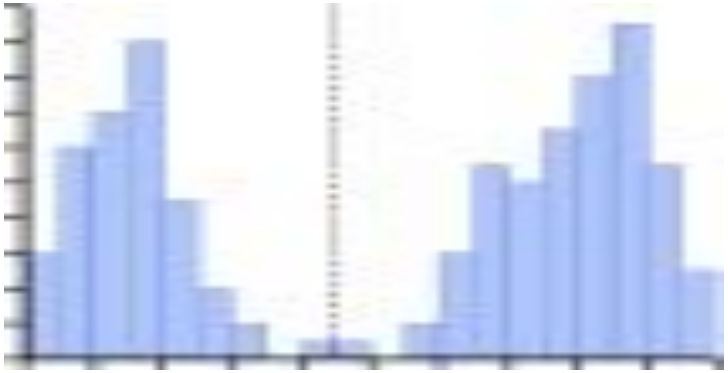
Otsu Method

Md. Khaliluzzaman

Dept. of Computer Science and Engineering, IIUC

Otsu thresholding

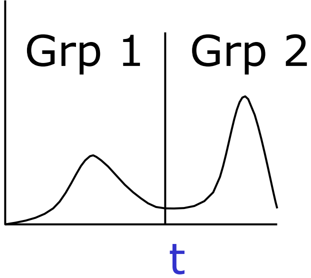
- Converting a grayscale image to monochrome
- Otsu's method, named after its inventor *Nobuyuki Otsu,* is one of many binarization algorithms



Assumption: the histogram is bimodal

Cont.

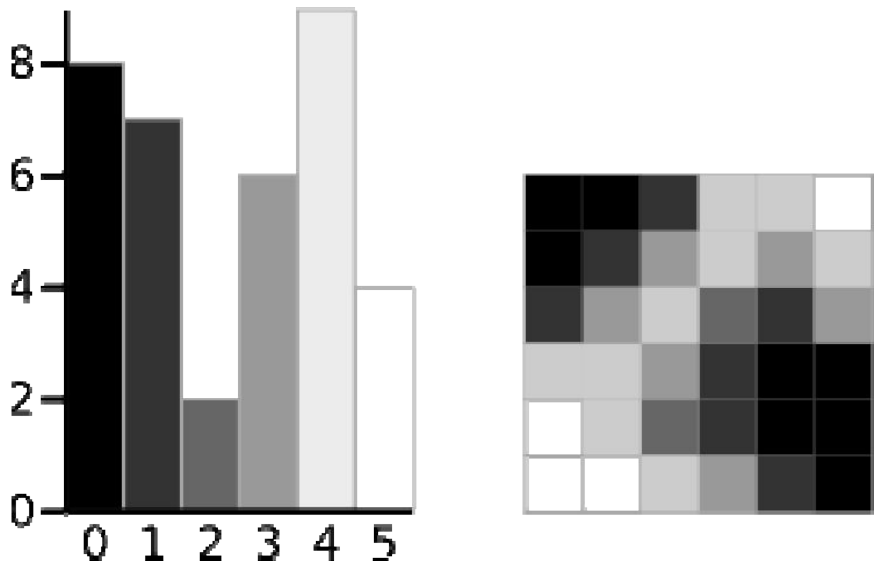
- This method *involves iterating* through all the possible threshold values and calculating a measure of spread for the pixel levels each side of the threshold, i.e. *the pixels that either fall in foreground or background*
  - The aim is to find the threshold value where the sum of foreground and background spreads is at its minimum



Assumption: the histogram is bimodal

Cont.

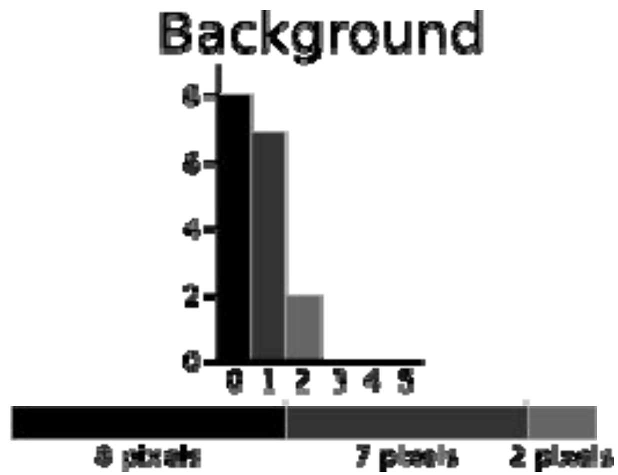
- Simple 6x6 image shown below and the histogram for the image is shown next
- To simplify the explanation, only 6 grayscale levels are used



A 6-level grayscale image and its histogram

Cont. (For Background)

- The calculations are bellow for finding the foreground & background variances *(the measure of spread)* for a single threshold

