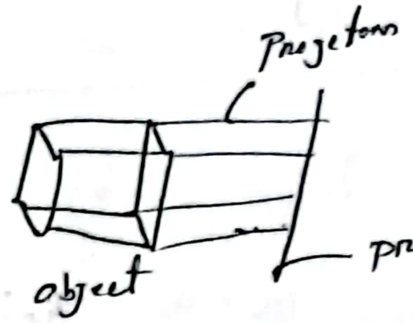
# Segment 6

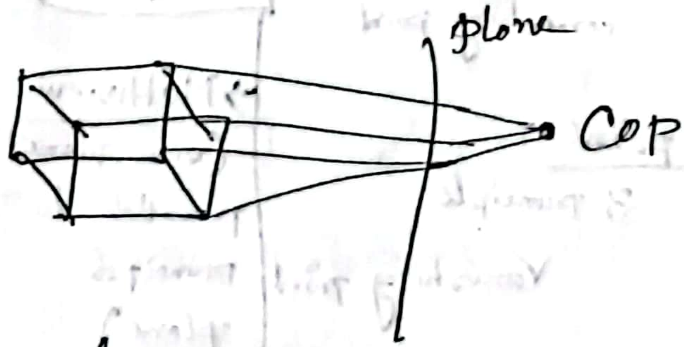
## Projection

It is a process of converting 3D object to 2D object



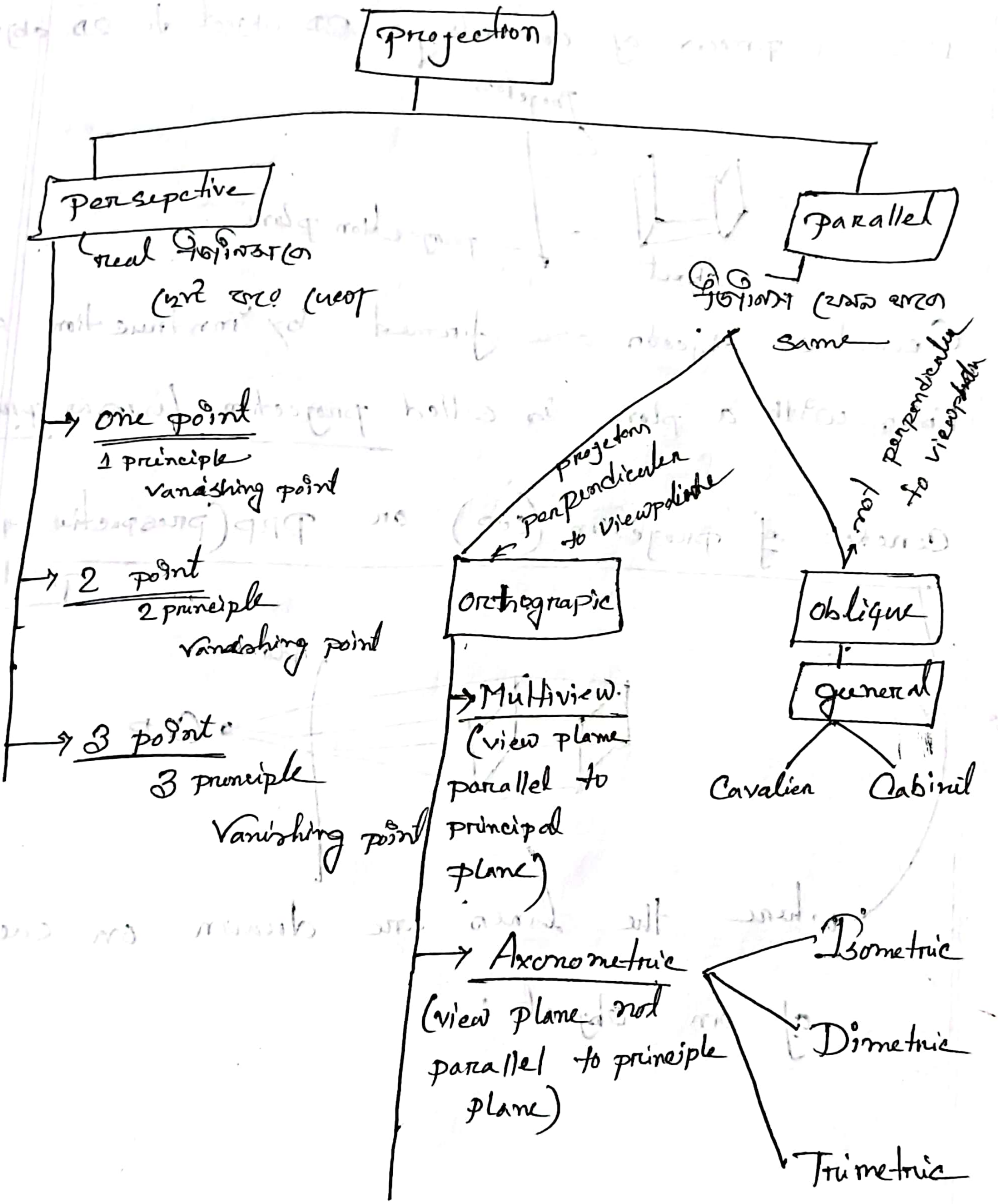

Geomatric objects are formed by intersection of Lines with a plane is called projection lines or projectors

Ceneter of projection (COP) or PRP (prospective reference point)



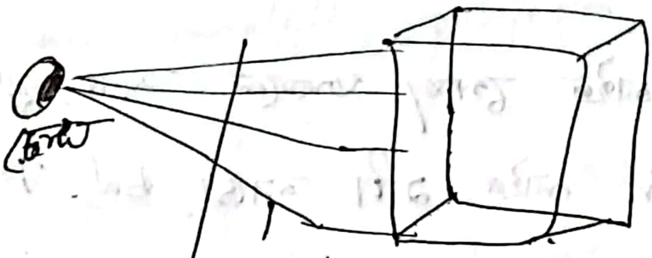

where the lines are drawn on each point of an object

# Toxynomy of projection



## Perspective projection

If center of the projection located at finite point in 3D space it will be perspective projection



projetor

Projetion plane.

## Prospective anomalies

anomalies distort the actual size and shape

Breakdown Some reason why anomalies occour.

**Prospective foreshortening**: when object are farther away they appear Smaller because their Size decrease with distance in prospactive view.

Ex → পাহাড় দূরে গেলে অনেক ছোট সামনে আসলে বিশাল but Size Same থাকে.

