This workbook contains the checks on the unit tests for the R4 Exterior Calculus classes.

$with(DifferentialGeometry)$

$[\&algmult, \&minus, \&mult, \&plus, \&tensor, \&wedge, Annihilator, ApplyTransformation, ChangeFrame, ComplementaryBasis, ComposeTransformations, DGIm, DGImageSpace, DGNullSpace, DGRe, DGbasis, DGconjugate, DGsetup, DGsolve, DGzip, DeRhamHomotopy, DualBasis, ExteriorDerivative, ExteriorDifferentialSystems, Flow, FrameData, GetComponents, GroupActions, Hook, InfinitesimalTransformation, IntegrateForm, IntersectSubspaces, InverseTransformation, JetCalculus, Library, LieAlgebras, LieBracket, LieDerivative, Preferences, Pullback, PullbackVector, Pushforward, RemoveFrame, Tensor, Tools, Transformation, evalDG]$ **(1)**

$with(VectorCalculus)$

$[\&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, ConvertVector, CrossProduct, Curl, Curvature, \mathrm{D}, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, \nabla, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series]$ **(2)**

$DGsetup([x, y, z, w], M, verbose)$

The following coordinates have been protected:

$[x, y, z, w]$

The following vector fields have been defined and protected:

$[D\_x, D\_y, D\_z, D\_w]$

The following differential 1-forms have been defined and protected:

$[dx, dy, dz, dw]$

frame name: M **(3)**

First, two vector (k=1) wedge product, yields a parallelogram with k = 2.

$a := e0 \cdot dx + e1 \cdot dy + e2 \cdot dz + e3 \cdot dw$

$$a := e0\, dx + e1\, dy + e2\, dz + e3\, dw \quad \textbf{(4)}$$

$b := rhs\_e0 \cdot dx + rhs\_e1 \cdot dy + rhs\_e2 \cdot dz + rhs\_e3 \cdot dw$

$$b := rhs\_e0\, dx + rhs\_e1\, dy + rhs\_e2\, dz + rhs\_e3\, dw \quad \textbf{(5)}$$

$c := a \ \&wedge\, b$

$$c := (e0\, rhs\_e1 - e1\, rhs\_e0)\, dx \wedge dy + (e0\, rhs\_e2 - e2\, rhs\_e0)\, dx \wedge dz + (e0\, rhs\_e3 \quad \textbf{(6)}$$

$- e3\ rhs\_e0)\ dx \wedge dw + (e1\ rhs\_e2 - e2\ rhs\_e1)\ dy \wedge dz + (e1\ rhs\_e3 - e3\ rhs\_e1)\ dy \wedge dw + (e2\ rhs\_e3 - e3\ rhs\_e2)\ dz \wedge dw$

$d1 := subs(\{e0 = 3, e1 = 4, e2 = 5, e3 = 1, rhs\_e0 = 6, rhs\_e1 = 7, rhs\_e2 = 8, rhs\_e3 = 1\}, c)$

$$d1 := -3\ dx \wedge dy - 6\ dx \wedge dz - 3\ dx \wedge dw - 3\ dy \wedge dz - 3\ dy \wedge dw - 3\ dz \wedge dw \quad (7)$$

A couple more k=2 examples used in unit testing the library follow :

$d2 := subs(\{e0 = 7, e1 = 4, e2 = 3, e3 = 1, rhs\_e0 = 11, rhs\_e1 = 17, rhs\_e2 = 13, rhs\_e3 = 1\}, c)$

$$d2 := 75\ dx \wedge dy + 58\ dx \wedge dz - 4\ dx \wedge dw + dy \wedge dz - 13\ dy \wedge dw - 10\ dz \wedge dw \quad (8)$$

$d3 := subs(\{e0 = -5, e1 = 6, e2 = 9, e3 = 1, rhs\_e0 = 13, rhs\_e1 = -17, rhs\_e2 = 4, rhs\_e3 = 1\}, c)$

$$d3 := 7\ dx \wedge dy - 137\ dx \wedge dz - 18\ dx \wedge dw + 177\ dy \wedge dz + 23\ dy \wedge dw + 5\ dz \wedge dw \quad (9)$$

Next, wedge product of a vector (k=1) and parallelogram (k=2), yielding a parallelepiped with k = 3.

