A heuristic approach to Arc length predictor & orthogonal trajectory corrector:

The residual $r(u\ \lambda) = 0$

Solve linearized residual

$$r(u+\Delta u, \lambda+\Delta\lambda) = r(u,\lambda) + \frac{\partial r}{\partial u}\Delta u + \underbrace{\frac{\partial r}{\partial \lambda}}_{= -q}\Delta\lambda = 0$$

$$r_n + K_n \Delta u - q\,\Delta\lambda = 0$$

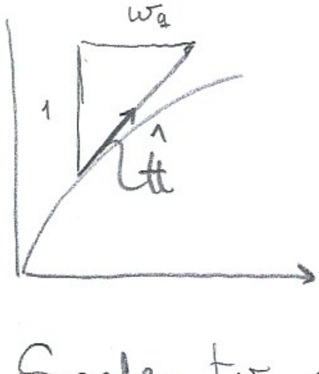
$$K\Delta u = q\,\Delta\lambda - r_n$$

$$\Delta u = \underbrace{K^{-1}q}_{w_q}\Delta\lambda \underbrace{- K^{-1}r}_{w_r}$$

<u>Predictor step</u> Assume residual is already zero

$$\Delta u = K^{-1}q\,\Delta\lambda \quad = w_{q0}\,\Delta\lambda$$

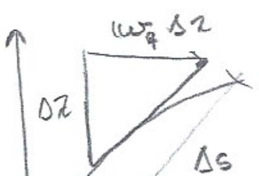
Length in augmented $\begin{bmatrix} u \\ \lambda \end{bmatrix}$ space:



$$\hat{t} = \frac{1}{f}\begin{bmatrix} w_q \\ 1 \end{bmatrix}$$

$$f = \sqrt{\begin{bmatrix} w_q^T & 1 \end{bmatrix}\begin{bmatrix} w_q \\ 1 \end{bmatrix}} = \sqrt{w_q^T w_q + 1}$$

Scale to arc length $\Delta s = \hat{t}^T \Delta\hat{u}$

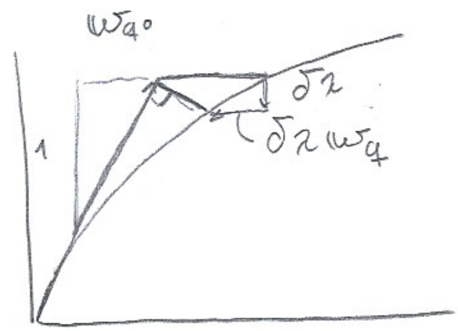


$$\Delta s = \frac{1}{f}\begin{bmatrix} w_q^T & 1 \end{bmatrix}\begin{bmatrix} w_q \\ 1 \end{bmatrix}\Delta\lambda$$

$$\Delta s = \frac{1}{f}(w_q^T w_q + 1)\Delta\lambda$$

$$\boxed{\Delta\lambda = \pm\frac{\Delta s \cdot f}{(1 + w_q^T w_q)}}$$

Corrector step:



$$r(u+\delta u, \lambda+\delta\lambda) = r(u,\lambda) + \frac{\partial r}{\partial u}\delta u + \underbrace{\frac{\partial r}{\partial \lambda}}_{-q}\delta\lambda = 0$$

$$r + K\delta u - q\,\delta\lambda = 0$$

$$K\delta u = q\,\delta\lambda - r$$

$$\delta u = K^{-1} q\,\delta\lambda - K^{-1} r$$

$$\delta u = w_q\,\delta\lambda + w_r$$

$$\delta\hat{u} = \begin{bmatrix} w_q \\ 1 \end{bmatrix}\delta\lambda - \begin{bmatrix} w_r \\ 0 \end{bmatrix}$$

Require ortogonality with $\hat{w}_{q0} = \begin{bmatrix} w_{q0} \\ 1 \end{bmatrix}$

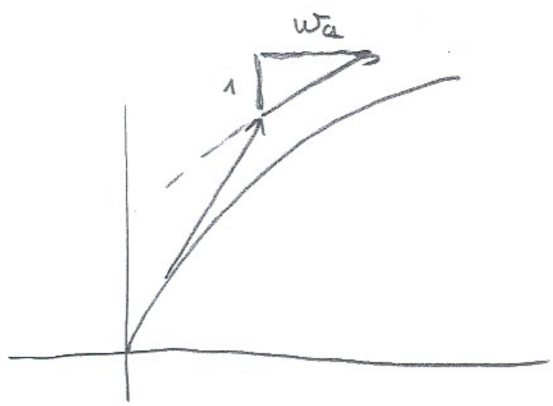
$$\begin{bmatrix} w_{q0}^T & 1 \end{bmatrix} \begin{bmatrix} w_q \delta\lambda + w_r \\ \delta\lambda \end{bmatrix} = 0$$

$$w_{q0}^T w_q \delta\lambda - w_{q0}^T w_r + \delta\lambda = 0$$

$$(1 + w_{q0}^T w_q)\delta\lambda + w_{q0}^T w_r = 0$$

$$\boxed{\delta\lambda = -\frac{w_{q0}^T w_r}{(1 + w_{q0}^T w_q)}}$$

When using updated normal plane:



w_{q0} gets substituted with w_q

$$\boxed{\delta\lambda = -\frac{w_q^T w_r}{(1 + w_q^T w_q)}}$$