

Problem # 1 — Short Questions (56 pts.)

- (2) What is the name of the “Father of Interactive Computer Graphics” (who was a guest in our Department) for the last two years ? Ivan Sutherland

- (4) Given the choices (voxels | B-rep mesh | CSG | sweep | instantiation), which is the preferred way to model :

a) The result of an MRI brain scan: Voxels

b) A curled-up garden hose: sweep

- (3) Given a perspective projection with the eye at $z = 0$, the image plane at $z = -1$, and the 3D scene object at $z = -100$, how does the perspective image change, if the z -distances are increased proportionally, so that the image plane is now at $z = -2$, and the scene object is now at $z = -200$? The transformation experienced by the image is (circle all that apply):

uniform scaling, non-uniform scaling, affine, sheared, none_of_the_previous

- (3) A colored light has the RGB components (0.4 0.3 0.1). Circle the term that best describes its perceptual color:

pink medium_grey brown purple bright_yellow light_green greenish_grey

- (6) The deCasteljau algorithm is used to subdivide a Bézier curve **with a cusp at $t = 0.5$** into two parts at **parameter value $t = 0.5$** . Circle **all** types of continuity that exist at the junction:

G0 C0 G1 C1 G2 C2 G3 C3 G4 C4

Circle **all** the types of continuity that hold for the two subdivided curve pieces:

G0 C0 G1 C1 G2 C2 G3 C3 G4 C4

- (3) A cross section with an area of 3 cm^2 is swept along a piecewise linear space path. What is the area of the cross section at a properly mitered joint that makes a 120 degree turn? 6 cm^2

- (3) How many degrees of freedom are associated with all possible **planes** in **R3** ? 3

- (3) How many DOF (degrees of freedom) are associated with all possible **cylinders** in **R3** with (variable) length L and radius R ? 7

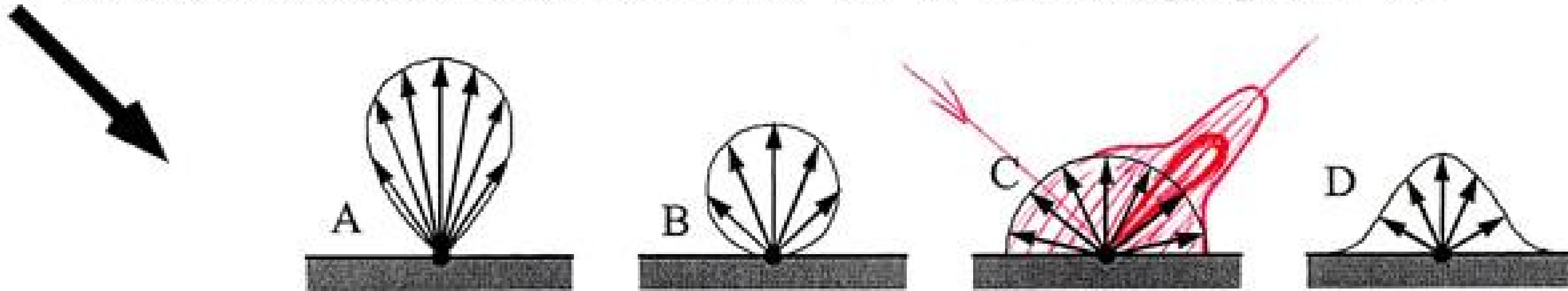
- (4) What are the **two** contributions discussed in this class that Mr. **Phong** has made to the field of computer graphics:

1.) Phong illumination model : highlights $\cos^n(\theta)$.
2.) Phong shading model : interpolating normals.

(4) What are the minimum and maximum number of vanishing points that can be obtained from a perspective projection of a **regular eight-sided prism** ?

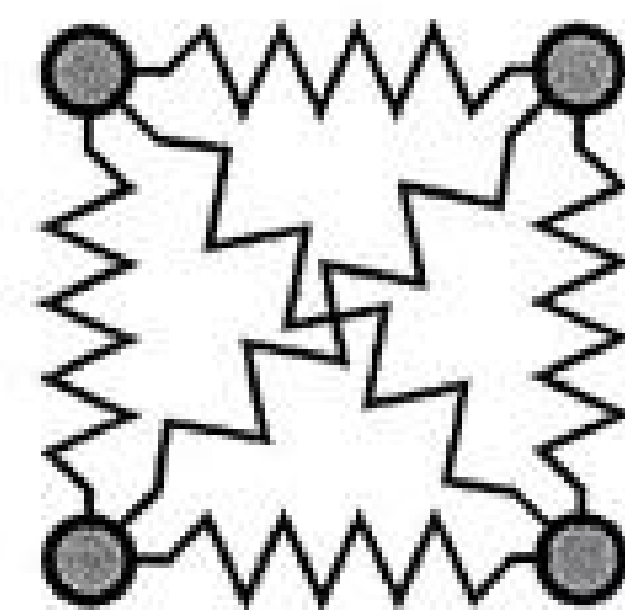
MIN: 1 MAX: 5

(4) Modify one of the four directional vector diagrams below to represent the perceived brightness observed in the directions opposite to the small arrows on an idealized **Phong** surface illuminated with a directional light coming from the upper left (fat arrow); the relevant surface characteristics are $k_d=0.4$, $k_s=0.6$, Phong exponent=20.



(3) Consider this planar spring-mass system in the 2D-plane with 4 masses and six springs. How many dimensions does the phase-space of this system have ?

16



(4) Which of the following effects can NOT be done with the type of ray-tracing renderer that you used in AS#5 and #6 ? (Check all that CANNOT be done).

- ☐ Phong highlights on a glossy surface
- ☐ Specular reflections of other objects in the scene
- ☒ Color-bleeding from one diffuse surface onto another diffuse surface
- ☒ Caustic lines
- ☐ Image enlargement seen through a convex glass lens
- ☐ Total internal reflection on a glass/air interface

(3) Two of Grassman's Laws state that perceptual color space is 3-dimensional and that metamers additively mix to form metamers. Write down the **third law**:

As physical color is varied continuously, perceived color also varies continuously.

(4) When raytracing a scene with fences or striped shirts by using a single ray per pixel, one will get bad aliasing. Name two conceptually different techniques to alleviate this problem:

