(a) If $v \notin S$, it must have never been chosen by the greedy algorithm. This means that it was deleted in some iteration by the selection of a node v': by the definition of the selection rule, this node v' must both be a neighbor of v, and have at least as much weight as v.

(b) Consider any other independent set T. For each node $v \in T$, we *charge* it to a node in S as follows. If $v \in S$, then we charge v to itself. Otherwise, by (a), v is a neighbor of some node $v' \in S$ whose weight is at least as large. We charge v to v'.

Now, if v is charged to itself, then no other node is charged to v, since S and T are independent sets. Otherwise, at most four neighboring nodes of no greater weight are charged to v. Either way, the total weight of all nodes charged to v is at most $4w(v)$. Since these charges account for the total weight of T, it follows that the total weight of nodes in T is at most four times the total weight of nodes in S.

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[1] ex727.874.96