

EJEMPLO 17

Usar la regla de L'Hopital para hallar los siguientes limites:

a) $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

b) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

c) $\lim_{x \rightarrow 0} \frac{x - \tan x}{x^3}$

d) $\lim_{x \rightarrow \infty} x \ln \frac{x-1}{x+1}$

e) $\lim_{x \rightarrow 0^+} x e^{1/x}$

Solucion

En cada caso, despues de verificar la indeterminacion, se aplica la regla de L'Hopital las veces necesarias.

a) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \left(\frac{0}{0} \right) \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{1}{1} = 1$

b) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \left(\frac{0}{0} \right) \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\sin x}{2x} = \left(\frac{0}{0} \right) \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{\cos x}{2} = \frac{1}{2}$

c) $\lim_{x \rightarrow 0} \frac{x - \tan x}{x^3} = \left(\frac{0}{0} \right) \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1 - (1 + \tan^2 x)}{3x^2} = \lim_{x \rightarrow 0} \frac{-\tan^2 x}{3x^2} =$
 $\left(\frac{0}{0} \right) \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{-2 \tan x (1 + \tan^2 x)}{6x} = \lim_{x \rightarrow 0} \frac{-\tan x}{3x} = \left(\frac{0}{0} \right) \stackrel{H}{=}$
 $\lim_{x \rightarrow 0} \frac{-(1 + \tan^2 x)}{3} = \frac{-1}{3}$

d) $\lim_{x \rightarrow \infty} x \ln \frac{x-1}{x+1} = (\infty \cdot 0) = \lim_{x \rightarrow \infty} \frac{\ln \frac{x-1}{x+1}}{\frac{1}{x}} = \left(\frac{0}{0} \right) \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x-1} - \frac{1}{x+1}}{\frac{-1}{x^2}} =$
 $= \lim_{x \rightarrow \infty} \frac{-2x^2}{x^2 - 1} = -2$

e) $\lim_{x \rightarrow 0^+} x e^{1/x} = (0 \cdot \infty) = \lim_{x \rightarrow 0^+} \frac{e^{1/x}}{\frac{1}{x}} = \frac{\infty}{\infty} \stackrel{H}{=} \lim_{x \rightarrow 0^+} \frac{\frac{-1}{x^2} e^{1/x}}{\frac{-1}{x^2}} = \lim_{x \rightarrow 0^+} e^{1/x} = \infty$