

We order the functions as follows.

- $g_1$  comes before  $g_5$ . This is like the solved exercise in which we saw  $2\sqrt{\log n}$ . If we take logarithms, we are comparing  $\sqrt{\log n}$  to  $\log n + \log(\log n) \geq \log n$ ; changing variables via  $z = \log n$ , this is  $\sqrt{z} = z^{1/2}$  versus  $z + \log z \geq z$ .
- $g_5$  comes before  $g_3$ , since  $(\log n)^3$  grows faster than  $\log n$ . (They're both polynomials in  $\log n$ , but  $(\log n)^3$  has the larger degree.)
- $g_3$  comes before  $g_4$ : Dividing both by  $n$ , we are comparing  $(\log n)^3$  with  $n^{1/3}$ , or (taking cube roots),  $\log n$  with  $n^{1/9}$ . Now we use the fact that logarithms grow slower than exponentials.
- $g_4$  comes before  $g_2$ , since polynomials grow slower than exponentials.
- $g_2$  comes before  $g_7$ : Taking logarithms, we are comparing  $n$  to  $n^2$ , and  $n^2$  is the polynomial of larger degree.
- $g_7$  comes before  $g_6$ : Taking logarithms, we are comparing  $n^2$  to  $2^n$ , and polynomials grow slower than exponentials.