

(a) The answer is Yes. A simple way to think about it is to break the ties in some fashion and then run the stable matching algorithm on the resulting preference lists. We can for example break the ties lexicographically — that is if a man  $m$  is indifferent between two women  $w_i$  and  $w_j$  then  $w_i$  appears on  $m$ 's preference list before  $w_j$  if  $i < j$  and if  $j < i$   $w_j$  appears before  $w_i$ . Similarly if  $w$  is indifferent between two men  $m_i$  and  $m_j$  then  $m_i$  appears on  $w$ 's preference list before  $m_j$  if  $i < j$  and if  $j < i$   $m_j$  appears before  $m_i$ .

Now that we have concrete preference lists, we run the stable matching algorithm. We claim that the matching produced would have no strong instability. But this latter claim is true because any strong instability would be an instability for the match produced by the algorithm, yet we know that the algorithm produced a stable matching — a matching with no instabilities.

(b) The answer is No. The following is a simple counterexample. Let  $n = 2$  and  $m_1, m_2$  be the two men, and  $w_1, w_2$  the two women. Let  $m_1$  be indifferent between  $w_1$  and  $w_2$  and let both of the women prefer  $m_1$  to  $m_2$ . The choices of  $m_2$  are insignificant. There is no matching without weak stability in this example, since regardless of who was matched with  $m_1$ , the other woman together with  $m_1$  would form a weak instability.

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<sup>1</sup>ex734.923.393