

*Path Selection* is in NP, since we can be shown a set of  $k$  paths from among  $P_1, \dots, P_c$  and check in polynomial time that no two of them share any nodes.

Now, we claim that  $3\text{-Dimensional Matching} \leq_P \text{Path Selection}$ . For consider an instance of  $3\text{-Dimensional Matching}$  with sets  $X$ ,  $Y$ , and  $Z$ , each of size  $n$ , and ordered triples  $T_1, \dots, T_m$  from  $X \times Y \times Z$ . We construct a directed graph  $G = (V, E)$  on the node set  $X \cup Y \cup Z$ . For each triple  $T_i = (x_i, y_j, z_k)$ , we add edges  $(x_i, y_j)$  and  $(y_j, z_k)$  to  $G$ . Finally, for each  $i = 1, 2, \dots, m$ , we define a path  $P_i$  that passes through the nodes  $\{x_i, y_j, z_k\}$ , where again  $T_i = (x_i, y_j, z_k)$ . Note that by our definition of the edges, each  $P_i$  is a valid path in  $G$ . Also, the reduction takes polynomial time.

Now we claim that there are  $n$  paths among  $P_1, \dots, P_m$  sharing no nodes if and only if there exist  $n$  disjoint triples among  $T_1, \dots, T_m$ . For if there do exist  $n$  paths sharing no nodes, then the corresponding triples must each contain a different element from  $X$ , a different element from  $Y$ , and a different element from  $Z$  — they form a perfect three-dimensional matching. Conversely, if there exist  $n$  disjoint triples, then the corresponding paths will have no nodes in common.

Since *Path Selection* is in NP, and we can reduce an NP-complete problem to it, it must be NP-complete.

*(Other direct reductions are from Set Packing and from Independent Set.)*

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<sup>1</sup>ex529.979.546