Project Part II: MIP Callbacks and TSP as a Service

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Outline

Introduction

MIP Callbacks

TSP

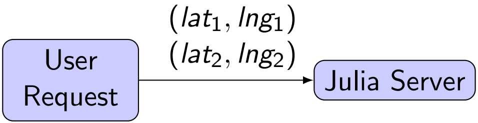
TSPaaS

Deployment

Project: TSP as a Service

User Request

Project: TSP as a Service



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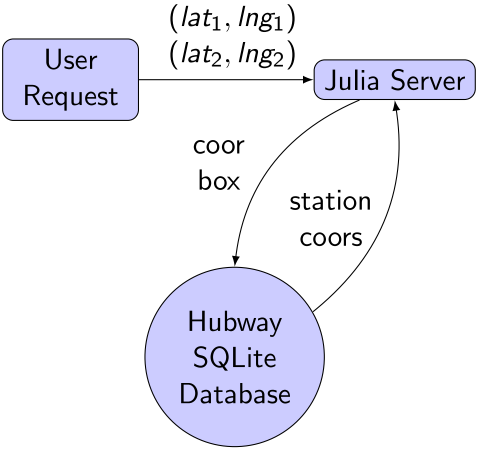

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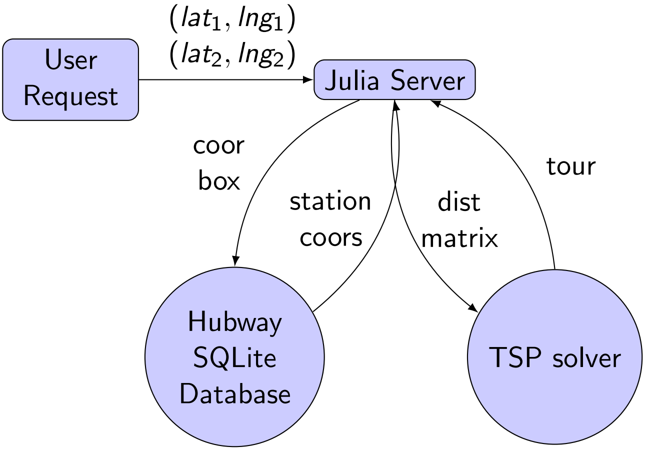

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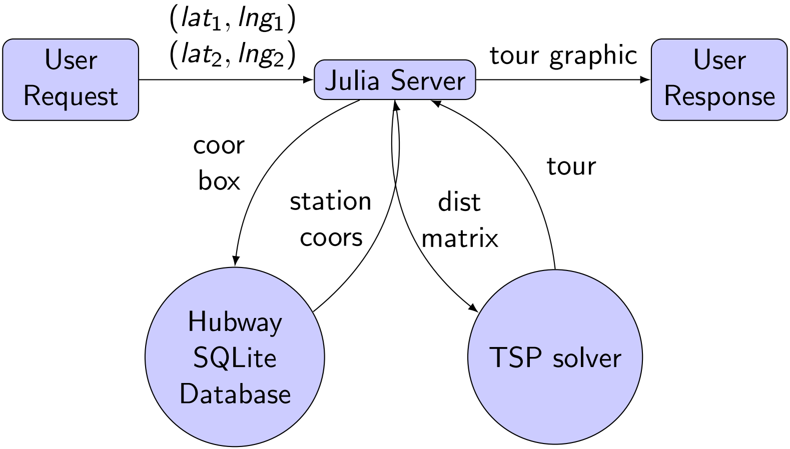

Last Time

- The internet

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- **Stations Service:** (lat, lng) for all hubway stations in a box

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- **Convert Stations Service to TSP Service**
- Deploying MIP solvers in the real world

What is a MIP Callback?

