Hitting Set is in NP: Given an instance of the problem, and a proposed set H, we can check in polynomial time whether H has size at most k, and whether some member of each set  $S_i$  belongs to H.

Hitting Set looks like a covering problem, since we are trying to choose at most k objects subject to some constraints. We show that  $Vertex\ Cover \leq_P Hitting\ Set$ . Thus, we begin with an instance of  $Vertex\ Cover$ , specified by a graph G=(V,E) and a number k. We must construct an equivalent instance of  $Hitting\ Set$ . In  $Vertex\ Cover$ , we are trying to choose at most k nodes to form a vertex cover. In  $Hitting\ Set$ , we are trying to choose at most k elements to form a hitting set. This suggests that we define the set K in the K in the K instance to be the K of nodes in the K in the K instance. For each edge K in the K we define a set K in the K in the K in the K instance.

Now we claim that there is a hitting set of size at most k for this instance, if and only if the original graph had a vertex cover of size at most k. For if we consider a hitting set H of size at most k as a subset of the nodes of G, we see that every set is "hit," and hence every edge has at least one end in H: H is a vertex cover of G. Conversely, if we consider a vertex cover G of G, and consider G as a subset of G, we see that each of the sets G is "hit" by G.

 $<sup>^{1}</sup>$ ex635.897.959