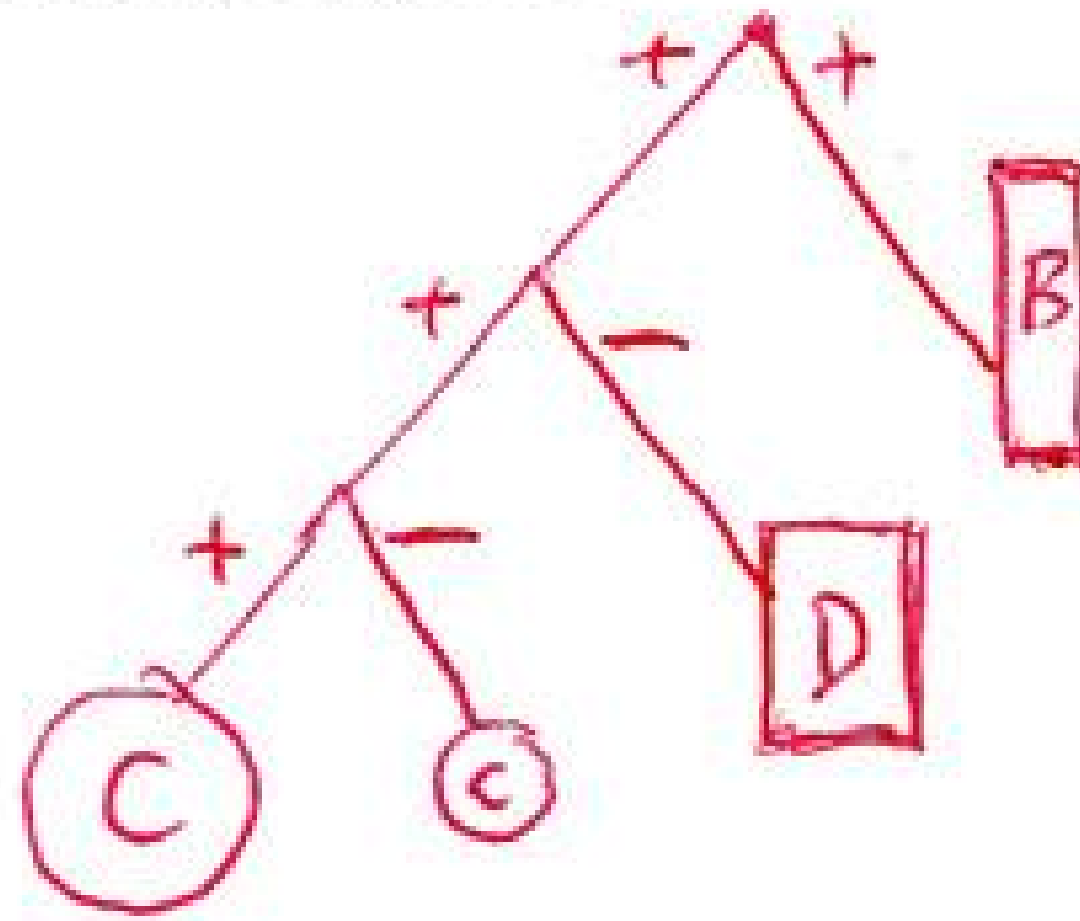
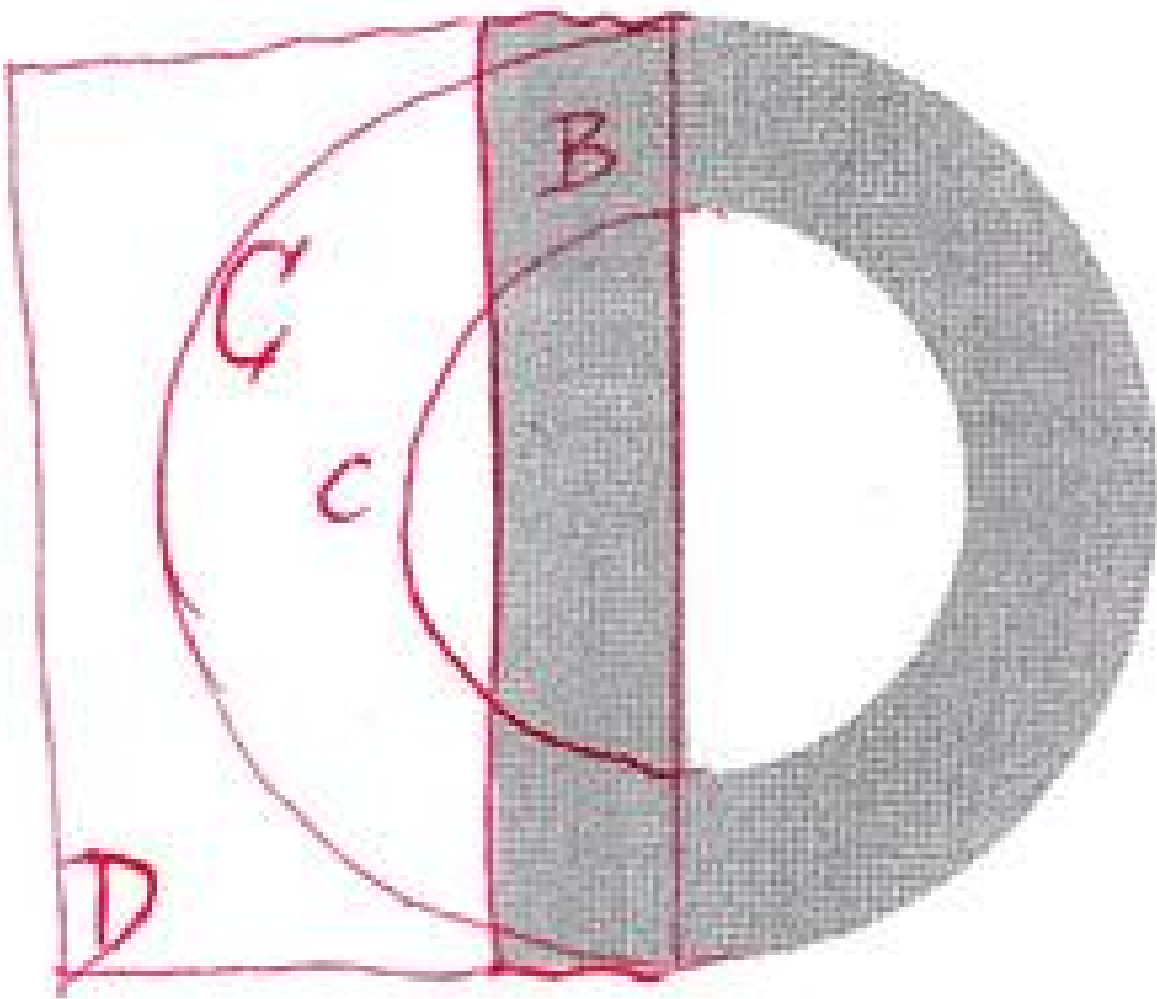
- 1.) low-pass filter the scene first.*
- 2.) super-sample in the rendering process.*

(3) What is the key difference between Photon-tracing and Path-tracing ? (Do **NOT** mention common features!).

Photon-tracing starts at the light source
Path-tracing starts at the eye/camera

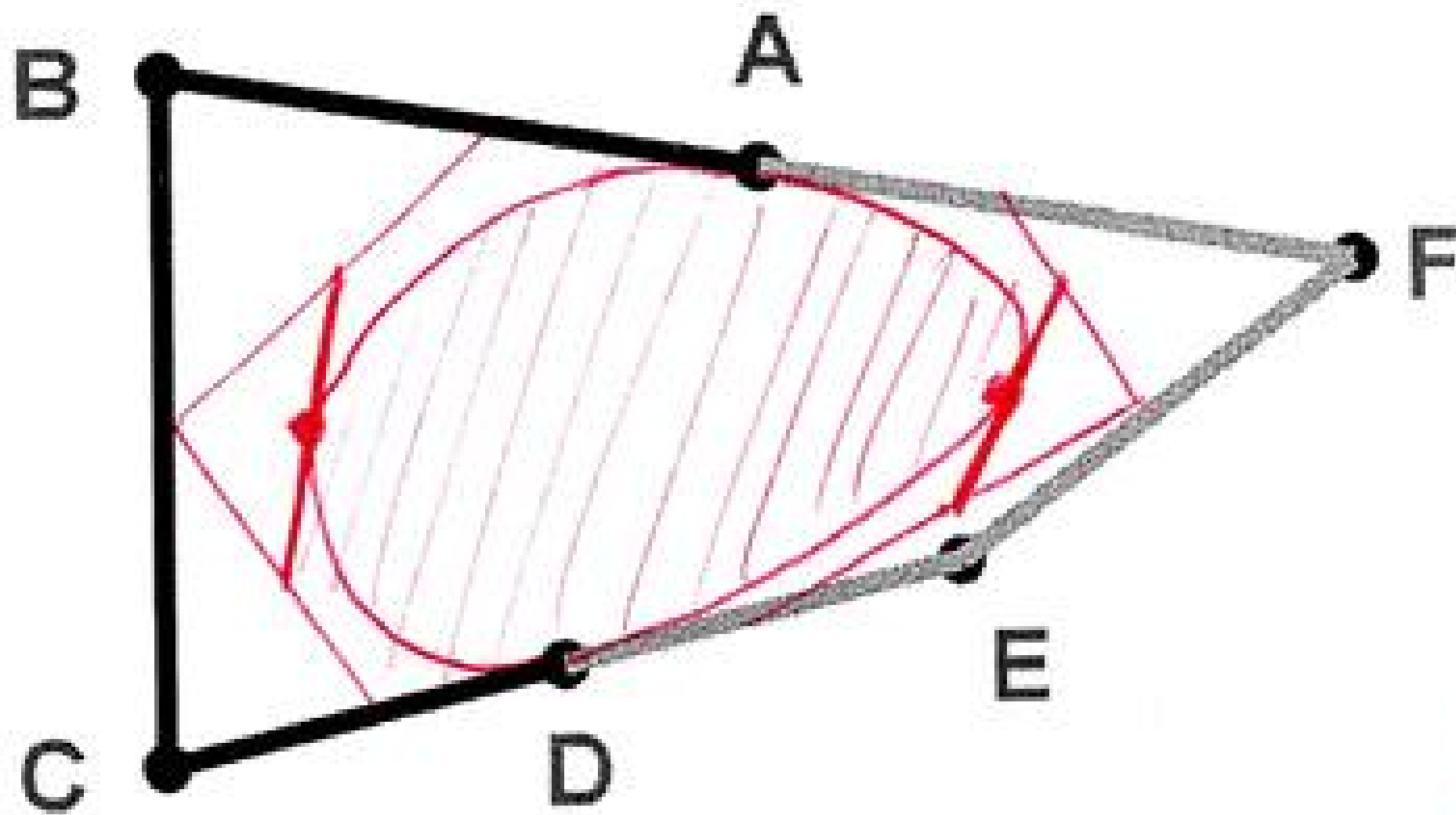
Problem #2 — CSG (8 pts.)

Given the 2-dimensional letter shape below and a 2-D computer graphics CSG system with only the primitives **unit-square** and **unit-circle**, draw a simple CSG tree that will model this letter D. Use a **minimal number** of elements and of Boolean operations. Ignore transformations. Also show the **transformed, instantiated leaf objects overlaid** on the picture of the letter D.



Problem # 3 — Bézier Curve (10 pts.)

