

We run BFS starting from node  $s$ . Let  $d$  be the layer in which node  $t$  is encountered; by assumption, we have  $d > n/2$ . We claim first that one of the layers  $L_1, L_2, \dots, L_{d-1}$  consists of a single node. Indeed, if each of these layers had size at least two, then this would account for at least  $2(n/2) = n$  nodes; but  $G$  has only  $n$  nodes, and neither  $s$  nor  $t$  appears in these layers.

Thus, there is some layer  $L_i$  consisting of just the node  $v$ . We claim next that deleting  $v$  destroys all  $s$ - $t$  paths. To see this, consider the set of nodes  $X = \{s\} \cup L_1 \cup L_2 \cup \dots \cup L_{i-1}$ . Node  $s$  belongs to  $X$  but node  $t$  does not; and any edge out of  $X$  must lie in layer  $L_i$ , by the properties of BFS. Since any path from  $s$  to  $t$  must leave  $X$  at some point, it must contain a node in  $L_i$ ; but  $v$  is the only node in  $L_i$ .

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<sup>1</sup>ex758.356.752