Propositional Logic
& first order logic

Propotional Logic. - 2টা output (true or false) ← 2টা উত্তর [illegible]

. sentence are called . world from formulae.

- ¬ Negation (Today is friday.) = (¬T) — not
- ∧ Conjunction (Me and you) = $P \land Q$
- ∨ Disconjuction (me or you) = $P \lor Q$
- → if then (if rain, then road wet) $P \to Q$
- ↔ iff (I will go iff i do shopping) $(P \leftrightarrow Q)$

and — or (একটা সত্য হলে ok)

P	Q	$P \land Q$	$P \lor Q$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

Ex. You access the internet from campus only if you are CSE student or you are not freshman.

P = You access the internet from campus; Q = are CSE student; R = you are not freshman

$\Rightarrow (P \to Q \lor \neg R)$ / $(Q \lor \neg R) \to P$

* if — then

w = you get full mark

q = i'll give you taka

w	q	w → q
0	0	1
0	1	1
1	0	0
1	1	1

w true হলে always true

হবে bcz [illegible] দিবে

* $P \leftrightarrow Q$ (p if and only if q) = p iff q

P	Q	$P \leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

দুইটাই হলে না হই the 1

" " হই তাহলে 1

একটা 1 আর না হলে 0

Example: you can't ride the roller coaster if you are under 4 feet tall unless you are older than 16.

$\neg r$ = you can't ride the roller coaster; u = you are under 4 feet tall; o = you are older than 16

$(u \wedge \neg o) \rightarrow \neg r$

unless মানে

$\neg Q \rightarrow P$

Quntifires (2 রকম বিভাজিত) থাকে) আলোচিত হবে

- for all $\forall x \; P(x)$

  $\forall$ — all

  $P(x)$ — pradicates Sub (সমূহ)

  আকার বাচক / বললে universal

- $\forall \Rightarrow$ universal Quntifier

(Ex) Student x got good manus in Quiz = $\boxed{\forall x \; E(x)}$.

$\boxed{\forall x \text{ থাকলে } (\rightarrow) \text{ আসবে}}$

(ii) Existenial Quanitification

$\exists(x) \; P(x)$ = Existinal Qualification

(কিছু কিছু বোঝায় (someone)

$$\boxed{\begin{aligned} \neg \forall x \; Px &\equiv \exists x \neg P(x) \\ \neg \exists x \; Q(x) &\equiv \forall x \neg Q(x) \end{aligned}}$$

Not থাকলে ulte হবে।

Ex

for every person x if x student in the class then x has studied calculus

$\underbrace{\text{x student in the class}}_{S(x)}$ $\underbrace{\text{has studied calculus}}_{C(x)}$

$$\forall x \left(S(x) \rightarrow C(x) \right)$$

* Babies are illogical. $\Rightarrow \forall x \left(P(x) \rightarrow \neg Q(x) \right)$
* Nobody is despised who can manage a crocodile $= \left(\forall x \left(R(x) \rightarrow \neg S(x) \right) \right)$

$P(x)$ = baby
Qx = logical
$R(x)$ = crocodile
$S(x)$ = despised.

* Illogical person are despised

$\rightarrow \forall x \left(\neg Qx \rightarrow Sx \right)$

FOL

Syntax of FOL → term; predicate, function

term ↓ Sub इकाई ← Like Emp, x, y, z. Constant एक टपका

x (3,2)
funs — constant

predicate ↓ true 4 अथवा false

function: 2 basic Quantifiers.

$\forall x$, $\exists x$.
all — someone

all dogs are faithful
(x = dogs, y = faithful)

$\forall x\ (x) \rightarrow (y)$

Example of FOL (continued)

- some dogs break.

$\exists(x)\ (dog(x) \rightarrow break(x))$

- All dogs 4 Legs.

$\forall x\ (dog(x) \rightarrow have\ 4\ leg(x))$

- all happy people smile.

$\forall x$: $people(x) \wedge happy \rightarrow smile(x)$

- every student loves some student

$= \forall x\ (S(x) \rightarrow \exists(y)\ (stu(y) \wedge love(x, y)))$

- every man respect his parent.

$\forall x\ man(x) \rightarrow respect\ (x, parent)$

Example of PL

[1] There is a books that are harvey.

$= \exists x\, (\text{Book}(x) \wedge \text{heavy}(x)))$

at least 1 ta book

যেটার বুক এবং সেটি harvey

এবং (and) এর $\wedge$ ব্যবহার হবে

[2] Each student must be registered for a degree program

student কে registered করতে হবে degree

student = x, degree = y

$\forall x, y\, (\text{registered for}(x, y) \rightarrow \text{student}(x) \wedge \text{degree}(y).)$

another Way

Each student registered for at least one degree.

Each student = x, registered = y, at least one degree = y.

$\forall x\, (\text{student}(x)) \rightarrow \exists y\, (\text{registered for}(x, y)) \wedge \text{degree}(y)$

$\forall$ = Each

$\exists$ = at least

$\boxed{\begin{array}{l} F = 0 \\ T = 1 \end{array}}$

Truth table of Connectivites

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
F	F	T	F	F	T	T
F	T	T	F	T	T	F
T	F	F	F	T	F	F
T	T	F	T	T	T	T

Rules of inference

Implication : $P \to Q$

* Converse : $Q \to P$

Contrapositive : $\neg Q \to \neg P$

* Inverse : $\neg P \to \neg Q$

$T = 1$

$F = 0$

P true হলে Q false হলে false হবে

Q true হলে P false হলে false হবে

P	Q	¬P	¬Q	P → Q	Q → P	¬Q → ¬P	¬P → ¬Q
0	0	1	1	1	1	1	1
0	1	1	0	1	0	1	0
1	0	0	1	0	1	0	1
1	1	0	0	1	1	1	1

Q 1, P 0 হলে 0 হবে.

এখন

$$P \to Q = \neg Q \to \neg P$$

$$Q \to P = \neg P - \neg Q$$

Types of inference

Modus ponens

$$\frac{(P \to Q,\ P)}{Q}$$

$(P \to Q, P)$ ← এগুলো same হলে

Q → Q same হবে।

P	Q	P→Q
0	0	1
0	1	1
1	0	0
1	1	1 — এটা $\boxed{\text{ধরেছ}}$

Modus tollens

$$\frac{P \to Q,\ \neg Q}{\neg P}$$

এগুলো Q same হলে

$\neg P$ same হবে

③ hypothetical Syllogism

$$\frac{P \to Q \quad Q \to R}{P \to R}$$

same হবে।

④ Disjunctive

$$\frac{P \vee Q,\ \neg P}{Q}$$

Inference in FOL

It's used to deduce new facts from existing facts.
অনুমান

Substitution: performed in terms and formulla.

$F(a/x)$ হলো substitute a constant in place of variable 'x'

Equality: 2টা same object হলে

Brother (John) = Smith / simillar 2টা কে বোঝাতে "=" used here

FOL inference rules for Quantifiers.