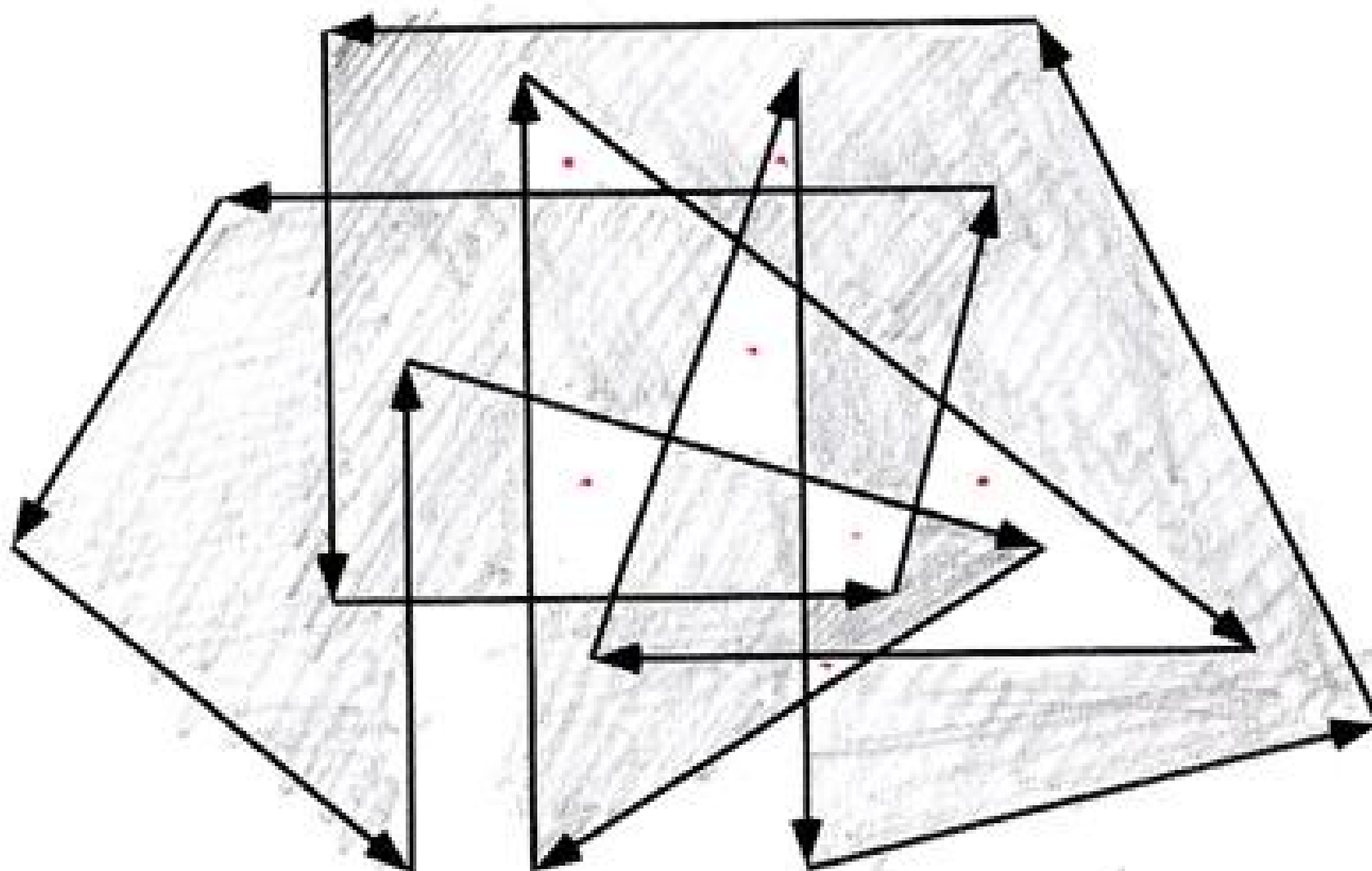
(A) Draw the **two** cubic Bézier segments defined by control polygons A,B,C,D, and D,E,F,A, respectively. Locate the mid-points and their tangent directions using the **deCasteljau method**.



(B) Now you want to use this set-up to draw a parameterized egg-shape in 2D composed of the two Bézier curves, joined into a single C1-continuous, closed curve. How many degrees of freedom does this system have (including the placement of the egg in the plane) ? -- **Explain your answer.**

8 : place A, B, C, D
E, F given by C1-constraints

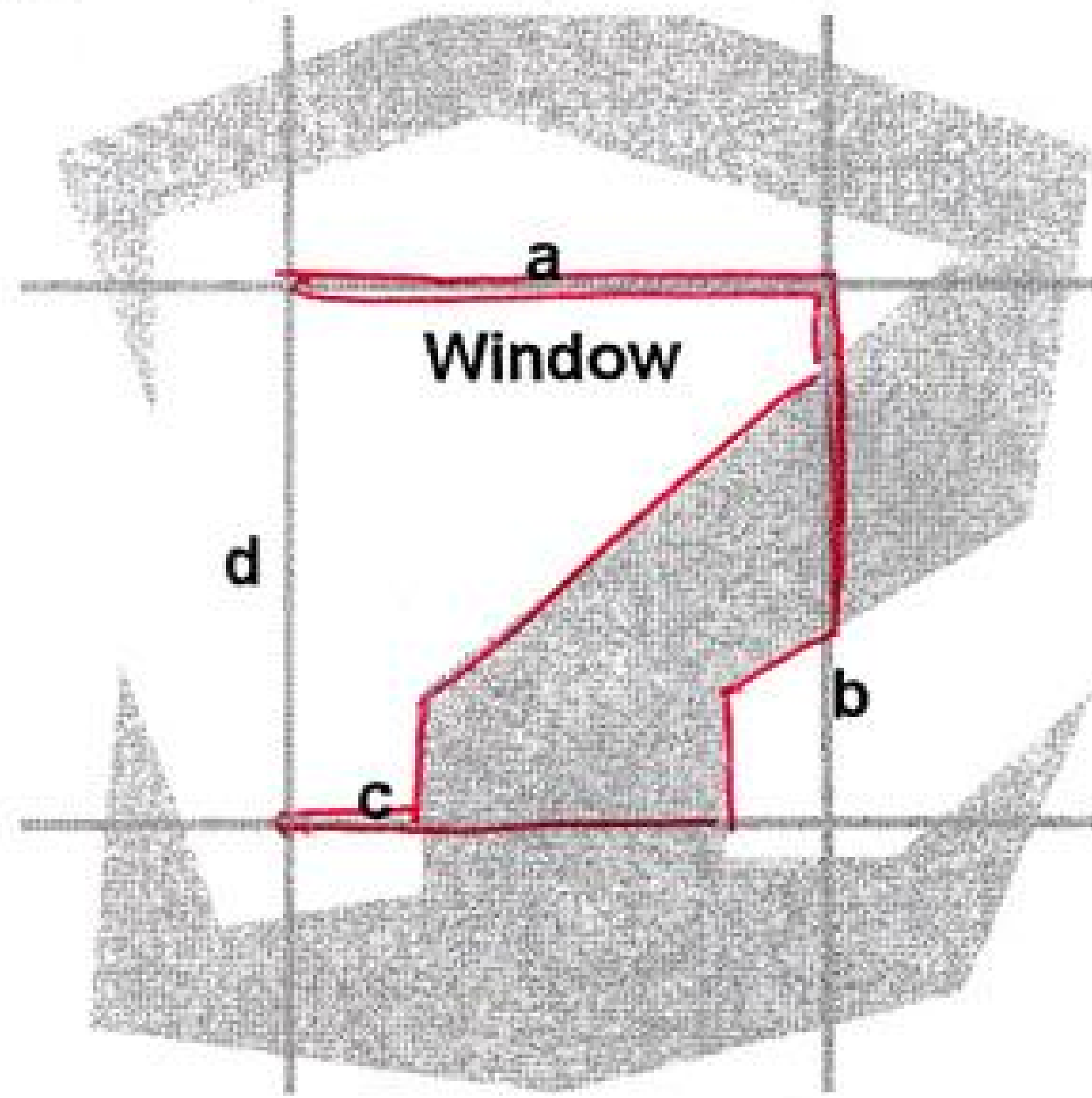
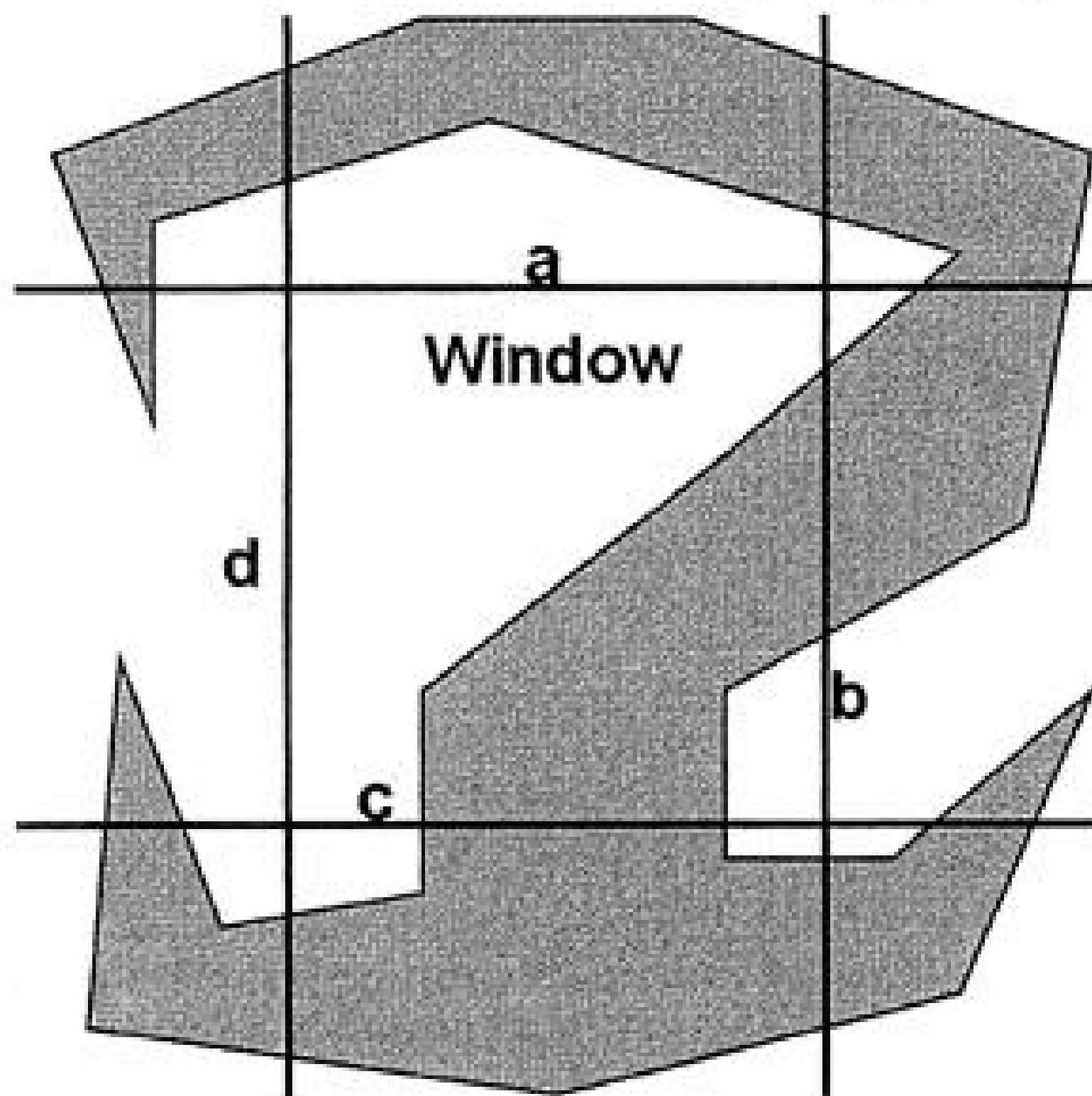
Problem # 4 — Polygon-fill (8 pts.)



