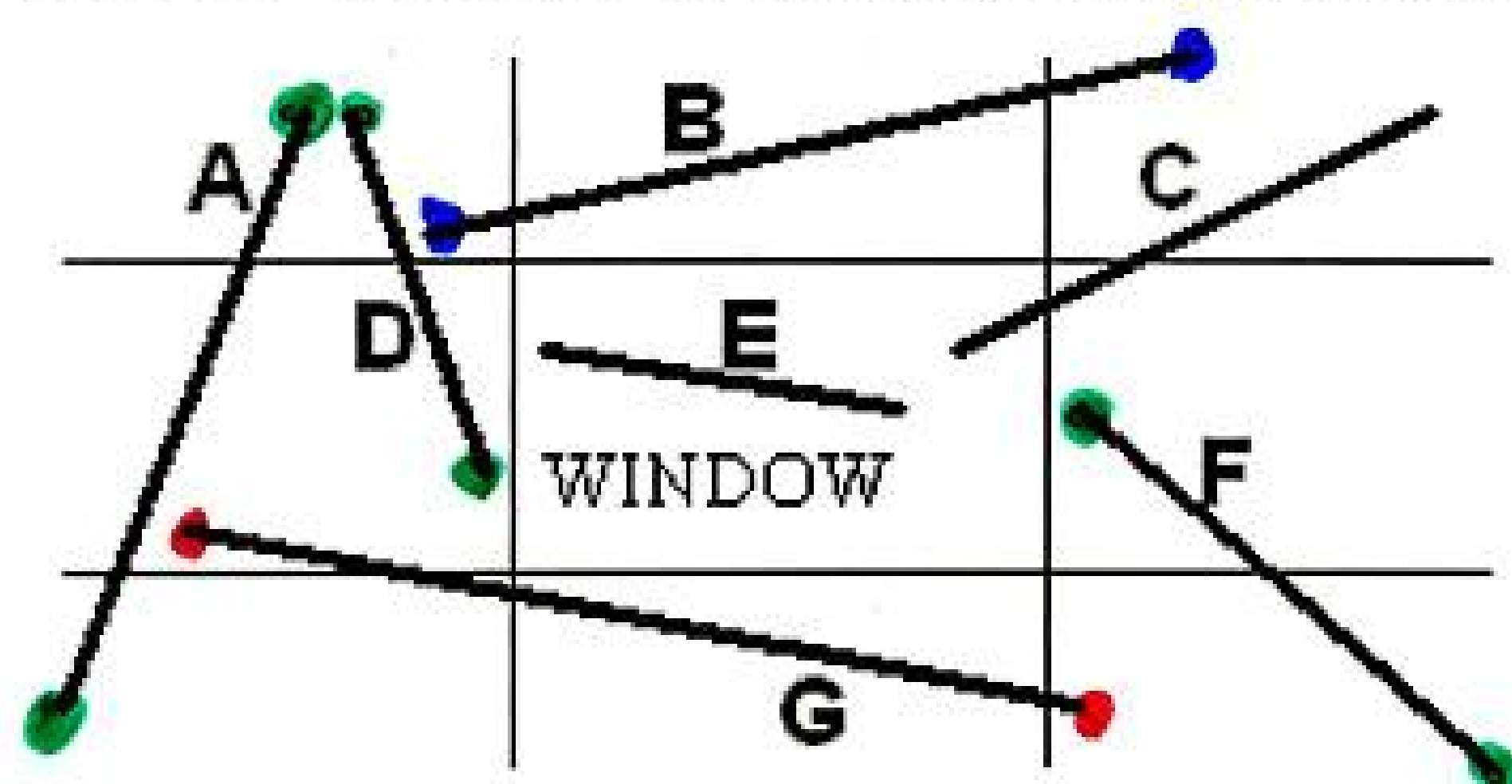
Parallel - Perspective - Can't tell!

Parallel - Perspective Can't tell!

Parallel - Perspective - Can't tell!

Problem # 3 — Clipping (8 pts.)

For the figure below list all the line segments that can be trivially culled away in the first step based on their "outcodes" in a Cohen-Sutherland line clipping algorithm.



