1. Describe how one can calculate the advanced heuristic value for any state of the puzzle.

A possible heuristic function could be:

h(state) = Manhattan_distance(Caocao_to_goal) + penalty for each piece blocking Caocao from reaching the goal state.

In this case, each 1x1 square that blocks Caocao from goal is counted as one point of penalty (1x2 piece counts as 2 points if blocking), and the total penalty is calculated to be the smallest possible points that is blocking Caocao from its goal.

For example, in this example, the smallest penalty for Caocao to go to goal state is 4, where Guanyu and two soldiers are blocking Caocao from going directly downward, which is 2 for Guanyu and 1 for each soldier.

2. Why is your advanced heuristic admissible?

The heuristic function is admissible because it never overestimates the actual cost. Thinking about the optimal moves, Caocao needs to at least penalty mentioned above to move all pieces away from Caocao's way to goal, with addition of at least Manhattan_distance(Caocao_to_goal) moves to reach its goal.

The actual cost is always equal or greater than this value because we does not take into account the additional moves to move away the pieces that blocks the pieces that blocks Caocao. In the above example, we cannot directly move away Guanyu to clean up way for Caocao, but we need to also move away Zhangfei or Zhaoyun, which takes extra steps.

3. Why does your advanced heuristic dominate the Manhattan distance heuristic?

The heuristic function dominates the Manhattan distance heuristic because the heuristic is adding a non-negative number to the Manhattan distance:

h(state) = Manhattan_distance(Caocao_to_goal) + penalty for each piece blocking Caocao from reaching the goal state.

Penalty could be zero when there is no piece blocking Caocao's way to escape, but it would never be negative. Therefore, h is always larger or equal to Manhattan distance. In cases where there is pieces blocking Caocao's way, penalty is positive such that h is greater than Manhattan distance.

Therefore, the heuristic function dominates the Manhattan distance heuristic.