BNCS1209: COMPUTER ORGANIZATION AND ARCHITECTURE

CHAPTER I

BINARY ARITHMETIC

Mr. Okello Jimmy Obira

INTRODUCTION TO NUMBER SYSTEMS

- A **number system** is a mathematical way of representing numbers using a consistent set of symbols and rules.
- It forms the foundation of arithmetic operations and computing.
- Number systems are classified based on the **base (or radix),** which represents the number of unique digits used, including zero.
- If base or radix of a number system is ‘r’, then the numbers present in that number system are ranging from zero to r-1.
- The total numbers present in that number system is ‘r’.

INTRODUCTION TO NUMBER SYSTEMS

- To define any number system we have to specify
- Base of the number system such as 2, 8, 10 or 16.
- The base decides the total number of digits available in that number system.
- First digit in the number system is always zero and last digit in the number system is always base-1.

INTRODUCTION TO NUMBER SYSTEMS

- **Types of Number Systems**
- There are several types of number systems, with the most commonly used ones being:
- Decimal Number System (Base-10)
- Binary Number System (Base-2)
- Octal Number System (Base-8)
- Hexadecimal Number System (Base-16)
- Binary Coded Decimal number(BCD) system

DECIMAL NUMBER SYSTEMS

- The base or radix of Decimal number system is 10.
- Its numbers ranging from 0 to 9 are used in this number system.
- The part of the number that lies to the left of the decimal point is known as integer part.
- Similarly, the part of the number that lies to the right of the decimal point is known as fractional part.
- In this number system, the successive positions to the left of the decimal point having weights of 10^0, 10^1, 10^2, 10^3 and so on.
- Similarly, the successive positions to the right of the decimal point having weights of 10^{-1}, 10^{-2}, 10^{-3} and so on.

DECIMAL NUMBER SYSTEMS

- That means, each position has specific weight, which is power of base 10
- Each digit in a decimal number has a place value based on powers of 10.
- For example, the decimal number 2735 can be expanded as:

$$2735 = (2 \times 10^3) + (7 \times 10^2) + (3 \times 10^1) + (5 \times 10^0)$$

- The decimal number system is the most widely used system in everyday life.
- It consists of ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

DECIMAL NUMBER SYSTEMS

Consider the **decimal number 1358.246.** Integer part of this number is 1358 and fractional part of this number is 0.246. The digits 8, 5, 3 and 1 have weights of 10^0, 10^1, 10^2 and 10^3 respectively. Similarly, the digits 2, 4 and 6 have weights of 10^{-1}, 10^{-2} and 10^{-3} respectively.

- **Mathematically**, we can write it as:

$1358.246 = (1 \times 10^3) + (3 \times 10^2) + (5 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (4 \times 10^{-2}) + (6 \times 10^{-3})$

- After simplifying the right hand side terms, we will get the decimal number, which is on left hand side.

BINARY NUMBER SYSTEM (BASE-2)

- The **binary number system** is the fundamental system used in computing and digital electronics.
- The base or radix of binary number system is 2
- It consists of only **two digits: 0 and 1**.
- All digital circuits and systems use this binary number system. The base or radix of this number system is 2.
- The part of the number, which lies to the left of the binary point is known as integer part.
- Similarly, the part of the number, which lies to the right of the binary point is known as fractional part.

BINARY NUMBER SYSTEM (BASE-2)

- In this number system, the successive positions to the left of the binary point having weights of 2^0, 2^1, 2^2, 2^3 and so on.
- Similarly, the successive positions to the right of the binary point having weights of 2^{-1}, 2^{-2}, 2^{-3} and so on.
- That means, each position has specific weight, which is power of base 2.
- Each digit (bit) in a binary number represents a power of **2.**
- For example, the binary number **1011** can be expanded as:

$$1011_2 = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$$
$$= 8 + 0 + 2 + 1 = 11_{10}$$

BINARY NUMBER SYSTEM (BASE-2)

- The left most bit, which has the greatest weight is called the Most Significant Bit (MSB).
- And the right most bit which has the least weight is called Least Significant Bit (LSB).
- **Applications**
- Used in computers and digital circuits.
- Represents data in **bits** (binary digits).
- Logical operations such as AND, OR, NOT, and XOR are performed using binary numbers.