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- Control branching & node selection
- Log **anything** about solver progress for offline analysis

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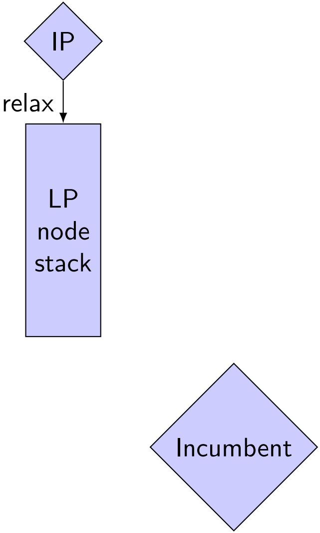

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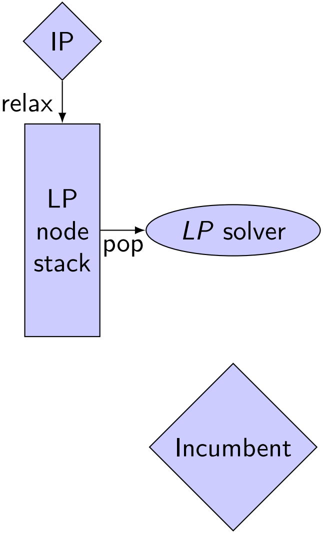

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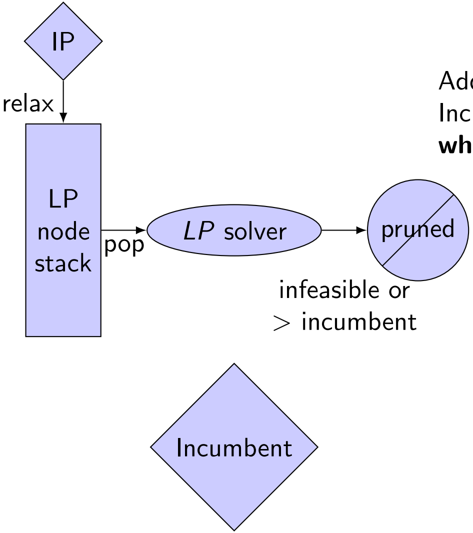

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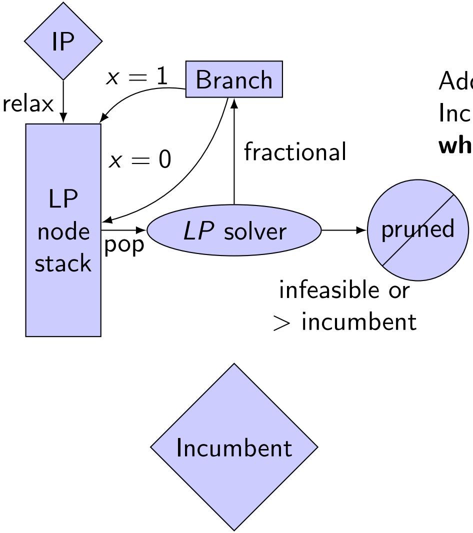

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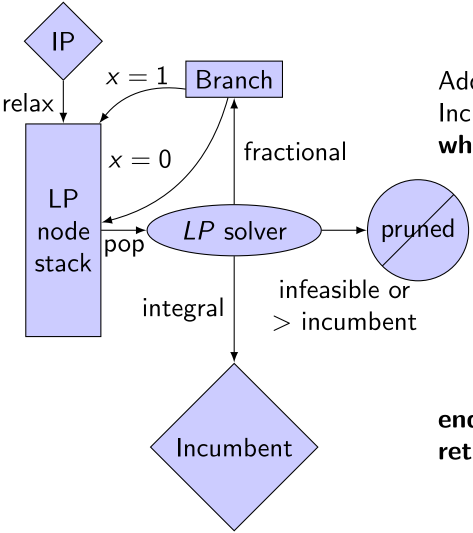

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  else if *Fractional* **then**
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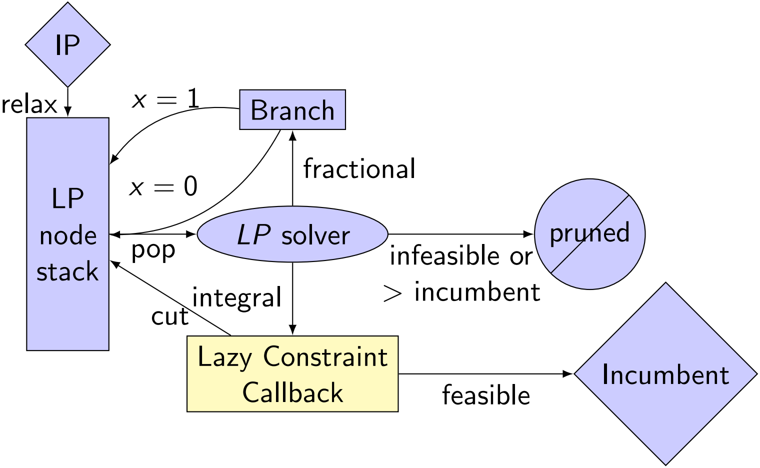
  else
    Set incumbent to solution
  end
end
return *incumbent*

The Lazy Constraint Callback

Add the "lazy" constraints only once they have been violated by an integer solution.



