

The problem is in \mathcal{NP} since we can exhibit a set of k nodes and check that the distance between all pairs is at least 3.

We now show *Independent Set* \leq_P *Strongly Independent Set*. Given a graph G and a number k , we construct a new graph G' in which we replace each edge $e = (u, v)$ by a path of length two: we add a new node w_e , and we add edges $(u, w_e), (w_e, v)$. We also include edges between every pair of new nodes.

Now suppose that G has an independent set of size k . Then in this new graph G' , all these k nodes are distance at least three from each other, so this is a strongly independent set of size k . Conversely, suppose G' has a strongly independent set of size k . Now, this set can't contain any of the new nodes, since all such nodes are within distance two of every node in the graph. Thus, it consists of nodes present in G . Moreover, no two of these nodes can be neighbors in G , since then they'd be at distance two in G' . Thus this set of nodes forms an independent set of size k in G .

¹ex900.39.43