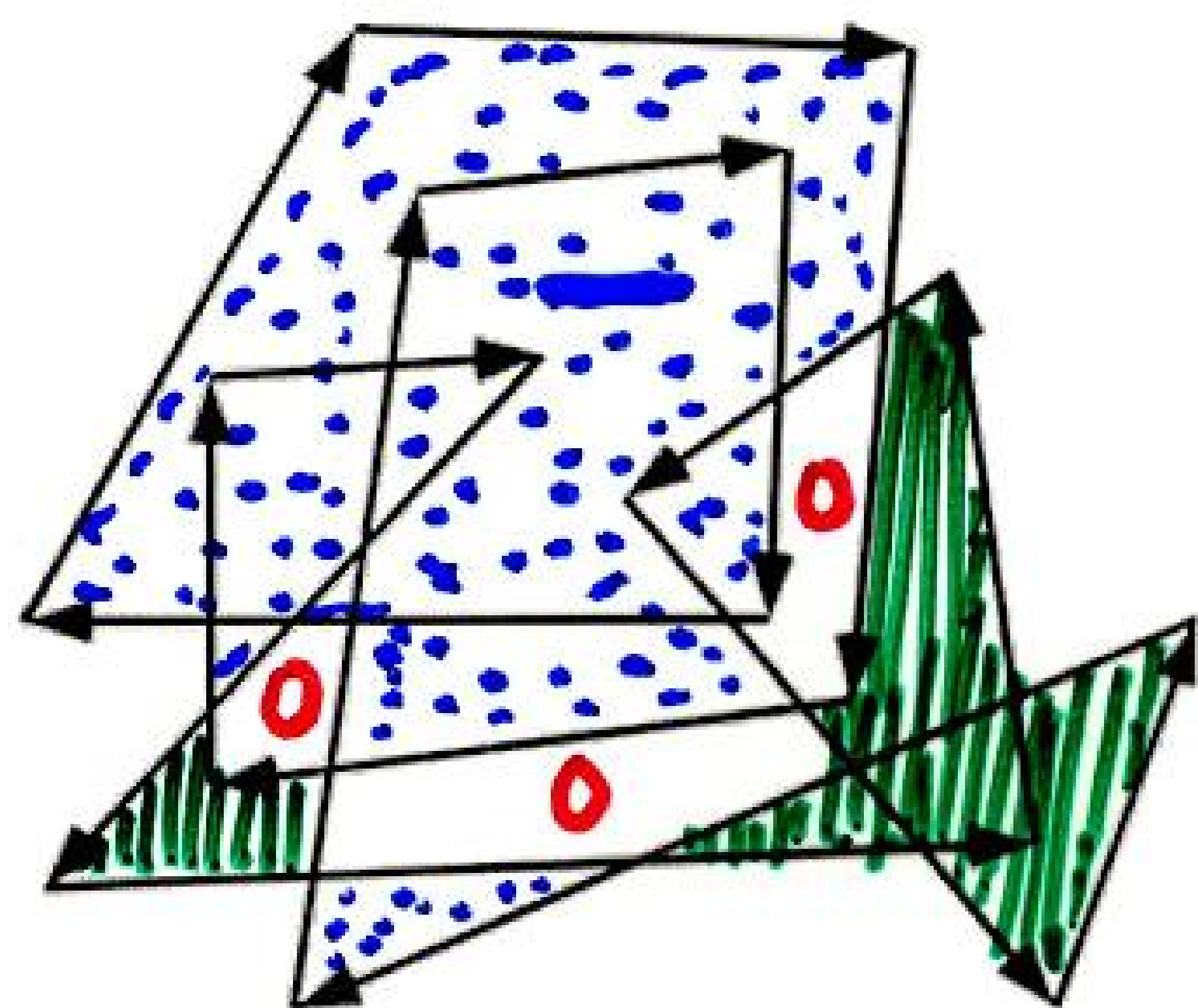
These line segments
can be trivially rejected:

A B D F

not G!

Problem # 4 — Polygon-fill (8 pts.)

Paint the "inside" areas according to the POSITIVE WINDING NUMBER MODEL.



