Exercise 18.1

Solution

$$\underbrace{r(\psi,\lambda)=\frac{1}{\cos(\alpha-\psi)}}_{r_1}\underbrace{\Big\{\Big(\underbrace{\frac{1}{\cos\alpha}(2+\lambda\sin\psi)-\frac{2}{\cos(\alpha-\psi)}}_{a_1}\Big)\underbrace{\tan(\alpha-\psi)}_{a_2}-\underbrace{\lambda\cos\psi\frac{1}{\cos\alpha}}_{a_3}\Big)}_{r_2}=0$$

$$r_1=\sec(\alpha-\psi)=\frac{1}{u}\quad,\quad u=\cos v\quad,\quad v=(\alpha-\psi)$$

$$\frac{\partial r_1}{\partial\psi}=\frac{\partial r}{\partial u}\frac{\partial u}{\partial v}\frac{\partial v}{\partial\psi}=-1\frac{1}{u^2}(-\sin v)(-1)$$

$$=-\frac{1}{\cos^2(\alpha-\psi)}\sin(\alpha-\psi)=-\frac{1}{\cos(\alpha-\psi)}\tan(\alpha-\psi)$$

$$a_1=\frac{1}{\cos\alpha}(2+\lambda\sin\psi)-\frac{2}{\cos(\alpha-\psi)}$$

$$\frac{\partial a_1}{\partial\psi}=\frac{1}{\cos\alpha}\lambda\cos\psi+2\Big(\underbrace{\frac{1}{\cos(\alpha-\psi)}\tan(\alpha-\psi)}_{\text{from }\frac{\partial r_1}{\partial\psi}}\Big)$$

$$a_2=\tan(\alpha-\psi)$$

$$\frac{\partial a_2}{\partial\psi}=\frac{\partial\tan(u)}{\partial u}\frac{\partial u}{\partial\psi}=-\frac{1}{\cos^2(\alpha-\psi)}$$

$$a_3=\lambda\cos\psi\frac{1}{\cos\alpha}$$

$$\frac{\partial a_3}{\partial\psi}=-\lambda\operatorname{sen}\psi\frac{1}{\cos\alpha}$$

$$r_2=a_1\cdot a_2-a_3$$

$$\frac{\partial r_2}{\partial\psi}=\frac{\partial a_1}{\partial\psi}a_2+a_1\frac{\partial a_2}{\partial\psi}-\frac{\partial a_3}{\partial\psi}$$

$$r=r_1\cdot r_2$$

$$\frac{\partial r}{\partial\psi}=\frac{\partial r_1}{\partial\psi}\cdot r_2+r_1\frac{\partial r_2}{\partial\psi}$$