Paint all “inside” areas according to the NON-ZERO WINDING NUMBER model.

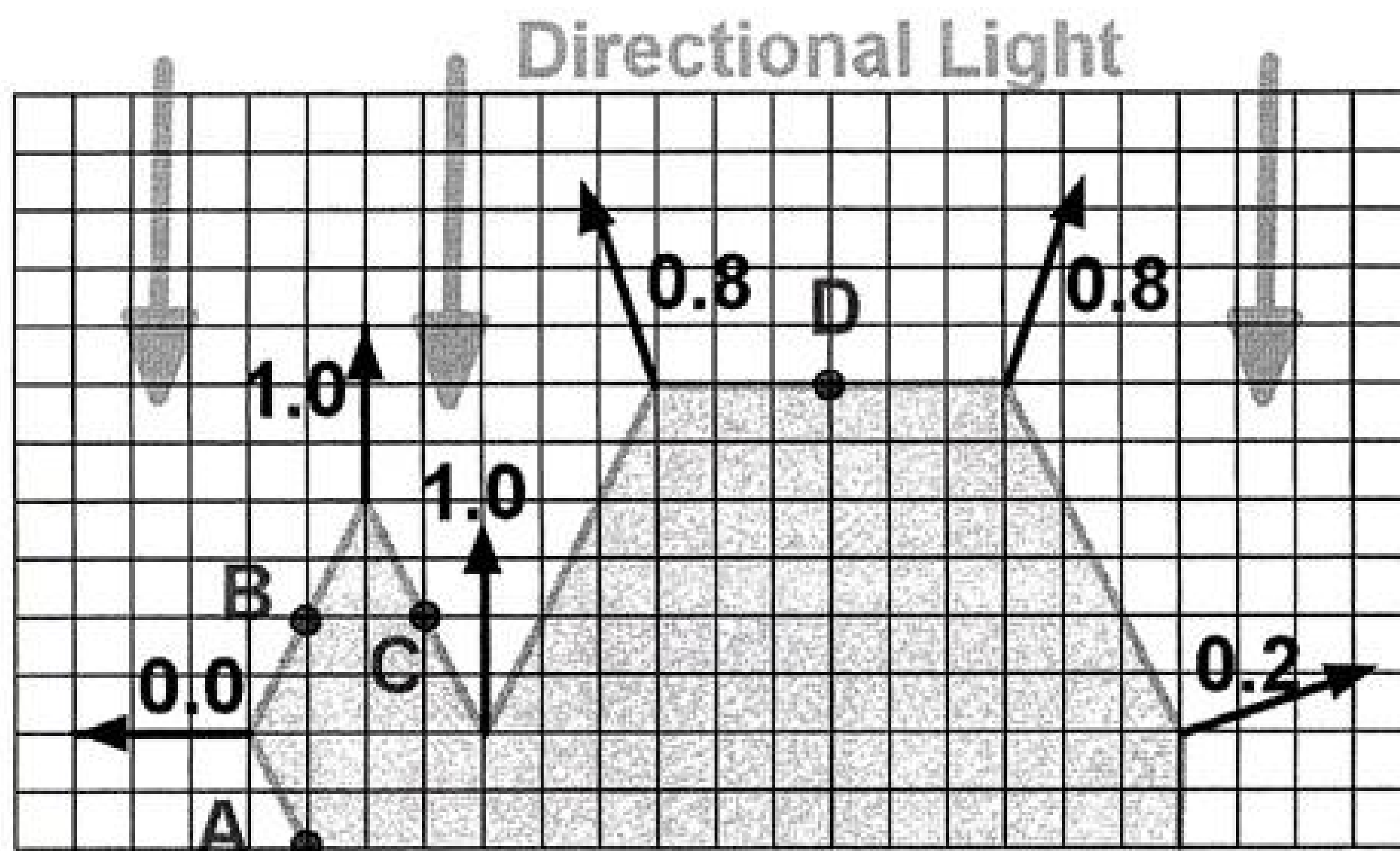
Problem # 5: — Polygon Clipping (8 points)

Show the polygon contour(s) including the spurious double segments on the Window frame that will be output from the Sutherland-Hodgman polygon clipping algorithm for the polygons shown below. Assume that the clipping sequence is : a, b, c, d. Do your draft on the left, and show the final result in the right figure by strongly tracing out all output line segments.



Problem # 6 — Phong Shading (8 pts.) .

You are processing the **Lambert** surface shown in profile below with **Phong** interpolation. The computed dot-products between the averaged vertex normals and the directional light of strength 1.0 are as indicated. Compute the resulting brightness values at the indicated points A, B, C, and D, assuming $k_s = 0$; $k_a = k_d = 0.2$.



Brightness @ A = 0

Brightness @ B = $0.2 \cdot \frac{\sqrt{2}}{2}$

Brightness @ C = 0.2

Brightness @ D = 0.2

Problem # 7 — Circle the correct answer (10 pts.; -2 each wrong)

☒ TRUE ☐ FALSE | Rotations in 2D are commutative.

☒ TRUE ☐ FALSE | A parallel projection from 3D to 2D is an affine transformation.

☒ TRUE ☐ FALSE | In 3-space, any number of uniform scalings and a single rotation around any axis through the origin can be applied in arbitrary order without affecting the result.

☒ TRUE ☐ FALSE | In 3-space, two elementary mirroring operations on two main coordinate planes can be applied in either order to get the same result.

☐ TRUE ☒ FALSE | The perspective projection of a line segment AB (without clipping) is identical to the line segment between the perspective projections of the two endpoints A and B.

☒ TRUE ☐ FALSE | In an oblique parallel projection, the midpoint of a straight line segment will always be mapped to the middle of the projected line segment.

☐ TRUE ☒ FALSE | A piece of surface will have the same preceptual color regardless of which of two metamers is used to illuminate it.

☐ TRUE ☒ FALSE | A spherical Lambert reflector illuminated with a uniform directional light will look like a disk of uniform brightness from the direction of that light.

☐ TRUE ☒ FALSE | In general, the Phong shading technique produces a planar $\{a*x + b*y + c\}$ brightness distribution on triangular faces of a convex polyhedral object.

☒ TRUE ☐ FALSE | Using the Frenet frame to define the orientation of the cross section swept along an arbitrary, smooth, closed space curve with no perfectly straight path segment, will guarantee that the cross sections will smoothly match up where the two curve ends join to close the loop.

Problem # 8 — Fill in the blanks with an appropriate answer (10 pts., 2 each)

The transpose of an orthonormal matrix is equal to its inverse.

Standard kinematic algorithms assume that articulated structures have the topology of a tree.