When to make a family of constraints lazy

Good idea when:

- Family of constraints is large (e.g. n^3, 2^n, $|\mathbb{R}|$)
- Integer solutions quickly *separated*
- Most constraints not violated

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Problems:

- Hard to find an integer solution
- Best bound improves slowly if many lazy constraints needed

Lazy Constraints in Julia

Simple Lazy IP:

$$\begin{aligned} \max \quad & x_1 + 2x_2 \\ \text{lazy:} \quad & x_1 + x_2 \leq 1 \\ & x_1, x_2 \in \{0, 1\} \end{aligned}$$

Julia Solution:

Lazy Constraints in Julia

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$$\max \quad x_1 + 2x_2$$

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Julia Solution:

```
m = Model(solver=GurobiSolver(LazyConstraints=1))
@defVar(m,x[1:2],Bin)
@setObjective(m,Max, x[1] + 2*x[2])
function lazy(cb)
    xVal = getValue(x)
    if xVal[1] + xVal[2] > 1 + 1e-4
        @addLazyConstraint(cb, x[1] + x[2] <= 1)
    end
end
setlazycallback(m,lazy)
solve(m)
```

Exercise: the feasible circle

Input: radius r, direction $\mathbf{c} = (c_1, c_2)$
Goal: maximize $\mathbf{c} \cdot \mathbf{x}$ on integer inside radius r circle.

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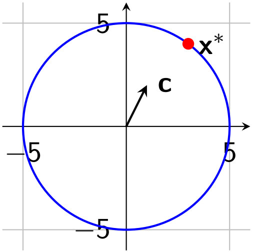
$$\begin{array}{ll} \max & \mathbf{cx} \\ \text{subject to:} & \|\mathbf{x}\|_2 \leq r \\ & \mathbf{x} \in \mathbb{Z}_2 \end{array}$$

$r = 5$, $\mathbf{c} = (2, 1)$



$\mathbf{x}^* = (3, 4)$

Solving the feasible circle with Lazy Constraint Callbacks

Lazy Formulation:

$$\begin{array}{lll} \text{max} & \mathbf{c} \cdot \mathbf{x} & \\ \text{s.t.} & -r \leq x_i \leq r & i = 1, 2 \\ \text{lazy:} & \mathbf{d} \cdot \mathbf{x} \leq r & \forall \|\mathbf{d}\|_2 = 1 \\ & \mathbf{x} \in \mathbb{Z}_2 & \end{array}$$

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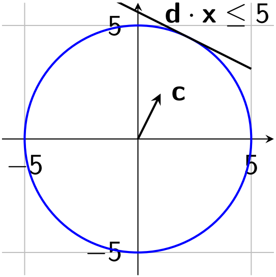

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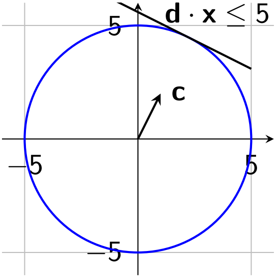

Implementing Lazy Constraints:

- Uncountably many!