## Vanashing point : Lines are not perfectly horizontal or vertical seem to meet at a distant point called vanashing point.

Ex – রেললাইন সোজা তাকালে অনেক দূরে দেখা যায় দুইটি লাইন, ১টা লাগে but parallel 2টাই থাকে

## View Confusion.

object behind your position get flipped upside down and backward when projected.

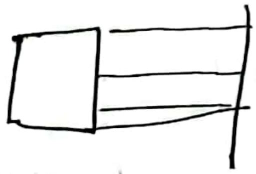
Exm → ছবি তুলতে গেলে যদি তার অনেক আমার পিছনে থাকে ছবির পড়ে আমার উপর

## Topological Distortion : Consider all points are parallel to view plane pass through the center of the point then there are broken line of infinite degree.

Ex → বিমান থেকে নিচে দেখলে আমরা রাস্তা দেখতে পাই, But Pie ও ভাগ ভেঙে যায় অসীম লাগে

## Parallel Projection:

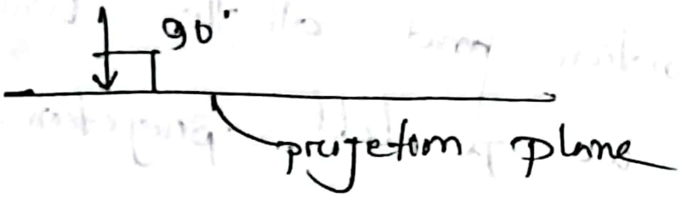
If center of the projetion is Located at infinite position and all the Lines are parallel. then it will we parallell projetion.