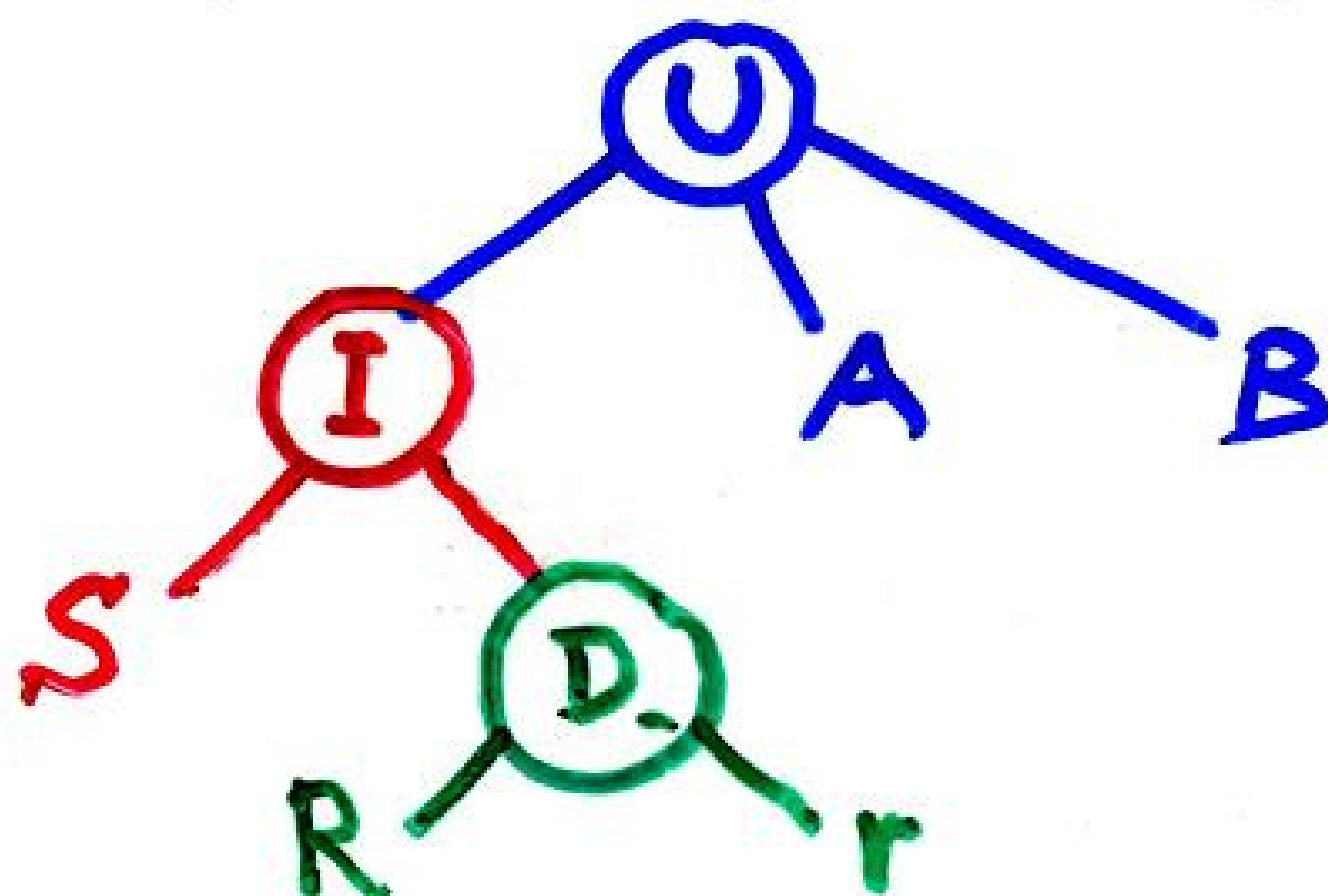
