

Ejercicio 1

$$f(x) = \begin{cases} c(1-x^2) & 0 \leq x \leq 1 \\ 0 & \text{Resto} \end{cases}$$

$$\int_0^1 c(1-x^2) dx = c \left(x - \frac{x^3}{3} \right) \Big|_0^1 =$$

$$= c \left(1 - \frac{1}{3} \right) = c \cdot \frac{2}{3}$$

$$\Rightarrow c \cdot \frac{2}{3} = 1 \Rightarrow c = \underline{\underline{\frac{3}{2}}}$$

$$\int_{0.5}^1 c(1-x^2) dx = c \left(x - \frac{x^3}{3} \right) \Big|_{0.5}^1 =$$

$$= c \left[\left(1 - \frac{1}{3} \right) - \left(\frac{1}{2} - \frac{1}{24} \right) \right] =$$

$$= c \left[\frac{2}{3} - \frac{11}{24} \right] = \frac{3}{2} \left(\frac{2}{3} - \frac{11}{24} \right) =$$

$$= 1 - \frac{33}{48} = \frac{15}{48} = \frac{5}{16} //$$

$$E(x) = \int_0^1 C x (1-x^2) dx =$$

$$= C \int_0^1 (x - x^3) dx =$$

$$= C \left(\frac{x^2}{2} - \frac{x^4}{4} \right) \Big|_0^1 =$$

$$= C \left(\frac{1}{2} - \frac{1}{4} \right) =$$

$$= \frac{3}{2} \cdot \frac{1}{4} = \frac{3}{8} //$$

$$E(x^2) = \int_0^1 C x^2 (1-x^2) dx =$$

$$= C \int_0^1 (x^2 - x^4) dx =$$

$$= C \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1 =$$

$$= C \left(\frac{1}{3} - \frac{1}{5} \right) =$$

$$= \frac{3}{2} \cdot \frac{2}{15} = 1/5 //$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$V(x) = \frac{1}{5} - \left(\frac{3}{8}\right)^2$$

$$V(x) = \frac{1}{5} - \frac{9}{64}$$

$$V(x) = \frac{19}{320} //$$

$$\sqrt{V(x)} = \sqrt{\frac{19}{320}} = 0.243669. //$$

Ejercicio 2

Función exponencial

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & \text{resto} \end{cases}$$

$$E(X) = \frac{1}{\lambda}$$

$$V(X) = \frac{1}{\lambda^2}$$

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & x > 0 \\ 0 & \text{resto.} \end{cases}$$

$$E(X) = 0.5 \text{ horas} \Rightarrow E(X) = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{0.5} = 2 //$$

X = tiempo estacionamiento

$$P(X > 3) = 1 - P(X \leq 3) \Rightarrow$$

$$\begin{aligned} 1 - \int_0^3 2e^{-2x} dx &= 1 - \left[-e^{-2x} \right]_0^3 \\ &= 1 - \left[-e^{-6} + e^0 \right] \\ &= 1 + e^{-6} - 1 = e^{-6} // \\ &= 0.002478 // \end{aligned}$$

$$P(X < 0.5) = \int_0^{0.5} 2e^{-2x} dx = -e^{-2x} \Big|_0^{0.5}$$

$$= -e^{-2 \cdot 0.5} + e^0 = \underline{\underline{0.6321}}$$

Otra forma de realizar los dos apartados es aplicando la función de distribución.

$$P(X > 3) = 1 - P(X \leq 3) = 1 - F(3) =$$

$$= 1 - [1 - e^{-3 \cdot 2}] =$$

$$= \cancel{1} - \cancel{1} + e^{-6} = e^{-6} //$$

$$P(X < 0.5) = F(0.5) = 1 - e^{-2 \cdot 0.5} =$$

$$= 1 - e^{-1} = \underline{\underline{0.6321}}$$

Exercício 3

$$P(X=k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

$$E(X) = V(X) = \lambda$$

$$E(X) = 24 \text{ clientes / hora.}$$

$$\lambda_0 = 24 \text{ clientes / hora.}$$

$$\Rightarrow 15 \text{ min} \rightsquigarrow 24 \text{ clientes} \longrightarrow 1 \text{ hora} = 60 \text{ min}$$
$$x \longrightarrow 15 \text{ min.}$$

$$\Rightarrow \underline{\underline{\lambda_1 = 6}}$$

$X =$ N° clientes utilizam caixa rápida.