Universal Generalization: $\dfrac{P(c)}{\forall x\,(P(x))}$

$P(c)$ true হলে $\forall x\,(P(x))$ true হবে।

যদি element some property হয় তবে সব এর হবে

2. universal instantiation : universal elimation বলে new sentence add করে

$$\frac{\forall x \; P(x)}{P c}$$

Ex: every person Like icecream — $\forall x \; (P(x))$
থেকে লিখতে পারি
Jhon Lives icecream ⇒ P(c).

3. Existential instantiation : (Ex. elimation.)
থেকে replace হবে

$$\frac{\exists x \; P(x)}{P(C)}$$ —— P(c) true হবে, c new term হতে হবে.

④ Existential introduction : $$\frac{P(C)}{\exists x \; P(x)}$$

Ex - ~~Progyal~~ ~~Enon~~ Mili got good markes.
→ Some one got good marks.
এভাবে convert হবে.

Resolution :

Example

1. All people who are graduating are happy.

2. All happy people smile.

3. Some one is graduating.

Convert into FOL : — (Q1)

1. $\forall x \,(Graduating(x) \rightarrow happy(x))$

2. $\forall (x)\; happy(x) \rightarrow Smile(x)$

3. $\exists (x)\; Graduating(x)$

Prove "someone smiling" : $(\exists x\; smiling(x))$ — (4)

Prove করলে Negative করে

so $[\neg \exists (x)\; smiling]$

Convert FOL to CNF (Qu2).

Step 1 : Eliminate Implication $(\alpha \rightarrow \beta \Rightarrow \neg \alpha \vee \beta)$ rules

- $\forall x\, [\neg Graduting(x) \vee happy(x)]$
- $\forall x\, [\neg happy(x) \vee Smile(x)]$
- $\exists x\; Graduating(x)$
- $\neg \exists x\; \cancel{Graduating}\; Smile(x)$

Step 2 Standardize variable ← 1 টা sentence এর 1 টা variable x, y, z এভাবে

1. $\forall x\ [\neg\ Graduating(x) \vee happy(x)]$

2. $\forall y\ [\neg\ happy(y) \vee smile(y)]$

3. $\exists z\ Graduating(z)$

4. $\neg\exists w\ smile(w)$

Step 3 move negation In words

এখানে 1, 2, 3 তাইলে কোনো Negation নাই So only 4 No. টা change হয়ে গিয়েছে আমাদের.

1, 2, 3 — as it is

4. $\forall w\ \neg smile(w)$ ← Negation থাকলে $\exists x$ থাকলে $\forall x$

$\forall x$ " $\exists x$,

এবং, ¬ আসিবেনা হবে না. object এর সাথে

Skolemization.

1 এখানে ∃ এর পরে যত (x), (y), (z) ← variable কে A, B, C তে Convert করে দিবা

1, 2, 4 = এগুলি তো হবে

3. Graduating (A) ← ($\exists x$) টাকে বাদ (x) → A তে convert

Drop universal Quantifier steps

1. ¬ Graduating (x) ∨ happy (x).
2. ¬ happy (y) ∨ smile (y)
3. Graduating (A)
4. ¬ smile (w)

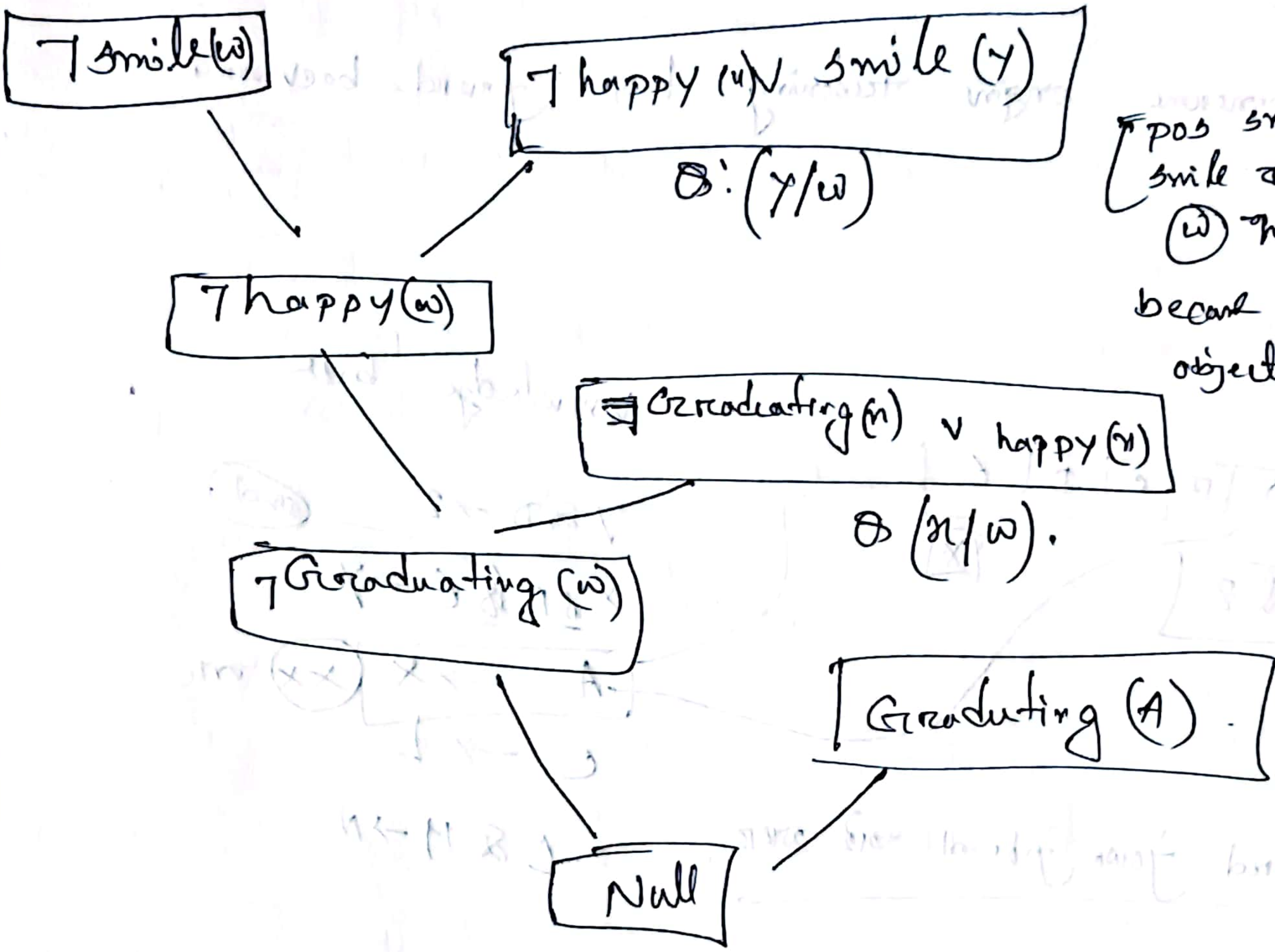
Resolution tree

* Prove করতে হবে (¬) দিয়ে শুরু করা। Ex F Prove করতে হলে শুরু করতে হবে ¬P

* Null clause আসলে prove.