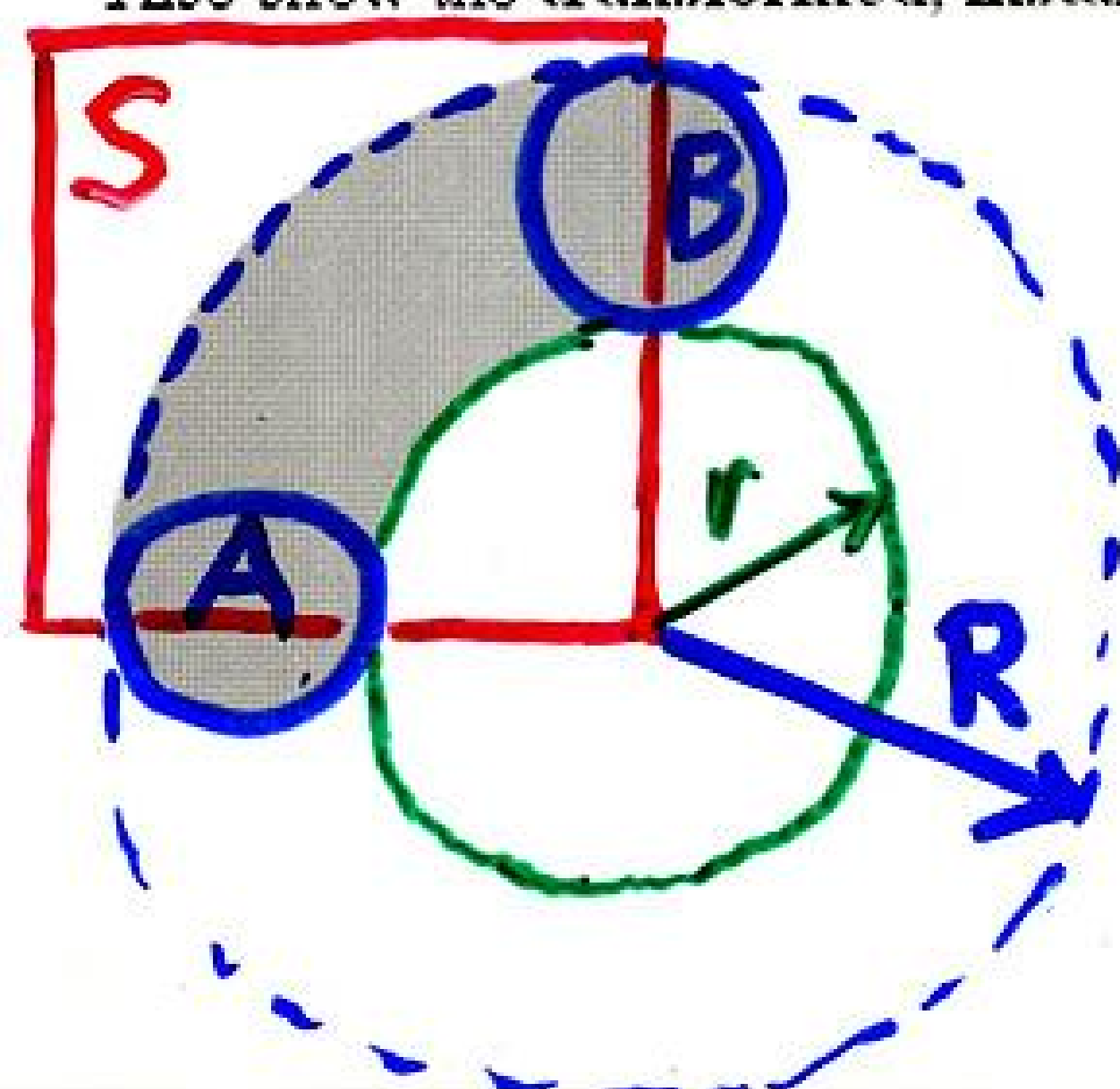
Problem #5 — CSG (8 pts.)