Difference between orthographic and oblique projection

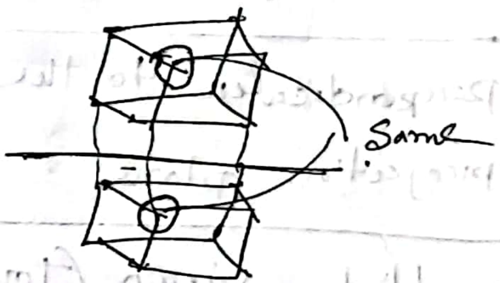
| Orthographic | Oblique |
| --- | --- |
| Perpendicular to the projection plane | At an angle to the projection plane |
| multiple views (Top, front side) | Single view with depth |
| Gives accurate ~~and~~ representation of the object | Less realistic due to angles |
| used in TW, blueprints | used sketches, architectural drawing |
| projection Lines from a 90° angle with projection plane | projection Lines from an angle commonly 45°, 30° with the projection line |

**Orthographic**: Direction of the projection is perpendicular (90°) to the projection plane




**Subcategories**

(i) **Isometric**: The direction of projection makes equal angle with all of the three principle axis (x, y, z), ($x = y = z$)




(ii) **Dimetric**: The direction of projection makes equal angle with exactly two of the principle axis.

$a = b \neq c$

(iii) **Trimetric**: The direction of projection makes unequal angles with three principal angles

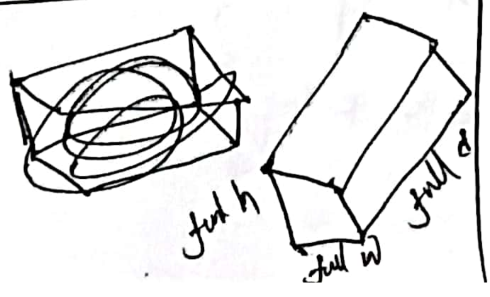
$a \neq b \neq c$

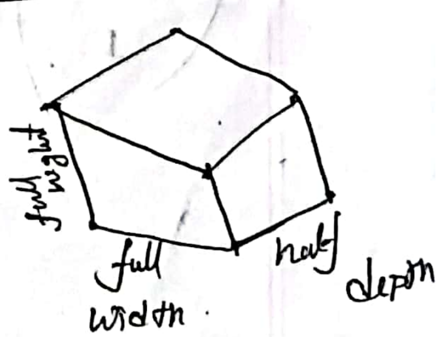
**Oblique:** The Direction of the projection is not perpendicular (নয়) to the projection plane

**Subcatagories:**

**Cavalier:** | **Cabinet**

| Cavalier | Cabinet |
|---|---|
| The direction of projection is choosen so that there is no foreshorting of lines perpendicular to my plane | projection is choosen so that lines perpendicular to the my plans are forshorted by their half Lengths (অর্ধ দৈর্ঘ্য) |
| Depth drawn full scale | Depth scaled to half |
| used in Two, Engineering Drawing | used furniture design, arcitectural sketchs. |
| used where measurement maintain accurate | used where visual realism is preferred |



## Mathematical Description of parallel projection

Perpendicular is in orthographic.

in 3D $(x,y,z)$ projected 2D $(x,y)$ so $z=0$

$$Par_k = \begin{cases} x' = x \\ y' = y \\ z' = 0 \end{cases}$$

Matrix

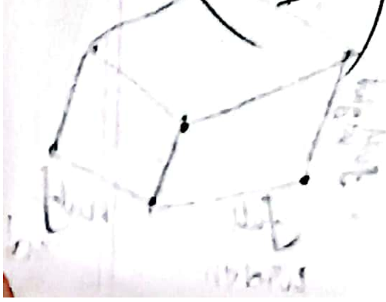
$$Par_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

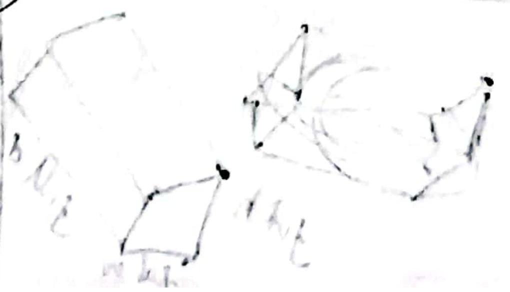
## Matical description of perspective projection

$P(x,y,z)$ 3D projected 2D $z=d$ when $d$ is distance of COP

$$x' = \frac{x \cdot d}{z}, \quad y = \frac{y \cdot d}{z}$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{pmatrix}$$





Math

7.

A unit cube is projected onto xy plane. x,y,z axes position. Draw projected img using standard perspective transformation with d = 1, b = 10 (d) distance of view plane.

