0.1 Trotterized Adiabatic Computing

Due to the constraint of two local channels, we cannot perform simultaneous local detuning on all qubits. This means the encoding scheme for QUBO matrices with non-identical diagonal terms should be modified. Considering adiabatic computing as the starting point, we propose a modified version of the adiabatic evolution called trotterized adiabatic computing, which is more suited for the current neutral atom device.

The Hamiltonian of an adiabatically evolving Rydberg atoms array can be expressed as a function of time t:

$$H(t) = (1 - \frac{t}{T})H_M + \frac{t}{T}H_c = (1 - \frac{t}{T})\frac{\hbar}{2}\Omega\sum_i \sigma_i^x - \frac{t}{T}\frac{\hbar}{2}\sum_i \delta_i\sigma_i^z + \frac{t}{T}\sum_{i<j} V_{ij}\sigma_i^z\sigma_j^z \tag{1}$$

The corresponding propagator of such Hamiltonian takes the following form:

$$\begin{aligned} U &= exp(-\frac{i}{\hbar}\int_0^T H(t)\,dt) \\ &= exp(-\frac{i}{\hbar}[\frac{\hbar}{2}\Omega\sum_i \sigma_i^x\int_0^T(1-\frac{t}{T})\,dt - \frac{\hbar}{2}\sum_i \delta_i\sigma_i^z\int_0^T\frac{t}{T}\,dt + \sum_{i<j} V_{ij}\sigma_i^z\sigma_j^z\int_0^T\frac{t}{T}\,dt]) \\ &= exp(-\frac{i}{\hbar}[\frac{T}{2}\frac{\hbar}{2}\Omega\sum_i \sigma_i^x - \frac{T}{2}\frac{\hbar}{2}\sum_i \delta_i\sigma_i^z + \frac{T}{2}\sum_{i<j} V_{ij}\sigma_i^z\sigma_j^z]) \end{aligned} \tag{2}$$

We can then trotterize this unitary operator to get an approximated adiabatic evolution.

$$U \approx (exp(-i\frac{T}{4r}(\sum_i \Omega\sigma_i^x))[\prod_i exp(-i\frac{T}{4r}\delta_i\sigma_i^z)]exp(-\frac{i}{\hbar}\frac{T}{2r}(\sum_{i<j} V_{ij}\sigma_i^z\sigma_j^z)))^r \tag{3}$$

As is shown in (3), the adiabatic evolution can be broken down into r trotterized steps with each step accommodating three operations. The first operation can be realized by a constant rabi pulse for a duration of $\Delta t = \frac{T\hbar}{4r}$. The second operation is a sequence of constant local detunings for a duration of Δt for each detuning. The last operation corresponds to the interactions between neutral atoms, which can be implemented by optimizing the qubits layout. With the help of local and global channels, we can in theory transform these operations into pulse sequences and repeat them for several trotterization steps.

In conclusion, for each trotterization step, we first apply a global rabi drive via the global channel. Then, we apply a sequence of local detunings to two qubits at a time via two local channels. We have to repeat these operations for several iterations and then make the measurement in the end. Note that the local detunings in the sequence correspond to the diagonal elements of the QUBO matrix with an additional minus sign. Figure 1 demonstrates an example sequence for a 9-qubit QUBO problem.

