Segment 4

Computer graphics

2D viewing & clipping

WCS

objects are palaced into the sence by modeling tranformation to a master Condinate Called world Coordinate System

NDCS

Monitor sizes differ from one system to another. Introduce a device independent tool to describe the display area tools called Normalized Device Coordinate System.

Viewpoint mapping & on Normalization transformation

Process of Converts object coordinates in WCS to NDCS is viewpoint mapping

Window to viewport Mapping

window specified WCS $W_{xmin}, W_{xmax}, W_{ymin}, W_{ymax}$.

Viewport " " V_{xmin} V_{xmax}, V_{ymin} V_{ymax}.

$$\frac{W_x - W_{xmin}}{W_{xmax} - W_{xmin}} = \frac{V_x - V_{xmin}}{V_{xmax} - V_{xmin}}$$

find v_x

$$= V_x = \frac{V_{xmax} - V_{xmin}}{W_{xmax} - W_{xmin}} (W_x - W_{xmin}) + V_{xmin}.$$

S_x = $S_x = \frac{V_{xmax} - V_{xmin}}{W_{xmax} - W_{xmin}}$.

$$S_y = \frac{V_{ymax} - V_{ymin}}{W_{ymax} - W_{ymin}}.$$

Math

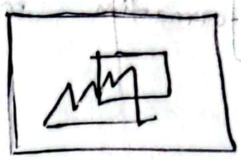
for window $X_{wmin} = 20$ $X_{wmax} = 80$, $Y_{min} = 40$, Y_{max} 80

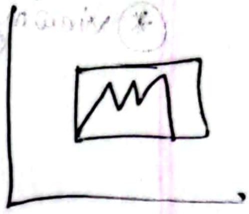
" Viewport " $= 30$ " $= 60$ " $= 40$ " $= 60$

$(\cancel{W_x, W_y = 30, 80})$ ans - ? $(W_x, W_y = 30, 80)$

$$S_x = \frac{V_{max} - V_{min}}{W_{max} - W_{min}}$$

$$= \frac{30 - 60}{80 - 20} = \frac{1}{2}$$





$$V_x = S_x \cdot (W_x - W_{xmin}) + V_{xmin}$$

$$= \frac{1}{2} \cdot (30 - 20) + 30 = 35$$

তিন matrix হবে:

$$N = \begin{pmatrix} 1 & 0 & V_{xmin} \\ 0 & 1 & V_{ymin} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -W_{xmin} \\ 0 & 1 & -W_{ymin} \\ 0 & 0 & 1 \end{pmatrix}$$

এই গুলো মান Put করে

$$= \begin{pmatrix} S_x & 0 & -S_x W_{xmin} + V_{xmin} \\ 0 & S_y & -S_y W_{ymin} + V_{ymin} \\ 0 & 0 & 1 \end{pmatrix}$$

(*) viewing tranformation mapped from WCS to Camra Coordinates.

(*) Viewing Coordinate System.

effect of moving & tilting Camera.

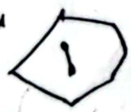
Clipping

It refers to removing objects, Lines, or line segment that are outside the viewing pane.

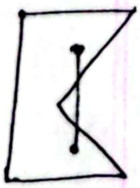
Line clipping → removing Lines, portions of Lines outside an area of interest

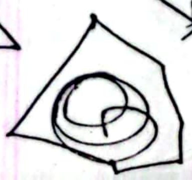
Polygon clipping → removed part of an outside a polygon.

Convex polygon. → polygon is convex, if Line joing any two interior points of the polygon lies completely inside the polygon.



Concave polygon. → outside the polygon Line joining any two interior points.





Line clipping Algorithm.

- Cohen Sutherland Line Clipping Algorithm.
- Midpoint Subdivision Line clipping Algorithm.
- Liang Barsky Line Clipping Algorithm

Cohen Sutherland

All lines fall into 3 categories.

Visible - Both endpoints of the line lie within the window

Not Visible: Line lies outside of window.

if $x_1, x_2 > x_{max}$ | $x_1, x_2 < x_{min}$.

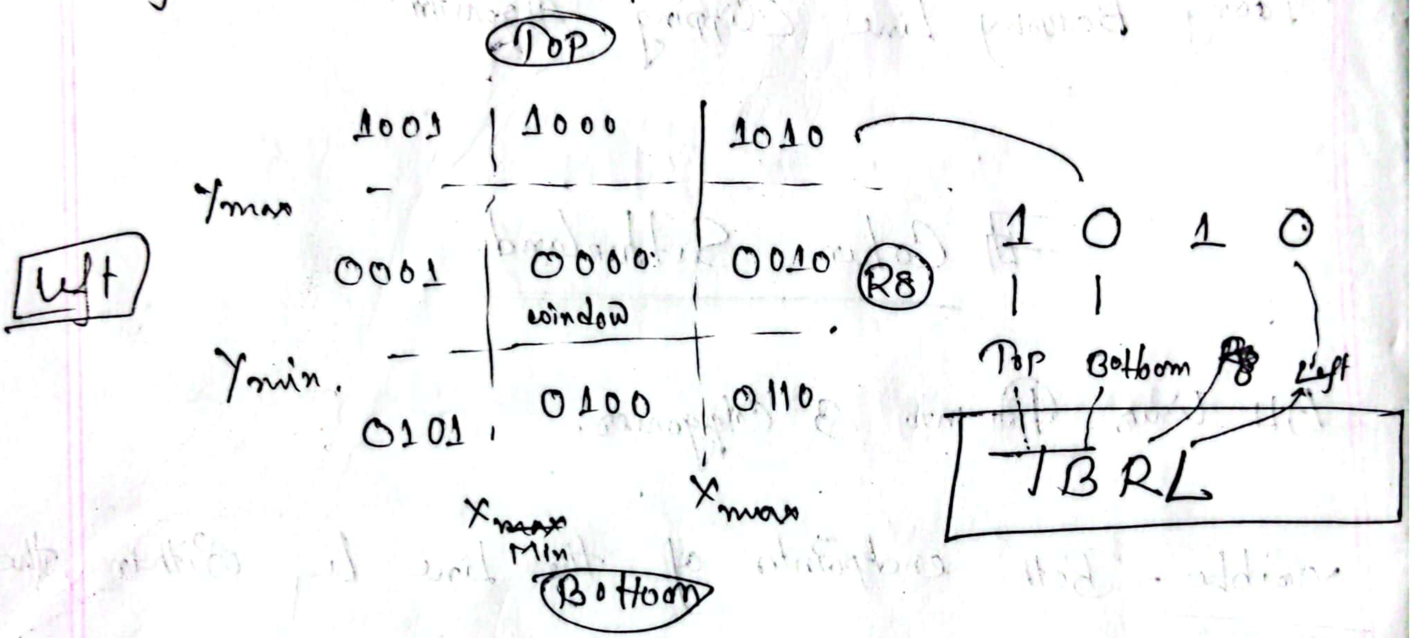
$y_1, y_2 > y_{max}$ | $y_1, y_2 < y_{min}$.