Prove smile (w) → শুরু করো ¬ smile (w)

<u>Prove</u> → smile প্রুভ করতে হবে



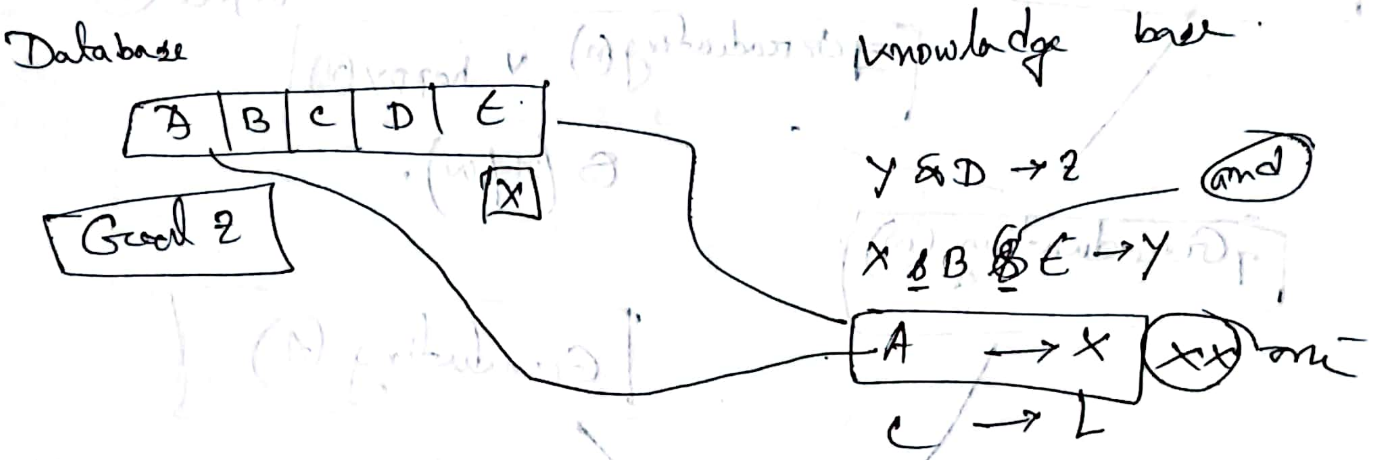
pos smile, neg smile বাদে
(ω) happy হয়েছে
because main object (ω).

So Null আসছে

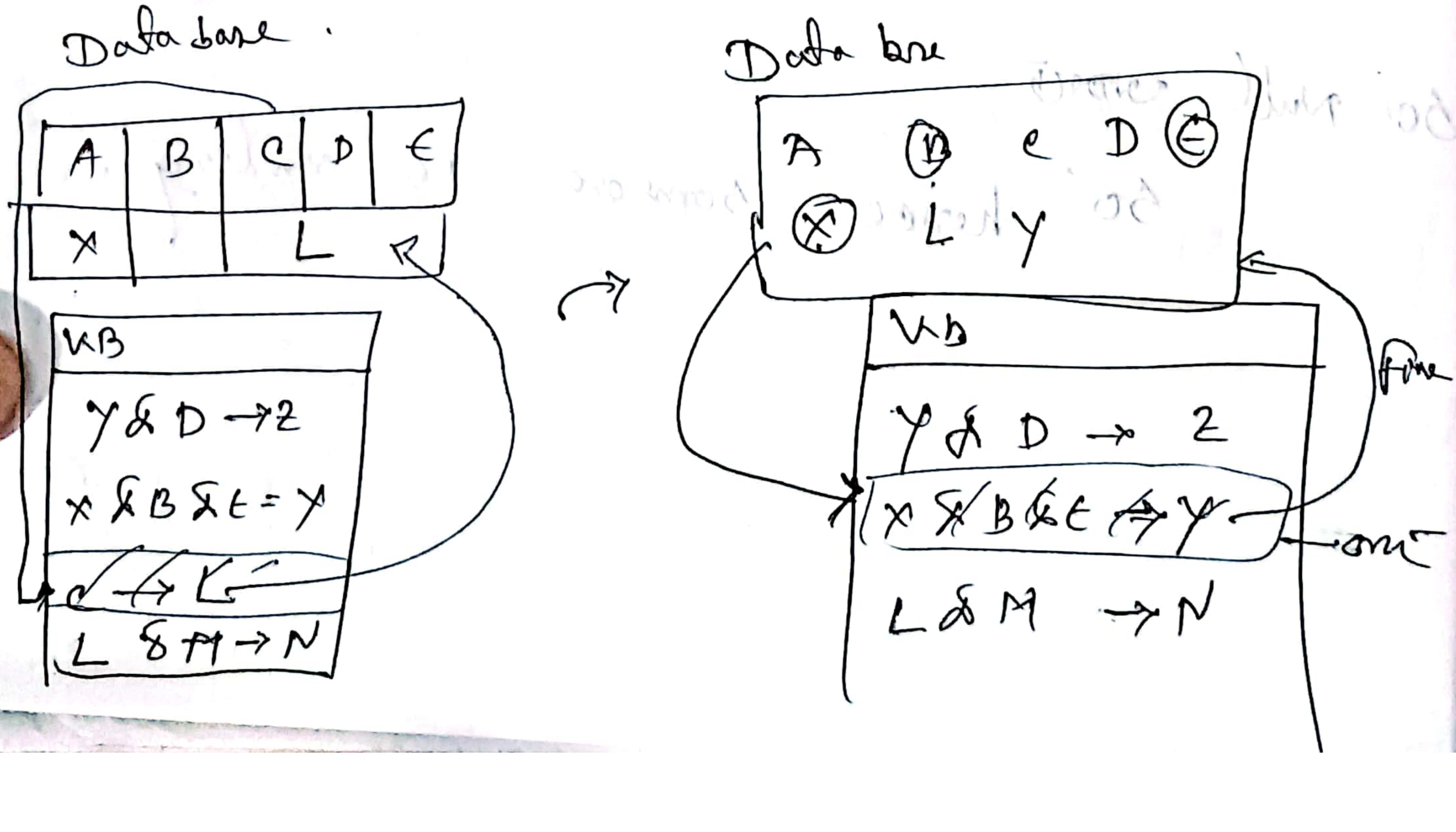
So hence someone is smiling.

Forward chaining

Infernerence engine reasoning তের forward, backword Chaining



backward jowa jabe না তাই আবার



Database

A	B	C	D	E
X	L	Y	Z	

Z পাইছে গেছি

KB
Y & B → Z
L & M → N

Backword Chain

A B C D E.

Z Y X

Goal = Z

KB

Y & D → Z

X & B & E → Y

A → X

C → L

L & M → N

এখানে Back যাওয়ার মানে 50 check করলাম এ Y point fire হয় না A → X database এ Y put করার পরে 50 Y, D X put করলাম

$\boxed{X, B, E}$ → Y put করার পরে Z পাওয়ার পরে

যেমন Match করলে Back যাওয়া যায়

Example

1. It is a crime for an $\underset{x}{\underline{\text{American}}}$ to $\underset{z}{\underline{\text{sell}}}$ $\underset{y}{\underline{\text{weapon}}}$. to the enemy of $\underset{z}{\underline{\text{amirica}}}$.

