Name________________________________ SID____________________________________

University of California, Berkeley – College of Engineering

Department of Electrical Engineering and Computer Sciences

Summer 2011 Instructor: Neel Shah 2011-07-27

Last Name	**Key**
First Name	**Solution**
Student ID Number	
Name of the person to your left	
Name of the person to your right	
All the work is my own. I had no prior knowledge of the exam contents nor will I share the contents with others in EE40 who have not taken it yet. I have not cleverly hidden notes and/or computational devices that give me an unfair advantage. (please sign)	

Instructions:

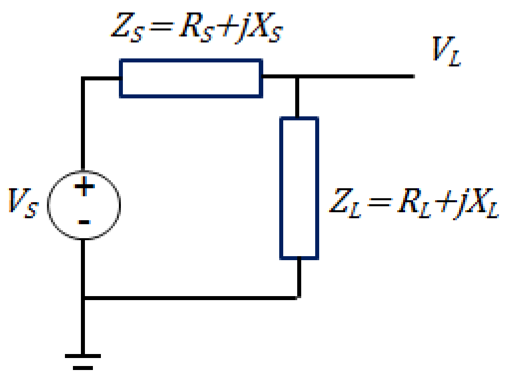
- You have 110 minutes to complete this exam, which we'll start at 12:05 PM. Once time is out at 1:55 PM, we'll announce that time is up, and you should stop writing immediately. The exact times will depend on when we can get everyone settled in.
- There are 12 pages on the exam. All work should be on this exam. Don't hand in random sheets of paper.
- Please turn off anything that might make noise, unless it is necessary for you to stay alive. Remove all hats and headphones. Allow at least one empty seat between your neighbor and you. All backpacks, laptops, and jackets should be up front, and nothing should be in the seats or on the floor between you and your neighbors.
- Calculators are allowed and recommended, but cell phone calculators are not.
- You may use 2 pages of notes (8.5" x 11" or A4), front and back, handwritten or typed, no magnifying glasses!
- Partial credit will be given for incomplete answers, so please show your work. The best possible thing you can do if you start running out of time is to write out things like the correct node voltage equations for problems you haven't completed.
- If you get stuck on an algebra problem, move on! The numerical part of a problem will be worth less than the conceptual part!
- This is a hard test, but hopefully not too hard! If the average is too low, remember that we're going to nudge grades up if we went too hard on you and wrote problems that were too difficult.

	Problem 1	Problem 2	Problem 3	Problem 4	Total
Earned					
Max	50	50	50	50	200

Name________________________________ SID____________________________________

Problem 1. Short Answer (a = 10pts, b = 10pts, c = 10pts, d = 20pts)

a) Given a non-ideal AC voltage source with source impedance $Z_S = R_S + jX_S$, what should the load impedance Z_L be in order to maximize the power factor of the circuit? Give your answer in the form $R_L + jX_L$, in terms of R_S and X_S.



$$Want \cos \tan^{-1} X/R = 1$$

$$\therefore X = 0$$

$$\therefore X_S = -X_L$$

$$R_S = R_L$$

Technically R_L can be anything as long as $X_L = -X_S$.

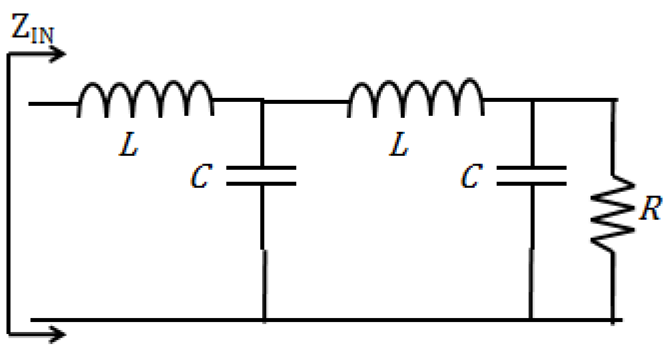
10/10 points for just stating the answer $R_L = R_S, X_L = -X_S$

5/10 points for showing some work involving the power factor

9/10 points if they get $X_L = -X_S$ and something else for R_L

5/10 points if they get lost in algebra somewhere

b) Given the transmission line circuit below, at what frequency (in terms of L, C, R) does the input impedance $Z_{IN} = R$?



