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ECE 3040 Homework 4

Question 1:

t	Velocity
0	22
15	24
18	37
22	25
24	123

Direct method:

$$v(t) = a_0 + a_1 (\sin(t)) + a_2 (\sin(2t))$$

Find $v(16)$

$v(15) = a_0 + a_1 (\sin 15) + a_2 (\sin 30) = 24$
$v(18) = a_0 + a_1 (\sin 18) + a_2 (\sin 36) = 37$ $\Rightarrow$
$v(22) = a_0 + a_1 (\sin 22) + a_2 (\sin 44) = 25$

$$\begin{bmatrix} 1 & \sin 15 & \sin 30 \\ 1 & \sin 18 & \sin 36 \\ 1 & \sin 22 & \sin 44 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 37 \\ 25 \end{bmatrix}$$

Inserting to MATLAB $\Rightarrow a_0 = 25.0079,$
$a_1 = -9.2637,$
$a_2 = -5.0770$

$v(t) = 25.0079 - 9.2637 (\sin t) - 5.0770 (\sin 2t)$
$v(16) = 19.7641$

Question 3:

t	velocity
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Newton's divided difference
linear approx.

Find $a(16)$

$$b_0 = v(15) = 362.78, \quad b_1 = \frac{v(20) - v(15)}{20 - 15} = \frac{517.35 - 362.78}{20 - 15} = 30.914$$

$$v(t) = b_0 + b_1(t - t_0) \Rightarrow v(16) = 362.78 + 30.914(16 - 15) = 393.694 \text{ m/s}$$

$$a(t) = \frac{d}{dt} v(t) = \frac{d}{dt}(362.78 + 30.914(t - 15))$$

$$a(t) = \frac{d}{dt}(30.914x - 100.93) = 30.914$$

$$a(16) = 30.914$$

Question 4:

t	velocity
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Lagrangian Method
Second-order

Find $a(9.5)$

$$v(t) = \sum_{i=0}^{2} L_i(t) v(t_i) = L_0(t) v(t_0) + L_1(t) v(t_1) + L_2(t) v(t_2)$$

$$t_0 = 0, \ t_1 = 10, \ t_2 = 15$$

$$L_0(t) = \prod_{\substack{j=0 \\ j \neq 0}}^{2} \frac{t - t_j}{t_0 - t_j} = \left(\frac{t - t_1}{t_0 - t_1}\right)\left(\frac{t - t_2}{t_0 - t_2}\right)$$

$$L_1(t) = \prod_{\substack{j=0 \\ j \neq 1}}^{2} \frac{t - t_j}{t_1 - t_j} = \left(\frac{t - t_0}{t_1 - t_0}\right)\left(\frac{t - t_2}{t_1 - t_2}\right)$$

$$L_2(t) = \prod_{\substack{j=0 \\ j \neq 2}}^{2} \frac{t - t_j}{t_2 - t_j} = \left(\frac{t - t_0}{t_2 - t_0}\right)\left(\frac{t - t_1}{t_2 - t_1}\right)$$

$$v(t) = \left(\frac{t-10}{0-10}\right)\left(\frac{t-15}{0-15}\right)(0) + \left(\frac{t-0}{10-0}\right)\left(\frac{t-15}{10-15}\right)(227.04)$$

$$+ \left(\frac{t-0}{15-0}\right)\left(\frac{t-10}{15-10}\right)(362.78)$$

$$v(t) = \frac{t}{10}\left(\frac{t-15}{-5}\right)(227.04) + \frac{t}{15}\left(\frac{t-10}{5}\right)(362.78)$$

$$a(t) = \frac{d}{dt} v(t) = (\text{Using diff}) \Rightarrow 0.5925t + 19.7413$$

$$a(16) = 0.5925(16) + 19.7413 = 29.2213 \ m/s^2$$

Question 5:

X	Y
3	2.5
4.5	1
7	2.5
9	0.5

Linear Spline Method

Find $y(5)$

$x_0 = 4.5$, $v(x_0) = 1$, $x_1 = 7$, $v(x_1) = 2.5$

$$y(x) = y(x_0) + \frac{y(x_1) - y(x_0)}{x_1 - x_0}(x - x_0) = 1 + \frac{2.5 - 1}{7 - 4.5}(x - 4.5)$$

$$y(5) = 1 + \frac{2.5 - 1}{7 - 4.5}(5 - 4.5) = 1.3$$

Question 6: For some reason, it's exactly the same as question 5?