Ans unit Cubes in homogeneous coordinates.

Cube সব নাম্বার বলে

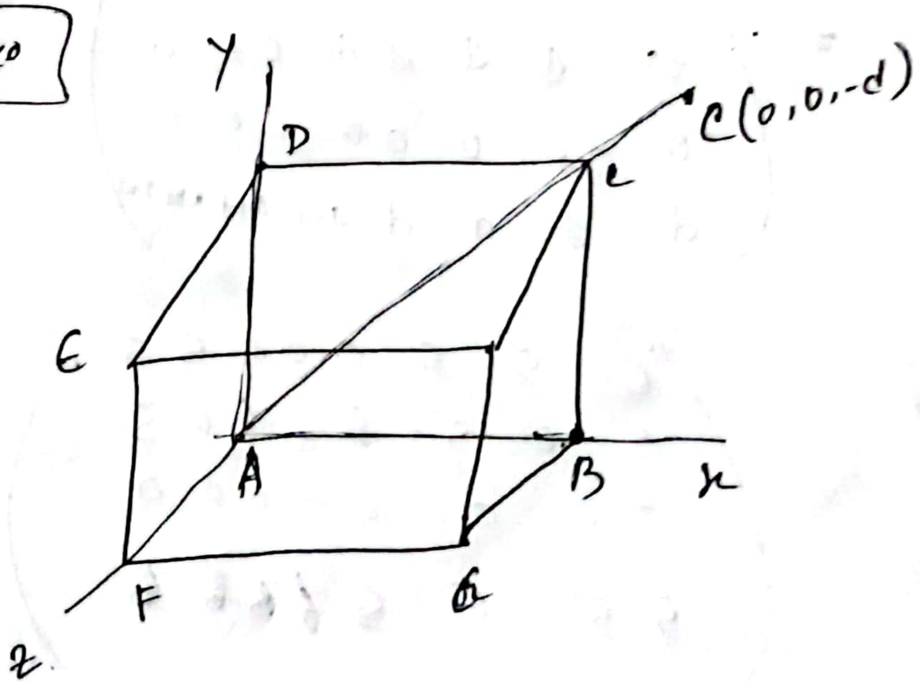
$$V = \begin{pmatrix} A & B & C & D & E & F & G & H \end{pmatrix} = \begin{array}{c} \\ x \\ y \\ z \\ 1 \end{array} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Origin, x অক্ষে, x,y অক্ষে, So x,y = 1 1, z = 0

তাহলে A 2'র মান 0

Same way

চিত্র আগে আঁকতে



## উত্তর

Standard perspective

$$Per_k = \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{pmatrix}$$

$d = 5$ হলে, Per K $\otimes$ V গুণ হবে

আগের page থেকে.

$$Per_K \times V = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d \end{pmatrix}$$

$$= \begin{pmatrix} 0 & d & d & 0 & 0 & 0 & d & d \\ 0 & 0 & d & d & d & 0 & 0 & d \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ d & d & d & d & 1+d & 1+d & 1+d & 1+d \end{pmatrix}$$

$$d = 5 \quad = \begin{pmatrix} 0 & 5 & 5 & 0 & 0 & 0 & 5 & 5 \\ 0 & 0 & 5 & 5 & 5 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 5 & 5 & 5 & 6 & 6 & 6 & 6 \end{pmatrix}$$

we know

$$x' = \frac{d \cdot x}{z+d} \qquad y' = \frac{d \cdot y}{z+d} \qquad z' = 0$$

অতএব যখন দিবে (1) লাইন দিয়ে ভাগ করে · Ans পাই যাই
$(d+1)$ দিয়ে

$A = (0, 0, 0)$ $\qquad E' = \left(0, \frac{5}{6}, 0\right)$

$B = (5, 0, 0)$ $\qquad F' = (0, 0, 0)$

$C = (5, 5, 0)$ $\qquad G' = \left(\frac{5}{6}, 0, 0\right)$

$D = (0, 5, 0)$ $\qquad H' = \left(\frac{5}{6}, \frac{5}{6}, 0\right)$

~~দিয়ে হবে Line~~

$G' = (5, 0, 0, 6)$

এখানে

$x' = \frac{d \cdot x}{z+d}$ $\qquad \frac{5}{1}$

Carry নিবে না বাচ্চা,

(1) → value গুলো (d+1) আছে last 4 টা

এইগুলোকে (d+1) মান দিয়ে ভাগ করবে।

Gx এর $(d+1) = 6$

$(x) = 5 = \frac{5}{6}$

## Describe

(a) one principal vanishing point perspective.