The answer is

$$f = \frac{1}{2\pi\sqrt{LC}}$$

10/10 points for just stating the right answer

5/10 points if they get lost in algebra somewhere

8/10 points if they forget the 2π

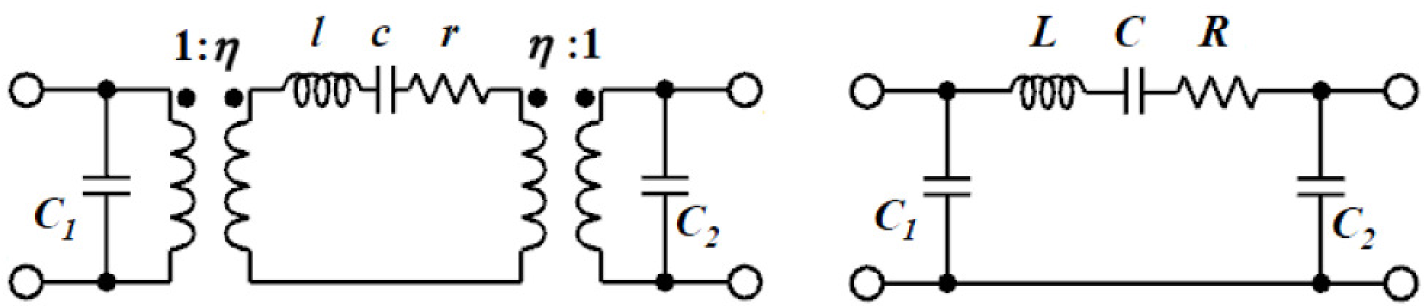
Also 10/10 pts for f = 0, I guess that works too! ☺

c) Function generators typically output sine waves by filtering square waves. Sketch a circuit that will do this. You don't need to provide values for your circuit elements so the frequencies do not matter. Just think of how to filter a square wave such that it is a sine wave, and then draw an example of such a circuit.

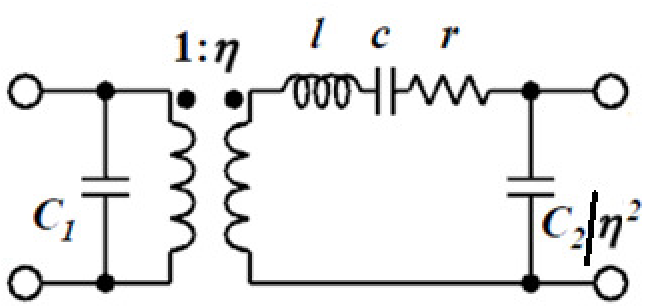
10/10 points for any low-pass filter, 0/10 points otherwise

Name________________________________ SID____________________________________

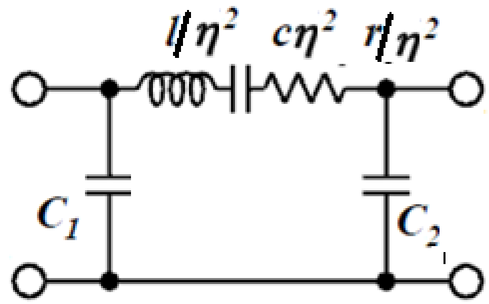
d) A typical model for a MEMS device uses transformers to convert energy between the electrical and mechanical domains. If the transformers on the left circuit have equal and opposite turns ratios, we can simplify it to the circuit on the right. Give the circuit parameters on the right (L, C, R) in terms of the parameters on the left (l, c, r, η).



Step 1: reduce the RH transformer



Step 2: reduce the LH transformer



20/20 points for just stating the answer:

$$L = \frac{l}{\eta^2}, C = c\eta^2, R = \frac{r}{\eta^2}$$

10/20 points if they tried to work out the impedances (basically on the right track)

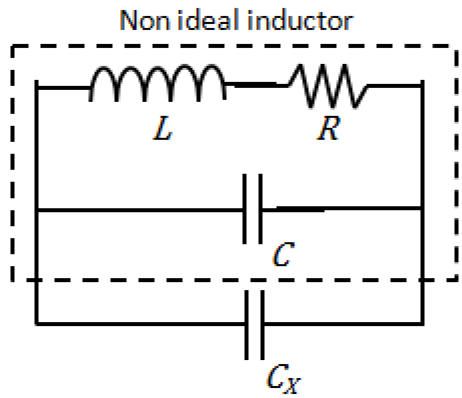
5/20 points if they got lost in algebra (basically on the wrong track)

15/20 points if they just got the turns ratio backwards on the answers

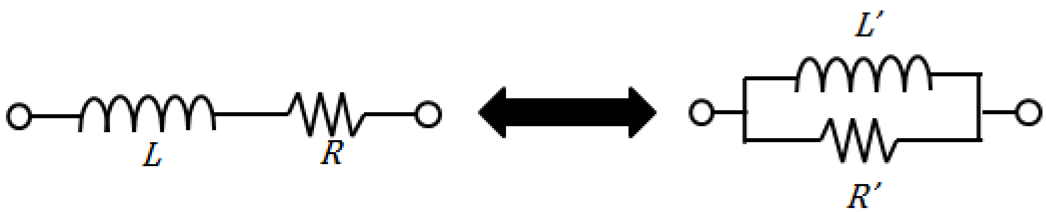
Name________________________________ SID____________________________________