Clipping: Neither 1, 2 then it's Clipping.

(partially inside or outside)

For find Catagory used 2 step

(1) 4 bit region code determines which region of the plane endpoint lies in.



(2) if 0000 then visible.

if bitwise Logical ~~AND~~ OR of the codes 0000

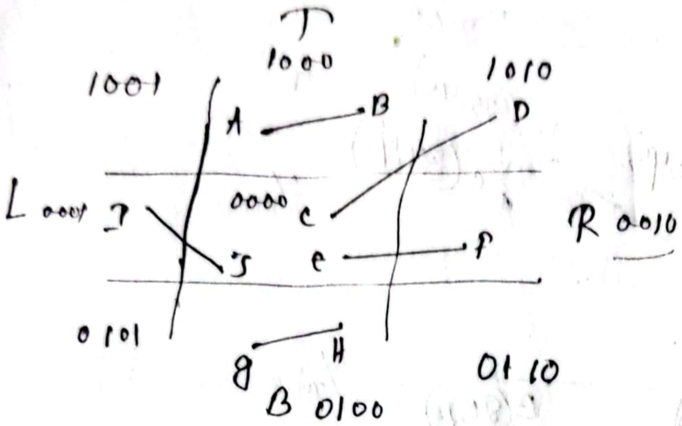
if bitwise Logical And of Codes ≠ 0000 হলে
then Not visible

if bitwise And 0000 the clipping Candidate.

কিছু widow কে ফিক্স করবো

Math

⑦



$\boxed{\text{TBRL}}$

Case1 = visible – 0000
Bite 2টাকে OR করে 0

② Not visible
Bit 2টাকে and করলে
≠0000 হলে

③ Line candidate
2 bit And করলে 0000 হলে

AB = 1000
1000

OR → 1000

And → 1000 ← completly outside

CD = 0000
1010

on → 1010 →

And → 0000 ← clipping condidet.

EF = 0000
0010

on → 0010

And → 0000 —— clipping Cond.

