

The problem *Decisive Subset (DS)* is in NP because we can check in polynomial time that a given subset of committee members is of size at most  $k$ , and that its voting outcome is the same as that of the whole committee.

Now we show that *Vertex Cover*  $\leq_P$  *DS*. Given a graph  $G = (V, E)$  and bound  $k$ , we create an issue  $I_e$  for each edge  $e$ , and a committee member  $m_v$  for each node  $v$ . If  $e = (u, v)$ , then we have members  $u$  and  $v$  vote “yes” on issue  $I_e$ , and all other committee members abstain. Note that the voting outcome by the whole committee on all issues is “yes.” We now ask whether there is a decisive subset of size at most  $k$ .

If there is a decisive subset  $S$  of size at most  $k$ , then it must lead to an affirmative decision for each issue. In particular, this means that for each edge  $e$ ,  $S$  must include at least one of the members  $m_u$  or  $m_v$ , and so the corresponding set of nodes will constitute a vertex cover in  $G$  of size at most  $k$ .

If there is a vertex cover  $C$  of size at most  $k$ , then for each edge  $e = (u, v)$ , at least one of  $u$  or  $v$  will be in  $C$ . For the set  $S$  of members corresponding to  $C$ , the voting outcome will thus be “yes” on each issue  $I_e$ , so  $S$  is decisive.

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<sup>1</sup>ex7.111.521