2. Country NONO is an enemy of amirica

3. NO NO has some missile.

4. all the mesille are sold to Nono by Colonel.

5. Missile is a ~~no~~ weaphon.

6. Connel is ~~a~~ amarican

[Prove is " Colonel is a criminal "]

Ans

Convert 1-4 into FOL

1. Aamarican (x) ∧ weapon(y) ∧ ~~Seller~~ enimy (z, america) ∧ Sells (x, y, z) ⇒ Criminal (x)

enemy of amirica.

2. Enemy (z, amirica)

Nono

3. own (nono, x)

Missile (x)

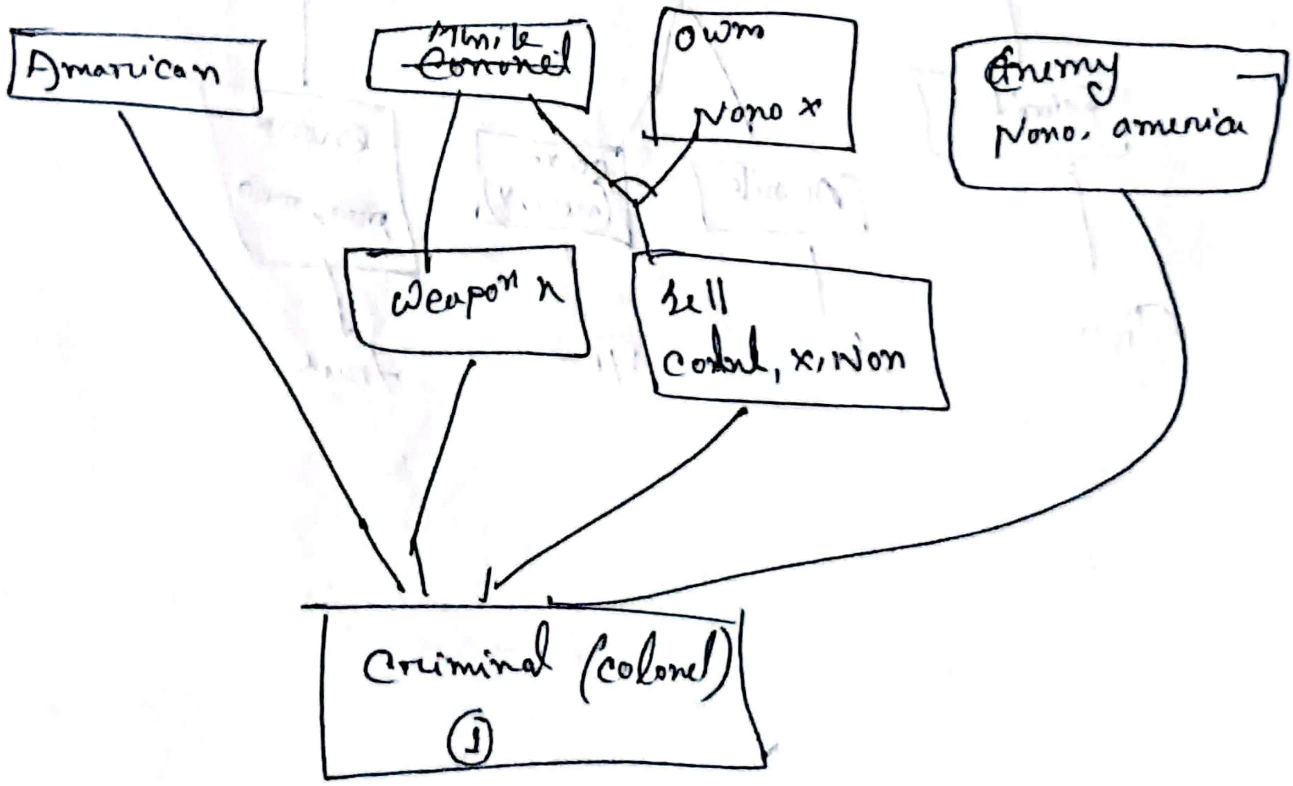
4 $\forall x$ Missile (x) $\wedge$ own (nono, x) $\Rightarrow$ sell (Conal, x, Nono)

5 Missile (x) $\Rightarrow$ weapon (x)

6 american (Colonel)

forward

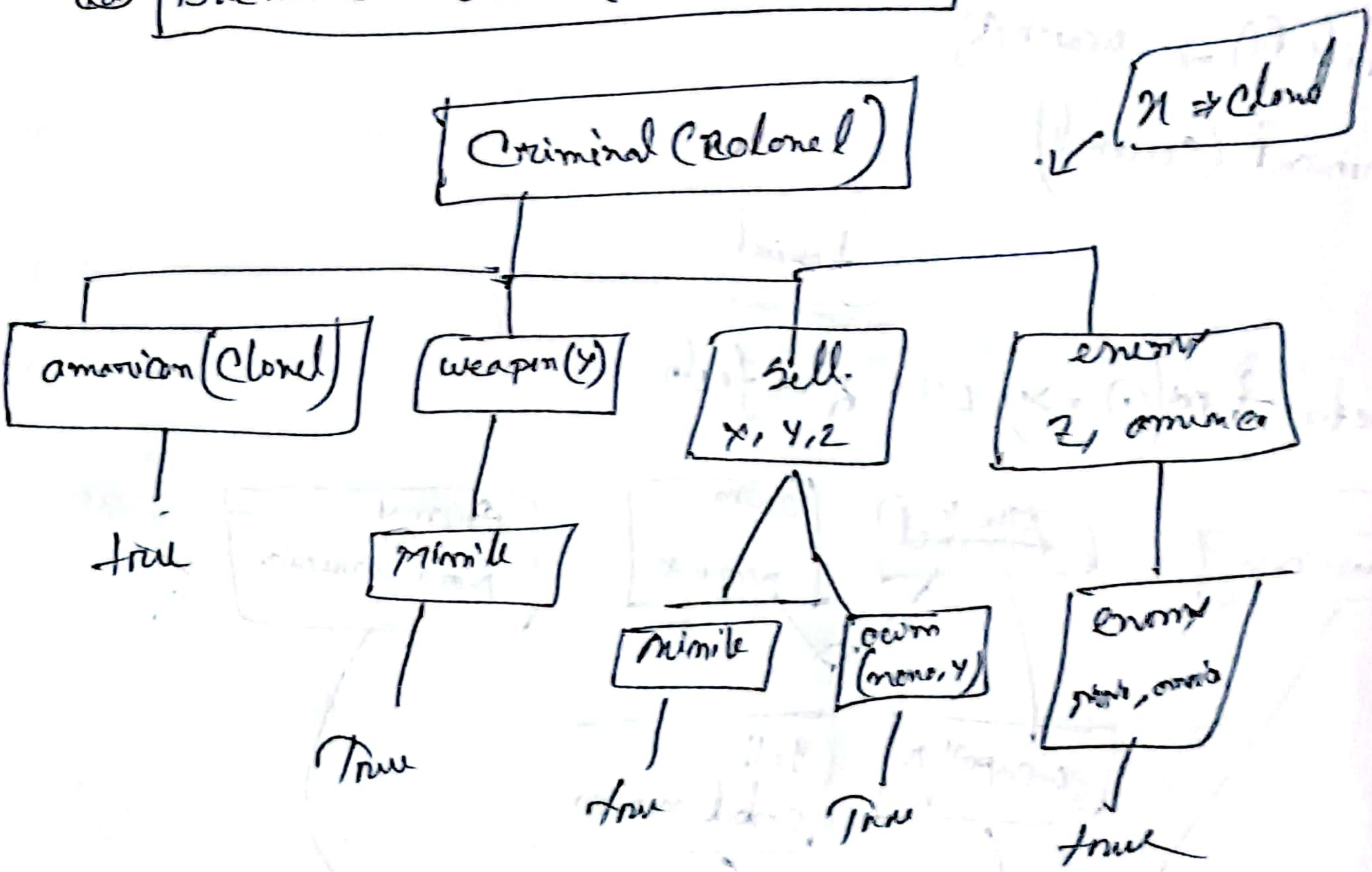
facts ... (1) $\Rightarrow$ LHS & init facts