GH = 0100
0100

on 0100

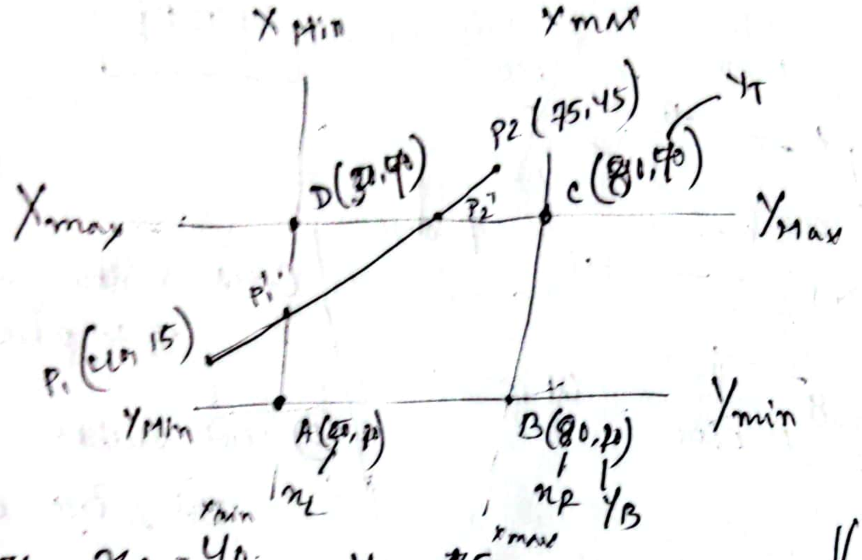
And 0100 ← Not Vasibl

IJ = 0001
0000

on 0001

And 0000 ← ~~not~~ Pantral vasible

Math 2



here $x_1 = 40$ $\quad y_1 = 15$

$x_2 = 75$ $\quad y_2 = 45$

$x_L = 50,\ x_R = 80,\ y_T = 40,\ y_B = 10$

here

$$
\begin{array}{r}
P_1 = 0001 \\
1000 \\
\hline
0000
\end{array}
$$

here

$$\text{Slop} = \frac{y_2 - y_1}{x_2 - x_1}$$

Window এ যা আছে
G find out করতে হবে

$$= \frac{45 - 15}{74 - 40} = 0.85 \simeq \frac{6}{7}$$

Left

$x_L = 50$, & $y = m \cdot (x_L - x_1) + y_1$

$= 0.85\,(50-40) + 13$

$= 23.57$

$x_R = 80$ $\quad y = m\,(x_R - x_1) + y_1$

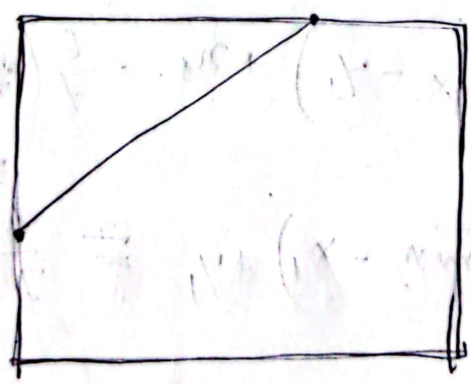
$= 69.16$

$y_T = 40$ $\quad x = \frac{1}{m}\,(y_T - y_1) + x_1$

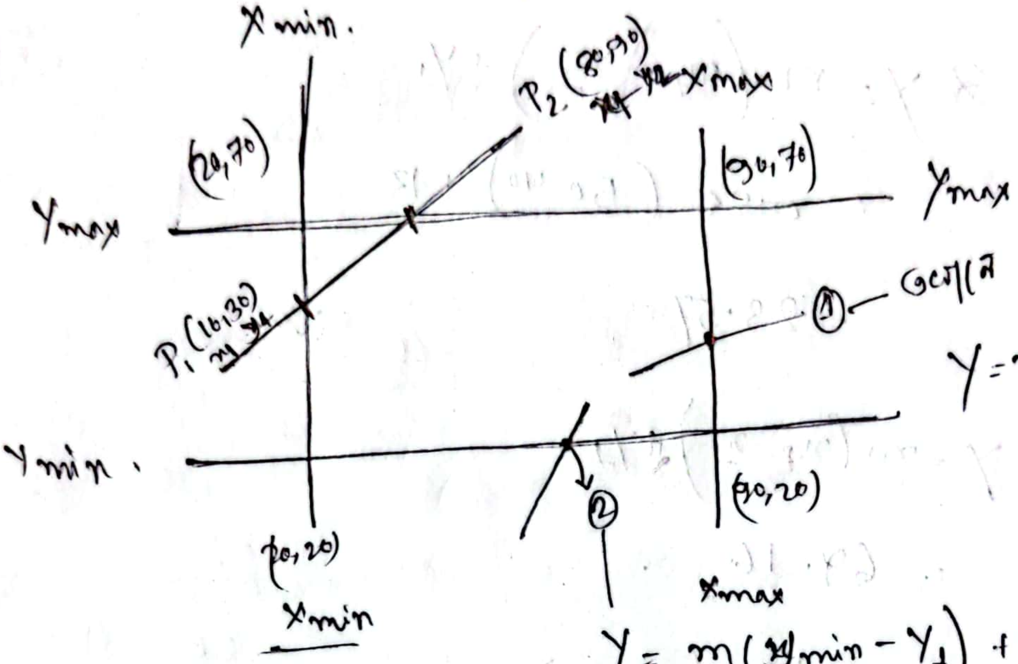
$= \frac{7}{6}\,(40-15) + 40 = 49.28$

$y_B = 10$ $\quad x = \frac{1}{m}\,(y_B - y_1) + x_1$

$= \frac{7}{6}\,(10-15) + 40 = \underline{34.16}$



tricks



যেখানে xmax intersect

$$y = m(x_{max} - x_1) + y_1$$

$$y = m(y_{min} - y_1) + x_1 .$$

যেটাকে intersect করবে তার উপর

$$m \{ \text{যেটাকে intersect} - x, y \text{ এর মান } x_1, y_1 \} + \text{তার offside}$$

here $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{60}{70} = \dfrac{6}{7}$.

here $y = m(y_{max} - y_1) + x_1 . = \frac{6}{7}(70 - 30) + 80$

$= 57$

$x = m(x_{min} - x_1) + y_1 = \frac{6}{7}(20 - 10) + 30$

$= 39.$

Math.

5.2

Left corner (1,1) upper right corner (3,5) onto

(a) Entire normalized device screen

$W_{x\,min} = 1$, $W_{x\,max} = 3$, $W_{y\,min}$ 1, $W_{y\,max} = 5$

$V_{x\,min} = 0$ $V_{x\,max} = 1$ $V_{y\,min} = 0$, $V_{y\,max} = 1$

↳ entire normalized তাহলে dist 01, 01

$$S_x = \frac{V_{x\,max} - V_{x\,min}}{W_{x\,max} - W_{x\,min}} = \frac{1-0}{3-1} = \frac{1}{2}$$

$$S_y = \frac{V_{y\,max} - V_{y\,min}}{W_{y\,max} - W_{y\,min}} = \frac{1-0}{5-1} = \frac{1}{4}$$

$$N = \begin{pmatrix} 1 & 0 & V_{x\,min} \\ 0 & 1 & V_{y\,min} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -W_{x\,min} \\ 0 & 1 & -W_{y\,min} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

CamScanner

Liang Barsky Line Clipping

$\boxed{4a} \rightarrow A\underset{x_1, y_1}{(2,3)}$ $B\underset{x_2, y_2}{(3,4)}$.

① $\Delta x, \Delta y$ মান বের করে নিতে হবে।

P_k ② then $P_1 = -\Delta x$, $P_2 = \Delta x$, $P_3 = -\Delta y$, $P_4 = \Delta y$.

q_k ③ $q_1 = x_1 - x_{wmin}$. $q_2 = x_{wmax} - x_1$.

$q_3 = y_1 - x_{wmin}$ $q_4 = y_{wmax} - y_1$.

Case 1

if $P_k = 0 \longrightarrow$ Parallel

$q_k < 0 \longrightarrow$ outside

Case 2

if $p_k < 0$

$$t_1 = \max\left(0, \frac{q_k}{p_k}\right)$$

if $p_k > 0$ then

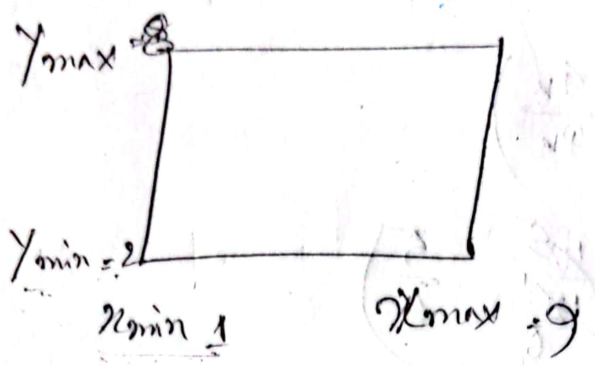
$$t_2 = \min\left(1, \frac{q_k}{p_k}\right)$$

$t_1 = 0, t_2 = 1 \Rightarrow$ inside

if $t_1 > t_2 \longrightarrow$ outside, না-হলে $x = x_1 + t\Delta x$
$y = y_1 + t\Delta y$

Math

here



$A = (-1, 7)$ $B(11, 1)$

$\Delta x = (-1 - (-11)) = 12$ $\Delta y = 1 - 7 = -6$

$P_1 = -12$,	$P_2 = 12$,	$P_3 = 6$	$P_4 = -6$
$q_1 = -1 - 1 = -2$	$q_2 = 9 - -1 = 10$	$q_3 = 7 - 2 = 5$	$q_4 = 8 - 7 = 1$

$P_k \neq 0$ so not paralel.

