

We label the vertices v_1, v_2, \dots, v_n according to a topological ordering. We now define $Win(j)$ to be equal to 1 if the player whose turn it is to move can force a win starting at node v_j , and define $Win(j)$ to be equal to 0 if the other (who isn't about to move) can force a win starting at node v_j .

We can initialize $Win(j) = 0$ for every node v_j with no out-going edges. In particular, this means that we will set $Win(n) = 0$. We now use dynamic programming to compute the values of $Win(j)$ in descending order of j . When we get to a particular value of j , we may assume that we have already computed $Win(k)$ for all $k > j$. Now, a player starting from v_j can force a win if and only if there is some node v_k for which (v_j, v_k) is an edge and a player starting from v_k has a forced loss. Thus, $Win(j) = 1$ if and only if $Win(k) = 0$ for some k with (v_j, v_k) an edge; and otherwise $Win(j) = 0$.

We thus compute all these values in $O(n)$ time per entry, for a total of $O(n^2)$. We then simply check the value of $Win(j)$ for the node v_j on which the game is designated to start.

¹ex701.675.797