Problem 2. Twisted iClicker Redux (a = 10pts, b = 20pts, c = 20pts, d = 10pts bonus)

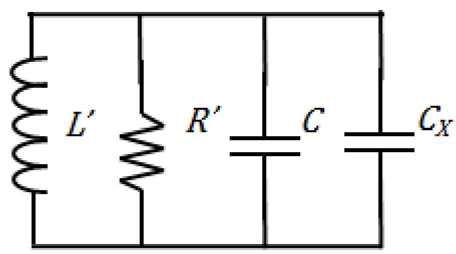
The following circuit consists of a non-ideal inductor with series resistance R and self-capacitance C in addition to the nominal inductance L. This non-ideal inductor is then placed in parallel with a capacitor C_X to form an LC tank circuit.



Brute force impedance analysis on this circuit is difficult, so we utilize a trick to transform the circuit, called the series-shunt transformation. At a particular frequency, the inductor and series resistor pair (L, R) is equal in impedance to a parallel inductor/resistor pair (L', R'). The quality factor for the series model is $Q_S = \omega_0 L/R$, and the quality factor for the transformed parallel model is $Q_P = R'/\omega_0 L'$. Both quality factors are equal to each other, i.e. $Q = Q_S = Q_P$.



Applied to the original, the resultant circuit looks like the one below.



$$L = 1\ \mu H$$
$$R = 6\ \Omega$$
$$C = 1\ pF$$
$$C_X = 8\ pF$$

a) What is the resonant frequency ω_0 of the circuit? Provide a formula and a numerical value.

$$\omega_0 = \frac{1}{\sqrt{L(C + C_X)}} = 333\frac{Mrad}{s} or\ f_0 = 53\ MHz$$

+4 points for recognizing C and CX are in parallel, +3 points for using the inductor L, +3 points for arithmetic

b) Assuming the resonant frequency is $\omega_0 = 2\pi 50\ Mrad/s$, what are the values of L' and R' at ω_0?

Impedance for the parallel combination:

$$\frac{j\omega_0 R'L'}{R' + j\omega_0 L'} = \frac{\omega_0^2 R' L'^2 + j\omega_0 R'^2 L'}{R'^2 + \omega_0^2 L'^2}$$

Equate this to the series impedance $R + j\omega_0 L$:

$$R = \frac{\omega_0^2 R' L'^2}{R'^2 + \omega_0^2 L'^2}\ and\ L = \frac{R'^2 L'}{R'^2 + \omega_0^2 L'^2}$$

Name________________________________ SID____________________________________

Divide the top and bottom of the R equation by $\omega_0^2 L'^2$:

$$R = \frac{R'}{Q^2+1} \therefore R' = R(Q^2+1) = 16.455\ k\Omega$$

Do the same thing to the L equation:

$$L = L'\left(\frac{Q^2}{Q^2+1}\right) = 0.99964L' \therefore L' \approx\ 1\ \mu H$$

Where $Q = \omega_0 L/R = 52.36$

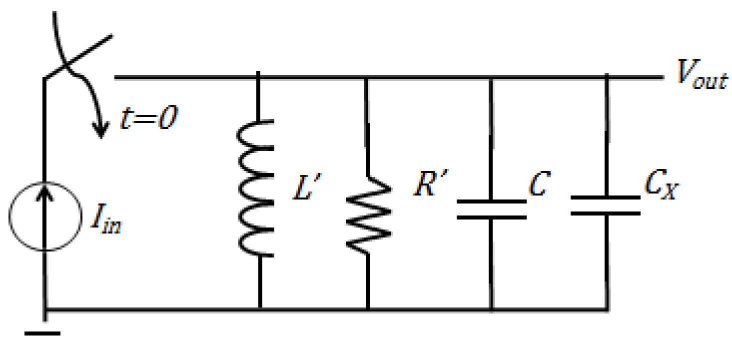
+5 points if they recognize they need to get the parallel combo of R' and L' → give partial credit +2 as needed

+5 points if they do the complex conjugate math → give partial credit +2/4 for major/minor errors

+6 points if they figure out how to substitute in Q → give partial credit +3 if they sub in Q but mess it up a bit

+4 points for correct answers (+2 for R', +2 for L')

c) Suppose we provide a burst of magnetic field energy that the inductor picks up and delivers to the circuit as a current source, as shown in the circuit below. Assuming $L' = 2\ \mu H$ and $R' = 250\ \Omega$, provide a sketch of the magnitude and phase Bode plots of the transfer function $H(j\omega) = V_{out}/I_{in}$. Mark the values of the magnitude and phase on the Bode plots at $\omega \to 0, \omega = \omega_0,$ and $\omega \to \infty$. Note you need to calculate a new resonant frequency for this circuit.