BINARY NUMBER SYSTEM (BASE-2)

- Consider the **binary number 1101.011**. Integer part of this number is 1101 and fractional part of this number is 0.011. The digits 1, 0, 1 and 1 of integer part have weights of 2^0, 2^1, 2^2, 2^3 respectively. Similarly, the digits 0, 1 and 1 of fractional part have weights of 2^{-1}, 2^{-2}, 2^{-3} respectively.
- **Mathematically**, we can write it as:

$$1101.011 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2}) + (1 \times 2^{-3})$$

OCTAL NUMBER SYSTEM

- Digital systems operate only on binary numbers.
- Since binary numbers are often very long, other shorthand notations; octal and hexadecimal, are used for representing large binary numbers.
- Octal systems use a **base or radix of 8.** It uses first eight digits of decimal number system (digits from 0 to 7).
- The part of the number that lies to the left of the octal point is known as integer part.
- Similarly, the part of the number that lies to the right of the octal point is known as fractional part.

OCTAL NUMBER SYSTEM

- In this number system, the successive positions to the left of the octal point having weights of 8^0, 8^1, 8^2, 8^3 and so on.
- Similarly, the successive positions to the right of the octal point having weights of 8^{-1}, 8^{-2}, 8^{-3} and so on.
- That means, each position has specific weight, which is **power of base 8**.
- It uses first eight digits of decimal number system (digits from 0 to 7).
- 0= 000, 1= 001, 2= 010, 3= 011, 4= 100, 5= 101, 6= 110, 7= 111

OCTAL NUMBER SYSTEM

- Consider the **octal number 1457.236.** Integer part of this number is 1457 and fractional part of this number is 0.236.
- The digits 7, 5, 4 and 1 have weights of 8^0, 8^1, 8^2 and 8^3 respectively.
- Similarly, the digits 2, 3 and 6 have weights of 8^{-1}, 8^{-2}, 8^{-3} respectively.
- **Mathematically**, we can write it as:

$$1457.236 = (1 \times 8^3) + (4 \times 8^2) + (5 \times 8^1) + (7 \times 8^0) + (2 \times 8^{-1}) + (3 \times 8^{-2}) + (6 \times 8^{-3})$$

OCTAL NUMBER SYSTEM

- Each digit in an octal number represents a power of 8.
- For example, the octal number 237 can be expanded as:

$$\begin{aligned} 237_8 &= (2 \times 8^2) + (3 \times 8^1) + (7 \times 8^0) \\ &= (2 \times 64) + (3 \times 8) + (7 \times 1) \\ &= 128 + 24 + 7 = 159_{10} \end{aligned}$$

Applications

- Used in older computing systems.
- Often utilized as a shorthand for binary numbers because one octal digit represents three binary digits.

HEXADECIMAL NUMBER SYSTEM

- The **base** or radix of Hexa-decimal number system is **16.**
- So, the numbers ranging from 0 to 9 and the letters from A to F are used in this number system.
- The decimal equivalent of Hexa-decimal digits from A to F are 10 to 15.
- The part of the number, which lies to the left of the **hexadecimal point** is known as integer part.
- Similarly, the part of the number, which lies to the right of the Hexa-decimal point is known as fractional part.

HEXADECIMAL NUMBER SYSTEM

- In this number system, the successive positions to the left of the Hexa-decimal point having weights of 16^0, 16^1, 16^2, 16^3 and so on.
- Similarly, the successive positions to the right of the Hexa-decimal point having weights of 16^{-1}, 16^{-2}, 16^{-3} and so on.
- That means, each position has specific weight, which is **power of base 16.**

HEXADECIMAL NUMBER SYSTEM

- The hexadecimal number system consists of 16 symbols:
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.
- (A = 10, B = 11, C = 12, D = 13, E = 14, F = 15)

Representation

Each digit in a hexadecimal number represents a power of 16.

