Regular Expression (RE)

⇒ RE are used for representing certain sets of string in an algebraic fashion.

1) Any terminal symbol i.e. symbols $\in \Sigma$ including Λ and Φ are regular expression.

2) $A \cup B$ = regular expression

3) $A \circ B$ = "

4) A^* the iteration (or closure) of a re is also a RE. $\boxed{a^* = \Lambda, a, aa, aaa - - -}$

5) এই operation গুলো এক সাথে form এ করলেও যেটা তৈরী হবে সেটাও একটি RE.

$\Lambda \rightarrow$ empty
$+ \rightarrow$ or

* Example of RE

① $\{0, 1, 2\} \rightarrow$ 0 or 1 or 2 $\rightarrow R = 0 + 1 + 2$

② $\{\Lambda, ab\} \rightarrow R = \Lambda ab$

③ $\{abb, a, b, bba\} \rightarrow$ abb or a or b or bba $\rightarrow R = abb + a + b + bba$

④ $\{\Lambda, 0, 00, 000, \ldots\} \rightarrow$ closure of 0, $R = 0^*$

⑤ $(1, 11, 111, 1111 \ldots) \rightarrow R = 1^+$

Indentities of R.E.

$+ \rightarrow$ Unioun

$R\phi$ - concatinate R, ϕ

1) $\phi + R = R$
2) $\phi R + R\phi = \phi$
3) $\epsilon R = R\epsilon = R$
4) $\epsilon^* = \epsilon$ and $\phi^* = \epsilon$
5) $R + R = R$ $\rightarrow$ unioun of 2 R.E is R.E
6) $R^* R^* = R^*$
7) $RR^* = R^* R$
8) $(R^*)^* = R^*$
9) $\epsilon + RR^* = \epsilon + R^* R = R^*$
10) $(PQ)^* P = P(QP)^*$
11) $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$
12) $(P+Q)R = PR + QR$ and $R(P+Q) = RP + RQ$

Designing R.E

Design R.E for the following language over $\{a, b\}$.

1) language accepting strings of length exactly 2

2) " " " " " atleast 2

3) " " " " " atmost 2

1) $L_1 = \{aa, ab, ba, bb\}$

$R = aa + ab + ba + bb$

$= a(a+b) + b(a+b)$

$= (a+b)(a+b)$

2) $L_1 = \{aa, ab, ba, bb, aaa, \ldots\}$

$R = (a+b)(a+b)(a+b)^*$

3) $L = \{\epsilon, a, b, aa, ab, ba, bb\}$

$R = \epsilon + a + b + aa + ab + ba + bb$

$= (\epsilon + a + b)(\epsilon + a + b)$

ARDEN'S THEOREM

* If P and Q are two R.E over Σ, and if P does not contain ϵ, the the following equation in R given by $R = Q + RP$ has a unique solution

i.e $R = QP^*$

If $R = Q + RP$
then, $R = QP^*$

$R = Q + RP$ ------ (1)

$= Q + QP^*P$ $R = QP^*$

$= Q(\epsilon + P^*P)$ $[\epsilon + R^*R = R^*]$

$= QP^*$
(proved)

$R = Q + RP$

$= Q + [Q + RP]P$

$= Q + QP + RP^2$

$= Q + QP + \boxed{Q + RP}P^2$

$= Q + QP + QP^2 + RP^3$

$\vdots$

$= Q + QP + QP^2 + \cdots QP^n + RP^{n+1}$ $[R = QP^*]$

$= Q + QP + QP^2 + \cdots QP^n + QP^*P^{n+1}$

$= Q[\epsilon + P + P^2 + \cdots P^n + P^*P^{n+1}]$

$R = QP^*$

An example proof using Identities of R.E

* Prove that $(1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1)$ is equal to $0^*1(0+10^*1)^*$

$$L.H.S = (1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1)$$

$$= (1+00^*1)\left[\epsilon+(0+10^*1)^*(0+10^*1)\right]$$

$$= (1+00^*1)(0+10^*1)^* \qquad \boxed{\epsilon+R^*R=R^*}$$

$$= (\epsilon\cdot 1+00^*1)(0+10^*1)^* \qquad \boxed{\epsilon\cdot R=R}$$

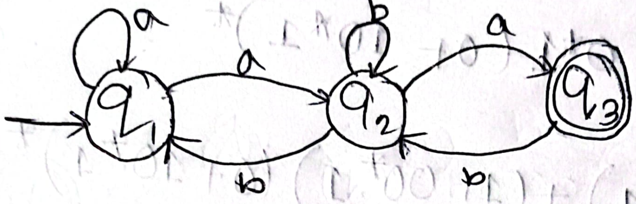
$$= (\epsilon+00^*)1(0+10^*1)^*$$

$$= 0^*1(0+10^*1)^*$$

$$= R.H.S$$

Designing R.E.