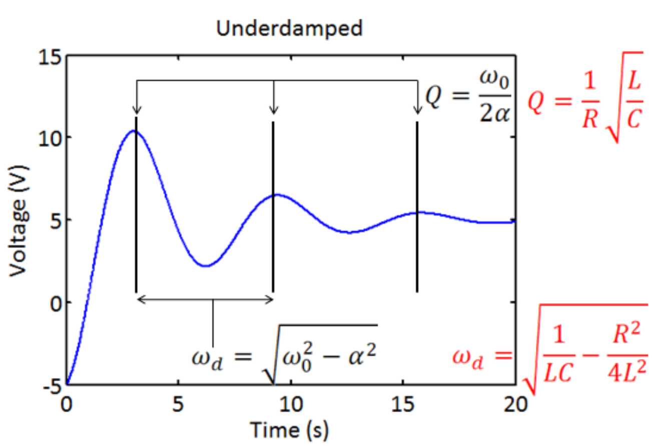
$L' = 2\ \mu H$
$R' = 250\ \Omega$
$C = 1\ pF$
$C_X = 8\ pF$
$\omega_0 = ??$

+10 points for the magnitude bode plot (+4 for the right shape – bandpass, +2 for each marked value)

+10 points for the phase bode plot(+4 for the right shape – mirrored S curve, +2 for each marked value)

Technically the transfer function is not asked for. It is possible to answer this question for full credit without deriving the transfer function by recognizing the BP filter and knowing what the mag/phase must be at the given frequencies. However if they do calculate the transfer function and are lacking in points for the rest of the problem (i.e. missing or incorrect mag/phase plots) give **up to** +10 partial credit points for doing this work. (up to +4/7/10 for getting the complex TF / mag response / phase response formulas)

d) Sketch the time domain waveform of V_{out}. Mark the resonant frequency, Q-factor, and steady state value. [Bonus]



+4 points for drawing the correct (underdamped) waveshape

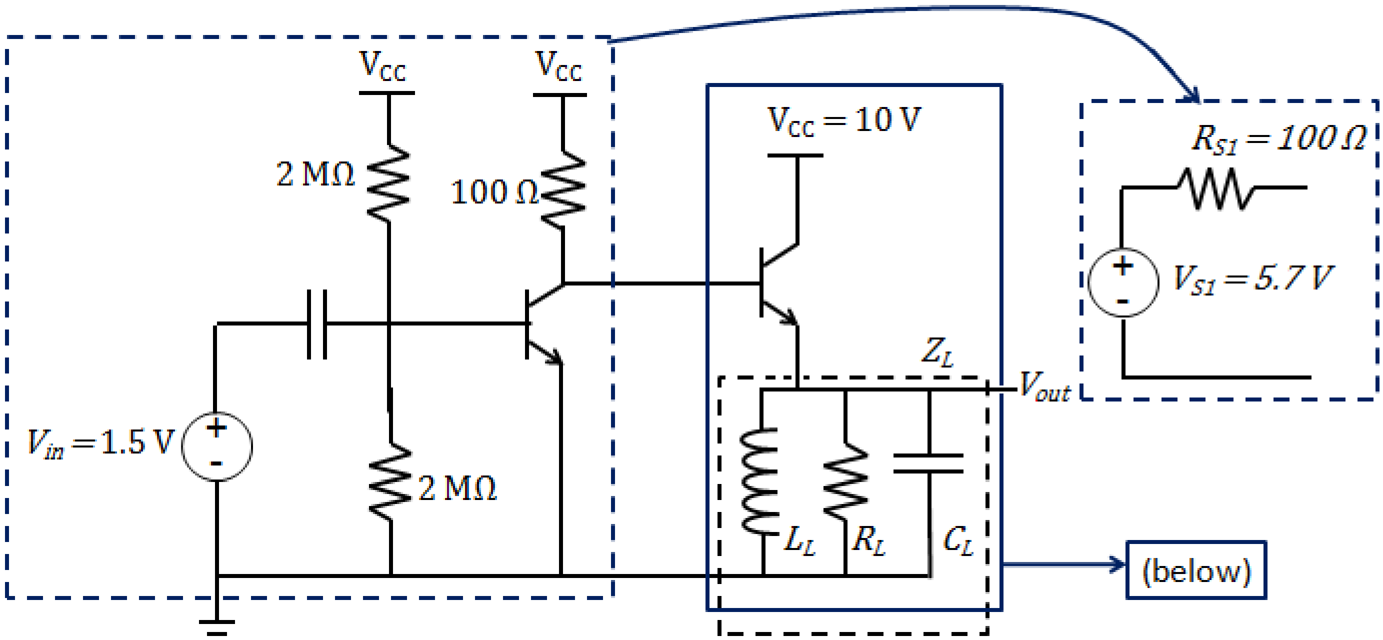
+1 point each for explaining the significance of or correctly marking each of ω_0, Q, V_{SS}