For example, the hexadecimal number 3F2 can be expanded as:

$$3F2_{16} = (3 \times 16^2) + (F \times 16^1) + (2 \times 16^0)$$

$$= (3 \times 256) + (15 \times 16) + (2 \times 1)$$

$$= 768 + 240 + 2 = 1010_{10}$$

HEXADECIMAL NUMBER SYSTEM

- **3F7A =** 0011 1111 0111 1010
- 53CE = 0101 0011 1100 1110

- **Applications**
- Widely used in computing and digital systems.
- Memory addresses and color codes in web design often use hexadecimal notation.
- Convenient because one hexadecimal digit represents four binary digits.

HEXADECIMAL NUMBER SYSTEM

- Consider the **Hexa-decimal number 1A05.2C4.** Integer part of this number is 1A05 and fractional part of this number is 0.2C4.
- The digits 5, 0, A and 1 have weights of 16^0, 16^1, 16^2 and 16^3 respectively.
- Similarly, the digits 2, C and 4 have weights of 16^{-1}, 16^{-2} and 16^{-3} respectively.
- **Mathematically**, we can write it as:

$$1A05.2C4 = (1 \times 16^3) + (10 \times 16^2) + (0 \times 16^1) + (5 \times 16^0) + (2 \times 16^{-1}) + (12 \times 16^{-2}) + (4 \times 16^{-3})$$

- **DECIMAL NUMBER TO OTHER BASES CONVERSION**
- If the decimal number contains both integer part and fractional part, then convert both the parts of decimal number into other base individually.
- Follow these steps for converting the decimal number into its equivalent number of any base ‘r’.
- Do division of integer part of decimal number and successive quotients with base ‘r’ and note down the remainders till the quotient is zero.

CONVERSION BETWEEN NUMBER SYSTEMS

- Consider the remainders in reverse order to get the integer part of equivalent number of base ‘r’.
- That means, first and last remainders denote the least significant digit and most significant digit respectively.
- Do **multiplication** of fractional part of decimal number and **successive fractions** with base ‘r’ and note down the carry till the result is zero or the desired number of equivalent digits is obtained.
- Consider the normal sequence of carry in order to get the fractional part of equivalent number of base ‘r’.

CONVERSION BETWEEN NUMBER SYSTEMS

- **1. Decimal to Binary Conversion**
- **Steps**
- Divide the decimal number by 2
- Write down the remainder.
- Repeat the process with the quotient until it becomes zero.
- The binary number is the remainders read from bottom to top.

- **1. Decimal to Binary Conversion**
- If the decimal number contains both integer part and fractional part, then convert both the parts of decimal number into other base individually.
- Follow these steps for converting the decimal number into its equivalent number of any base ‘r’.
- Do **division** of integer part of decimal number and **successive quotients** with base ‘r’ and note down the remainders till the quotient is zero.

CONVERSION BETWEEN NUMBER SYSTEMS

- Conversion of decimal to binary
- Consider the remainders in reverse order to get the integer part of equivalent number of base ‘r’.
- That means, first and last remainders denote the least significant digit and most significant digit respectively.
- Do **multiplication** of fractional part of decimal number and **successive fractions** with base ‘r’ and note down the carry till the result is zero or the desired number of equivalent digits is obtained.
- Consider the normal sequence of carry in order to get the fractional part of equivalent number of base ‘r’.

CONVERSION BETWEEN NUMBER SYSTEMS

- Conversion of decimal to binary

Example: Convert 13 to binary.

$$13 \div 2 = 6, \quad \text{Remainder} = 1$$

$$6 \div 2 = 3, \quad \text{Remainder} = 0$$

$$3 \div 2 = 1, \quad \text{Remainder} = 1$$

$$1 \div 2 = 0, \quad \text{Remainder} = 1$$

So, 13 in decimal = 1101 in binary.