$P_k < 0 \Rightarrow P_1, P_4$ less than 0

$t_1 = \max\left(0, \frac{-2}{-12}, \frac{1}{-6}\right)$

$= \max\left(0, \frac{1}{6} - \frac{1}{6}\right) = \frac{1}{6}$ ← max

$P_k > 0$ then.

$$t_2 = \min\left(1, \frac{q_k}{p_k}\right)$$

$$= \min\left(1, \frac{\cancel{10}5}{\cancel{12}6}, \frac{5}{6}\right)$$

$$= \left(\frac{5}{6}\right) \text{min}.$$

if $t_1 > t_2 \rightarrow$ outside

here $t_1 < t_2$, so outside না

clip করতে হবে

A

$x = x_1 + t\Delta x$. $\Rightarrow x = -1 + \frac{1}{6} \times 12$

$y = y_1 + t\Delta y$. $= 1$.

$\rightarrow y = 7 + \frac{1}{6} \times (-6)$

$= 6$

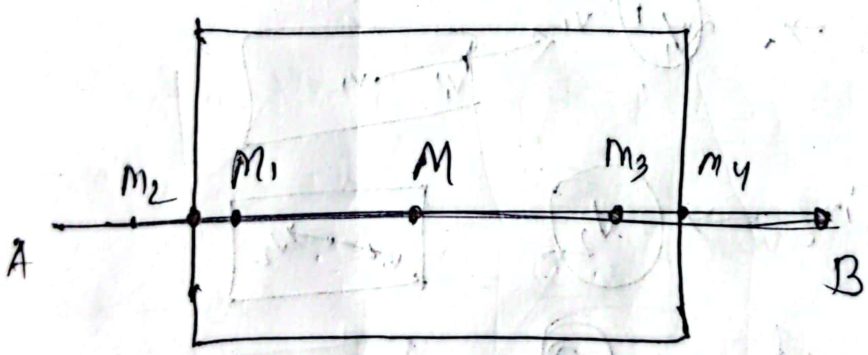
$A' = (1, 6)$

B',

$x = -1 + \frac{5}{\cancel{6}} \times \cancel{12}^{2} = 9$ $\qquad$ $y = 7 + \frac{5}{\cancel{6}} \times (\cancel{-6}) = 7 - 5 = 2.$

$B'(9, 2)$.

Mid point Submission



Step 1 find A B and ... clipping

(M) Midpoint $\frac{A^{x_1 y_1}}{2}, \frac{B^{x_2 y_2}}{2}$.

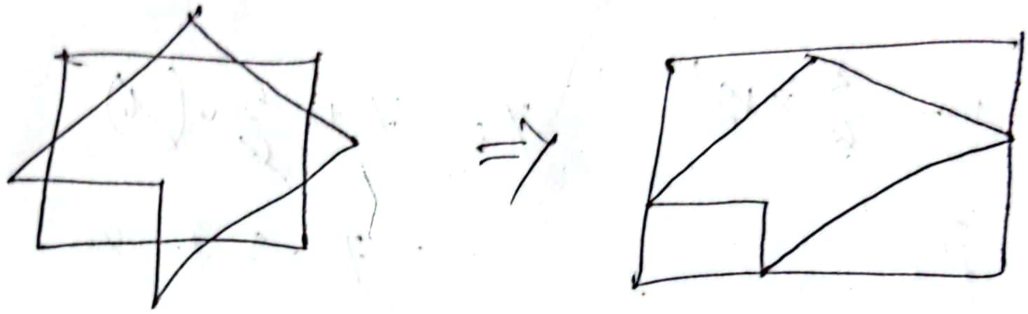
then (A, M) এর same process. M_1

" (A, M_1) " " "

Middle point যদি border পরে so clip হবে সেখানে

Polygon

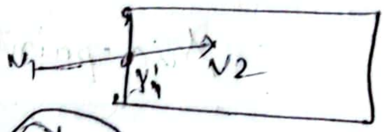
'Sutherland Hodgan polygon'



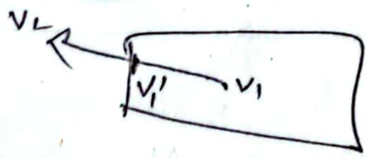
বাদে হবে

Case 4

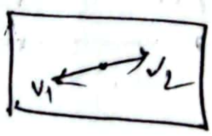
(1) → out → in. হলে (v_1' v_2)



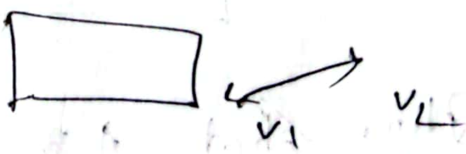
(2) → in → out →. v_1'



(3) → in to in → v_2



(4) → out to out → 0

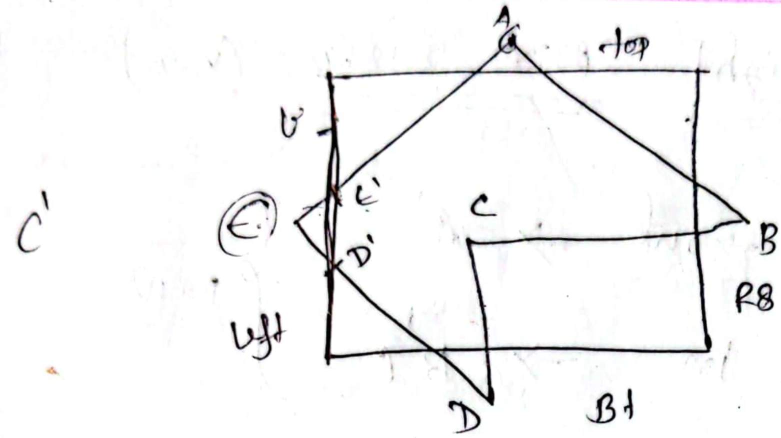


v_2

v_1'

v_1

Math



Clipping Left edge.

