EE 123 DIGITAL SIGNAL PROCESSING, Fall 2009

Midterm # 1, October 9, Friday, 9:30-10:50 am

Name ___________________

Closed book. Two letter-size crib-sheets are allowed. Show all your work. Credit will be given for partial answers.

Problem	Points	Score
1	30	
2	30	
3	40	
Total	100	

1. When the input to a causal LTI system is:

$$x[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] - \frac{4}{3}2^n u[-n-1],$$

the z-transform of the output is:

$$Y(z) = \frac{1 + z^{-1}}{(1 - z^{-1})(1 + \frac{1}{2}z^{-1})(1 - 2z^{-1})}.$$

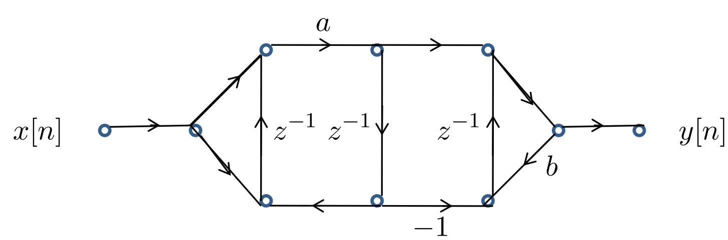
a) (10 points) Find the z-transform of $x[n]$.

b) (10 points) Determine the region of convergence for $Y(z)$.

c) (10 points) Find the impulse response $h[n]$.

Additional workspace for Problem 1

2. a) (15 points) Find the transfer function implemented by the flow diagram:



b) (5 points) Determine the ranges of the parameters a and b that guarantee BIBO stability.

c) (10 points) Assume that this system is implemented with $(B+1)$-bit two's complement fixed point arithmetic, and the products are rounded to $(B+1)$ bits. Ignoring the quantization noise due to the coefficient a, write an expression for the output noise variance introduced by the coefficient b.

Additional workspace for Problem 2

3. Consider the length-8 sequence:

$$x[n] = \{-3,\ 5,\ 4,\ -1,\ -9,\ -6,\ -8,\ 2\}$$

and let $X[k]$ be its 8-point DFT.

a) (10 points) Evaluate $\sum_{k=0}^{7}(-1)^k X[k]$.

b) (10 points) Evaluate $\sum_{k=0}^{7} |X[k]|^2$.

c) (10 points) Determine the DFT of $X[n]$; that is, the sequence obtained by applying the DFT twice to $x[n]$.

d) (10 points) Find the length-4 sequence $w[n]$ whose 4-point DFT $W[k]$ is given by:

$$W[k] = X[2k] \quad k = 0, 1, 2, 3.$$

Additional workspace for Problem 3.

TABLE 2.3 FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n-n_0]$	$e^{-j\omega n_0}$
3. $1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4. $a^n u[n] \quad (\|a\| < 1)$	$\dfrac{1}{1 - ae^{-j\omega}}$
5. $u[n]$	$\dfrac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6. $(n+1)a^n u[n] \quad (\|a\| < 1)$	$\dfrac{1}{(1 - ae^{-j\omega})^2}$
7. $\dfrac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\|r\| < 1)$	$\dfrac{1}{1 - 2r\cos\omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8. $\dfrac{\sin\omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \|\omega\| < \omega_c, \\ 0, & \omega_c < \|\omega\| \le \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\omega} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)]$

TABLE 3.1 SOME COMMON z-TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\dfrac{1}{1 - z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\dfrac{1}{1 - z^{-1}}$	$\|z\| < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $a^n u[n]$	$\dfrac{1}{1 - az^{-1}}$	$\|z\| > \|a\|$
6. $-a^n u[-n-1]$	$\dfrac{1}{1 - az^{-1}}$	$\|z\| < \|a\|$
7. $na^n u[n]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\|z\| > \|a\|$
8. $-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1 - az^{-1})^2}$	$\|z\| < \|a\|$
9. $[\cos\omega_0 n]u[n]$	$\dfrac{1 - [\cos\omega_0]z^{-1}}{1 - [2\cos\omega_0]z^{-1} + z^{-2}}$	$\|z\| > 1$
10. $[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1 - [2\cos\omega_0]z^{-1} + z^{-2}}$	$\|z\| > 1$
11. $[r^n \cos\omega_0 n]u[n]$	$\dfrac{1 - [r\cos\omega_0]z^{-1}}{1 - [2r\cos\omega_0]z^{-1} + r^2 z^{-2}}$	$\|z\| > r$
12. $[r^n \sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1 - [2r\cos\omega_0]z^{-1} + r^2 z^{-2}}$	$\|z\| > r$
13. $\begin{cases} a^n, & 0 \le n \le N-1, \\ 0, & \text{otherwise} \end{cases}$	$\dfrac{1 - a^N z^{-N}}{1 - az^{-1}}$	$\|z\| > 0$