CONVERSION BETWEEN NUMBER SYSTEMS

- Conversion of decimal to binary
- The following two types of operations take place, while converting decimal number into its equivalent binary number.
- Division of integer part and successive quotients with base 2.
- Multiplication of fractional part and successive fractions with base 2.

Example

Consider the **decimal number 58.25**.

Convert it to binary

CONVERSION BETWEEN NUMBER SYSTEMS

- Consider the **decimal number 58.25.**
- Convert it to binary

Here, the integer part is 58 and fractional part is 0.25.

Step 1 – Division of 58 and successive quotients with base 2

CONVERSION BETWEEN NUMBER SYSTEMS

- Consider the **decimal number 58.25.**

Step 1 – Division of 58 and successive quotients with base 2

Operation	Quotient		Remainder
58/2	29		0 LSB
29/2	14		1
14/2	7		0
7/2	3		1
3/2	1		1
1/2		⇒	1 MSB

Therefore, the **integer part** of equivalent binary number is **111010**.

Step 2 − Multiplication of 0.25 and successive fractions with base 2.

Operation	Result	Carry
0.25 x 2	0.5	0
0.5 x 2	1.0	1
-	0.0	-

$$\Rightarrow .25_{10} = .01_2$$

Therefore, the **fractional part** of equivalent binary number is **.01**

$$\Rightarrow 58.25_{10} = \mathbf{111010.01}_2$$

Therefore, the **binary equivalent** of decimal number 58.25 is 111010.01.

2. Decimal to Octal Conversion

- The following two types of operations take place, while converting decimal number into its equivalent octal number.
- Division of integer part and successive quotients with base 8.
- Multiplication of fractional part and successive fractions with base 8.

2. Decimal to Octal Conversion

Consider the **decimal number 58.25**. Here, the integer part is 58 and fractional part is 0.25.

Step 1 – Division of 58 and successive quotients with base 8.

Operation	Quotient	Remainder
58/8	7	**2**
7/8	0	**7**

$$\Rightarrow 58_{10} = 72_{8}$$

Therefore, the **integer part** of equivalent octal number is **72**.

2. Decimal to Octal Conversion

Step 2 – Multiplication of 0.25 and successive fractions with base 8.

Operation	Result	Carry
0.25 x 8	2.00	2
-	0.00	-

$$\Rightarrow .25_{10} = .2_8$$

Therefore, the **fractional part** of equivalent octal number is .2

$$\Rightarrow 58.25_{10} = 72.2_8$$

Therefore, the **octal equivalent** of decimal number 58.25 is 72.2.

- **3. Decimal to Hexa-Decimal Conversion**
- The following two types of operations take place, while converting decimal number into its equivalent hexa-decimal number.
- Division of integer part and successive quotients with base 16.
- Multiplication of fractional part and successive fractions with base 16.

3. Decimal to Hexa-Decimal Conversion

Consider the **decimal number 58.25**. Here, the integer part is 58 and decimal part is 0.25.

Step 1 – Division of 58 and successive quotients with base 16.

Operation	Quotient	Remainder
58/16	3	10=A
3/16	0	3

$$\Rightarrow 58_{10} = 3A_{16}$$

Therefore, the **integer part** of equivalent Hexa-decimal number is 3A.

3. Decimal to Hexa-Decimal Conversion

Step 2 – Multiplication of 0.25 and successive fractions with base 16.

Operation	Result	Carry
0.25 x 16	4.00	4
-	0.00	-

$$\Rightarrow .25_{10} = .4_{16}$$

Therefore, the **fractional part** of equivalent Hexa-decimal number is .4.

$$\Rightarrow 58.25_{10} = 3A.4_{16}$$

Therefore, the **Hexa-decimal equivalent** of decimal number 58.25 is 3A.4.

