The problem is in $\mathcal{NP}$ since we can exhibit a set X and check the size of its intersection with every set A_i.

We now show that *3-Dimensional Matching*$\leq_P$ *Intersection Inference*. Suppose we are given an instance of *3-Dimensional Matching*, consisting of sets X, Y, and Z, each of size n, and a set T of m triples from $X \times Y \times Z$. We define the following instance of *Intersection Inference*. We define $U = T$. For each element $j \in X \cup Y \cup Z$, we create a set A_j of these triples that contain j. We then ask whether there is a set $M \subseteq U$ that has an intersection of size 1 with each set A_j.

Such sets are precisely those collections of triples for which each element of $X \cup Y \cup Z$ appears in exactly one: in other words, they are precisely the perfect three-dimensional matchings. Thus, our instance of *Intersection Inference* has a positive answer if and only if the original instance of *3-Dimensional Matching* does.

[1] ex803.795.220