

Suppose  $m \leq n$ , and let  $L$  denote the maximum length of any string in  $A \cup B$ . Suppose there is a string that is a concatenation over both  $A$  and  $B$ , and let  $u$  be one of minimum length. We claim that the length of  $u$  is at most  $n^2 L^2$ .

For suppose not. First, we say that position  $p$  in  $u$  is of *type*  $(a_i, k)$  if in the concatenation over  $A$ , it is represented by position  $k$  of string  $a_i$ . We define *type*  $(b_i, k)$  analogously. Now, if the length of  $u$  is greater than  $n^2 L^2$ , then by the pigeonhole principle, there exist positions  $p$  and  $p'$  in  $u$ ,  $p < p'$ , so that both are of type  $(a_i, k)$  and  $(b_j, k)$  for some indices  $i, j, k$ . But in this case, the string  $u'$  obtained by deleting positions  $p, p+1, \dots, p'-1$  would also be a concatenation over both  $A$  and  $B$ . As  $u'$  is shorter than  $u$ , this is a contradiction.

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<sup>1</sup>ex690.144.299