BINARY NUMBER TO OTHER BASES CONVERSION

- The process of converting a number from binary to decimal is different to the process of converting a binary number to other bases.
- Now, let us discuss about the conversion of a binary number to decimal, octal and Hexa-decimal number systems one by one.
- **1. Binary to Decimal Conversion**
- For converting a binary number into its equivalent decimal number, first multiply the bits of binary number with the respective positional weights and then add all those products.

BINARY NUMBER TO OTHER BASES CONVERSION

1. Binary to Decimal Conversion

Mathematically, we can write it as

$$1101.11_2 = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2})$$

$$\Rightarrow 1101.11_2 = 8 + 4 + 0 + 1 + 0.5 + 0.25 = 13.75$$

$$\Rightarrow 1101.11_2 = 13.75_{10}$$

Therefore, the **decimal equivalent** of binary number 1101.11 is 13.75.

BINARY NUMBER TO OTHER BASES CONVERSION

- **1. Binary to Octal Conversion**

We know that the bases of binary and octal number systems are 2 and 8 respectively. Three bits of binary number is equivalent to one octal digit, since $2^3 = 8$.

Follow these two steps for converting a binary number into its equivalent octal number.

- Start from the binary point and make the groups of 3 bits on both sides of binary point. If one or two bits are less while making the group of 3 bits, then include required number of zeros on extreme sides.
- Write the octal digits corresponding to each group of 3 bits

BINARY NUMBER TO OTHER BASES CONVERSION

- **1. Binary to Octal Conversion**
- Write the octal digits corresponding to each group of 3 bits

$$\Rightarrow 101\ 110.011\ 010$$

Step 2 – Write the octal digits corresponding to each group of 3 bits.

$$\Rightarrow 101110.011010_2 = 56.32_8$$

Therefore, the **octal equivalent** of binary number 101110.01101 is 56.32.

BINARY NUMBER TO OTHER BASES CONVERSION

- **1. Binary to Hexa-Decimal Conversion**
- We know that the bases of binary and Hexa-decimal number systems are 2 and 16 respectively.
- Four bits of binary number is equivalent to one Hexa-decimal digit, since $2^4 = 16$.
- Follow these two steps for converting a binary number into its equivalent Hexa-decimal number.

BINARY NUMBER TO OTHER BASES CONVERSION

- **1. Binary to Hexa-Decimal Conversion**
- Start from the binary point and make the groups of 4 bits on both sides of binary point.
- If some bits are less while making the group of 4 bits, then include required number of zeros on extreme sides.
- Write the Hexa-decimal digits corresponding to each group of 4 bits.

BINARY NUMBER TO OTHER BASES CONVERSION

1. Binary to Hexa-Decimal Conversion

Example

Consider the **binary number 101110.01101**

Step 1 − Make the groups of 4 bits on both sides of binary point.

10 1110.0110 1

Here, the first group is having only 2 bits. So, include two zeros on extreme side in order to make it as group of 4 bits. Similarly, include three zeros on extreme side in order to make the last group also as group of 4 bits.

$$\Rightarrow 0010\ 1110.0110\ 1000$$

Step 2 − Write the Hexa-decimal digits corresponding to each group of 4 bits.

$$\Rightarrow 00101110.01101000_2 = 2E.68_{16}$$

Therefore, the **Hexa-decimal equivalent** of binary number 101110.01101 is $2E.68$.

BINARY ARITHMETIC

- Binary arithmetic is essential part of all the digital computers and many other digital system.

Binary Addition

- It is a key for binary subtraction, multiplication, division. There are four rules of binary addition.

Case	A	+	B	Sum	Carry
1	0	+	0	0	0
2	0	+	1	1	0
3	1	+	0	1	0
4	1	+	1	0	1

BINARY ARITHMETIC

In fourth case, a binary addition is creating a sum of,

(1 + 1 = 10) i.e. 0 is written in the given column and a carry of 1 over to the next column.

