

- (i) This is false in general, since it could be that  $g(n) = 1$  for all  $n$ ,  $f(n) = 2$  for all  $n$ , and then  $\log_2 g(n) = 0$ , whence we cannot write  $\log_2 f(n) \leq c \log_2 g(n)$ .

On the other hand, if we simply require  $g(n) \geq 2$  for all  $n$  beyond some  $n_1$ , then the statement holds. Since  $f(n) \leq cg(n)$  for all  $n \geq n_0$ , we have  $\log_2 f(n) \leq \log_2 g(n) + \log_2 c \leq (\log_2 c)(\log_2 g(n))$  once  $n \geq \max(n_0, n_1)$ .

- (ii) This is false: take  $f(n) = 2n$  and  $g(n) = n$ . Then  $2^{f(n)} = 4^n$ , while  $2^{g(n)} = 2^n$ .
- (iii) This is true. Since  $f(n) \leq cg(n)$  for all  $n \geq n_0$ , we have  $(f(n))^2 \leq c^2(g(n))^2$  for all  $n \geq n_0$ .

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<sup>1</sup>ex66.350.972