Backward

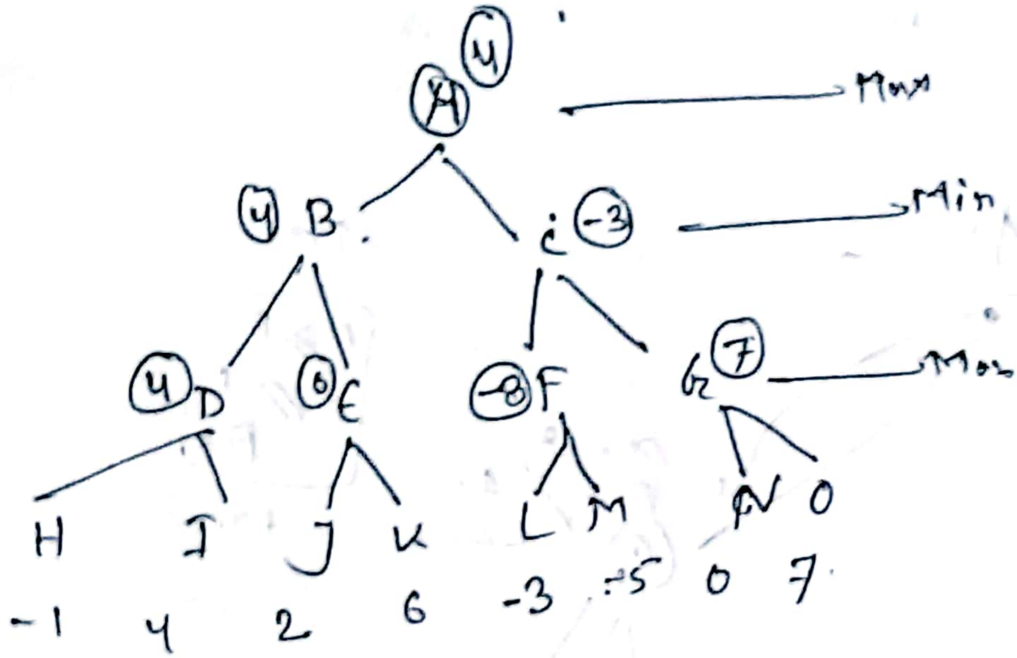
Previous exam by

Backward Goal থেকে St ধরে

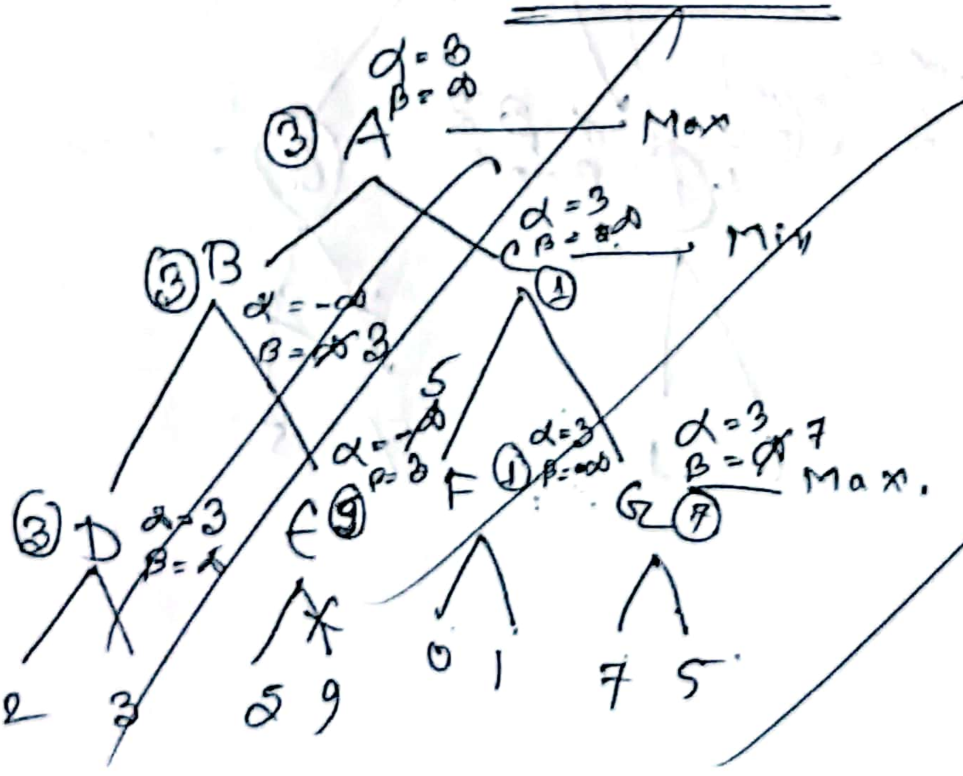


Game playing

Minimax



ALpa beta -Depth for sean



alpha = Max = $-\infty$

Beta = Min = $-\infty$

purning

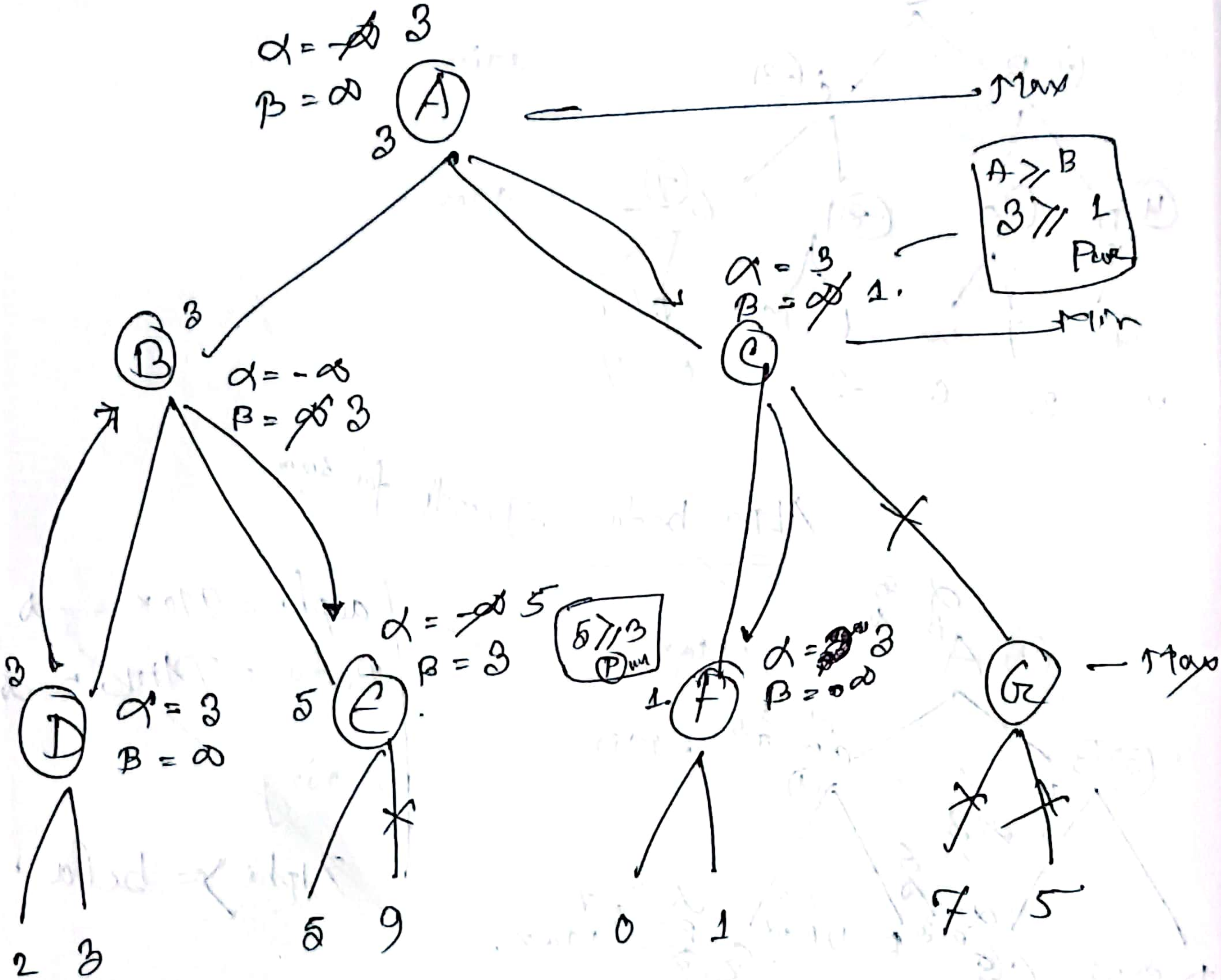
Alpha >= beta

Alpha beta purning

