Sec. 1.5.7. Ejercicio 2.

a) para $\phi = \phi(r) = \phi(x, y, z)$

$\psi = \psi(r) = \psi(x, y, z)$

$$\rightarrow \nabla(\phi\psi) = \frac{\partial(\phi\psi)}{\partial x} + \frac{\partial(\phi\psi)}{\partial y} + \frac{\partial(\phi\psi)}{\partial z}$$

$$= \phi\frac{\partial\psi}{\partial x} + \psi\frac{\partial\phi}{\partial x} + \phi\frac{\partial\psi}{\partial y} + \psi\frac{\partial\phi}{\partial y} + \phi\frac{\partial\psi}{\partial z} + \psi\frac{\partial\phi}{\partial z}$$

$$= \phi\frac{\partial\psi}{\partial x} + \phi\frac{\partial\psi}{\partial y} + \phi\frac{\partial\psi}{\partial z} + \psi\frac{\partial\phi}{\partial x} + \psi\frac{\partial\phi}{\partial y} + \psi\frac{\partial\phi}{\partial z}$$

$$= \phi\left(\frac{\partial\psi}{\partial x} + \frac{\partial\psi}{\partial y} + \frac{\partial\psi}{\partial z}\right) + \psi\left(\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y} + \frac{\partial\phi}{\partial z}\right)$$

$$= \phi\nabla\psi + \psi\nabla\phi$$

//

Sec. 1.5.7. Ejercicio 2.

d) para $\vec{a} = \vec{a}(r) = a(x,y,z) = a^i(x,y,z)\,\hat{i}_i$

$$\rightarrow \nabla\cdot(\nabla\times\vec{a}) = \nabla\cdot\left(\frac{\partial a^3}{\partial y} - \frac{\partial a^2}{\partial z},\ \frac{\partial a^1}{\partial z} - \frac{\partial a^3}{\partial x},\ \frac{\partial a^2}{\partial x} - \frac{\partial a^1}{\partial y}\right)$$

$$= \frac{\partial^2 a^3}{\partial y\partial x} - \frac{\partial^2 a^2}{\partial z\partial x} + \frac{\partial^2 a^1}{\partial z\partial y} - \frac{\partial^2 a^3}{\partial x\partial y} + \frac{\partial^2 a^2}{\partial x\partial z} - \frac{\partial^2 a^1}{\partial y\partial z}$$

$$= \underbrace{\frac{\partial^2 a^3}{\partial y\partial x} - \frac{\partial^2 a^3}{\partial x\partial y}}_{0} + \underbrace{\frac{\partial^2 a^1}{\partial z\partial y} - \frac{\partial^2 a^1}{\partial y\partial z}}_{0} + \underbrace{\frac{\partial^2 a^2}{\partial x\partial z} - \frac{\partial^2 a^2}{\partial z\partial x}}_{0}$$

Por teorema de Clairaut de la igualdad de las derivadas cruzadas, se tiene que $\nabla\cdot(\nabla\times\vec{a}) = 0$

ya que se asume, las derivadas cruzadas existen y son continuas.

Sec. 1.5.7. Ejercicio 2.

d) para $\vec{a} = \vec{a}(r) = a(x,y,z) = a^i(x,y,z)\,\hat{\imath}_i$

la operación $\nabla \times (\nabla \cdot \vec{a})$ no se puede realizar pues de $\nabla \cdot \vec{a}$ se obtiene un escalar que no se puede operar con un producto vectorial.

Sec. 1.5.7. Ejercicio 2.

f) para $\vec{a} = \vec{a}(r) = a(x, y, z) = a^i(x, y, z)\,\hat{\imath}_i$

$$\rightarrow \nabla \times (\nabla \times \vec{a}) = \left(\nabla \times (\nabla \times \vec{a})\right)^i = \varepsilon^{ijk} \nabla_j (\nabla \times \vec{a})$$

$$= \varepsilon^{ijk} \nabla_j \varepsilon_{kmn} \nabla^m a^n = \varepsilon^{ijk} \varepsilon_{kmn} \nabla_j \nabla^m a^n$$

$$= (\delta^i_m \delta^j_n - \delta^j_m \delta^i_n) \nabla_j \nabla^m a^n$$

$$= \delta^i_m \delta^j_n \nabla_j \nabla^m a^n - \delta^j_m \delta^i_n \nabla_j \nabla^m a^n$$

$$= \delta^i_m \nabla^m \delta^j_n \nabla_j a^n - \delta^j_m \nabla_j \nabla^m \delta^i_n a^n$$

$$= \nabla^i \underbrace{\nabla_n a^n} - \underbrace{\nabla_m \nabla^m} a^i$$

$$= \underbrace{\nabla^i (\nabla \cdot \vec{a})_n} - \underbrace{(\nabla \cdot \nabla)_m} a^i$$

$$= \nabla (\nabla \cdot \vec{a}) - \nabla^2 \vec{a}$$