AB = in → in = B

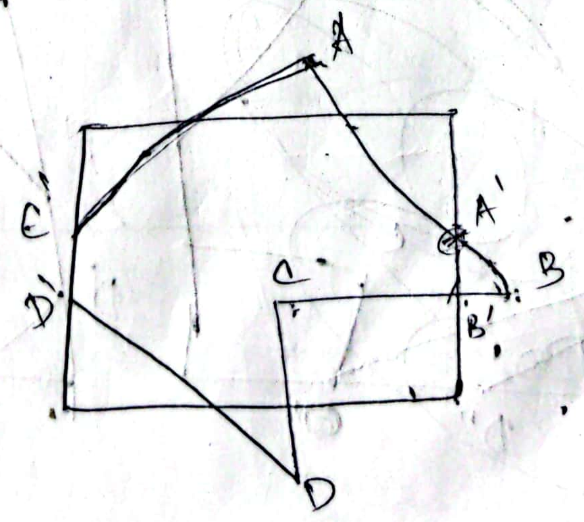
BC = " = C

CD = " = D

DE = in out ⇒ D'

EA = out in = E'A

new vertex.

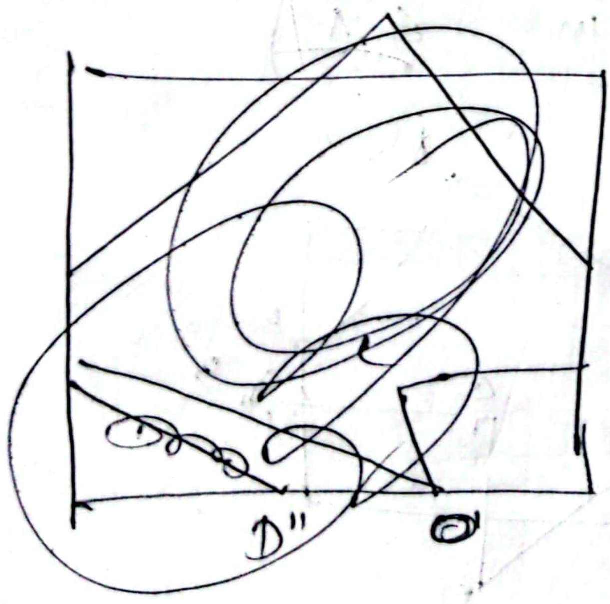


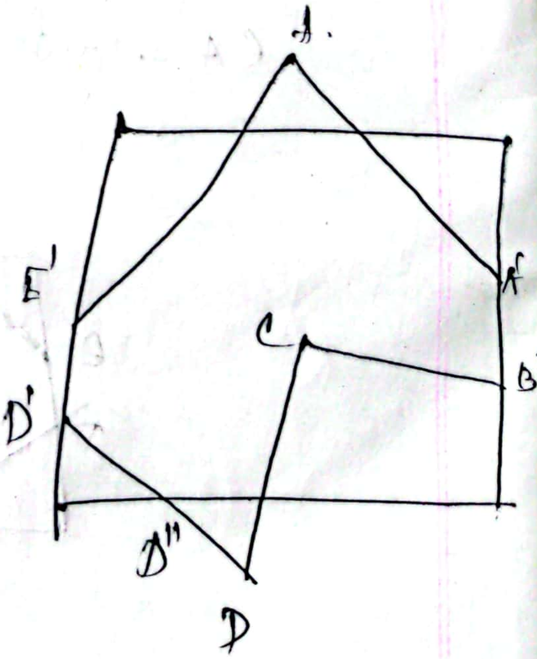
New diagram.

Clipping for Right ডান সাইডের (new)

A	B	=	in → out	⇒	A'
B	C	=	out in	→	B'C
C	D	=	in - in	→	D
D	D'	=	"	→	D'
E'	A	=	"	→	A
D'	E'	=	"	→	E'

new





Now Bottom (RS 7(2P))

A A' → in in → A'

A' B' → in in → B'

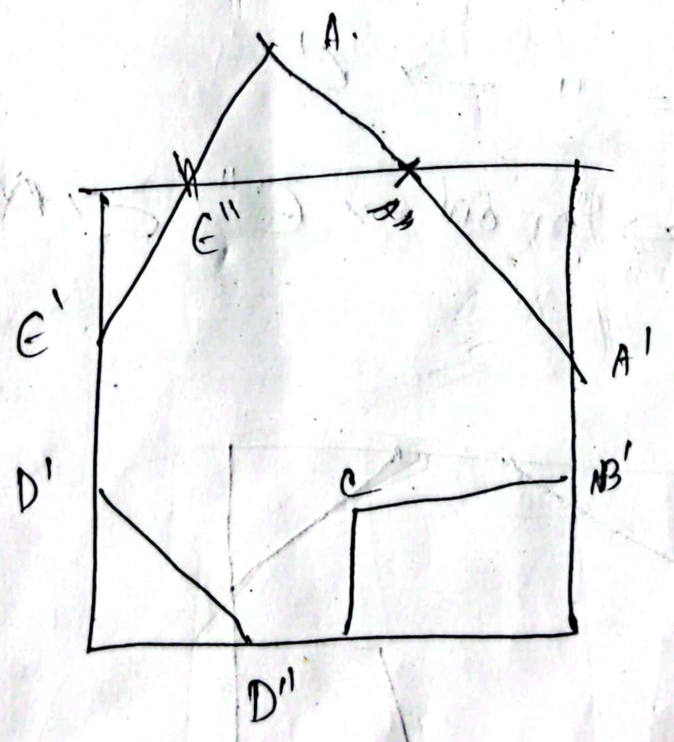
B C → in in → C

C D → in - out → C'