+3 points for showing any work related to determining the type of damping of the system (second order DE, $\alpha < \omega_0$)

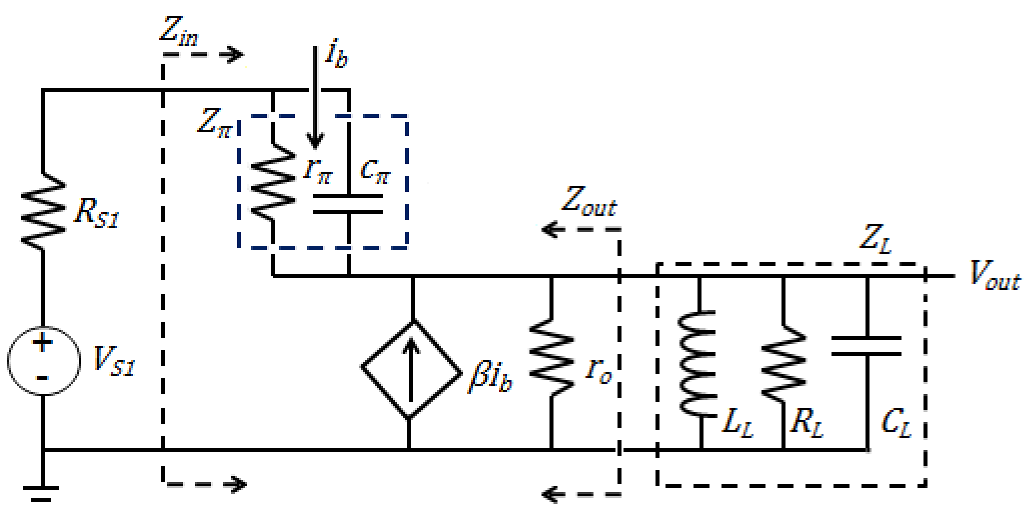
Name________________________________ SID____________________________________

Problem 3. I/O Impedance of a Tuned BJT Receiver (a = 20pts, b = 20pts, c = 10pts)

Consider the following two-stage radio receiver, which picks up a small signal from an antenna and amplifies it. Using the tuned load Z_L, the amplifier effectively ignores all signals out of the band specified by the resonant frequency of Z_L. Assume for this problem we are operating in the GSM-II frequency band where $f_c = 900\ MHz$.



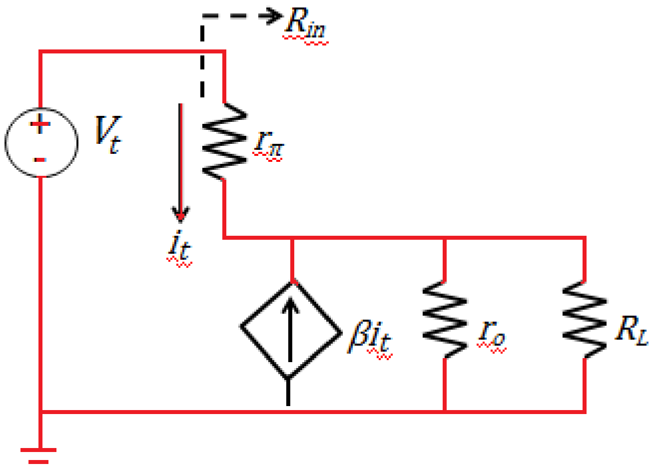
The properties of the first stage are distilled into the Thévenin model above. The second stage equivalent circuit is shown below. Due to the high-frequency nature of radio operation we have to include parasitic capacitive effects of the transistor in Z_π, a combined resistor/capacitor pair between the input and the output. Note the current i_b is the net current into Z_π; that is, $i_b = i_{r_\pi} + i_{c_\pi}$.



$r_\pi = 700\ \Omega$
$c_\pi = 1\ pF$
$r_o = 100\ k\Omega$
$\beta = 100$
$R_L = 1\ k\Omega$
$L_L = 2.5\ nH$
$C_L = 12.5\ pF$
$V_{S1} = 5.7\ V$
$R_{S1} = 100\ \Omega$
$f_c = 900\ MHz$

Name________________________________ SID____________________________________

a) Calculate the input impedance at f_c seen by the driving stage $S1$. Hint: Apply a test voltage v_{in} at the point marked Z_{in}, and find the test current i_{in}. Give your answer in the form $Z_{in} = R_{in} \pm jX_{in}$ and provide numerical values for R_{in} and X_{in}. The majority of points go to correct derivation, not arithmetic. Remember to include the load impedance!