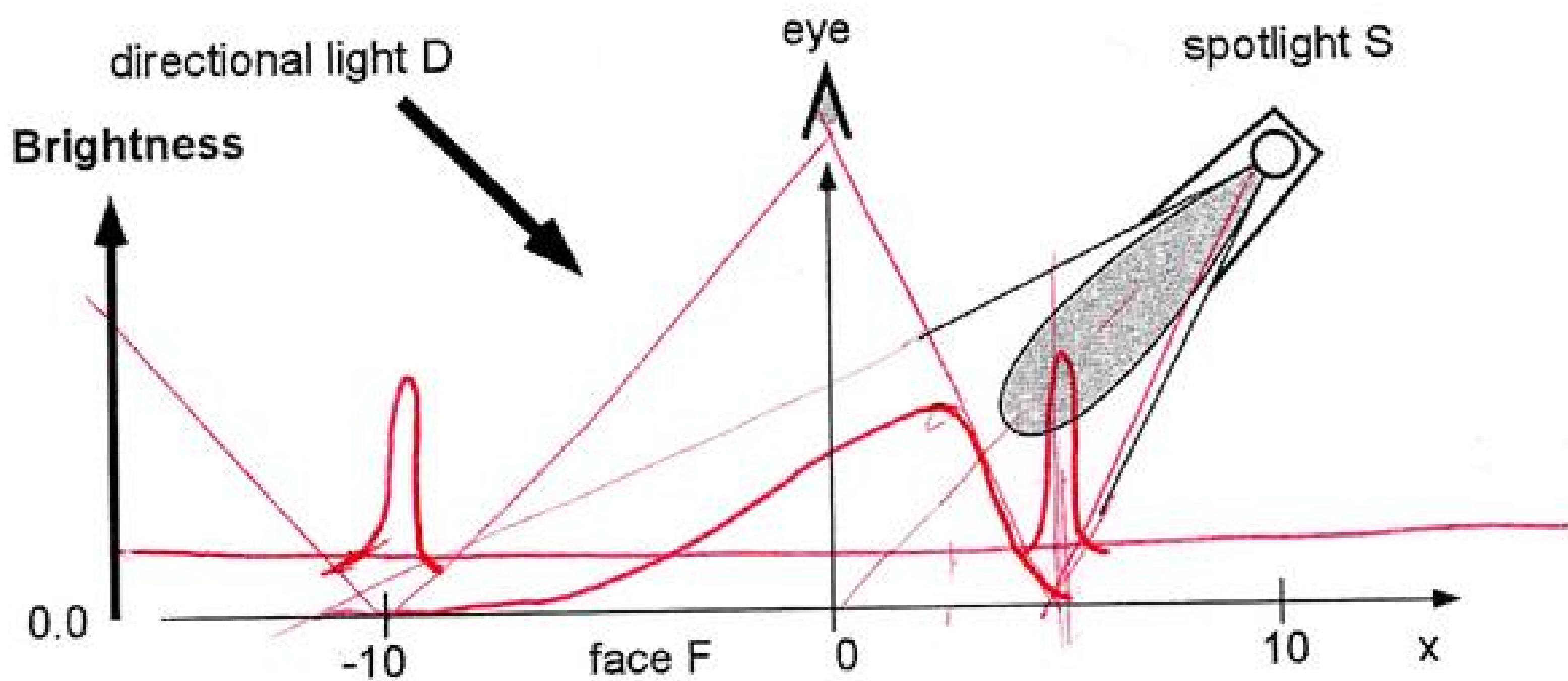
The BRDF is a function that describes how much a material reflects incoming light from one direction out in another direction.

In Catmull-Clark subdivision, the number of quads grows by a factor of 4 for each level of subdivision.

A bump map is used to change the normal vectors when shading an object.

Problem # 9 — Illumination (12 pts.)

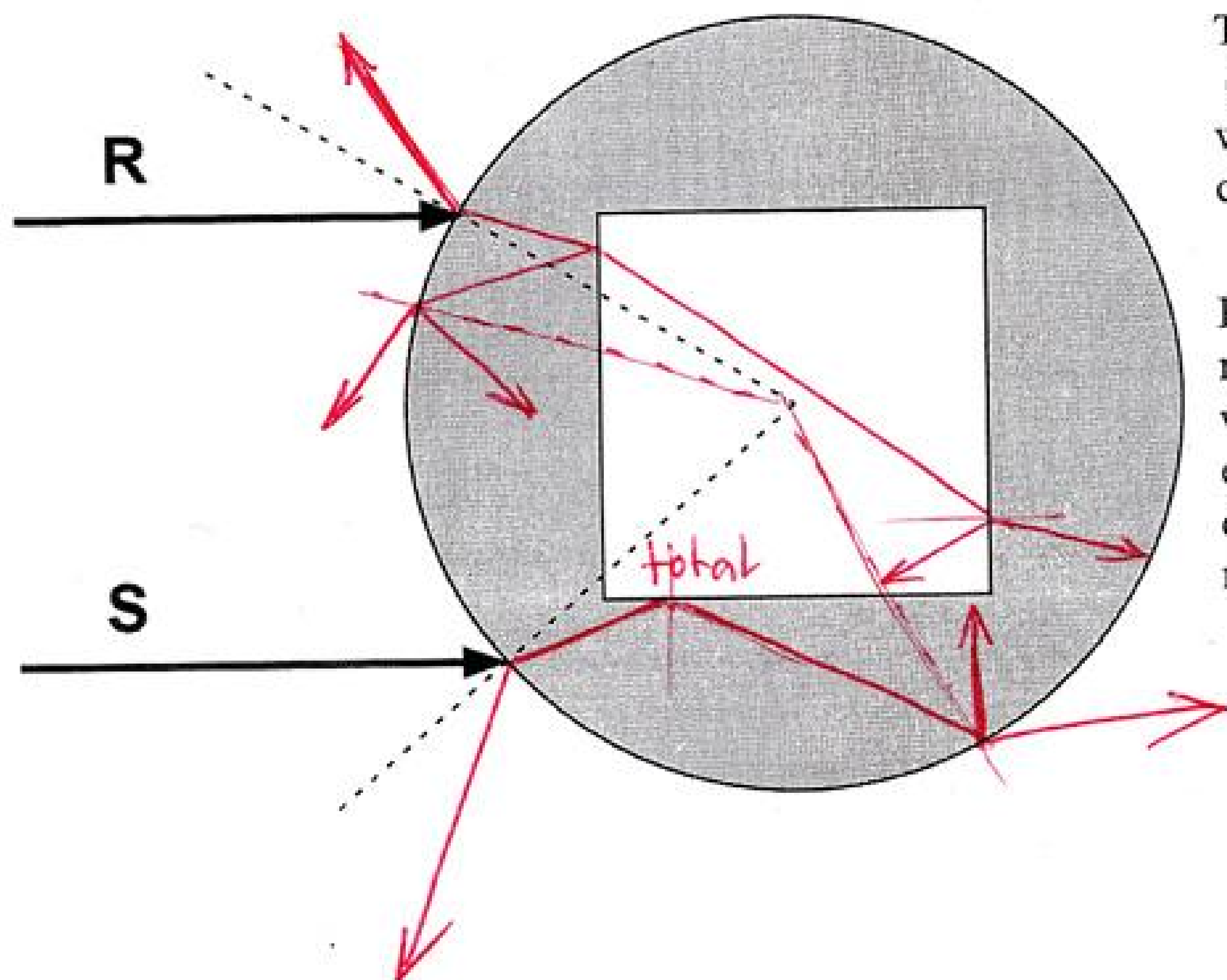
Sketch observed brightness B , as seen from camera eye at $(0, 10, 0)$, along real face F (Phong model, $K_{\text{amb}} = K_{\text{diff}} = K_{\text{spec}} = 0.5$, $N_{\text{phong}} = 50$), illuminated by directional light D and spotlight S . Directional light D is of intensity 1 and shines from direction $(-1000, 1000, 0)$; Spotlight S is of intensity 10 and shines from point $(10, 10, 0)$ towards $(0, 0, 0)$; its angular falloff is 4.



At what x -values (approximately) does "eye" observe a peak in Lambert reflection? $x = \underline{\sim +2}$

At what x -values (approximately) does "eye" observe a peak in Phong reflection? $x = \underline{\frac{-10}{+5}}$

Problem # 10 — Refraction and Reflection (10 pts.)