D D' → out → in → D'' D'

D' E' → in in → E'

E' A → in in → A



TOP

A A' → out → in → A''A'

A' B' = in - in → B'

B' C = " → C

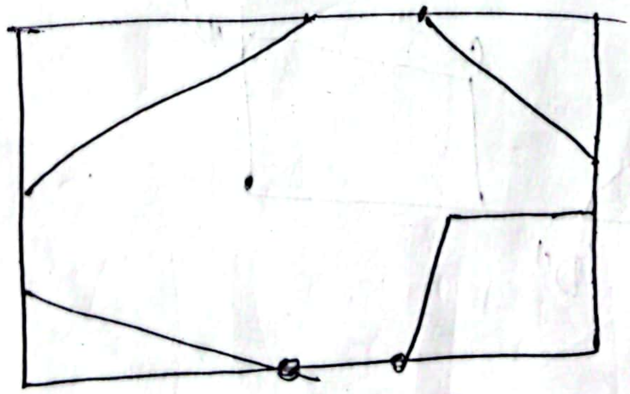
C C' = " → C'

C' D'' = " → D''

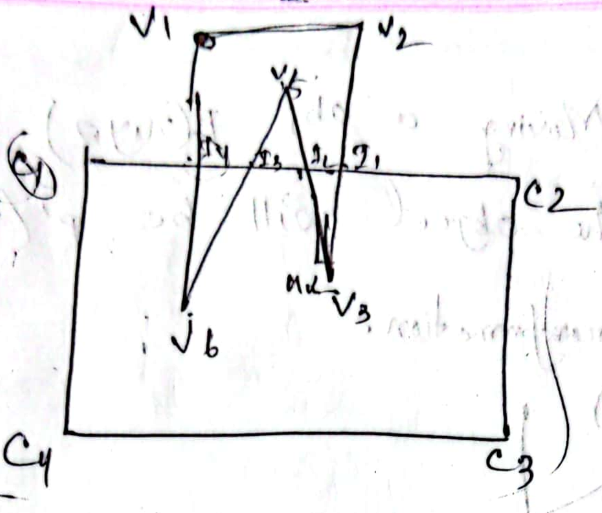
D'' D' = " → D'

D' E' = " → E'

E' A ⇒ in out → E'' → New Ver



Weiler Atherton.

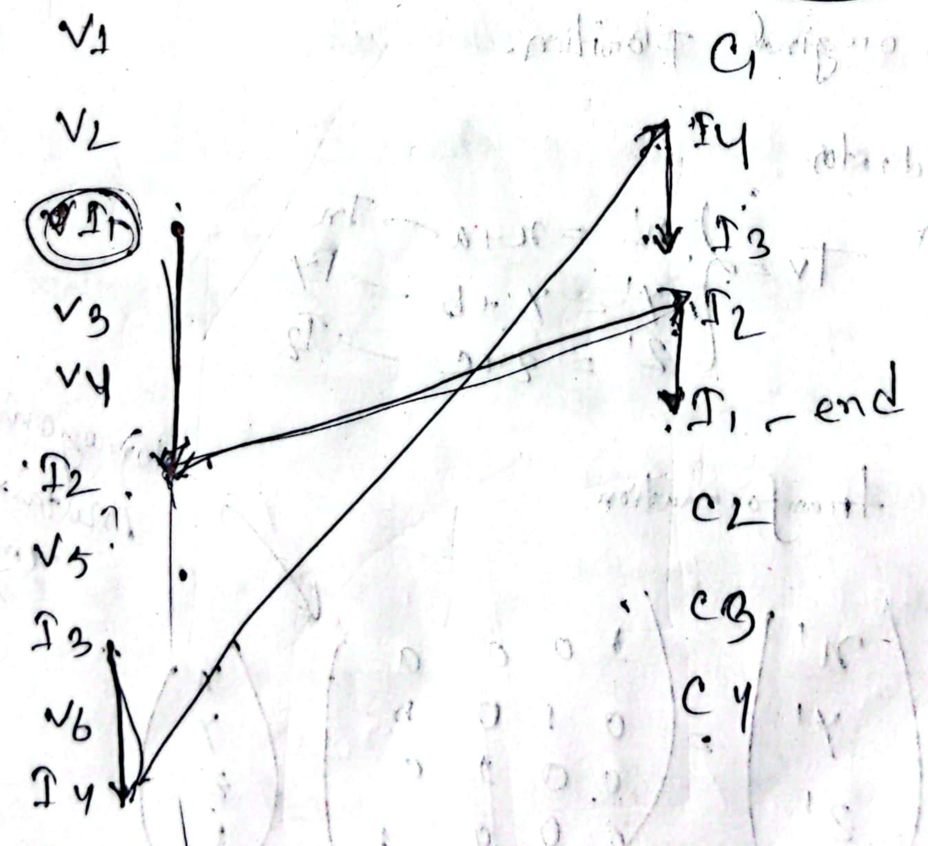


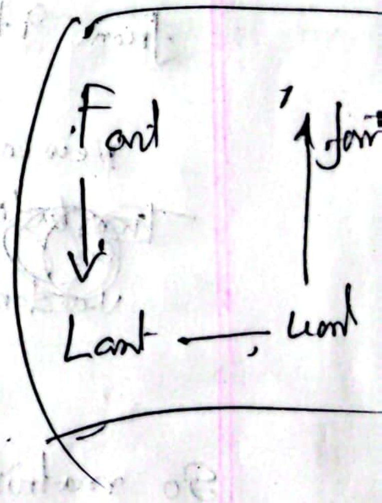
① intersect
দিয়ে বাহির হবে

Clockwise

Sub list

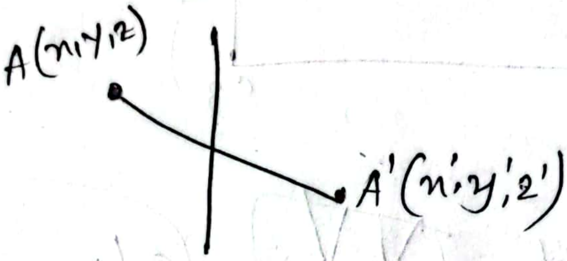
Clip list





3D Transformation
(Seg 5)

⇒ Transformation: Moving a obj $P(x,y,z)$ to a new position and the object will be $P'(x',y',z')$ this is called 3D transformation.



object is displaced a given distance and direction from its original position.