Given the 2-dimensional sausage below and a 2-D computer graphics CSG system with only the primitives unit-square and unit-circle, draw a simple CSG tree that will model the sausage.

Use a minimal number of elements and of Boolean operations. Ignore transformations.

Also show the transformed, instantiated leaf objects overlaid on the picture of the sausage.

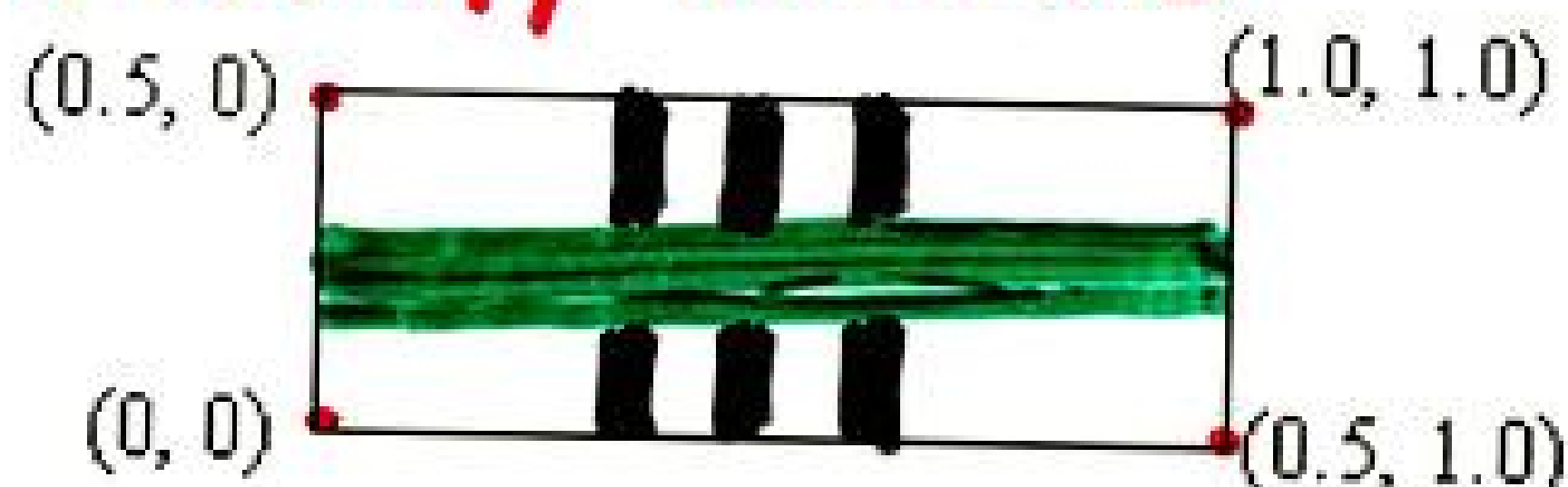
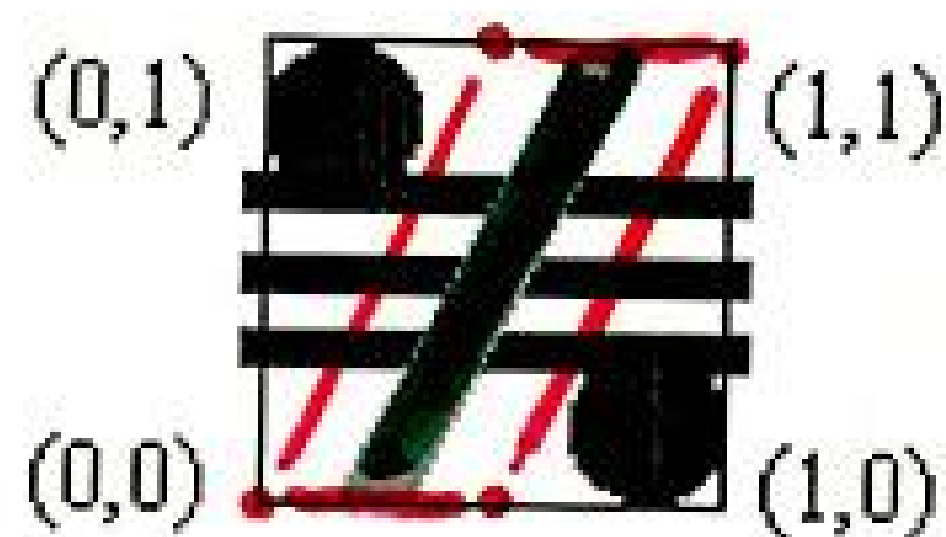
not a "sweep"!



Problem #6 — Texture Mapping (10 pts.)