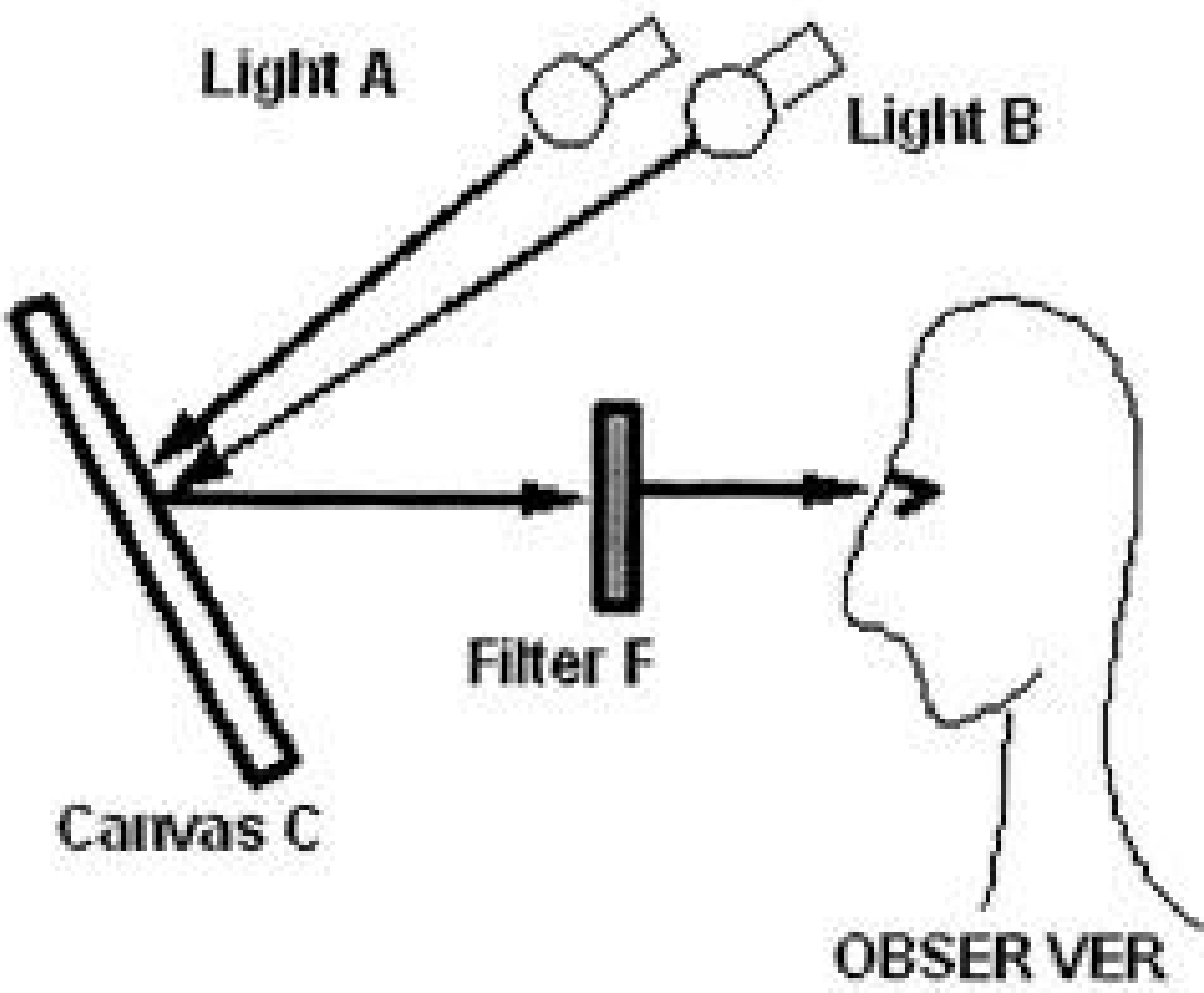


Two rays enter a glass sphere (refractive index $n = 1.5$) with a cubical evacuated cut-out as shown.

Ray-trace each of the two rays through the interactions with the **first 3** glass surfaces encountered, and show the directions of the emerging rays after that.

Problem # 11 — Reflection & Filtering (10 points)

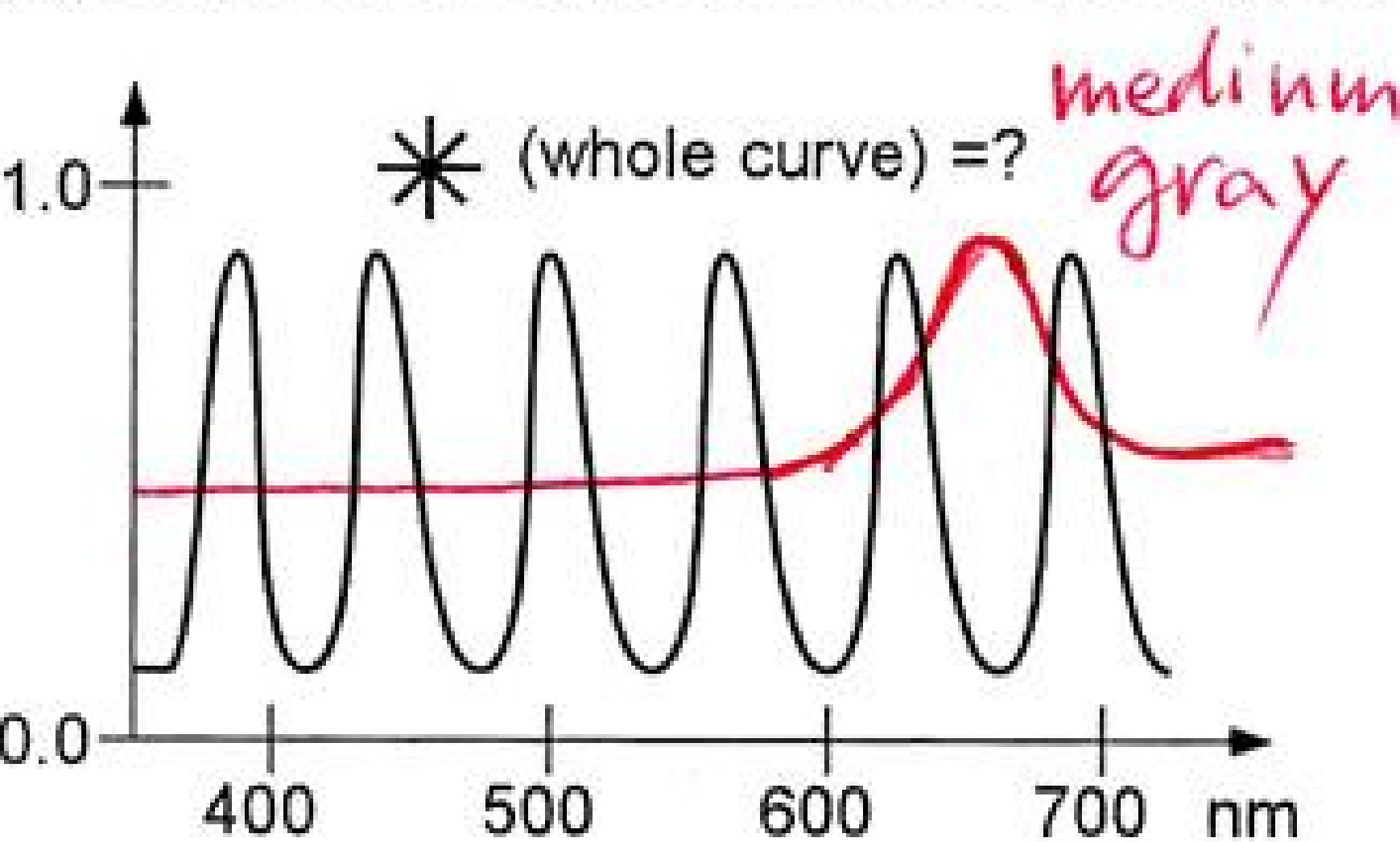
For the set-up illustrated below, in which two lights (A & B) shine on painted canvas C, which is observed through Filter (that passes color F), -- specify what OBSERVER sees (Fill in the table !):



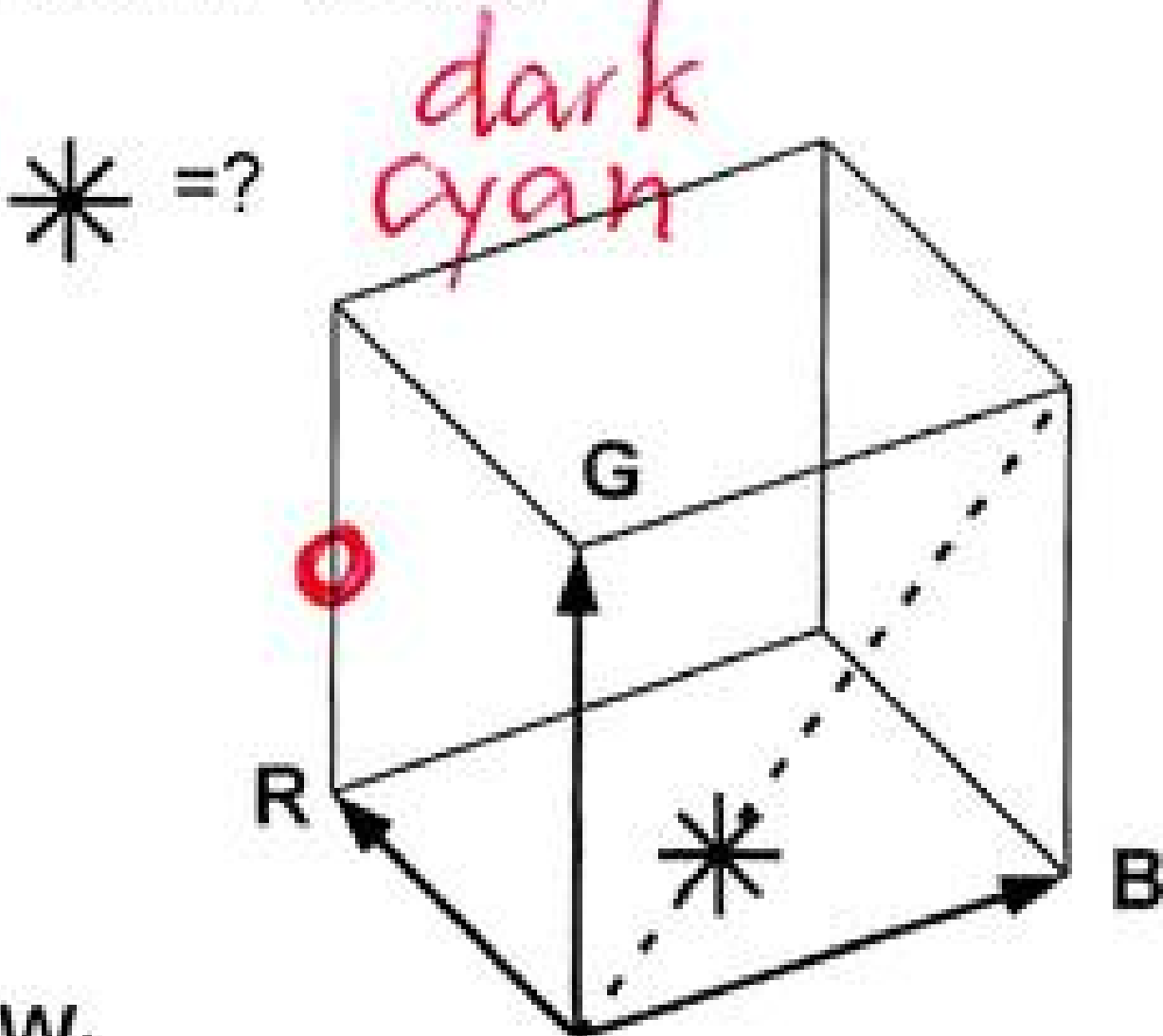
Light A	Light B	Painting	Filter F	OBSERVED
Cyan	none	Yellow	Green	? <i>Green</i>
Yellow	Blue	Cyan	Magenta	? <i>Blue</i>
Green	Red	Magenta	Green	? <i>Black</i>

Problem # 12 — Color Spaces (12 pts.)