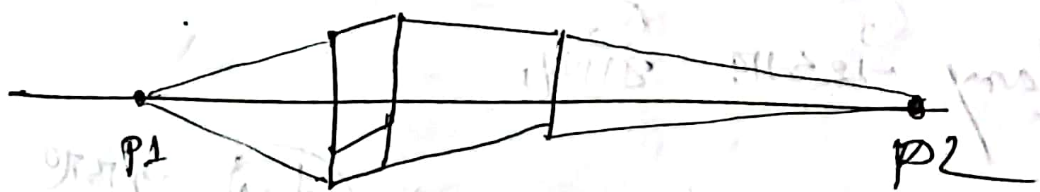
↳ when projection plane is perpendicular to one of the principal axes $(x, y, z)$.

$x = a$
$y = b$
$z = c + \frac{d}{n_3}$

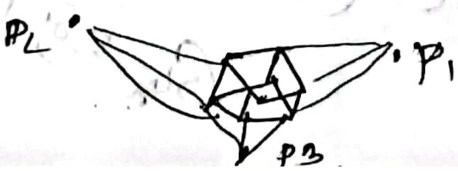
(b) 2 principal vanishing point perspective.

↳ Occours when the projection plane intersects exactly two of the principal axes

$$P_1 \begin{cases} x_1 = a + \frac{d}{n_1} \\ y_1 = b \\ z_1 = c \end{cases} \qquad P_2 = \begin{cases} x_2 = a \\ y_2 = b + \frac{d}{n_2} \\ z_2 = c. \end{cases}$$



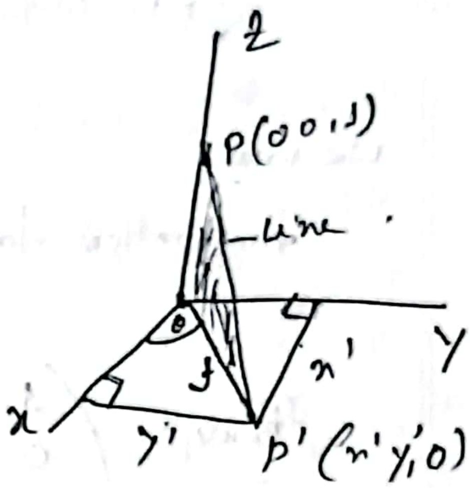

* 3 vanishing point intersect 3 all principal.



## Math 7.12

* find the general form of an oblique projection onto xy plane

here $x' = f\cos\theta$, & $y' = f\sin\theta$




$$Par_V = \begin{pmatrix} 1 & 0 & f\cos\theta & 0 \\ 0 & 1 & f\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

oblique projection tfm

## Math 7.13

find transformation.

a) Cavilere with $\theta = 45'$

b) Cabinet projection with $\theta = 30'$.

c) Draw projection of the unit cube for each transformation

(a) Cavalier is an oblique projection.

$f = 1$. with $\theta = 45^\circ$

we know:

mathmatical form oblique / cavilier is

$$Par V_1 = \begin{pmatrix} 1 & 0 & f\cos\theta & 0 \\ 0 & 1 & f\sin\theta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\left[ \begin{array}{l} \therefore f\cos\theta \\ 1 \times \cos 45^\circ \\ = \frac{\sqrt{2}}{2} \end{array} \right.$

$\left[ \begin{array}{l} f\sin\theta \\ 1 \sin 45 \end{array} \right.$

(b) Cabinet projection is an oblique.

here $f = \frac{1}{2}$ with $\theta = 30^\circ$

Same way (c) এর Par V আসবে

$\left| \begin{array}{l} f\cos\theta \\ \frac{1}{2} \cdot \cos 30^\circ \\ = \sqrt{3}/4 \end{array} \right.$

$$Par V_2 = \begin{pmatrix} 1 & 0 & \frac{\sqrt{3}}{4} & 0 \\ 1 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\left| \begin{array}{l} f\sin\theta \\ \frac{1}{2}\sin 30 \\ = \frac{1}{4} \end{array} \right.$