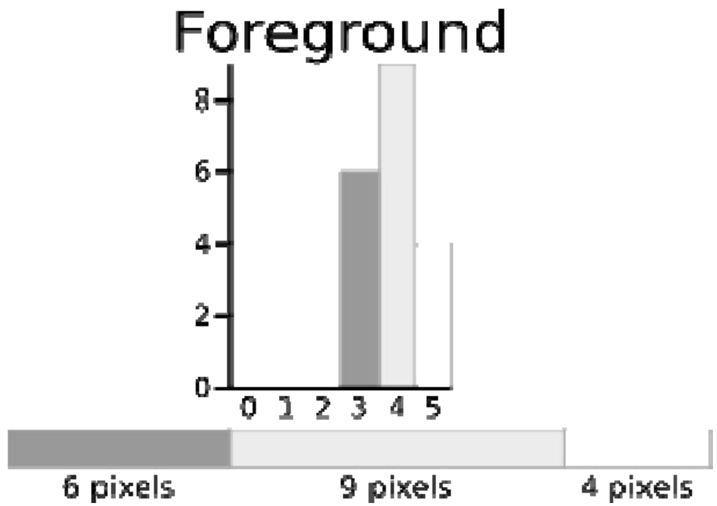


$$\text{Weight} \quad W_b = \frac{8+7+2}{36} = 0.4722$$

$$\text{Mean} \quad \mu_b = \frac{(0\times 8)+(1\times 7)+(2\times 2)}{17} = 0.6471$$

$$\begin{aligned}\text{Variance} \quad \sigma_b^2 &= \frac{((0-0.6471)^2\times 8)+((1-0.6471)^2\times 7)+((2-0.6471)^2\times 2)}{17} \\ &= \frac{(0.4187\times 8)+(0.1246\times 7)+(1.8304\times 2)}{17} \\ &= 0.4637\end{aligned}$$

Cont. (For Foreground)



$$\text{Weight} \quad W_f = \frac{6+9+4}{36} = 0.5278$$

$$\text{Mean} \quad \mu_f = \frac{(3\times 6)+(4\times 9)+(5\times 4)}{19} = 3.8947$$

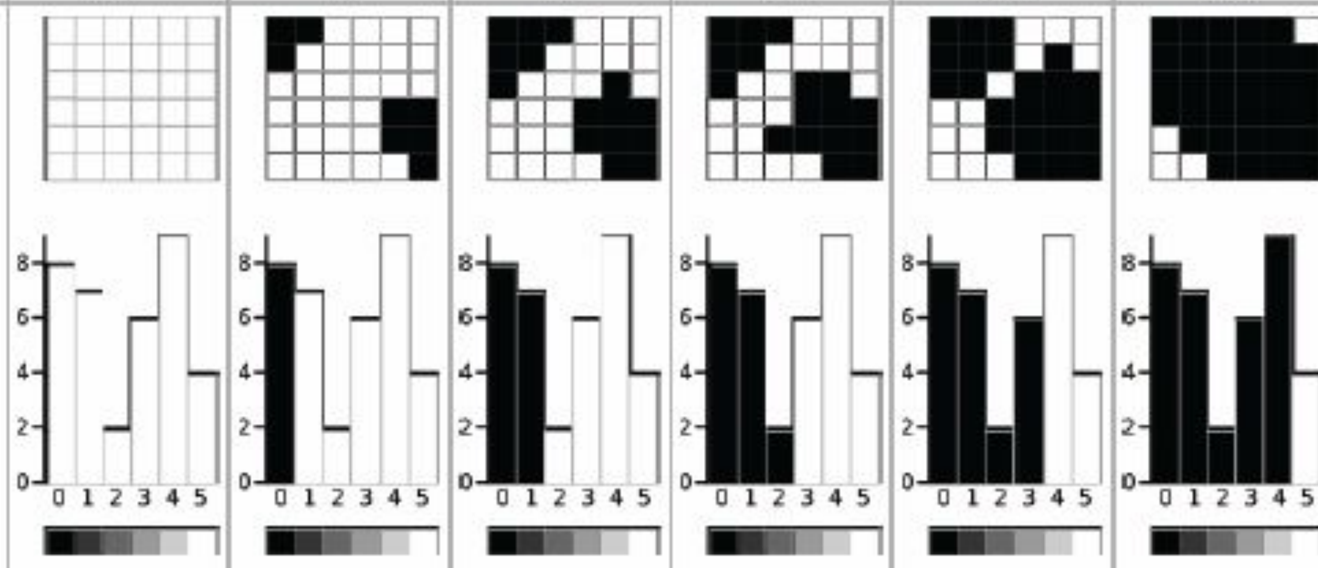
$$\begin{aligned}\text{Variance} \quad \sigma_f^2 &= \frac{((3-3.8947)^2\times 6)+((4-3.8947)^2\times 9)+((5-3.8947)^2\times 4)}{19} \\ &= \frac{(4.8033\times 6)+(0.0997\times 9)+(4.8864\times 4)}{19} \\ &= 0.5152\end{aligned}$$

Cont.