$f1 := a\ \&wedge\ d1$

$$f1 := -(3\ e2 - 6\ e1 + 3\ e0)\ dx \wedge dy \wedge dz - (3\ e3 - 3\ e1 + 3\ e0)\ dx \wedge dy \wedge dw - (6\ e3 - 3\ e2 + 3\ e0)\ dx \wedge dz \wedge dw - (3\ e3 - 3\ e2 + 3\ e1)\ dy \wedge dz \wedge dw \quad (10)$$

$g1 := subs(\{e0 = 14, e1 = -10, e2 = 5, e3 = 1\}, f1)$

$$g1 := -117\ dx \wedge dy \wedge dz - 75\ dx \wedge dy \wedge dw - 33\ dx \wedge dz \wedge dw + 42\ dy \wedge dz \wedge dw \quad (11)$$

Another, wedge product of a parallelogram (k=2) and a vector (k=1), yielding a parallelepiped with k = 3.

$f2 := d2\ \&wedge\ b$

$$f2 := (rhs\_e0 - 58\ rhs\_e1 + 75\ rhs\_e2)\ dx \wedge dy \wedge dz - (13\ rhs\_e0 - 4\ rhs\_e1 - 75\ rhs\_e3)\ dx \wedge dy \wedge dw - (10\ rhs\_e0 - 4\ rhs\_e2 - 58\ rhs\_e3)\ dx \wedge dz \wedge dw - (10\ rhs\_e1 - 13\ rhs\_e2 - rhs\_e3)\ dy \wedge dz \wedge dw \quad (12)$$

$g2 := subs(\{rhs\_e0 = 14, rhs\_e1 = -10, rhs\_e2 = 5, rhs\_e3 = 1\}, f2)$

$$g2 := 969\ dx \wedge dy \wedge dz - 147\ dx \wedge dy \wedge dw - 62\ dx \wedge dz \wedge dw + 166\ dy \wedge dz \wedge dw \quad (13)$$

Inner Product of two vectors (k=1).

$dp1 := DotProduct(\langle e0, e1, e2, e3\rangle, \langle rhs\_e0, rhs\_e1, rhs\_e2, rhs\_e3\rangle)$

$$dp1 := e0\ rhs\_e0 + e1\ rhs\_e1 + e2\ rhs\_e2 + e3\ rhs\_e3 \quad (14)$$

$$subs(\{e0 = 3, e1 = 4, e2 = 5, e3 = 1, rhs\_e0 = 6, rhs\_e1 = 7, rhs\_e2 = 8, rhs\_e3 = 1\}, dp1)$$

$$87 \tag{15}$$

Inner Producr of two parallelograms (k=2).

$$dp2 := DotProduct(\langle e0, e1, e2, e3, e4, e5\rangle, \langle rhs\_e0, rhs\_e1, rhs\_e2, rhs\_e3, rhs\_e4, rhs\_e5\rangle)$$

$$dp2 := e0\ rhs\_e0 + e1\ rhs\_e1 + e2\ rhs\_e2 + e3\ rhs\_e3 + e4\ rhs\_e4 + e5\ rhs\_e5 \tag{16}$$

$$subs(\{e0 = 75, e1 = 58, e2 = -4, e3 = 1, e4 = -13, e5 = -10, rhs\_e0 = 7, rhs\_e1 = -137, rhs\_e2 = -18, rhs\_e3 = 177, rhs\_e4 = 23, rhs\_e5 = 5\}, dp2)$$

$$-7521 \tag{17}$$