alpha = Max = $-\infty$

Beta = Min = ∞

alpha $\geq$ beta = Purning

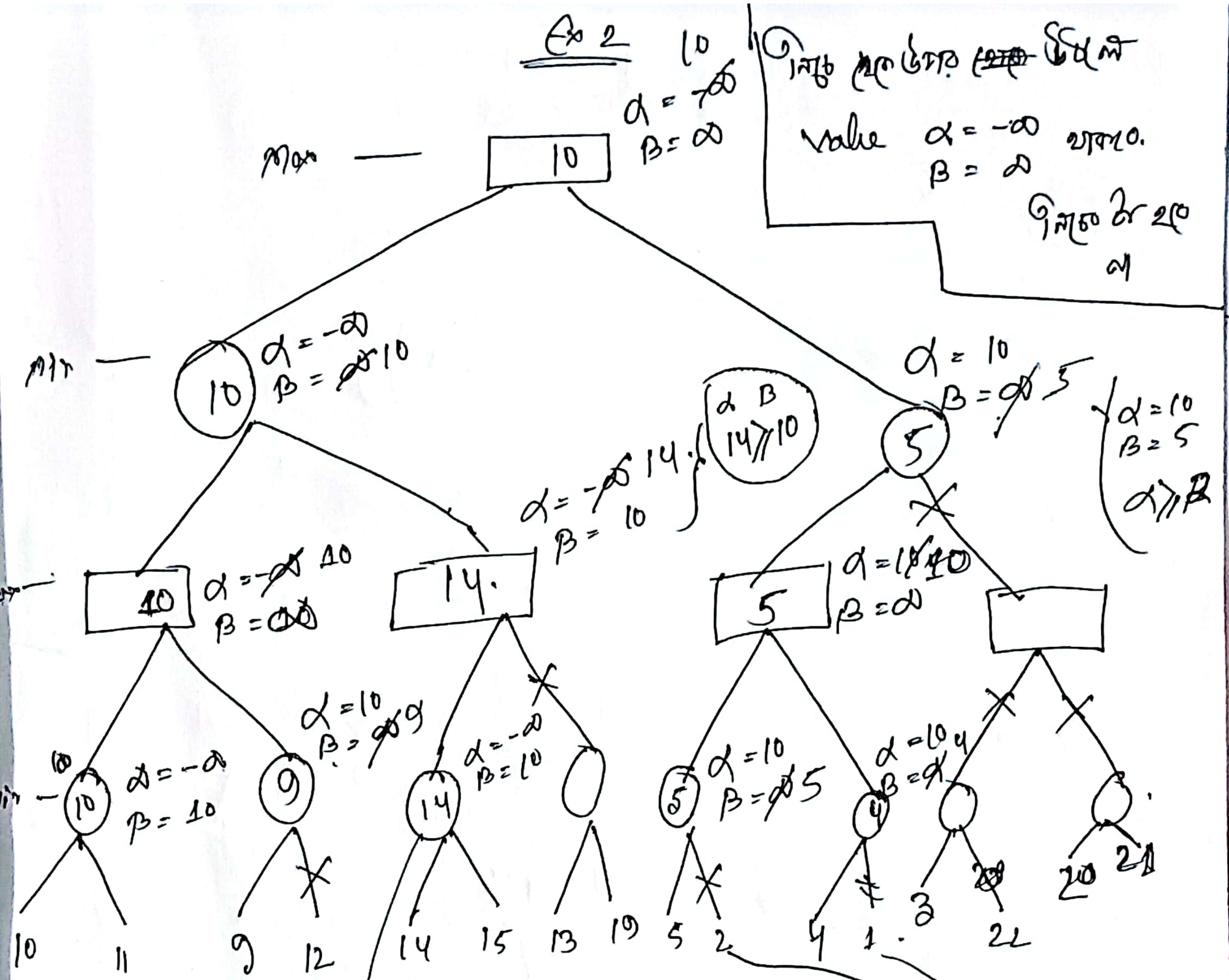


Ex 2

নিচে থেকে উপরে ~~[illegible]~~ উঠবে

value $\alpha = -\infty$ থাকবে.
$\beta = \infty$

নিচে দিবে না



অর্থাৎ (10, 14, 15) ছাড়া সব

10 থেকে 50 point 10 থেকে যাবে

Same case

এটার tread এ কারণ

নাই 30 টা Min.