New coordinates

~~Translation~~

$$T_v = \begin{cases} x' = x + a \rightarrow T_x \\ y' = y + b \rightarrow T_y \\ z' = z + c \rightarrow T_z \end{cases}$$

So matrix tranformation.

Homogonious Matrix

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} x+a \\ y+b \\ z+c \\ 1 \end{pmatrix}$$

CamScanner

(*) 3D object coordinates point, $T_x = 1,\ T_y = 1,\ T_z = 2$

we know that

$A(0,3,1) = (1, 4, 3)$

$B(3,3,2) = (4, 4, 4)$

$C(3,0,0) = (4, 1, 2)$

$D(0,0,0) = (1, 1, 2)$

$T_v = \begin{matrix} x + t_x \\ y + t_y \\ z + t_z \end{matrix}$

Geometric

3D Scaling

It refers to the process of resizeing an object by increasing or decreasing its dimensions.

Scaling factor > 1 [size বড় হবে]

" " < 1 [size ছোট হবে]

$x' = x * S_x$ scaling factor

$y' = y * S_y$

$z' = z * S_z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(*) 3D object Coordinates point. $S_x = 2$, $S_y = 3$, $S_z = 3$

$A' = 0, 3, 3 = (0, 9, 9)$

$B' = 3, 3, 6 = (6, 9, 18)$

$C' = 3, 0, 1 = (6, 0, 3)$

$D' = (0, 0, 0) = (0, 0, 0)$

$$\begin{pmatrix} A = x^{*} S_x \\ B\ y^{*} S_y \\ C\ z^{*} \cdot S_z \end{pmatrix}$$

Rotation → object Rotate

transformation that pivots points or object around a fixed axis on center by specified angle.

For x axis

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

For y axis

$x' = z\sin\theta + x\cos\theta$

$y' = y$

$z' = z\cos\theta - x\sin\theta$

For z axis

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$z' = z$

tricks ① যেটার ও axis সেটার cosθ

$y' = y\cos\theta$; $z' = z\cos\theta$

x axis = y, z find ulta হবে

y " = x, z "

z " = x, y "

(sin, cos +, −)

(*) a point (1,2,3) rotation 90° degree towards X, Y, Z axis find out new Co-ordinates point.

Ans

for x axis

$x' = 1$

$y' = y\cos\theta - z\sin\theta = 2\cos 90 - 3\sin 90 = -3$

$z' = y\sin\theta + z\cos\theta = 2\sin 90 - 3\cos 90' = 2.$

for y axis

$x' = z\sin\theta + x\cos\theta = 3\sin 90 + 1\cos 90 = 3$

$y' = y = 2$

$z' = z\cos\theta - x\sin\theta = 3\cos 90' - 1\sin 90' = -1.$

for z axis.

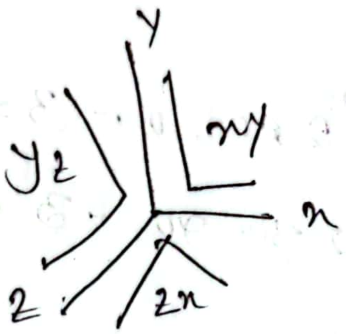
$x' = x\cos\theta - y\sin\theta = -2$

$y' = x\sin\theta + y\cos\theta = 1$

$z' = z = 3$

Reflection — mirror reflect
ফলন

It is a types of rotation where the angle is 180°



অবস্থান
⊖ হবে

— এখানে

xy -অক্ষ $\Rightarrow x' = x,\ y' = y\ \boxed{z' = -z}$

yz " $\Rightarrow \boxed{x' = -x}\ y' = y\ \ z' = z$

zx " $\Rightarrow x' = x\ \boxed{y' = -y}\ \ z' = z.$

(*) A Coordinate $A(3,4,1)$, Apply the reflection on ~~x~~xy

So.

$x' = x$ ———— 3

$y' = y$ ———— 4

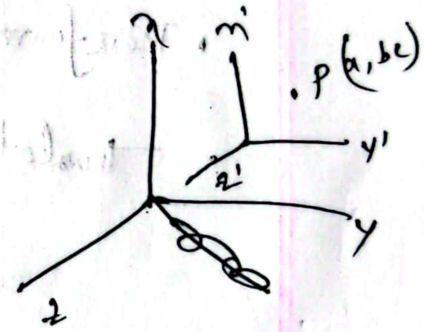
$z' = -z$ ———— -1 Ans

Coordinate Transformation

It involve moving the observer's coordinate system (not object) and recalculating object coordinates relative to the new system.

Translation Transformation $V = ai + bj + ck$.

$$x' = x - a$$
$$y' = y - b$$
$$z' = z - c$$



Matrix relationship

Translation $T_v \longleftrightarrow T_{-v}$

Rotation $R_\theta \longleftrightarrow R_{-\theta}$

Scaling $S_{x,y,z} \longleftrightarrow S_{1/x, 1/y, 1/z}$

Coordinate inverse of geometric translation

$T_x, t_y, t_z = -t_x, -t_y, -t_z$

$T_v = T_{-v}$	$S_{sx,sy,sz} = S_{1/sx, 1/sy, 1/sz}$	$R_{\theta,p} = R_{-\theta,p}$	$M_{plane} = M_{-p}$

Composite tranformation

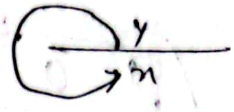
It combines multiple tranformation (translation, scaling rotation) into a single operation using matrix multiplication.

- Translate the fixed point to orgin (- -RO back tree) $-t_x, -t_y$.
- Perform Rotation (anticlockwise)
- translate the point back to orginal position.