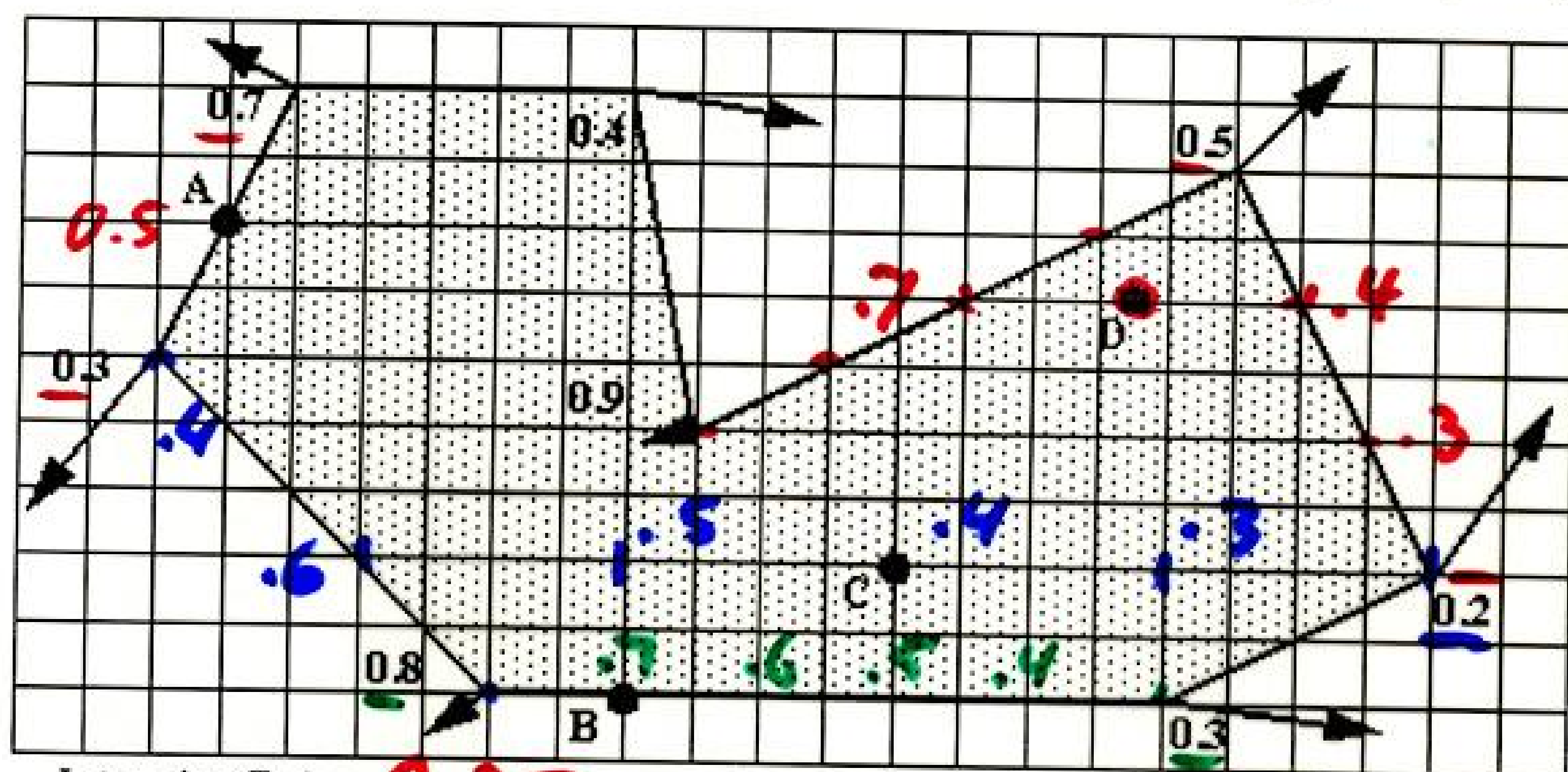
Use the texture map below and apply it to the rectangular surface on the right, carefully observing the given texture coordinates (u,v).

1. identify corners



Problem #7 — Gouraud Shading (12 pts.)

You are scan-line processing (in the usual way) the polygon below using Gouraud interpolation. The x- and y- components of the normalized vertex normals are drawn at each corner, and the cosine values of the z-components are given as decimal numbers (since you have no calculators). Parallel light of intensity 1.0 is incident perpendicularly to the plane of the paper. Compute the brightness values displayed at the indicated points A, B, C, D, assuming $k_s = k_r = 0$; $k_d = 0.5$.



Intensity @ A = 0.25

Intensity @ C = 0.2

Intensity @ B = 0.35

Intensity @ D = 0.275

$$\frac{.7 + .4}{2} = .55$$

Problem #8 — Parametric Representation (14 pts.)

- (6) Give a parametric representation of a ray that starts at point A, passes through point B, and then goes off to infinity.