Example – Addition

0011010 + 001100 = 00100110

$$\begin{array}{rl} 1\,1 & \text{carry} \\ 0\,0\,1\,1\,0\,1\,0 & = 26_{10} \\ +\,0\,0\,0\,1\,1\,0\,0 & = 12_{10} \\ \hline 0\,1\,0\,0\,1\,1\,0 & = 38_{10} \end{array}$$

BINARY SUBTRACTION

Subtraction and Borrow, these two words will be used very frequently for the binary subtraction. There are four rules of binary subtraction.

Case	A - B	Subtract	Borrow
1	0 - 0	0	0
2	1 - 0	1	0
3	1 - 1	0	0
4	0 - 1	0	1

Example – Subtraction

0011010 - 001100 = 00001110

1 1 borrow

0 0 ~~1 1~~ 0 1 0 = 26_{10}

- 0 0 0 1 1 0 0 = 12_{10}

0 0 0 1 1 1 0 = 14_{10}

BINARY MULTIPLICATION

Binary multiplication is similar to decimal multiplication. It is simpler than decimal multiplication because only 0s and 1s are involved. There are four rules of binary multiplication.

Case	A x B	Multiplication
1	0 x 0	0
2	0 x 1	0
3	1 x 0	0
4	1 x 1	1

Example – Multiplication

Example:

0011010 x 001100 = 100111000

```
        0 0 1 1 0 1 0    = 26₁₀
      x 0 0 0 1 1 0 0    = 12₁₀
  -----------------------
        0 0 0 0 0 0 0
      0 0 0 0 0 0 0
    0 0 1 1 0 1 0
  0 0 1 1 0 1 0
  -----------------------
  0 1 0 0 1 1 1 0 0 0    = 312₁₀
```

BINARY DIVISION

- **Steps for Binary Division**
- Align the dividend and divisor as in long division.
- Check if the divisor fits into the current portion of the dividend:
- If YES, write 1 in the quotient and subtract.
- If NO, write 0 in the quotient and bring down the next bit.
- Repeat until the entire dividend is processed.
- If any remainder is left, write it down separately.

BINARY DIVISION

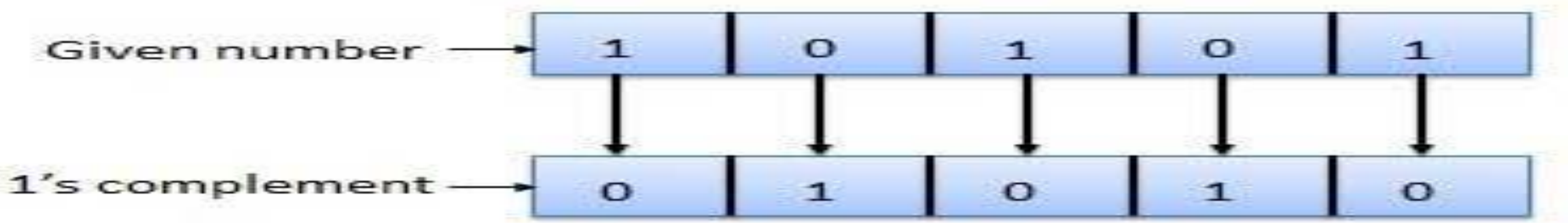
```
          00100111
    101 ) 11000011
         -101
          ----
            1000
           -101
            ----
             0111
            -101
             ----
              0101
             -101
              ----
               000
```

BINARY SYSTEM COMPLEMENTS

- As the binary system has base r = 2. So the two types of complements for the binary system are 2's complement and 1's complement.