Find the regular expression for the followin NFA



Soln:

$q_3 = q_2 a \longrightarrow (1)$

$q_2 = q_1 a + q_2 b + q_3 b \longrightarrow (2)$

$q_1 = \cancel{q_1 a + q_2 b}\ \epsilon + q_1 a + q_2 b \longrightarrow (3)$

⇒ incoming transition

(1) ⇒ $q_3 = q_2 a$

$= (q_1 a + q_2 b + q_3 b)\, a$

$= q_1 aa + q_2 ba + q_3 ba \longrightarrow (4)$

(2) ⇒ $q_2 = q_1 a + q_2 b + q_3 b$ putting value of q_3 from (1)

$= q_1 a + q_2 b + (q_2 a) b$

$= q_1 a + q_2 b + q_2 ab$

$\underset{R}{q_2} = \underset{Q}{q_1 a} + \underset{R}{q_2}\, \underset{P}{(b + ab)}$

$R = Q + RP$ Arden's theorem
$R = QP^*$

$q_2 = (q_1 a)(b + ab)^* \longrightarrow (5)$

③ $\Rightarrow q_1 = \epsilon + q_1 a + q_2 b$

putting value of q_2 from ⑤

$q_1 = \epsilon + q_1 a + ((q_1 a)(b+ab)^*)b$

$\underbrace{q_1}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_1}_{R} \underbrace{(a + a(b+ab)^*)b}_{P}$ | $R = Q + RP$, $R = QP^*$

$q_1 = \epsilon((a + a(b+ab)^*)b)^*$ | $\epsilon \cdot R = R$

$= ((a + a(b+ab)^*)b)^* \rightarrow$ ⑥

Final State (q_3)

$q_3 = q_2 a$

$= q_1 a(b+ab)^* a$ putting value of q_2 from ⑤

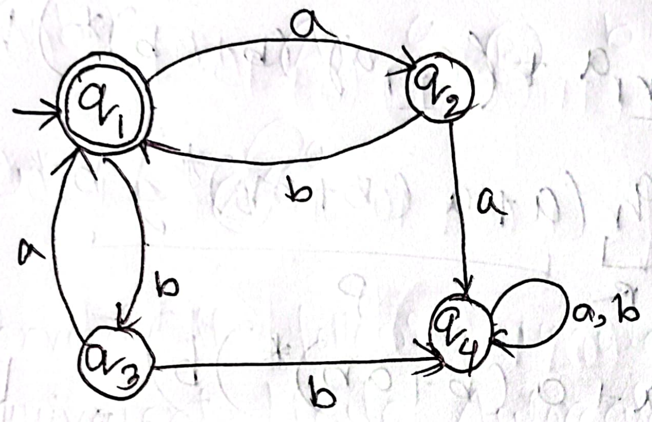
$= (a + a(b+ab)^* b)^* a(b+ab)^* a$; putting value of q_1 from ⑥

$$\boxed{q_3 = (a + a(b+ab)^* b)^* a(b+ab)^* a}$$

Required R.E for given NFA

⇒ DFA to Regular expression

* Find the regular expression for the following DFA.



Soln:

$$q_1 = \epsilon + q_2 b + q_3 a \longrightarrow ①$$

$$q_2 = q_1 a \longrightarrow ②$$

$$q_3 = q_1 b \longrightarrow ③$$

$$q_4 = q_2 a + q_3 b + q_4 a + q_4 b \longrightarrow ④$$

① ⇒

$$q_1 = \epsilon + q_2 b + q_3 a$$

putting values of q_2 and q_3 from ② and ③

$$q_1 = \epsilon + q_1 ab + q_1 ba$$

$$\underset{R}{q_1} = \underset{Q}{\epsilon} + \underset{R}{q_1} \underset{P}{(ab + ba)}$$

$R = Q + RP$

$R = QP^*$

$$q_1 = \epsilon\,(ab + ba)^*$$

$$q_1 = (ab + ba)^* \rightarrow \text{Regular expression}$$

* Find the R.E for the following DFA

$q_1 = \epsilon + q_1 0 \longrightarrow ①$

$q_2 = q_1 1 + q_2 1 \longrightarrow ②$

$q_3 = q_2 0 + q_3 0 + q_3 1 \longrightarrow ③$

Final State q_1

$$① \Rightarrow \underset{R}{\underline{q_1}} = \underset{Q}{\underline{\epsilon}} + \underset{R}{\underline{q_1}}\,\underset{P}{\underline{0}}$$

$R = Q + RP$
$R = QP^*$

$$q_1 = \epsilon \cdot 0^*$$
$$= 0^* \quad \epsilon \cdot R = R$$
$$q_1 = 0^* \longrightarrow ④$$

Final State q_2

$$q_2 = q_1 1 + q_2 1$$

putting value of q_1 from ④

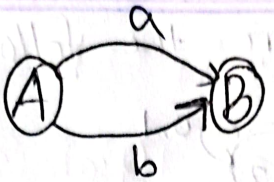
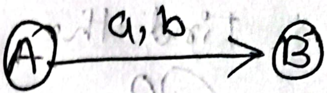
$$\underset{R}{\underline{q_2}} = \underset{Q}{\underline{0^* 1}} + \underset{R}{\underline{q_2}}\,\underset{P}{\underline{1}}$$