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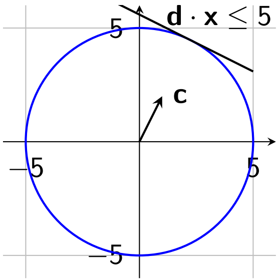

Implementing Lazy Constraints:

- Uncountably many!
- If $\|\mathbf{x}\|_2 > r$, take $\mathbf{d} = \frac{1}{\|\mathbf{x}\|_2}\mathbf{x}$, as

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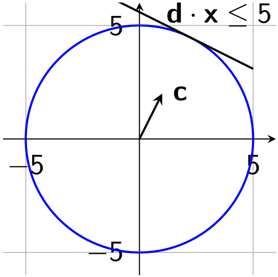

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- Psuedo code:

if $\|\mathbf{x}\|_2 > r$ **then**
    $\mathbf{d} = \frac{1}{\|\mathbf{x}\|_2}\mathbf{x}$;
    add lazy constraint $\mathbf{d} \cdot \mathbf{x} \leq r$;
end

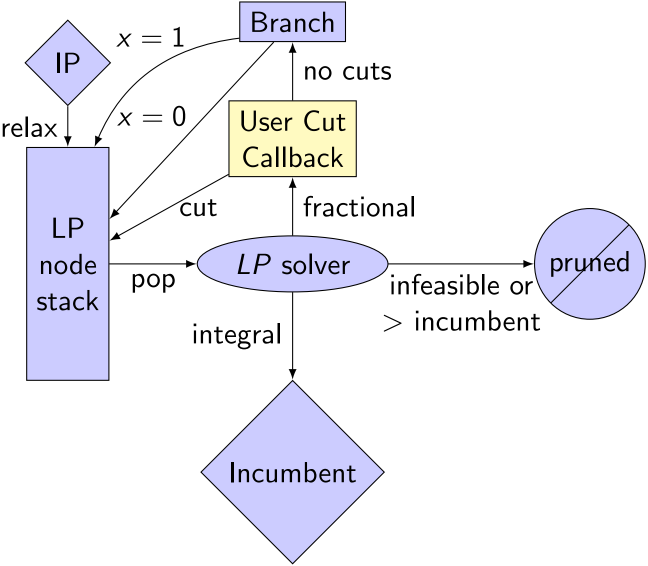
Lazy constraints vs. Lazy constraint callbacks

Some solvers support lazy constraints without callbacks. However, they have the following limitations:

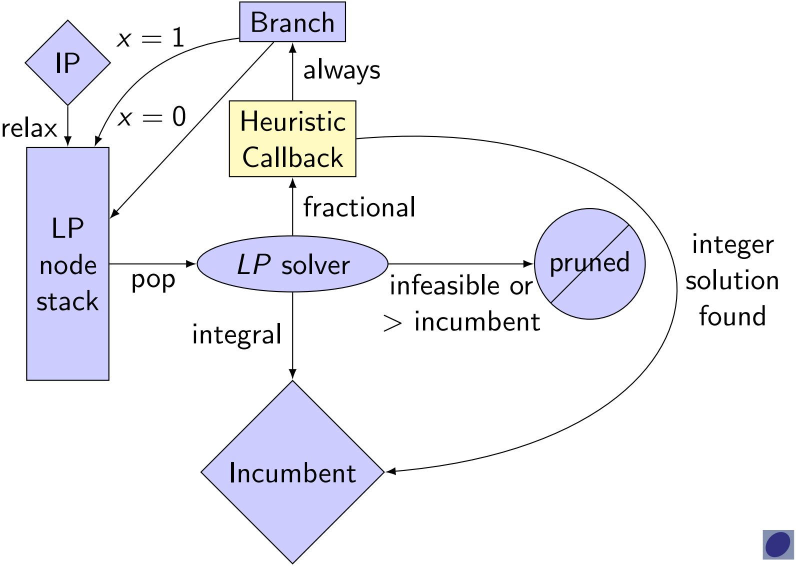
- All constraints must be generated at start
- Each constraint is checked manually against integer solutions.

Wouldn't work for the "feasible circle."

User Cut Callback



Heuristic Callback



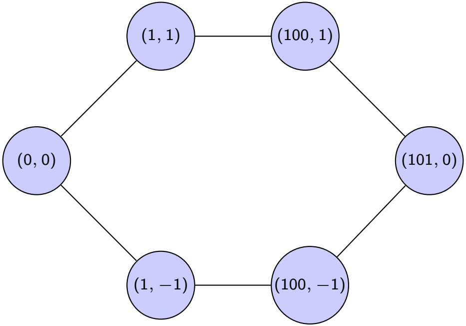
Other callbacks

- **Incumbent Callback:** optionally reject solutions, improvement heuristics
- **Branching Callback:** select variable/constraint to branch on
- **Node Selection Callback:** select node from node stack

The Traveling Salesman Problem

The TSP:

- n cities
- c_{ij} cost between cities
  - (Euclidean distance)
- Visit each city once
- Minimize total cost



Optimal tour, geometric c_{ij}.

