

(a) If  $v \notin S$ , it must have never been chosen by the greedy algorithm. This means that it was deleted in some iteration by the selection of a node  $v'$ : by the definition of the selection rule, this node  $v'$  must both be a neighbor of  $v$ , and have at least as much weight as  $v$ .

(b) Consider any other independent set  $T$ . For each node  $v \in T$ , we *charge* it to a node in  $S$  as follows. If  $v \in S$ , then we charge  $v$  to itself. Otherwise, by (a),  $v$  is a neighbor of some node  $v' \in S$  whose weight is at least as large. We charge  $v$  to  $v'$ .

Now, if  $v$  is charged to itself, then no other node is charged to  $v$ , since  $S$  and  $T$  are independent sets. Otherwise, at most four neighboring nodes of no greater weight are charged to  $v$ . Either way, the total weight of all nodes charged to  $v$  is at most  $4w(v)$ . Since these charges account for the total weight of  $T$ , it follows that the total weight of nodes in  $T$  is at most four times the total weight of nodes in  $S$ .

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<sup>1</sup>ex727.874.96