The problem is in NP since, given a schedule for message transmissions, one can verify that it obeys the switching time constraints, and that all nodes in the target set receive the message by the desired time.

We now show how to reduce 3-*Dimensional Matching* to *Broadcast Time*. Consider an instance of 3-*Dimensional Matching*, with triples from $X \times Y \times Z$, where $n = |X| = |Y| = |Z|$. We construct a graph G with a root r, a node v_t for each triple t, and a node w_a for each element $a \in X \cup Y \cup Z$. There are directed edges (r, v_t) for each t, and (v_t, w_a) when a belongs to triple t. The target set T is $\{w_a : a \in X \cup Y \cup Z\}$. The root r has switching time 4, and all other nodes have switching time 1.

We claim that there exists a set of n triples covering $X \cup Y \cup Z$ if and only if there is a broadcast scheme with completion time at most $4n - 1$. Indeed, if there are n such triples, then r sends messages to each corresponding node v_t at times $0, 4, 8, \ldots, 4n - 4$; and if node v_t receives a message from r at time $4j$, it sends messages to its three elements at times $4j + 1, 4j + 2, 4j + 3$. Conversely, if there is a broadcast scheme with completion time at most $4n - 1$, then r must have sent at most n messages, since its switching time is 4. Now, if R is the set of nodes that received messages from r, then $M = \{t : v_t \in R\}$ must cover $X \cup Y \cup Z$, since every node w_a has received the message. Thus M forms a solution to the instance of 3-*Dimensional Matching*.

[1] ex597.881.577