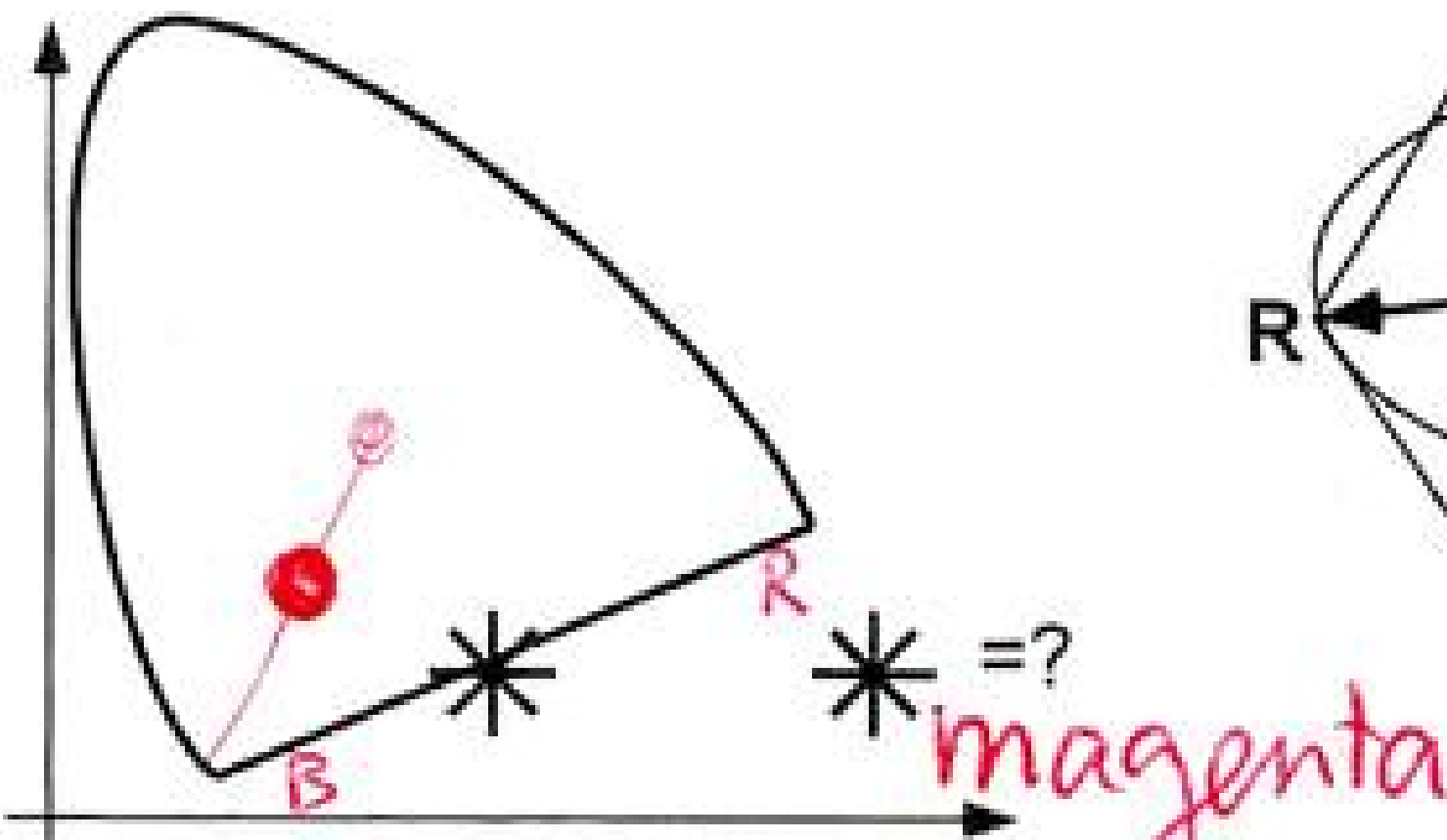
For each of the diagrams of a color spaces: (A) Name the perceptual color indicated by: * =? (B) Represent the requested color in the diagram (SHOW: "color")