This is the exact same circuit set up as Midterm 1.
Replace Rin with Zin, Rπ with Zπ, RL with ZL.
Also ZL is tuned such that at 900 MHz, ZL = RL.

KCL at V_{out} to get it in terms of V_t:

$$i_t + \beta i_t = V_{out}/(r_o || R_L)$$

$$\frac{(\beta+1)(V_t - V_{out})}{Z_\pi} = V_{out}\left(\frac{1}{r_o} + \frac{1}{R_L}\right)$$

$$\frac{\beta+1}{Z_\pi}V_t = V_{out}\left(\frac{1}{r_o} + \frac{1}{R_L} + \frac{\beta+1}{Z_\pi}\right)$$

$$V_{out} = V_t\left(\frac{1}{r_o} + \frac{1}{R_L} + \frac{\beta+1}{Z_\pi}\right)^{-1}\left(\frac{\beta+1}{Z_\pi}\right) = AV_t$$

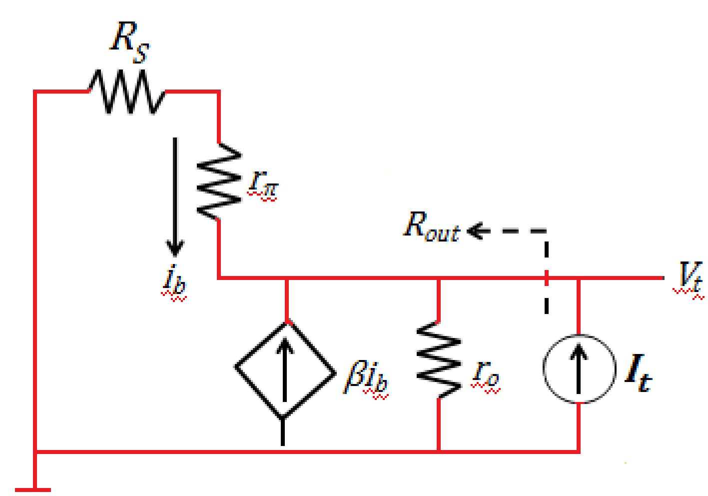
$$A = \frac{\frac{\beta+1}{Z_\pi}}{\frac{1}{r_o} + \frac{1}{R_L} + \frac{\beta+1}{Z_\pi}} = \frac{\frac{101}{700 - j177}}{.00001 + .001 + \frac{101}{700 - j177}} = 0.9930 + j0.0017$$

Now combine to get $Z_{in} = v_t/i_t$:

$$i_t = \frac{(1-A)}{Z_\pi}V_t \rightarrow Z_{in} = \frac{Z_\pi}{1-A} = \frac{700 - j177}{0.0070 + 0.0017j} = 100700 - j177\ \Omega$$

+3/3 points for defining the test current and circuit appropriately
(+1/1 point partial credit if an attempt is made)
+8 points for setting up KCL correctly (+4 pts partial credit if some of the terms are right)
+2 points for getting the right numerical answer for Z_{in}
+4 partial credit points for stuff like noticing $Z_L \approx R_L$ → be liberal with these four points

b) Calculate the output impedance and Thévenin voltage seen by the tuned load Z_L at f_c. Hint: Apply a test current source i_{out} at the point marked Z_{out}, and find the test voltage v_{out}. Give your answer in the form $Z_{out} = R_{out} \pm jX_{out}$, and provide numerical values for R_{out} and X_{out}. The majority of points go to correct derivation, not arithmetic. Remember to include the source resistance!



All we need is KCL at the output node:

$$I_t + \beta i_b = \frac{V_t}{r_o} + \frac{V_t}{Z_\pi + R_S}$$

$$I_t - \frac{\beta V_t}{r_\pi + R_S} = V_t\left(\frac{1}{r_o} + \frac{1}{Z_\pi + R_S}\right)$$

$$I_t = V_t\left(\frac{1}{r_o} + \frac{\beta+1}{Z_\pi + R_S}\right)$$

$$Z_{out} = \frac{V_t}{I_t} = \left(\frac{1}{r_o} + \frac{\beta+1}{Z_\pi + R_S}\right)^{-1}$$

Plugging in numbers:

$$Z_{out} = \left(\frac{1}{100000} + \frac{101}{800 - j177}\right)^{-1} = 7.92 - j1.75\ \Omega$$

+3/3 points for defining the test current and circuit appropriately
(+1/1 point partial credit if an attempt is made)
+8 points for setting up KCL correctly (+4 pts partial credit if some of the terms are right)
+2 points for getting the right numerical answer for Z_{out}
+4 partial credit points for other stuff → be liberal with these four points

c) Suppose the receiver instead picks up interference from the nearby $915\ MHz$ ISM (industrial, scientific, medical) band. Would you expect the input impedance to be higher or lower? What about the output impedance? Support your answer with a brief analysis of Z_{in} and Z_{out}.
The output impedance magnitude will drop slightly (i.e. be less capacitive).

