On the surface, *Monotone Satisfiability with Few True Variables* (which we'll abbreviate *Monotone Satisfiability with Few True Variables*) is written in the language of the Satisfiability problem. But at a technical level, it's not so closely connected to *SAT*; after all no variables appear negated, and what makes it hard is the constraint that only a few variables can be set to true.

Really, what's going on is that one has to choose a small number of variables, in such a way that each clause contains one of the chosen variables. Phrased this way, it resembles a type of covering problem.

We choose *Vertex Cover* as the problem X, and show *Vertex Cover* $\leq_P$ *Monotone Satisfiability with Few True Variables.* Suppose we are given a graph $G = (V, E)$ and a number k; we want to decide whether there is a vertex cover in G of size at most k. We create an equivalent instance of *Monotone Satisfiability with Few True Variables* as follows. We have a variable x_i for each vertex v_i. For each edge $e_j = (v_a, v_b)$, we create the clause $C_j = (x_a \vee x_b)$. This is the full instance: we have clauses $C_1, C_2, \ldots, C_m$, one for each edge of G, and we want to know if they can all be satisfied by setting at most k variables to 1.

We claim that the answer to the *Vertex Cover* instance is "yes" if and only if the answer to the *Monotone Satisfiability with Few True Variables* instance is "yes." For suppose there is a vertex cover S in G of size at most k, and consider the effect of setting the corresponding variables to 1 (and all other variables to 0). Since each edge is covered by a member of S, each clause contains at least one variable set to 1, and so all clauses are satisfied. Conversely, suppose there is a way to satisfy all clauses by setting a subset X of at most k variables to 1. Then if we consider the corresponding vertices in G, each edge must have at least one end equal to one of these vertices — since the clause corresponding to this edge contains a variable in X. Thus the nodes corresponding to the variables in X form a vertex cover of size at most k.

[1] ex799.396.989