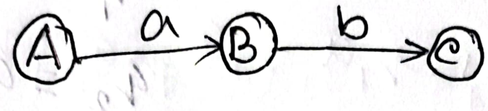
$$q_2 = 0^* 1 (1)^*$$

R = Union of both final state

$$= 0^* + 0^* 1\ 1^*$$
$$= 0^* (\epsilon + 1 1^*) \quad [\epsilon + RR^* = R^*]$$
$$= 0^* 1^* \rightarrow \text{Regular expression}$$

Conversion of Regular Expression to Finite Automata

① $(a+b) \Rightarrow$  or 

② $(a \cdot b) \Rightarrow$ 

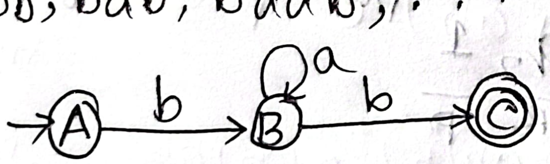
③ $a^* \Rightarrow$ 

④ Convert the following regular expressions to their equivalent Finite Automata:

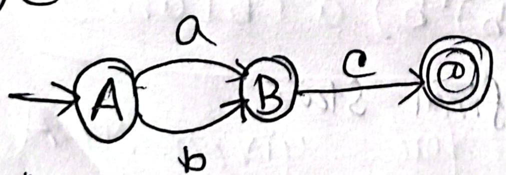
1) ba^*b
2) $(a+b)c$
3) $a(bc)^*$

1) ba^*b

$\Rightarrow$ bb, bab, baab, . . .



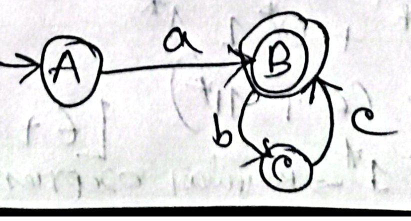
2) $(a+b)c$



ac ✓
bc ✓

3) $a(bc)^*$

$\Rightarrow$ a, abc, abcbc, . . .

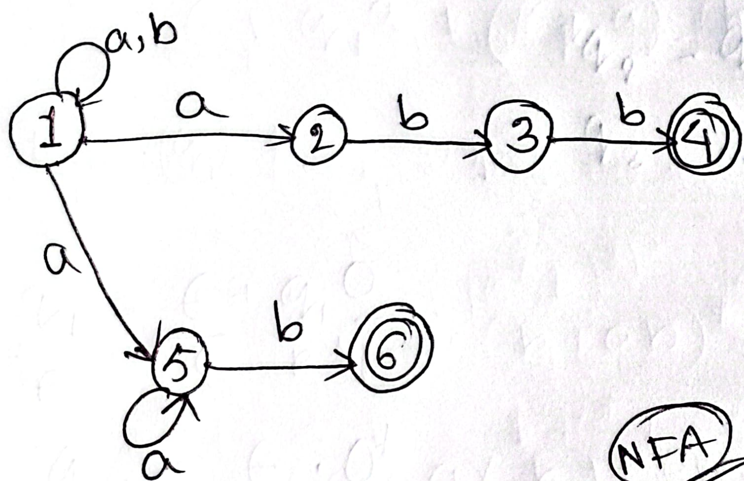


| = OR same as +

$\boxed{| = +}$

* Convert the following R.E. to its equivalent finite Automata.

$(a|b)^* (abb|a^+b)$



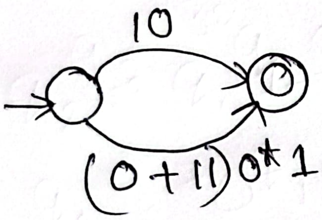
$a^+ = \{a, aa, aaa, \dots\}$

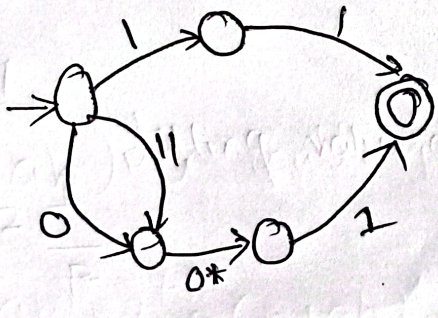
$a^* = \{\epsilon, a, aa, aaa \dots$

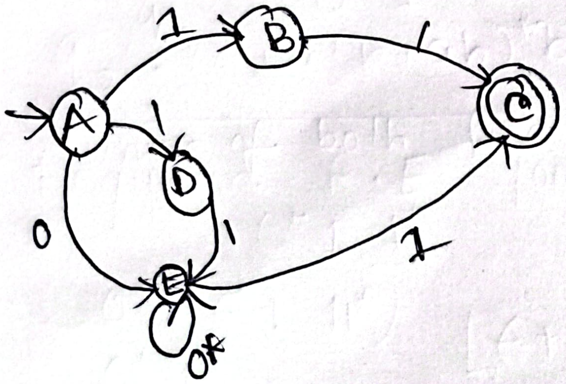
| পাইলে আলাদা শুরু থেকে শুরু করবো

NFA

* $10 + (0+11)\,0^*1$







✓