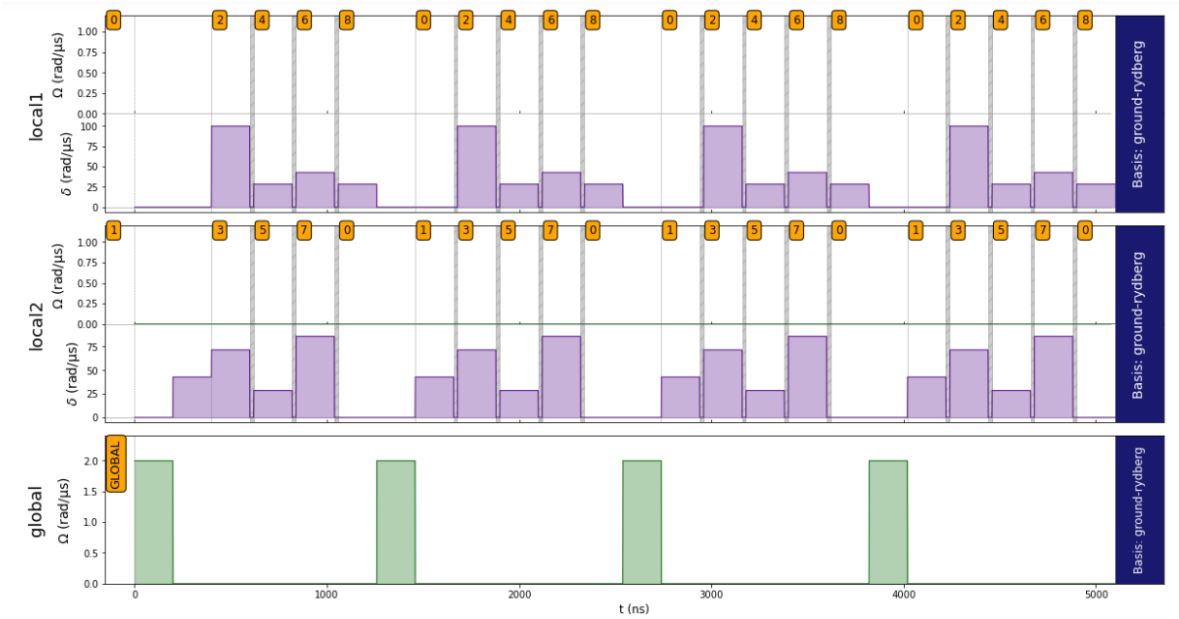

Figure 1: The pulse sequence for a trotterized adiabatic evolution with 4 trotterization steps and 9 qubits. The local detunings in the sequence correspond to the diagonal elements of the QUBO matrix with an additional minus sign ($-Q_{ii}$)

0.2 Formulation of QUBO matrices for our project

In this section, we present the QUBO formulation of two example problems from our project for which we can perform the trotterized adiabatic computing on the Pasqal's quantum computer.

0.2.1 Stations Positioning

As described in the presentation, we attempt to find the best locations for potential stations by solving a Maximum Weighted Independent Set problem (MWIS). Given the weights of each node W_i, the objective Hamiltonian to be minimized can be formulated as:

$$H_c = -\sum_i W_i X_i \quad , \quad X_i \in 0, 1 \tag{4}$$

The independent-set constraint is intrinsically imposed by the layout out of the physical qubits. The distances between nearest-neighbor qubits on the grid are set to be smaller than the Rydberg radius, while the distances between the next-nearest-neighbor qubits are longer than the radius. Figure 2. shows an example of a 9-qubit grid.

For implementation, we first recognize that the diagonal elements of the corresponding QUBO matrix $Q_{ii} = -W_i$. Then the conversion to local detunings $\delta_i = -Q_{ii} = W_i$ is trivial. Before we execute the trotterized adiabatic computing, we have to rescale the detunings by mapping them onto the domain between 0 and the maximum allowed detuning frequency.

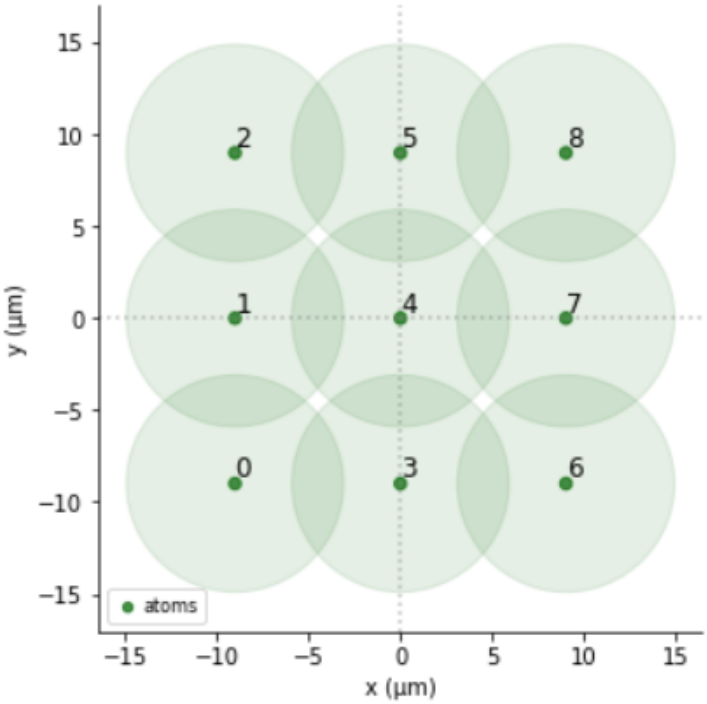

Figure 2: The 10-qubit layout directly reflects the locations of interest. The side of the grid is set to be shorter than the Rydberg radius.