IP for TSP

- Graph $G = (V, E)$
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- For $S \subset V$, $\delta(S) =$ edges with **one** endpoint in S

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\min \quad & \sum_{e \in E} c_e x_e \\
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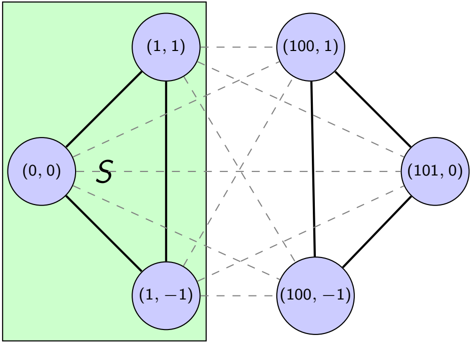

A violated cutset constraint

Exercise: Solving TSP in Julia w/o cutset constraints

Input: a symmetric 2d matrix c_{ij} with $c_{ii} = 0$
Output: the optimal cost, and an array with the city indices in order
Step 1: Ignore the cutset constraints, make sure it works.

- Create variables x_{ij} for $i = 1, \ldots, n$, $j = 1, \ldots, n$
- Add constraints

$$x_{ii} = 0$$
$$x_{ij} = x_{ji}$$

- Add degree $= 2$ constraints & objective
- Use `extractCycle` to get the optimal tour when you are done

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Exercise: Check your work

- 3 cities (in `testTspSolver.jl`)
- 6 cities with subtours (in `testTspSolver.jl`)
- Try `plotTour`

Exercise: Add cutset constraints with lazy constraints

Don't check each of $2^V - 2$ cutsets! Use *separation*!

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- Check your work on 6 cities
- Try a larger instance from TSPLIB (in `tsplib.jl`)

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Aside: the TSP and separation

The *separation problem* for a polyhedron P:

- Given $\mathbf{x}$ show $\mathbf{x} \in P$, or find a violated constraint

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The *separation problem* for a polyhedron P:

- Given $\mathbf{x}$ show $\mathbf{x} \in P$, or find a violated constraint

For lazy constraints, we assumed in addition that $\mathbf{x}$ was integer, a *much easier* problem!

Use the separation problem for *user cut callbacks*.

For TSP, the separation by n Max-Flow Min-Cut computations.

Challenge: use Julia (make n LPs in each callback)!

Aside: more callbacks for TSP

Easy **heuristic callback**:

- Sort edges by LP relaxation value
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Two-Opt: given an integer TSP solution:

- Find a better solution by changing at most two edges
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Two-Opt: given an integer TSP solution:

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Incumbent callback + heuristic callback + two-opt:

- Use incumbent callback to grab integer solutions
- Run two-opt
- If solution improves, add with heuristic callback

Software-as-a-Service (SaaS)

Sending a solver is hard

- Licenses
- Compiling
- Data/Databases
- One time revenue

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Bring problem to the solver!

TSP-as-a-Service

Code is ready. Lets try it!

- Navigate to `cd winstonWorks`
- Run `julia tsp_service_winston.jl`
- Wait for terminal output `Listening on 8000...`
- Go to `http://localhost:8000/stationservice/42.3/42.4/-71.2/-71.0`

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It worked! Now lets look at `tsp_service_winston.jl`

Considerations for deploying MIPs

- Real time or offline MIP solving
- Number of simultaneous users
- Solver licenses
- Reliability/maximum solve time

Deployment Options

Option	Pros	Cons
Personal computer	Easy	Spilled coffee Two concurrent users Blackouts International lag Data Safety
Rent a box	Pretty Easy	Two concurrent users International lag Data safety
Cloud (Amazon/Google)	Scale up for many users Data safety Low latency Monitoring/uptime	Set up time Learn a system Can be $$
Gurobi a la cart (Amazon) (*Solver only*)	Pay for what you use Scale up for many users	Can be $$ No callbacks