$$P(X > 6) = 1 - P(X \leq 6)$$

$$= 1 - \left[e^{-6} \left(\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!} + \frac{6^4}{4!} + \frac{6^5}{5!} + \frac{6^6}{6!} \right) \right]$$

$$= 1 - \left[e^{-6} \left(1 + 6 + 18 + 36 + 54 + \frac{324}{5} + \frac{324}{5} \right) \right]$$

$$= 0'3937 //$$

$$\begin{array}{lcl} 5 \text{ min} & \Rightarrow & 24 \text{ clientes} \rightarrow 1 \text{ hora} = 60 \text{ min} \\ & & x \rightarrow 5 \text{ min} \end{array}$$

$$\Rightarrow \underline{\underline{\lambda_2 = 2}}$$

$$\begin{aligned} P(X < 3) &= e^{-2} \left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} \right) \\ &= e^{-2} (1 + 2 + 2) = \underline{\underline{0'6767}} \end{aligned}$$

Ejercicio 4

$$f(x,y) = \begin{cases} \frac{2-x-y}{8} & -1 \leq x \leq 1 \quad ; \quad -1 \leq y \leq 1 \\ 0 & \text{en otro caso.} \end{cases}$$

$$f_X(x) = \int_{-\infty}^{+\infty} f(x,y) dy$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y) dx$$

$$f_X(x) = \int_{-1}^1 \frac{2-x-y}{8} dy \quad (-1 \leq x \leq 1)$$

$$f_X(x) = \int_{-1}^1 \left(\frac{2}{8} - \frac{x}{8} - \frac{y}{8} \right) dy$$

$$= \left[\frac{2}{8} y - \frac{xy}{8} - \frac{y^2}{16} \right]_{-1}^1 =$$

$$= \frac{2}{8} - \frac{x}{8} - \frac{1}{16} - \left(-\frac{2}{8} + \frac{x}{8} - \frac{1}{16} \right)$$

$$= \frac{4}{8} - \frac{2x}{8} = \underline{\underline{\frac{1}{2} - \frac{x}{4}}} \quad (-1 \leq x \leq 1)$$

$$f_Y(y) = \int_{-1}^1 \left(\frac{2}{8} - \frac{x}{8} - \frac{y}{8} \right) dx \quad (-1 \leq y \leq 1)$$

$$= \left[\frac{2}{8}x - \frac{x^2}{16} - \frac{xy}{8} \right]_{-1}^1 =$$

$$= \frac{2}{8} - \frac{1}{16} - \frac{y}{8} - \left(-\frac{2}{8} - \frac{1}{16} + \frac{y}{8} \right)$$

$$= \frac{4}{8} - \frac{2y}{8} = \frac{1}{2} - \frac{y}{4} \quad (-1 \leq y \leq 1)$$

Si x e y son indep \Rightarrow

$$f(x,y) = f_x \cdot f_y$$

$$\left(\frac{1}{2} - \frac{x}{4} \right) \left(\frac{1}{2} - \frac{y}{4} \right) \neq \frac{2-x-y}{8}$$

\Rightarrow no son indep.

Ejercicios

$$X \sim N(\mu, 0.5)$$

$$n = ?$$

$$E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$0.05 = 1.96 \cdot \frac{0.5}{\sqrt{n}}$$

$$\sqrt{n} = 1.96 \cdot \frac{0.5}{0.05}$$

$$1 - \alpha = 0.95$$

$$\alpha = 0.05$$

$$\frac{\alpha}{2} = 0.025$$

$$1 - 0.025 = 0.975$$

↓
Miramos tabla
 $N(0,1)$

$$\underline{\underline{z_{\alpha/2} = 1.96}}$$

$$n = \left(1.96 \cdot \frac{0.5}{0.05} \right)^2 = 384.16 \quad \text{aprox al entero mayor}$$

$$\underline{\underline{n \simeq 385}}$$

$$X \sim N(2, 0.5) \quad n=16$$

$$\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \Rightarrow \bar{X} \sim N\left(2, \frac{0.5}{\sqrt{16}}\right)$$

$$P(S > 30) = P(\bar{X} > \frac{30}{16})$$

$$\downarrow$$

$$X_1 + \dots + X_{16}$$

\Rightarrow

$$\hookrightarrow Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \quad \text{typiquement de la médiane.}$$

$$P\left(Z > \frac{\frac{30}{16} - 2}{0.5/\sqrt{16}}\right) = P(Z > -1) =$$

$$= P(Z < 1) = 0.8413 //$$

