

We assume the graph  $G$  is connected; otherwise we work with the connected components separately (after computing them in  $O(m + n)$  time).

We run BFS starting from an arbitrary node  $s$ , obtaining a BFS tree  $T$ . Now, if every edge of  $G$  appears in the BFS tree, then  $G = T$ , so  $G$  is a tree and contains no cycles. Otherwise, there is some edge  $e = (v, w)$  that belongs to  $G$  but not to  $T$ . Consider the least common ancestor  $u$  of  $v$  and  $w$  in  $T$ ; we obtain a cycle from the edge  $e$ , together with the  $u$ - $v$  and  $u$ - $w$  paths in  $T$ .