(c)

এখানে সবার প্রথমে cube draw করতে হবে transforma-
tion matrix (V) আগে Par v. Par v₁ দিয়ে

transformation matrix

$$V = \begin{array}{c|c} & \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{array}{c} x \\ y \\ z \\ w \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

$$Par\,v_1 \cdot V = \begin{array}{c|c} & \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{array}{c} x \\ y \\ z \\ \\ \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 & \frac{\sqrt{2}}{\sqrt{2}} & \frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} \\ 0 & 0 & 1 & 1 & 1+\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

(A B C D এর point ও একই)
এর same আছে just x এর g h (1+) হয়ে became
1 1 আছে

" " " " y ", E, H (1+) হয়ে bez
(1 0 0 1)
আছে

z এর 0 হয়ে যাবে (Par v₁ . V) তে became
(x,y) plane বলতে z = 0 .

[A B C D তে z = 0 এর point ও একই থাকে
e, f, g, h (1 1 1 1) so x,y (1) একই থাকে
(যোগ)]

here

$A' = (0\ 0\ 0)$ $\qquad E' = \left(\frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}, 0\right)$

$B' = (1, 0, 0)$ $\qquad F' = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$C' = (1, 1, 0)$ $\qquad G' = \left(1 + \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$D' = (0, 1, 0)$ $\qquad H' = \left(1 + \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}, 0\right)$.

Same cabinet point $\gamma$ - এ E, H ① হয়ে যাবে

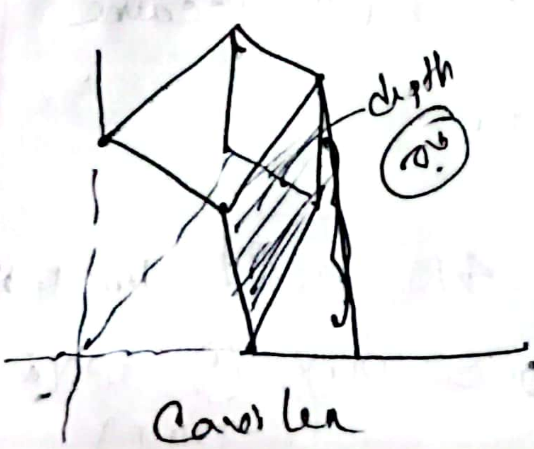
$$P_{an\,1/2} \cdot V = \begin{pmatrix} 0 & 1 & 1 & 0 & \frac{\sqrt{3}}{4} & \frac{\sqrt{3}}{4} & 1+\frac{\sqrt{3}}{4} & 1+\frac{\sqrt{3}}{4} \\ 0 & 0 & 1 & 1 & 1+\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 1+\frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$A = (0\ 0\ 0)$ $\qquad E' = \left(\frac{\sqrt{3}}{4}, 1+\frac{1}{4}, 0\right)$

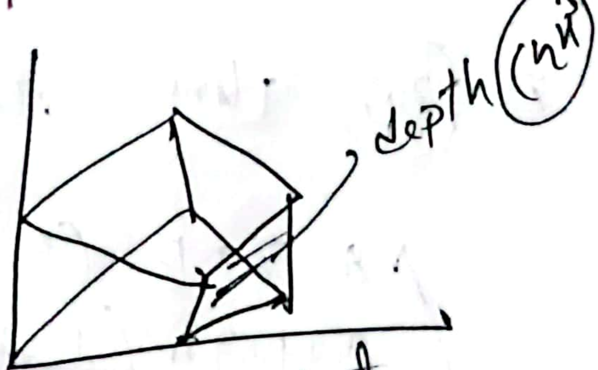
$B = (1, 0, 0)$ $\qquad F' = \left(\frac{\sqrt{3}}{4}, \frac{1}{4}, 0\right)$

$C = (1\ 1\ 0)$ $\qquad G' = \left(1+\frac{\sqrt{3}}{4}, \frac{1}{4}, 0\right)$

$D = (0, 1, 0)$ $\qquad H' = \left(1+\frac{\sqrt{3}}{4}, 1+\frac{1}{4}, 0\right)$




Cavalier




cabinet

## cp-2. 3D Graphics pipeline