

The conjecture is true. Consider the assignment of jobs to machines in an arbitrary optimal solution, and order the jobs arbitrarily on each machine. We say that the *base height* of a job j is the total time requirements of all jobs that precede it on its assigned machine.

We order all jobs by their base heights (breaking ties arbitrarily), and we feed them to the Greedy-Balance algorithm in this order. (We will label the jobs $1, 2, \dots, n$ according to this order.)

We claim the following by induction on r : after the first r jobs have been processed by Greedy-Balance, the set of machine loads is the same as the set of machine loads if we consider the assignment of these r jobs made by the optimal solution.

This is clearly true for $r = 1$, since one machine will have load t_1 , and all others will have load 0. Now suppose it is true up to some r , with loads T_1, \dots, T_m , and consider job $r + 1$. Because we have sorted jobs by base height, job $r + 1$ comes from the machine that, in the optimal solution, has load $\min_i T_i$. By the definition of Greedy-Balance, this is the machine on which job $r + 1$ will be placed, giving it a load of $t_{r+1} + \min_i T_i$. This completes the induction step.

¹ex286.347.713