

We construct a directed graph  $G$ ; the test for consistency will turn out to be equivalent to testing whether  $G$  is acyclic.

For each person  $P_i$ , we define nodes  $b_i$  and  $d_i$ , representing their (unknown) birth and death dates respectively. Edges will correspond to one event preceding another. So to start, we include edges  $(b_i, d_i)$  for each  $i$ . When we are told that

- for some  $i$  and  $j$ , person  $P_i$  died before person  $P_j$  was born,

we include an edge  $(d_i, b_j)$ . When we are told that

- for some  $i$  and  $j$ , the lifespans of  $P_i$  and  $P_j$  overlapped at least partially,

we include edges  $(b_i, d_j)$  and  $(b_j, d_i)$ . This completes the construction of  $G$ .

Now suppose  $G$  has a cycle. Then each of the events corresponding to nodes in this cycle must precede the next; but this means that there is no event among these that can be put first in time, consistent with the given information. Hence the information is not internally consistent.

On the other hand, suppose  $G$  has no cycle. Then it has a topological ordering. If we use this as the order of the birth and death dates of all people, then we have an ordering that is consistent with all the given pieces of information.

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<sup>1</sup>ex92.401.128