0.2.2 Partition of Stations to Minimize Overall length

In this step, the cost Hamiltonian is formed by considering the overall connection length of two subnetworks with W_{ij} being the distance between the stations and X_i being the binary variable representing the class of the station.

$$\begin{aligned} H_c &= \sum_{i<j}[(1-X_i)(1-X_j)+X_iX_j]W_{ij} \\ &= \sum_{ij} W_{ij}X_iX_j - \sum_i(\sum_j W_{ij})X_i \end{aligned} \tag{5}$$

We also have to consider the constraint of equal station numbers as the penalty Hamiltonian with N being the total number of stations.

$$\begin{aligned} H_p &= (\sum_i X_i - N/2)^2 \\ &= \sum_{ij} X_iX_j + \sum_i(1-N)X_i \end{aligned} \tag{6}$$

Here we ignore the constant term since it does not affect the optimal solution.

Hence, the Hamiltonian of concern is

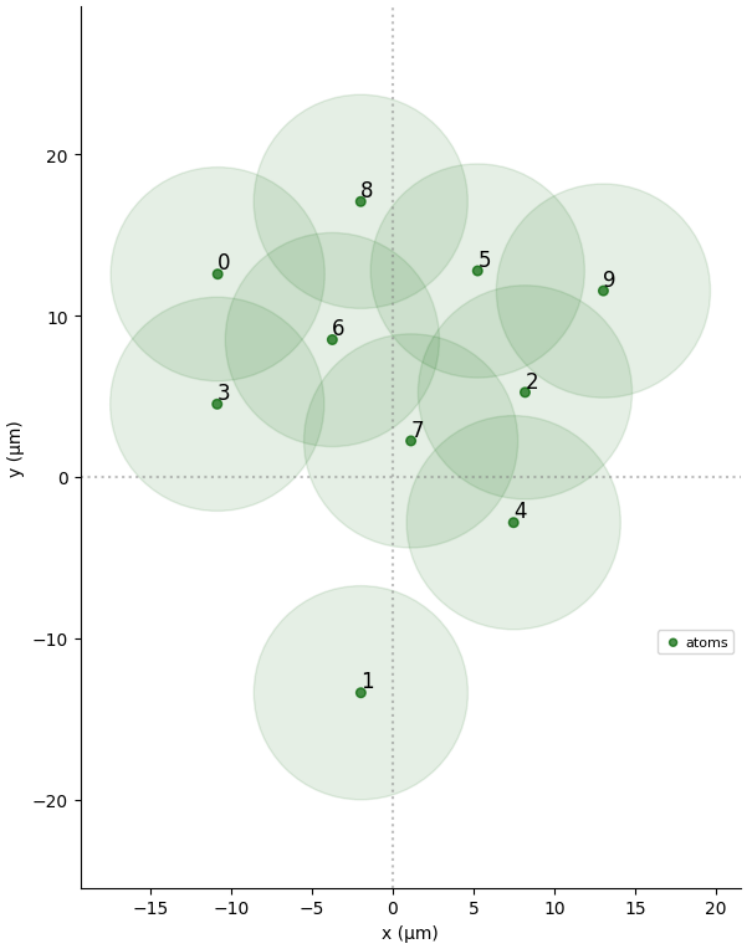

Figure 3: The 10-qubit layout for the partition of stations.All the stations are initially connected and the distance matrix is mapped to the Rydberg interactions, which in turn determine the optimal physical qubits layout.

$$H = H_c + \alpha H_p = \sum_{ij}(W_{ij} + \alpha)X_i X_j + \sum_i [(-\sum_j W_{ij}) + \alpha(1 - N)]X_i \tag{7}$$

It's then not hard to construct Q and the local detunings with $Q_{ii} = -\delta_i = (-\sum_j W_{ij}) + \alpha(1 - N)$, $Q_{ij} = W_{ij} + \alpha$. The optimized qubits layout is shown in Figure 3.