

The problem is in  $\mathcal{NP}$  since we can exhibit a subset  $E'$  of the edges, and it can be checked in polynomial time that at most  $a$  nodes in  $X$  are incident to an edge in  $E'$ , and at least  $b$  nodes in  $Y$  are incident to an edge in  $E'$ .

We now show that *Set Cover* is reducible to this problem. Given a collection of sets  $S_1, \dots, S_m$  over a ground set  $U$  of size  $n$ , we define a bipartite graph in which the nodes in  $X$  correspond to the sets  $S_i$ , and the nodes in  $Y$  correspond to the elements in  $U$ . We join each set node to the nodes corresponding to elements that it contains. We also set  $a = k$  and  $b = n$ . (In particular, this means that our  $(a, b)$ -skeleton must touch every node in  $Y$ .)

Now, if  $G$  has an  $(a, b)$ -skeleton  $E'$ , then the  $k$  nodes in  $X$  incident to edges in  $E'$  correspond to  $k$  sets that collectively contain all elements, so they form a set cover. Conversely, if there is a set cover of size  $k$ , then taking  $E'$  to be the set of all edges incident to the corresponding set nodes yields an  $(a, b)$ -skeleton.

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<sup>1</sup>ex748.182.100