2 দিলে

$\alpha = 5\quad \beta = 2$ $\quad \alpha \geq \beta$

Probabilistic Reasoning

Bayes' theorem ← concept of probability

[Describe probabiliability of occurance of an event related any condition"

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

A, B = events

$P(A|B)$ = probability of A/B

$P(B/A)$ = " " B/A

$P(A), P(B)$ = indipendent probability

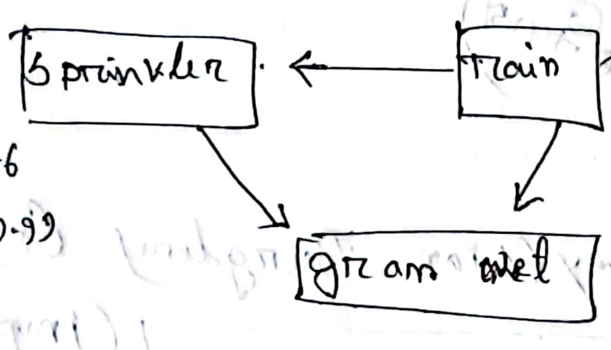
[illegible] $= \frac{P(A \cap B)}{P(B)}$

Bayesian Network

* DAG হতে হবে,
* cycle হতে পারবে না

Ex

Rain	S	~S
F	0.4	0.6
T	0.01	0.99



Dependent না

Rain	
R	~R
0.2	0.8

S	R	W
$\sim S$	$\sim R$	0.0
$\sim S$	R	0.8
S	$\sim R$	0.9
S	R	0.99

Prove that

$P(W \cap S \cap R)$

We know

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore P(A \cap B) = P(A/B)\, P(B)$$

$P(W \cap S \cap R)$

Depend করে S,R এর

$= P(W/S,R) \times P(S/R) \times P(R)$

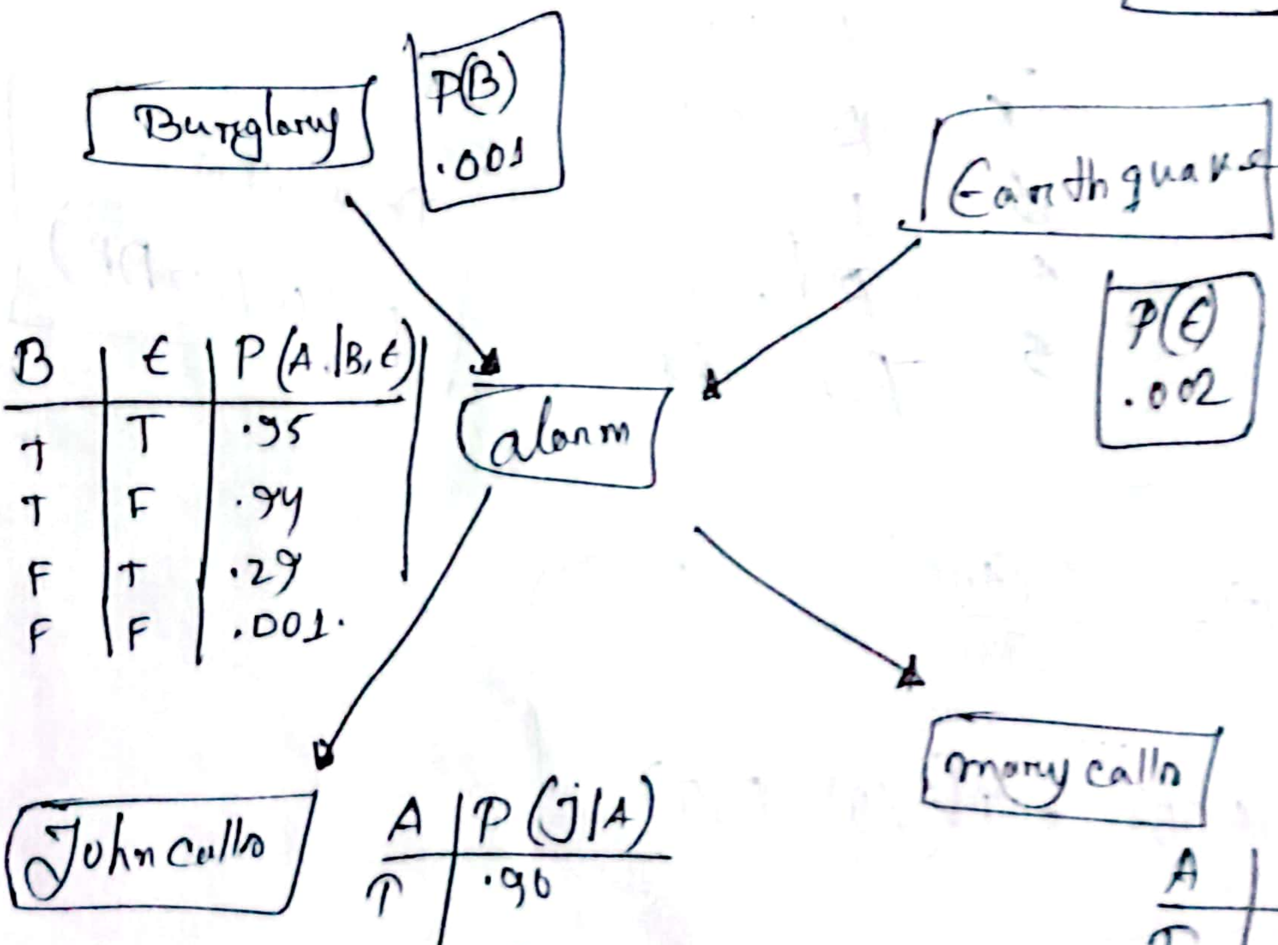
dep্d করে Rain এ

$= 0.99 \times 0.01 \times 0.2$

$= 3$

(No [illegible] করে [illegible] করে 1 এর উপর)

Bayesian Burglary Ex (imp)



P(B)
.001

P(E)
.002

B	E	P(A | B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.50

A	P(M|A)
T	.70
F	.01

Probability alarm sound but neither burglary nor an.

(e) has occured and both jon, Mary call.

$\Rightarrow P(J \wedge M \wedge a \wedge \neg b \wedge \neg e)$ Not e and b ঘটেনি

$= P(J/A) \wedge P(M/A) \wedge P(a/\neg b, \neg e) \cdot \underbrace{P(\neg b)\,P(\neg e)}_{\text{Depends করে না}}$

$= (.90 \times .70 \times 0.001 \times 0.99 \times 0.998)$

$= 0.00062.$

~~Probability that John call?~~ a burglary given that John and Mary call

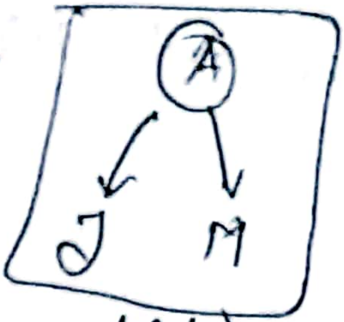
Conditional probability

$P(A \cap B) = P(A/B)\,P(B)$

$P(A \wedge B) = P(B/A) \times P(A)$

$P(B/A) = \dfrac{P(A/B) * P(B)}{P(A)}$

Joint probability Distribution.