Since Z_L detunes, the input impedance will drop much more significantly (because ZL is maximized at the resonant frequency).

+3/3 points for getting the directions right on each impedance

+2/2 points for an explanation (i.e. for output, talk about Zpi, for input, talk about ZL)

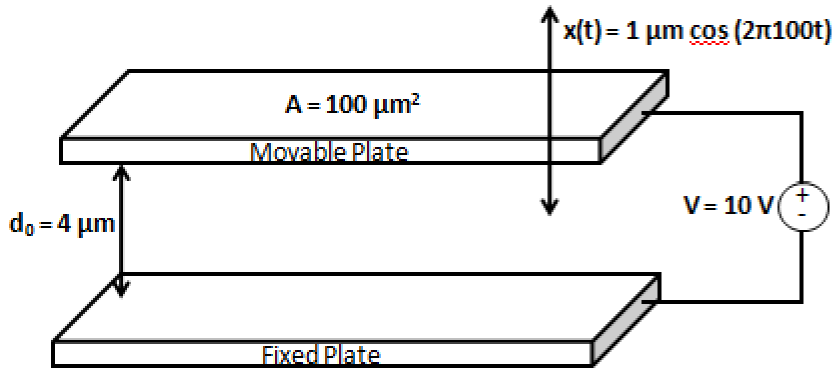
(+1 point if for input impedance they talk about Zpi but not ZL)

Name_______________________________ SID___________________________________

Problem 4. MEMS Capacitance Amplifier (a = 15pts, b = 15pts, c = 10pts)

In your homework you solved for the capacitance equations for a differential MEMS accelerometer with movable capacitive plates. Consider now just one of those capacitive plates:

$$C(t) = \frac{\epsilon_0 A}{d_0 - vt} \quad \text{and} \quad \frac{dC}{dt} = \frac{-\epsilon_0 A}{(d_0 - vt)^2} \approx \frac{\epsilon_0 A}{d_0^2}$$



In MEMS it is common to separate the capacitance time derivative dC/dt into a spatial derivative dC/dx and a velocity $v = dx/dt$ which is usually a constant. So if the gap of a capacitor is changing…

$$C(x) = \frac{\epsilon_0 A}{d_0 - x} \quad \text{and} \quad \frac{dC}{dx} = \frac{-\epsilon_0 A}{(d_0 - x)^2} \approx -\frac{\epsilon_0 A}{d_0^2}$$

For a sinusoidal input $x(t) = X_0 \cos \omega t$, the derivative is simply

$$\frac{dx}{dt} = -\omega X_0 \sin \omega t = \omega X_0 \cos(\omega t + \frac{\pi}{2}) = \omega x(t) e^{j\pi/2}$$

Given a constant voltage, the motional current can be calculated by

$$I(t) = \frac{d}{dt}(CV) = V\frac{dC}{dt} = V\frac{dC}{dx}\frac{dx}{dt} = V\frac{dC}{dx}\,\omega x(t) e^{j\pi/2}$$

Where $e^{j\pi/2}$ represents the introduction of a 90° phase shift. This is just another way of writing $(j\omega = \omega e^{j\pi/2})$.

Assume the following values for this problem and use the approximation for dC/dx:

$$\boldsymbol{\epsilon_0 = 8.854\ pF/m, \qquad A = 100\ \mu m^2, \qquad d_0 = 4\ \mu m, \qquad V = 10\ V}$$

Name________________________________ SID____________________________________

a) Calculate the motional current when $x(t) = 1\ \mu m \cdot \cos(2\pi 100 t)$. Give your answer as $I(t) = I_0 \cos(\omega t + \phi)$.

