

ECE 3040 Homework 3

Question 2: $x_1 + x_2 + x_3 = 4$, $x_3 = -4$, $(a^2 - 4)x_1 + x_3 = a - 2$

$$x_1 + x_2 + x_3 = 4 \Rightarrow x_1 + x_2 = 8 \Rightarrow x_1 = 8 - x_2$$

$$(a^2 - 4)x_1 + x_3 = a - 2 \Rightarrow a^2 - 4 = \frac{a + 2}{8 - x_2} \Rightarrow a^2 - 4 = \frac{a + 2}{8 - x_2}$$

Now, if $a = 2$, then: $a^2 - 4 = \frac{a + 2}{8 - x_2} \Rightarrow 0 = 0 \Rightarrow$ infinite solutions

Question 3: $x + y = 2 \Rightarrow x = 2 - y$
 $6x + 6y = 12 \Rightarrow 6(2 - y) + 6y = 12$
 $\Rightarrow 12 - 6y + 6y = 12 \Rightarrow 12 = 12$
 \Rightarrow infinite solutions

Question 4: $4a + b - c = -2$
 $5a + b + 2c = 4$
 $6a + b + c = 6$
 $\Rightarrow \begin{bmatrix} 4 & 1 & -1 \\ 5 & 1 & 2 \\ 6 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 4 & 1 & -1 \\ 0 & -0.2 & 2.6 \\ 6 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 5.2 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 1 & -1 \\ 0 & -0.2 & 2.6 \\ 0 & -0.33 & 1.66 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 5.2 \\ 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 1 & -1 \\ 0 & -0.2 & 2.6 \\ 0 & 0 & -1.6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 5.2 \\ -1.6 \end{bmatrix} \left\{ \begin{array}{l} \therefore -1.6c = -1.6 \Rightarrow c = 1 \\ -0.2b + 2.6c = 5.2 \Rightarrow b = -13 \\ 4a + b - c = -2 \Rightarrow a = 3 \end{array} \right.$$

Question 6: LU decomposition

$$\begin{aligned} 4x_1 + x_2 - x_3 &= -2 \\ 5x_1 + x_2 + 2x_3 &= 4 \\ 6x_1 + x_2 + x_3 &= 6 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 4 & 1 & -1 \\ 5 & 1 & 2 \\ 6 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}$$

From Question 4 \Rightarrow
$$\begin{bmatrix} 4 & 1 & -1 \\ 0 & -0.2 & 2.6 \\ 0 & 0 & -1.6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 5.2 \\ -1.6 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 4 & 1 & -1 \\ 0 & -0.2 & 2.6 \\ 0 & 0 & -1.6 \end{bmatrix}$$

Ratios used: $\frac{4}{5}$, $\frac{4}{6}$, $\frac{-0.2}{-0.33}$

$$\therefore [L] = \begin{bmatrix} \frac{4}{5} & 0 & 0 \\ 5 & \frac{4}{6} & 0 \\ 6 & 1 & \frac{-0.2}{-0.33} \end{bmatrix}$$

Question 7: Gauss-Seidel

$$\begin{bmatrix} 3 & 6 & 2 \\ 12 & 7 & 3 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 17 \\ 49 \end{bmatrix}$$

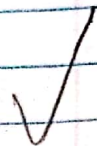
Rewrite to make it diagonally dominant:

$$\begin{bmatrix} 12 & 7 & 3 \\ 3 & 6 & 2 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ 9 \\ 49 \end{bmatrix}$$

$$|12| \geq |7| + |3|$$

$$|6| \geq |3| + |2|$$

$$|-11| \geq |7| + |2|$$



$$12x_1 + 7x_2 + 3x_3 = 17 \Rightarrow x_1 = \frac{17 - 7x_2 - 3x_3}{12}$$

$$3x_1 + 6x_2 + 2x_3 = 9 \Rightarrow x_2 = \frac{9 - 3x_1 - 2x_3}{6}$$

$$2x_1 + 7x_2 - 11x_3 = 49 \Rightarrow x_3 = \frac{49 - 2x_1 - 7x_2}{-11}$$

initial guesses: $x_1 = 1.1$, $x_2 = 2.1$, $x_3 = -2.9$

After 2nd iteration, $x_1 = 0.02799$

$$x_1 = \frac{17 - 7(2.1) - 3(-2.9)}{12} = \frac{11}{12}, \quad |\epsilon_{x_1}| = \left| \frac{\frac{11}{12} - 1.1}{\frac{11}{12}} \right| \times 100 = \%20$$

$$x_2 = \frac{9 - 3(1.1) - 2(-2.9)}{6} = \frac{11.5}{6}, \quad |\epsilon_{x_2}| = \left| \frac{\frac{11.5}{6} - 2.1}{\frac{11.5}{6}} \right| \times 100 = \%9.6$$

$$x_3 = \frac{49 - 2(1.1) - 7(2.1)}{-11} = \frac{32.1}{-11}, \quad |\epsilon_{x_3}| = \left| \frac{\frac{32.1}{-11} - (-2.9)}{\frac{32.1}{-11}} \right| \times 100 = \%0.62$$

2nd iteration

$$x_1 = \frac{17 - 7(\frac{11.5}{6}) - 3(\frac{32.1}{-11})}{12} = 1.0282, \quad |\epsilon_{x_1}| = \left| \frac{1.0282 - \frac{11}{12}}{1.0282} \right| \times 100 = \%10.85$$

$$x_2 = \frac{9 - 3(\frac{11}{12}) - 2(\frac{32.1}{-11})}{6} = 2.0144, \quad |\epsilon_{x_2}| = \left| \frac{2.0144 - \frac{11.5}{6}}{2.0144} \right| \times 100 = \%4.85$$

$$x_3 = \frac{49 - 2(\frac{11}{12}) - 7(\frac{11.5}{6})}{-11} = \frac{33.75}{-11}, \quad |\epsilon_{x_3}| = \left| \frac{\frac{33.75}{-11} - \frac{32.1}{-11}}{\frac{33.75}{-11}} \right| \times 100 = \%4.88$$