- The next step is to calculate the *'Within-Class Variance‘ i.e.* this is simply the sum of the two variances multiplied by their associated weights

$$\text{Within Class Variance} \quad \sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2 = 0.4722 * 0.4637 + 0.5278 * 0.5152$$
$$= 0.4909$$

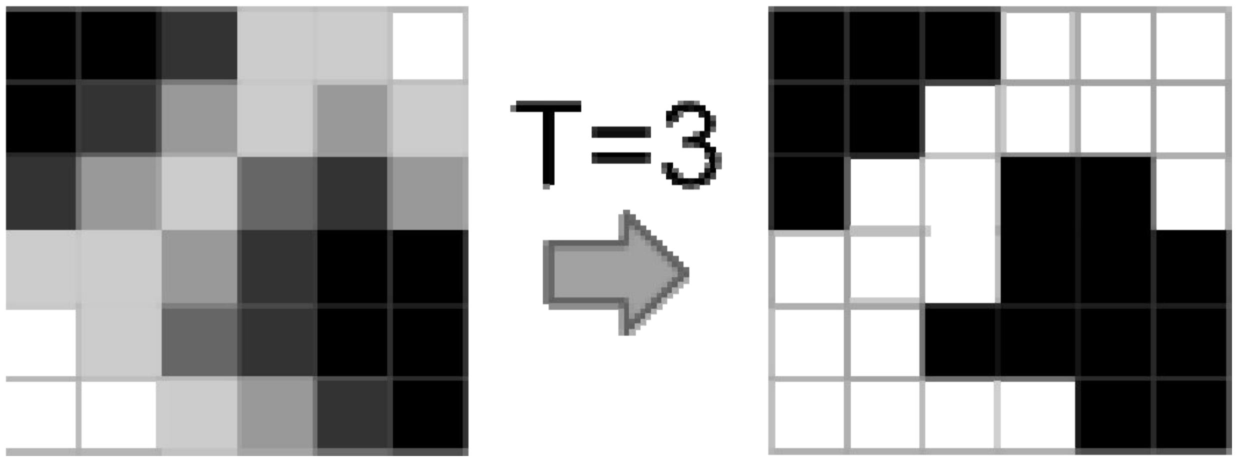
Cont.



Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Weight, Background	$W_b = 0$	$W_b = 0.222$	$W_b = 0.4167$	$W_b = 0.4722$	$W_b = 0.6389$	$W_b = 0.8889$
Mean, Background	$M_b = 0$	$M_b = 0$	$M_b = 0.4667$	$M_b = 0.6471$	$M_b = 1.2609$	$M_b = 2.0313$
Variance, Background	$\sigma^2_b = 0$	$\sigma^2_b = 0$	$\sigma^2_b = 0.2489$	$\sigma^2_b = 0.4637$	$\sigma^2_b = 1.4102$	$\sigma^2_b = 2.5303$
Weight, Foreground	$W_f = 1$	$W_f = 0.7778$	$W_f = 0.5833$	$W_f = 0.5278$	$W_f = 0.3611$	$W_f = 0.1111$
Mean, Foreground	$M_f = 2.3611$	$M_f = 3.0357$	$M_f = 3.7143$	$M_f = 3.8947$	$M_f = 4.3077$	$M_f = 5.000$
Variance, Foreground	$\sigma^2_f = 3.1196$	$\sigma^2_f = 1.9639$	$\sigma^2_f = 0.7755$	$\sigma^2_f = 0.5152$	$\sigma^2_f = 0.2130$	$\sigma^2_f = 0$
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\boldsymbol{\sigma^2_W = 0.4909}$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$

the threshold equal to 3, as well as being used for the example, also has the lowest sum of weighted variances

Conclusion



Result of Otsu method

Between class variance

$$\text{Within Class Variance} \quad \sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2 \quad \text{(as seen above)}$$

$$\begin{aligned}\text{Between Class Variance} \quad \sigma_B^2 &= \sigma^2 - \sigma_W^2 \\ &= W_b(\mu_b - \mu)^2 + W_f(\mu_f - \mu)^2 \quad (\text{where } \mu = W_b\,\mu_b + W_f\,\mu_f) \\ &= W_b\,W_f\,(\mu_b - \mu_f)^2\end{aligned}$$

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\sigma^2_W = 0.4909$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$
Between Class Variance	$\sigma^2_B = 0$	$\sigma^2_B = 1.5928$	$\sigma^2_B = 2.5635$	$\sigma^2_B = 2.6287$	$\sigma^2_B = 2.1417$	$\sigma^2_B = 0.8705$