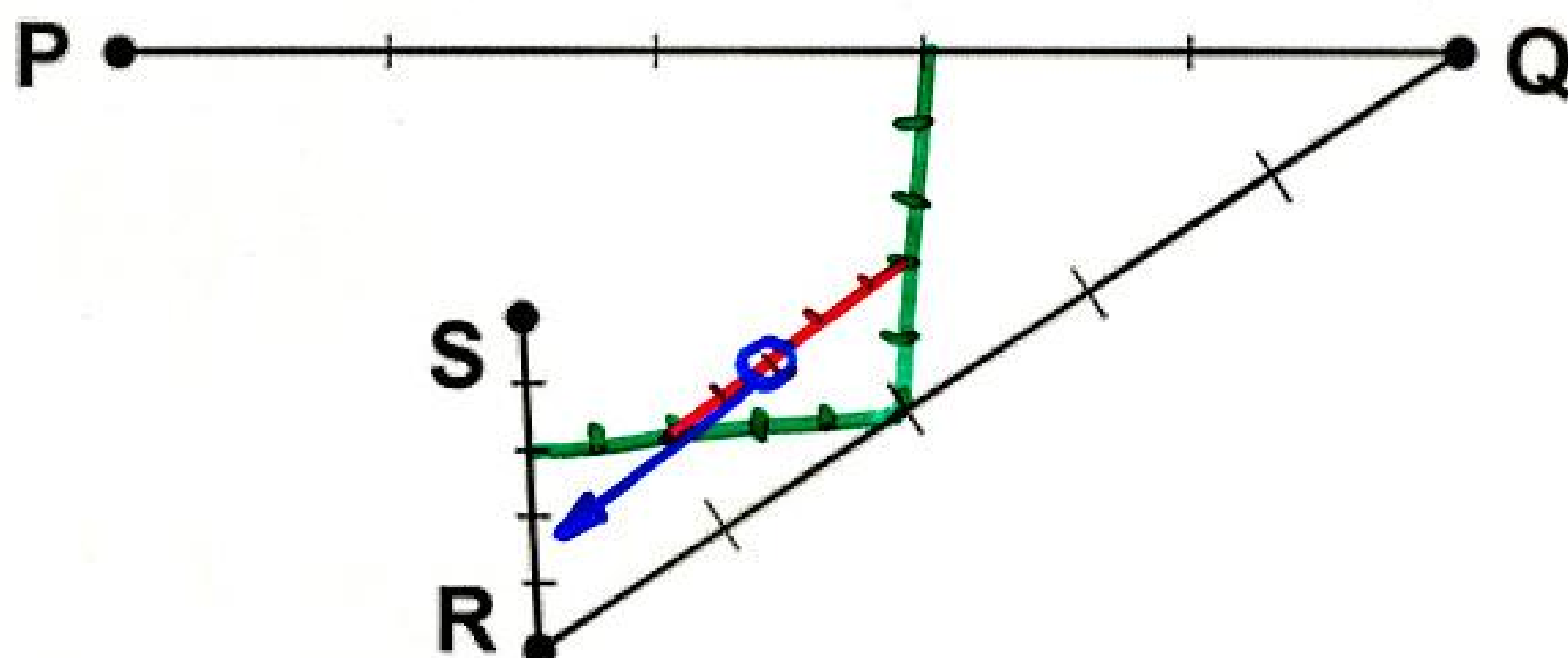
$$r(t) = A + (B - A)t; \quad t \in [0, \infty)$$

- (8) Give two reasons why a parametric curve representation: $x = F_x(t)$, $y = F_y(t)$, $z = F_z(t)$ is preferable to the form: $y = f_y(x)$, $z = f_z(x)$.

- can draw multivalued fct. w.r.t. x
- can control speed at which curve is traversed

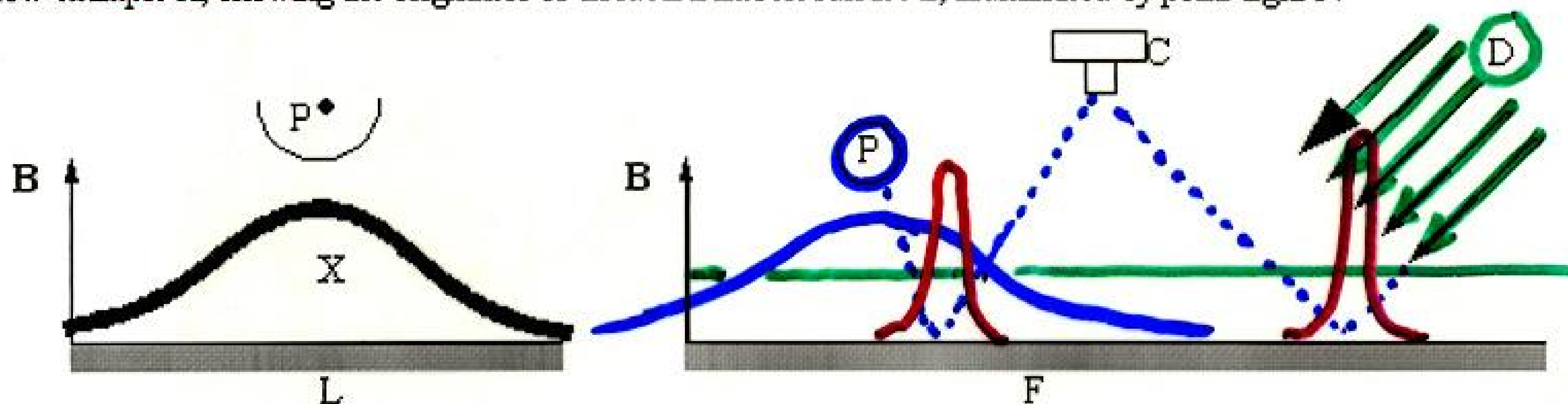
Problem # 9 — Bézier Curve (10 pts.)

For the given cubic Bezier segment (P,Q,R,S), find the point at $t = 0.60$ and its tangent direction using the deCasteljau method.

