We build the following bipartite graph G. There is a node u_i for each variable x_i, a node v_j for each clause C_j, and an edge (u_i, v_j) whenever x_i appears in C_j.

We first note that since each variable appears three times, and each clause has length three, the number of nodes on the left side of G equals the number of nodes on the right side. More strongly, we claim that G in fact has a perfect matching.

This is a consequence of a more general claim: that if every node in a bipartite graph G has the same degree d, then G has a perfect matching. (Here $d = 3$.) Indeed, if G does not have a perfect matching, then by Hall's Theorem it has a set A on the left side for which $|A| < |\Gamma(A)|$, where $\Gamma(A)$ denotes the set of neighbors of any node in A. But A has $d|A|$ nodes coming out of it, and $\Gamma(A)$ can only absorb $d|\Gamma(A)|$ of them, so this is not possible.

Consider the perfect matching in the graph G we constructed from the *3-SAT* instance. For each variable, we set it in a way that satisfies the clause it is matched to. In this way, we satisfy the full collection of clauses.

[1] ex592.206.332