Homework Exercises for Chapter 18
Overview of Solution Methods

EXERCISE 18.1

[C:20] Consider the following residual equilibrium equation:

$$r(\psi,\lambda) = \sec(\alpha-\psi)\Big(\big(\sec\alpha\,(2+\lambda\,\sin\psi) - 2\,\sec(\alpha-\psi)\big)\tan(\alpha-\psi) - \lambda\,\cos\psi\,\sec\alpha\Big) = 0, \qquad \text{(E18.1)}$$

in which α is a problem parameter, λ the control parameter, and ψ the only degree of freedom. This r comes from the 2-bar arch problem already studied in Exercise 6.2. Here α is the initial arch rise angle whereas $\psi = \alpha - \theta$ is the angle change from the reference state, at which $\psi_0 = 0$ and $\lambda_0 = 0$. A plot of the exact $\lambda(\psi)$ for $\psi = [0, 60^\circ] = [0, \pi/3]$ is shown in Figure E18.1(a); the fundamental path ends at limit point L.

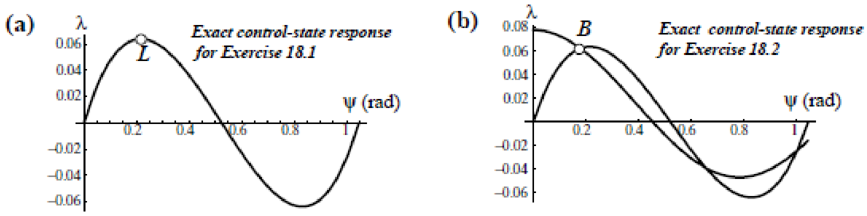

FIGURE E18.1. Exact responses for Exercises E18.1 and E18.2.

(a) Derive the first-order rate form $K\,\dot\psi = q\,\dot\lambda$ by taking $t \equiv \lambda$, and convert to $\dot\psi = du/d\lambda = v$. (Recall that $K = \partial r/\partial\psi$, $q = -\partial r/\partial\lambda$, and $v = K^{-1}q$.)

(b) Integrate numerically the rate equation $\dot\psi = v$ found in (a) by the purely incremental, forward-Euler method with load control (FELC) over $\lambda = [0, 0.1]$. Start from $\lambda_0 = 0$ and $\psi_0 = 0$. (All angles should be in radians.) Use $\alpha = 30^\circ = \pi/6$ as arch rise angle and take 10 load increments of $\ell_n = 0.01$ (same for all steps). Are you able to detect and traverse the limit point L?

(c) Repeat the run twice, each time cutting ℓ_n by 1/4 and quadrupling the number of steps. Is limit point detection and traversal improved?