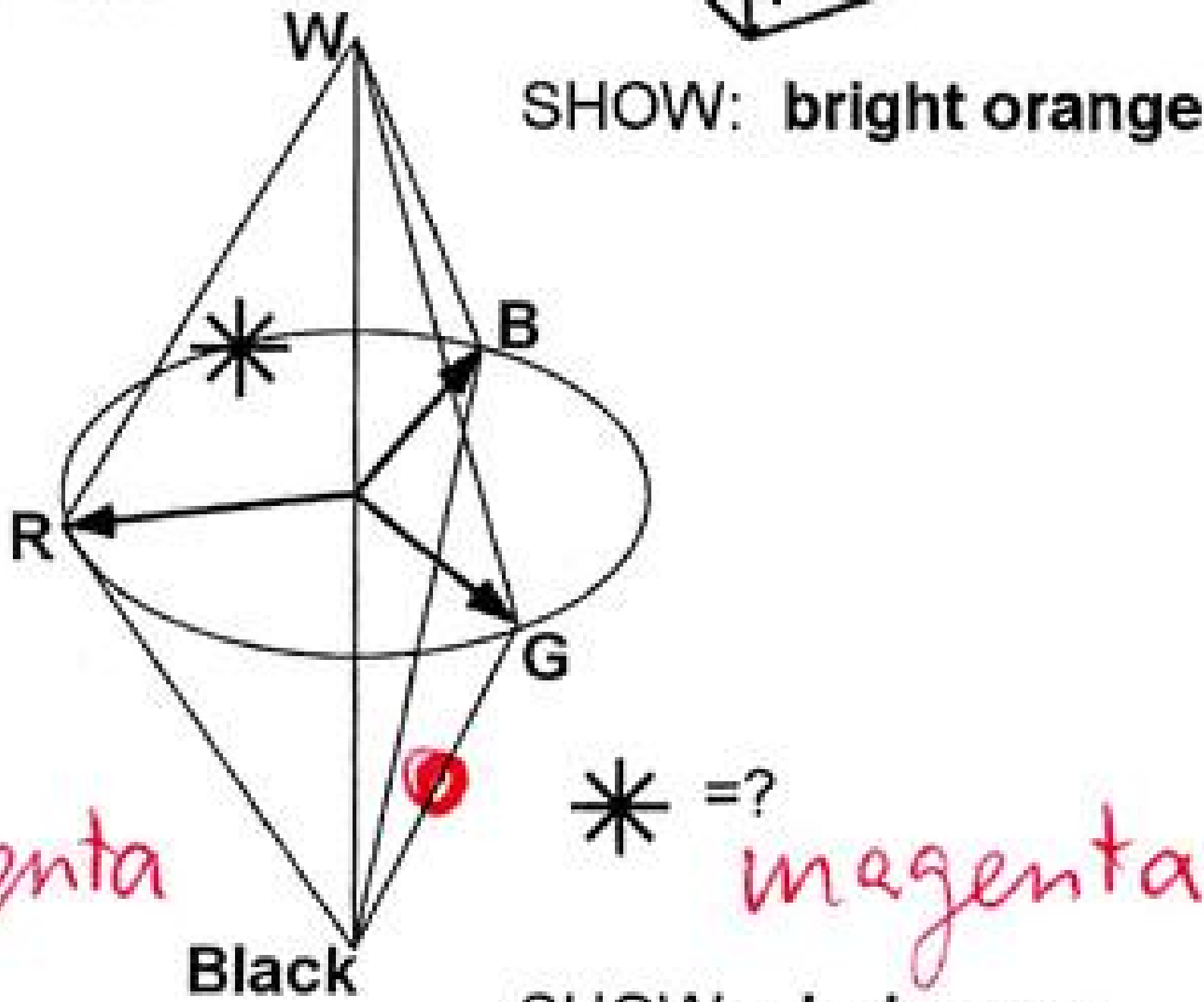
SHOW: light pink



SHOW: bright orange



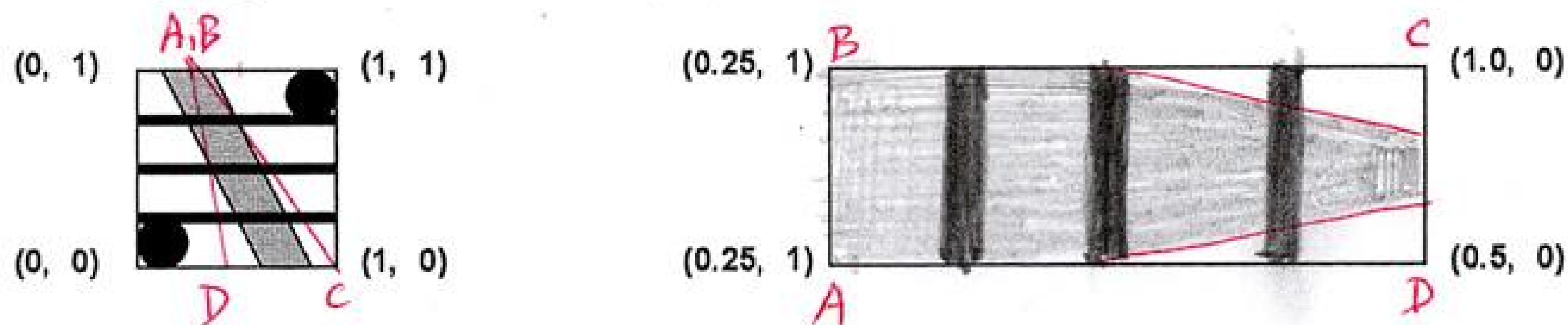
SHOW: whiteish blue



SHOW: dark green

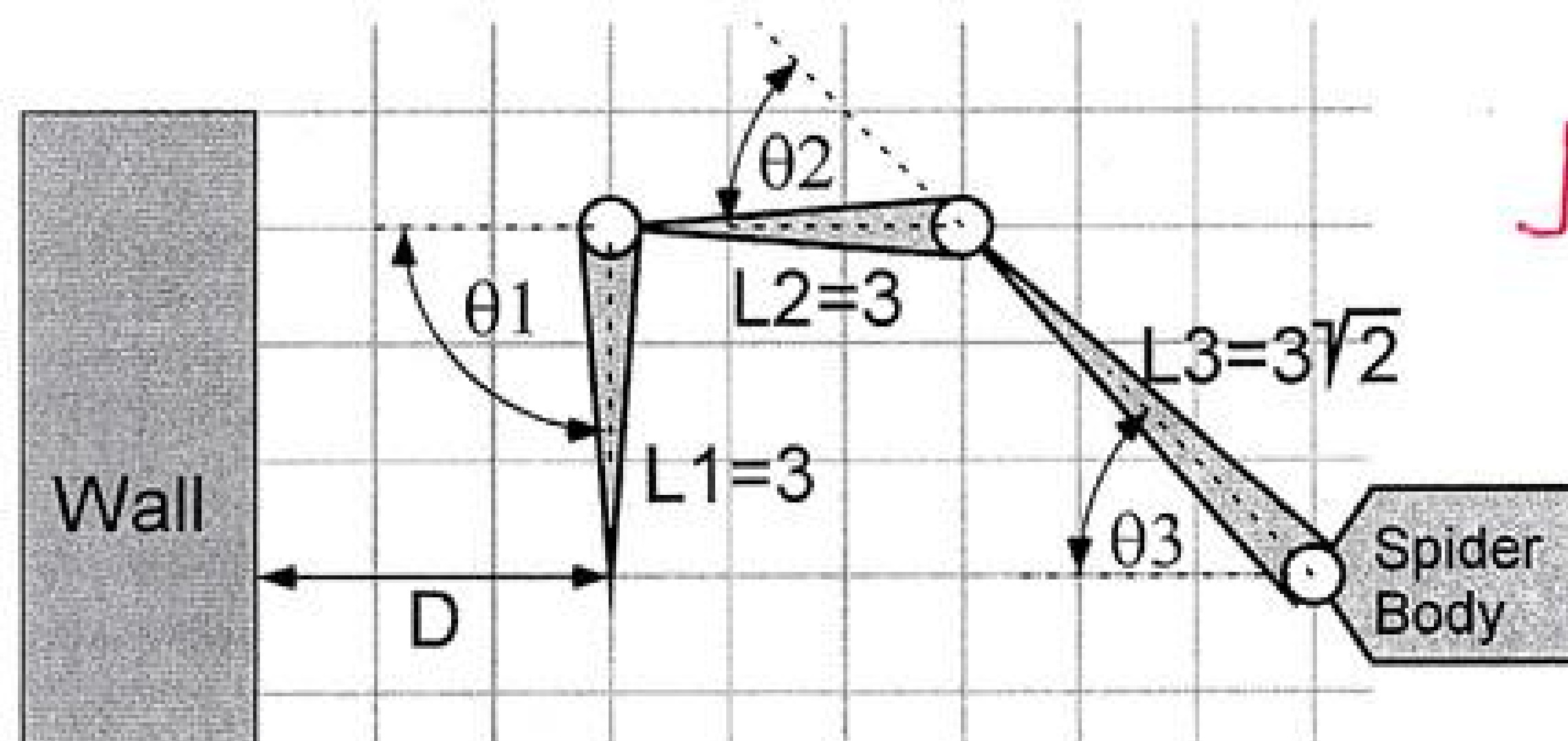
Problem # 13 — Texture Mapping (8 pts.)

Use the texture map below and apply it to the rectangular surface on the right, carefully observing the given texture coordinates (u,v).



Problem # 14 — Inverse Kinematics (10 pts.)

Write out the (numerical entries in the) Jacobian for the 3-link spider leg with respect to the distance D of its end-effector from the Wall, based on the three actuator angles $\theta_1, \theta_2, \theta_3$, (measured in radians). $\{ \theta_1 = \pi/2; \theta_2 = \theta_3 = \pi/4 \}$ $\{ \sin \pi/4 = \cos \pi/4 = 0.707 \}$



$$J = \begin{bmatrix} 3 & 3 & 0 \end{bmatrix}$$

Problem # 15 — Perspective Warp (10 pts.)

Where in 3-space do all the lines that were originally parallel to the z-axis of the VRCS intersect after the {Shirley} perspective transform ? (Here is the relevant homogeneous matrix):

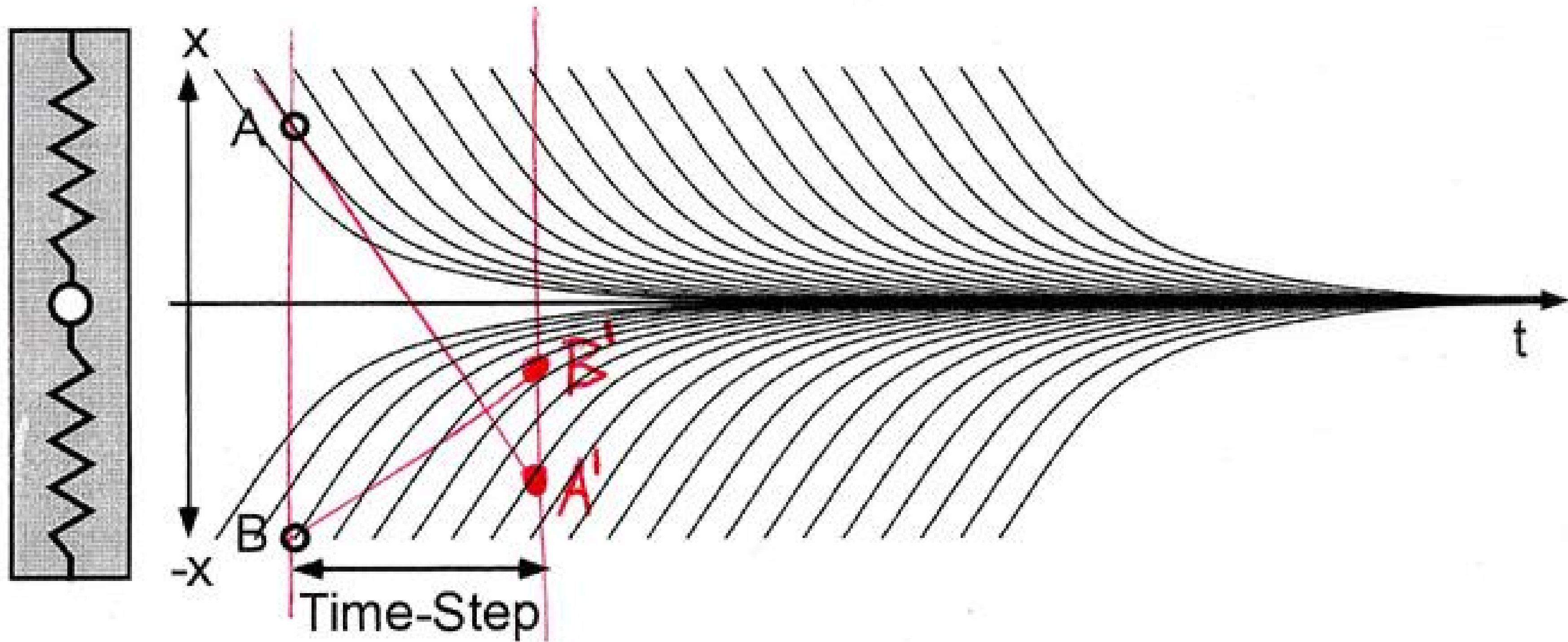
$$M_p = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$x = \underline{0} \quad y = \underline{0} \quad z = \underline{n+f}$$

Problem # 16 — Simulation (10 pts.)

Consider the field of trajectories for the simple, one-dimensional, heavily damped spring-mass system discussed in class. Consider two different starting states and simulation methods:

For start state A of the mass use forward Euler simulation; for start state B use implicit Euler. For both cases advance the simulation by the same “Time-Step” shown below and draw where the state will be at the end of the simulation step.



Problem # 17 — Catmull-Clark Subdivision (8 pts.)

Draw the result of applying one iteration of Catmull-Clark subdivision to the mesh below. Then **circle strongly** all vertices (both original and new ones created) that are extraordinary.