~~P(…)~~ …

$P(A, J, M) = P(A) * P(J/A) * P(M/A)$.

* ~~Prob. jhon calls~~

$\Rightarrow P(J)$

DS theory (evidence theory)

It combines all possible outcomes of the problem.

Ex:

	ω_i	Belief $B_{1,i}$	$B_{2,i}$	$B_{3,i}$
Accuracy	.35	0.4	0.5	0
reputation	.65	0.1	0.75	0.15

$\tilde{M}_{1,1} = \omega_1 (1 - \sum B_{n,1})$

$= 0.35(1 - (0.5 + 0.4 + 0)$

$= 0.035$

$M_{1,2} = \omega_2 (1 - \sum B_{n,1})$

Calculate belief and probability mass.

Grade, weight, Belief, probability mass

Grade	ω_i	B_1	B_2	B_3	BH	$M_{1,i}$	$M_{2,i}$	$M_{3,i}$	$M_{H,i}$	$\bar{M}_{H,i}$	$\tilde{M}_{H,i}$
Accuracy	0.35	0.4	0.5	0	0.1	0.14	0.175	0	0.685	0.65	0.035
reputation	0.65	0.1	0.75	0.15	0	0.065	0.487	0.0975	0.35	0.35	0

accuracy and reputation

BH

$(1 - (B_1 + B_2 + B_3))$

$= 1 - 0.4 + 0.5 + 0$

$= 1 - 0.9 = 0.1$

$m_{1,1} = \omega_i \times B_1$

$= 0.35 \times 0.4$

$= 0.14$

Same sol.

ω আলাদা হবে

$M_{H,i}$

$= \tilde{M}_{H,i} + \bar{M}_{,i}$

$= 0.065 + 0.35$

$= 0.685$

$\bar{M}_{H,i}$

$= 1 - \omega_1$

$= 1 - 0.35 = 0.65$

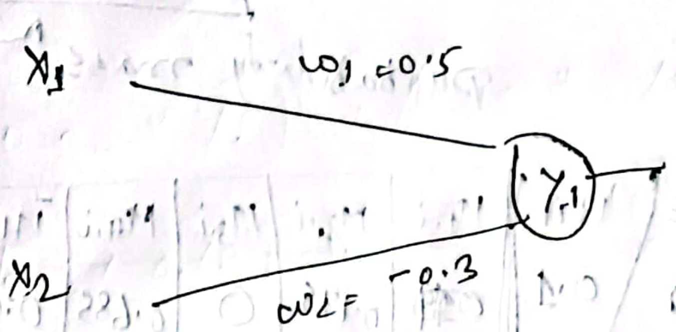
$1 - \omega_2$

$1 - 0.65 = 0.35$

Artificial Neural Network

Components

- Neuron (nodes) — (value রাখে)
- Layers — input Layer, hiddin Layer
- Connection — Layer গুলোকে এক connect
- Activation fun — 0,1 try করে
  — transform function



$b = 0.1$

$x_1 = 2, 8, 0$

$x_2 = 8, 25$

$Y = x_1 w_1 + x_2 w_2 + b$

activation জন্য sigmoid use for 0,1 আনা যায়

$Y = \frac{1}{1+e^{-Y}}$ Y মানে বের করেছো

$Y = x_1 w_1 + x_2 w_2 + b$

$= 2 \times .5 + 8 \times -.3 + 0.1$

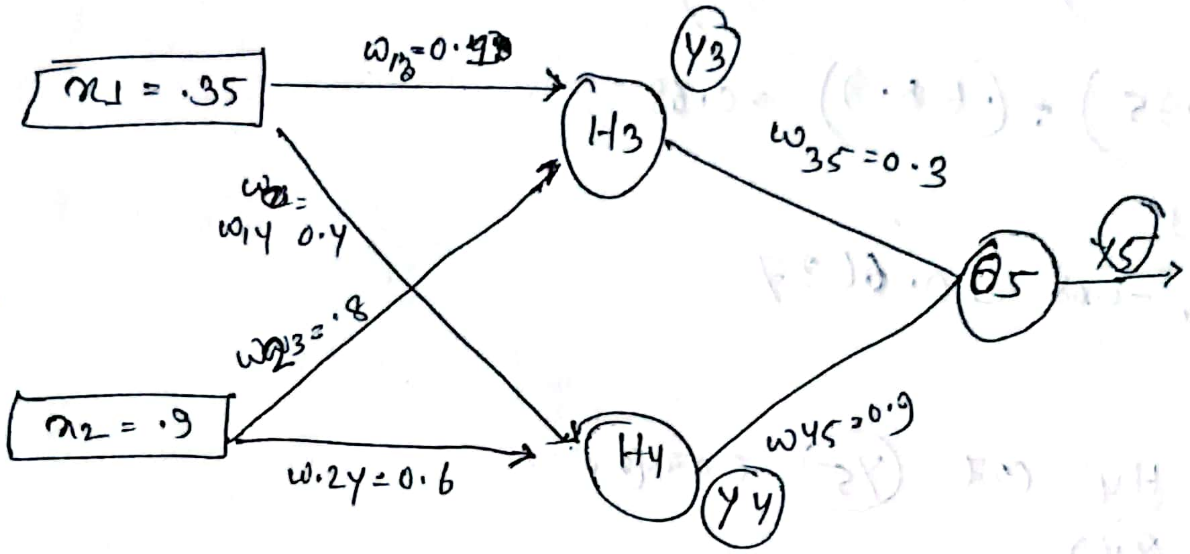
$= -1.3$

$Y = \frac{1}{1+e^{-Y}} = \frac{1}{1+e^{+1.3}}$

$= .2$

ANN (MultiLayer) example

Back propagation



find y_3, y_4, y_5 (forward pass)

$$a_j = \sum w_i * x_i$$

$$= (w_{13} * x_1) + (x_2 * w_{23})$$

$$= (0.1 * .35) + (.9 * .8)$$

$$= 0.755$$

এখানে Y_3 বাহির করতে হবে
Y_3 কে x_1 x_2 Parent হবে.
তাদের হবে তাদের (W) ধরে

used transfer/activitation.

$$a\ y_j = \frac{1}{1 + e^{-a}}$$

$$= \frac{1}{1 + e^{-0.755}} = 0.68$$

H4

$a_2 = (w_{14} * x_1) + (w_{24} * x_2)$

$= (0.4 * .35) + (.6 * .9) = 0.68$

$$Y_4 = \frac{1}{1 + e^{-0.68}} = 0.6637.$$

এবার H_3, H_4 থেকে Y_5 করবো।
Y_3 Y_4

$a_3 = (w_{35} * Y_3) + (w_{45} * Y_4)$

$= (0.3 * 0.68) + (0.9 * 0.66) = 0.801.$

$Y_5 = 1/1 + e^{-0.801} = 0.69$ (Network Output)

(Output মান দিতে expect করে 0.5)

So error

$Y_{target} - Y_5$

$= 0.50 - 0.69 = (-0.19)$