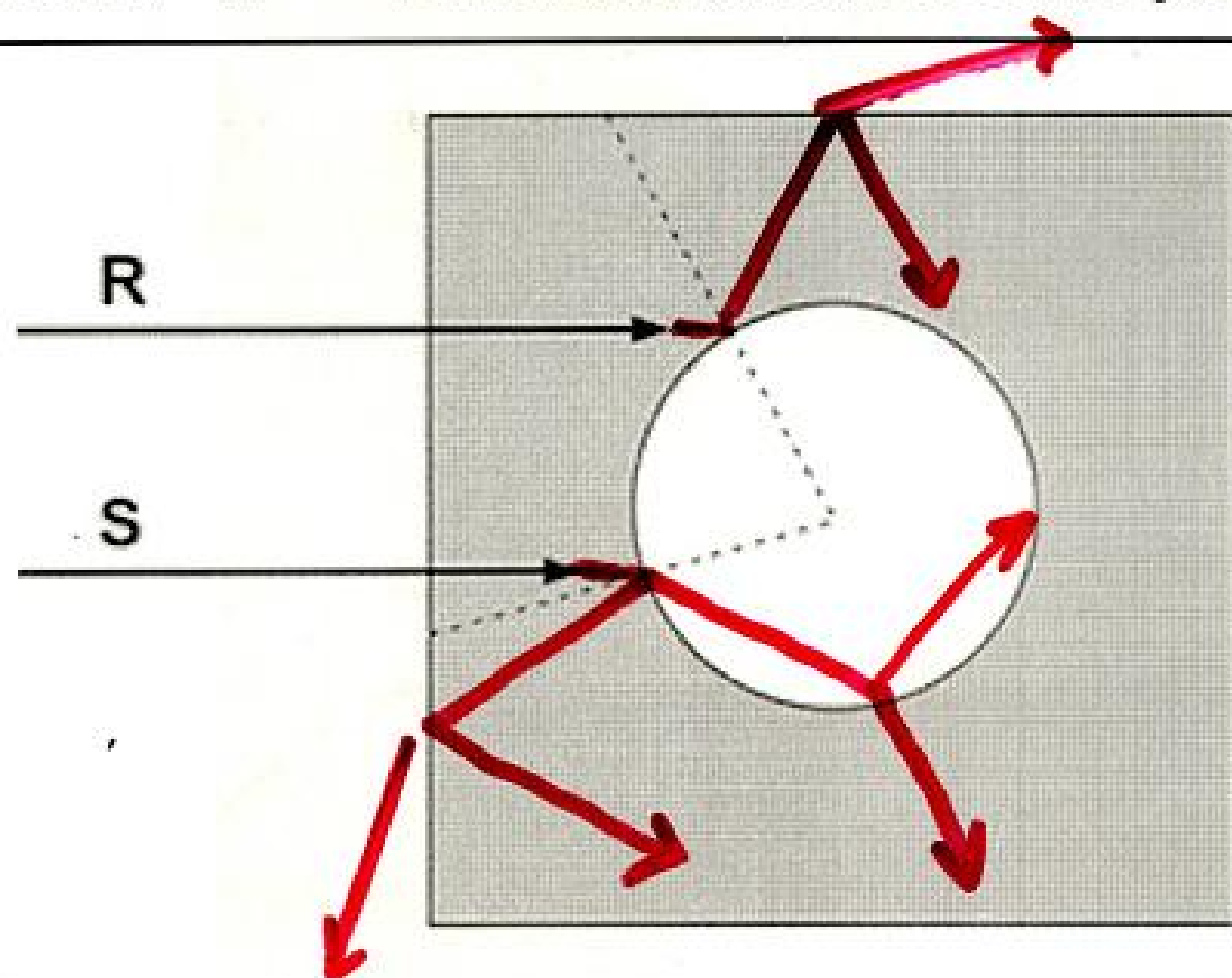


Problem # 10 — Illumination (10 pts.)

Sketch apparent brightness B , as seen from camera C , along real face F (Phong model, $K_{amb}=K_{diff}=K_{spec}=0.5$, $N_{phong}=50$), illuminated by point-light p and directional light D . Follow example X , showing the brightness of an ideal Lambert surface L , illuminated by point-light P .



Problem # 11 — Refraction and Reflection (10 pts.)



Two rays enter a glass cube (refractive index $n=1.5$) with a spherical evacuated hollow as shown. Ray-trace each of the two rays through the interactions with the next 2 glass surfaces encountered, and show the directions of the emerging rays after that.