(6.1)

* tilting rotation x axis followed rotation y axis

(a) find tilting matrix.



$$T = R_{\theta_y} \times R_{\theta_x} \cdot I$$

$$= \begin{array}{c c} & \begin{matrix} x & \quad y & \quad z & \end{matrix} \\ \begin{matrix} x \\ y \\ z \\ \\ \end{matrix} & \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \times \begin{array}{c c} & \begin{matrix} x & \quad y & \quad z & \end{matrix} \\ \begin{matrix} x \\ y \\ z \\ \\ \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} \cos\theta_y & \sin\theta_y \cdot \sin\theta_x & \sin\theta_y \cos\theta_x & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ -\sin\theta_y & \sin\theta_x \cos\theta_y & \cos\theta_y \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Does the order of performing the rotation matter. (x, y) (y, x) ?

$T = R_{\theta_x, j} \cdot R_{\theta_x, I}$

$$= \begin{pmatrix} \boxed{1 \quad 0 \quad 0 \quad 0} \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y & 0 \\ -\sin\theta_x \cdot -\sin\theta_y & \cos\theta_x & -\sin\theta_y \cos\theta_y & 0 \\ -\cos\theta_x \cdot \sin\theta_y & \sin\theta_x & \cos\theta_x \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

So Not Same

Rotation method.

6.2 find transformation A_v which aligns a given vector V with the verctor K along the positive Z axis.

① Rotate about the x axis by θ_1 so V rotets onto upper half xz plane

$$\sin\theta_1 = \frac{b}{\sqrt{b^2+c^2}} \qquad \cos\theta_1 = \frac{c}{\sqrt{b^2+c^2}}$$

rotation.

$$R_{\theta,I} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

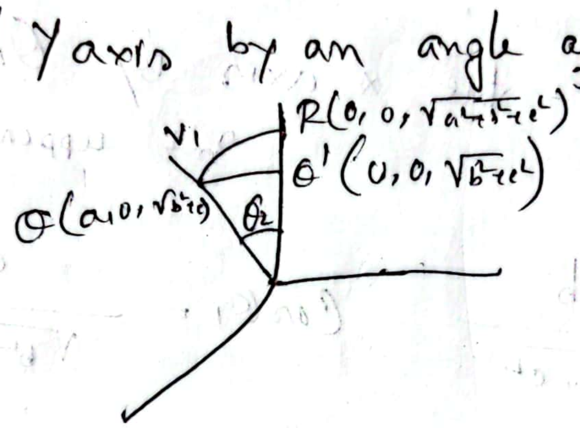
$$R_{\theta_1,I}(P) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\sqrt{b^2+c^2}} & -\frac{b}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ 1 \end{pmatrix}$$

গুণ করলে টের পাবে

V_1 Components $(a, 0, \sqrt{b^2+c^2})$

$$= \begin{bmatrix} a \\ 0 \\ \sqrt{b^2+c^2} \\ 1 \end{bmatrix}.$$

② v' about Y axis by an angle of $-\theta_2$.



$$\sin(-\theta_2) = -\sin\theta_2$$

$$= \frac{-a}{\sqrt{a^2+b^2+c^2}}$$

$$\cos(-\theta_2) = \cos\theta_2$$

$$= \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}}$$

$$R_{-\theta_2, y} = \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} V'$$

$$= \begin{pmatrix} \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 & \frac{-a}{\sqrt{a^2+b^2+c^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{\sqrt{a^2+b^2+c^2}} & 0 & \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} a \\ 0 \\ \sqrt{b^2+c^2} \\ 1 \end{pmatrix}$$

$$= \begin{bmatrix} \frac{\cancel{a\sqrt{b^2+c^2}} - \cancel{a\sqrt{b^2+c^2}}}{\sqrt{a^2+b^2+c^2}} \\ 0 \\ \frac{a^2+b^2+c^2}{\sqrt{a^2+b^2+c^2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{a^2+b^2+c^2} \\ 1 \end{bmatrix}$$

→ now Composite matrix A.K can find.

$$R' = R_{-\theta_2 J} * R_{\theta_1 I}(P)$$

$$\begin{bmatrix} \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 & -\frac{a}{\sqrt{a^2+b^2+c^2}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{\sqrt{a^2+b^2+c^2}} & 0 & \frac{\sqrt{b^2+c^2}}{\sqrt{a^2+b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\sqrt{b^2+c^2}} & -\frac{b}{\sqrt{b^2+c^2}} & 0 \\ 0 & \frac{b}{\sqrt{b^2+c^2}} & \frac{c}{\sqrt{b^2+c^2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let $\sqrt{b^2+c^2} = \lambda$

$\sqrt{a^2+b^2+c^2} = |V|$

$$= \begin{bmatrix} \frac{\lambda}{|V|} & 0 & -\frac{a}{|V|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{V} & 0 & \frac{\lambda}{V} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{c}{\lambda} & -\frac{b}{\lambda} & 0 \\ \frac{a}{|V|} & \frac{b}{|V|} & \frac{c}{|V|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

গুণ করলে

$$= \begin{bmatrix} \frac{\lambda}{|V|} & \frac{-ab}{\lambda|V|} & -\frac{ac}{\lambda|V|} & 0 \\ 0 & c/\lambda & -b/\lambda & 0 \\ \frac{a}{|V|} & \frac{b}{|V|} & \frac{c}{|V|} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If $b = c = 0$

then $\lambda = 0$ $|V| = a$ $\theta_2 = \pm 90$

$$A_{v,k} = \begin{bmatrix} 0 & 0 & -\frac{a}{|a|} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{a}{|a|} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= R_{\theta_2 J}$$

$$A^{-1}_{v,k} = \left(R_{-\theta_2 J} \cdot R_{\theta_1 I}\right)^{-1}.$$

6.3 an axis of rotation L be specified by a direction vector V and a loc point P. find transformation for rotation of θ° about L.

Ans

Translate p to the origin $= T_{-P_0}$

Align vector V with K $= A_{v,k}$

Rotate object about z angle $\theta = R_{\theta,k}$

Realign vector V back $= A^{-1}_{v,k}$

Retranslate P_0 back $= T^{-1}_{-P_0}$

3D viewing pipeline