Magnitude:

$$I_0 = V \cdot \frac{\epsilon_0 A}{d_0^2} \cdot \omega X_0 = 10 \cdot \frac{8.854e-12 \cdot 100e-12}{(4e-6)^2} \cdot 2\pi 100 \cdot 1e-6 = 0.347\ pA$$

Phase:

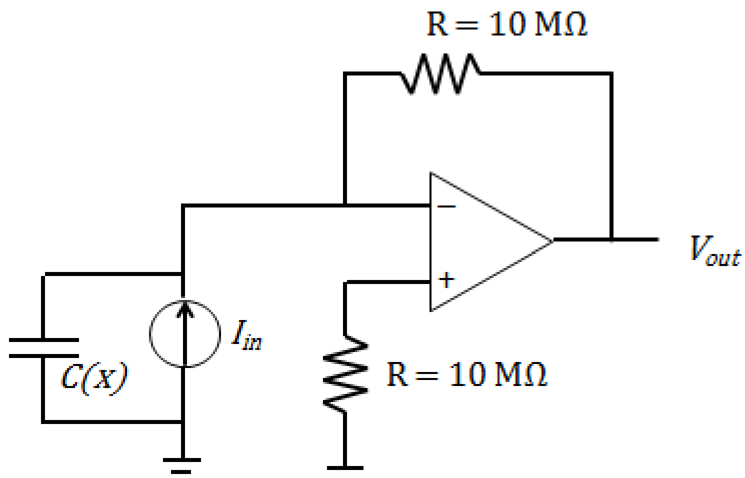
$$\phi = \frac{\pi}{2}\left(from\ the\ \frac{dx}{dt} discussion\right) - \pi\left(from\ the - sign\ in \frac{dC}{dx}\right) = -\frac{\pi}{2}$$

Phase: +5 points for correct answer,(+3 points if they get $+\frac{\pi}{2}$, +2 points if they get $\pm\pi$)

Magnitude: +5 points for correct answer, (+3 points for correct equation use but math error)

Frequency: +5 points for stating $\omega = 2\pi 100\ or\ 626\ rad/s$ (freebie points basically)

b) We use a transimpedance amplifier like the one in the diagram below to convert the output current into a voltage. Calculate the output voltage v_{out} of the transimpedance amplifier when $x(t)$ is as in (a). Give your answer in the form $v_{out}(t) = V_0 \cos(\omega t + \phi)$.



For a transimpedance amplifier $v_{out} = -I_{in} R$

The phase is now $+\pi/2$ and the magnitude of $v_{out} = 3.47\ \mu V$

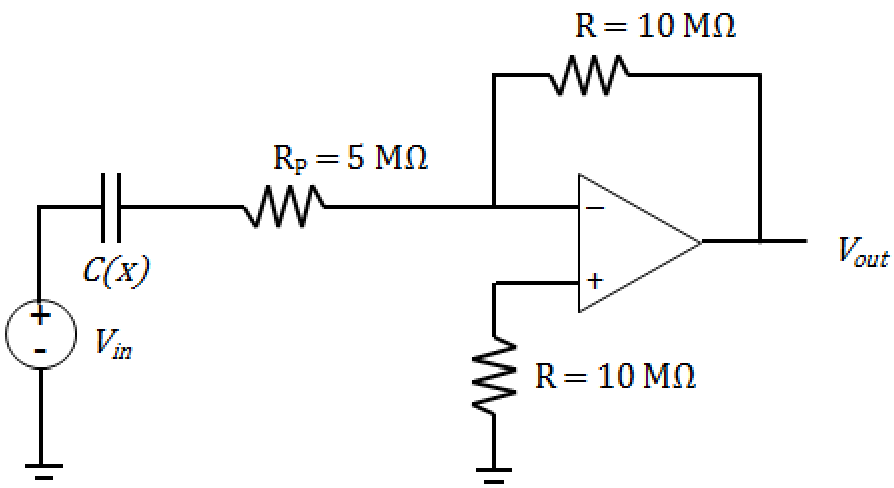
+5 points for the transfer function (+3 points if they get confused about C(x) but set up KCL right etc.)

+5 points for new phase (+3 points if they get $-\frac{\pi}{2}$, +2 points if they get 0)

+2 points for new magnitude numerical answer

+3 points → apply liberally if they seem on the right track

c) Due to non-idealities, the parasitic resistance of the accelerometer forms a first order filter with its capacitance. In reality the circuit looks more like this. (The parallel current source +RC has been converted to a series voltage source + RC for you, using source and series-shunt transformations.) State the kind of filter this circuit forms (i.e. low-pass or high-pass). Briefly explain why.



High pass filter.

Explanation: Capacitor in series blocks low frequencies.

Transfer function:

$$V_{out} = -\frac{R}{R_P + C(x)} = -\frac{R}{R_P + \frac{1}{j\omega C(x)}} = -\frac{j\omega R C(x)}{1 + j\omega R_P C(x)}$$

Solving for the transfer function and concluding it is a high pass (plugging in $\omega = 0$ and $\omega = \infty$) suffices as an explanation as well.

+5 points for correct filter answer, +5 points for explanation