4-Dimensional Matching is in NP, since we can check in $O(n)$ time, using an $n \times 4$ array initialized to all 0, that a given set of n 4-tuples is disjoint.

We now show that *3-Dimensional Matching* $\leq_P$ *4-Dimensional Matching*. So consider an instance of *3-Dimensional Matching*, with sets X, Y, and Z of size n each, and a collection C of ordered triples. We define an instance of *4-Dimensional Matching* as follows. We have sets W, X, Y, and Z, each of size n, and a collection C' of 4-tuples defined so that for every $(x_j, y_k, z_\ell) \in C$, and every i between 1 and n, there is a 4-tuple (w_i, x_j, y_k, z_ℓ). This instance has a size that is polynomial in the size of the initial *3-Dimensional Matching* instance.

If $A = (x_j, y_k, z_\ell)$ is a triple in C, define $f(A)$ to be the 4-tuple (w_j, x_j, y_k, z_ℓ); note that $f(A) \in C'$. If $B = (w_i, x_j, y_k, z_\ell)$ is a 4-tuple in C', define $f'(B)$ to be the triple (x_j, y_k, z_ℓ); note that $f'(B) \in C$. Given a set of n disjoint triples $\{A_i\}$ in C, it is easy to show that $\{f(A_i)\}$ is a set of n disjoint 4-tuples in C'. Conversely, given a set of n disjoint 4-tuples $\{B_i\}$ in C', it is easy to show that $\{f'(B_i)\}$ is a set of n disjoint triples in C. Thus, by determining whether there is a perfect 4-Dimensional matching in the instance we have constructed, we can solve the initial instance of *3-Dimensional Matching*.

[1] ex420.972.30