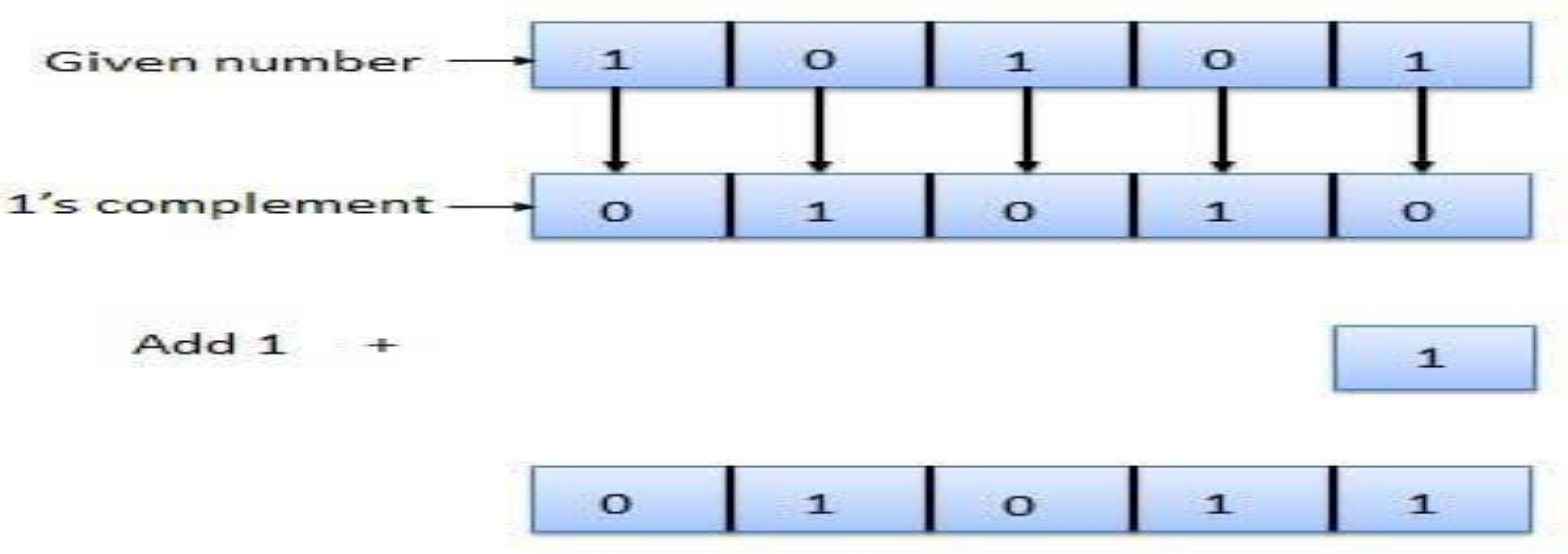
1's complement

- The 1's complement of a number is found by changing all 1's to 0's and all 0's to 1's. This is known as taking complement or 1's complement. Example of 1's Complement is as follows.



BINARY SYSTEM COMPLEMENTS

- The 2's complement of binary number is obtained by adding 1 to the Least Significant Bit (LSB) of 1's complement of the number.
- 2's complement = 1's complement + 1
- Example of 2's Complement is as follows.



BINARY SYSTEM COMPLEMENTS

1. *Express the following numbers in sign magnitude 1's and 2's complement :*
i) –56 ii)107

Solution : i) – 56

	56 =	0 1 1 1 0 0 0	
	– 56 =	1 0 0 0 1 1 1	1's Complement
		+ 1	
	=	1 0 0 1 0 0 0	2's Complement
ii) 107	107 =	0 1 1 0 1 0 1 1	
	–107 =	1 0 0 1 0 1 0 0	1's Complement
		+ 1	
	=	1 0 0 1 0 1 0 1	2's Complement.

BINARY SYSTEM COMPLEMENTS

2. *Find 2's complement of* $(1\ 0\ 0\ 1)_2$

Solution :

1	0	0	1	number
0	1	1	0	1's complement
+			1	
0	1	1	1	2's complement

3. *Find 2's complement of* $(1\ 0\ 1\ 0\ \ 0\ 0\ 1\ 1)_2$

Solution :

1 0 1 0 0 0 1 1	number
0 1 0 1 1 1 0 0	1's complement
+ 1	
0 1 0 1 1 1 0 1	2's complement

Thank You