

Magnets is in NP. A witness can be a multiple set of words. We can count the number of each kind of magnets used in these words, and verify whether that is equal to the given number of that kind of magnets.

Now we will show that *Magnets* is NPC by reducing from *3D Matching*. We have an instance of *3D Matching*, which consists of three sets X, Y, Z , s.t. $|X| = |Y| = |Z| = n$, and a set of tuples M . We want to find n tuples from M , s.t. each element is covered by exactly one tuple. Create an instance of *Magnets* as follows: each element in X, Y , or Z becomes a magnet with a unique letter, (so our alphabet will have $3n$ letters), and every tuple (x_i, y_j, z_k) becomes a word that Madison knows. Solving this instance of *Magnets* will solve the instance of *3D Matching*.

We must now show that there is a perfect matching in *3D Matching* problem iff all the magnets can be used up in *Magnets* problem. If all the magnets are used up, we must have got exactly n words, since each word has 3 letters, and there are totally $3n$ letters, and therefore we have the desired n tuples. If there is a perfect matching in *3D Matching*, then those words that are corresponding to the tuples in the matching will exactly use up all the magnets without overlapping.

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