CAT2025

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$K = 500$

$\beta = 0{,}5$

$\alpha = 50°$

$J = 400$

$\theta_E = 100°$

$$J\dot{\omega} = \tau(\theta)\,C_m - \beta\omega - K\theta$$

$$\tau(\theta) = \frac{\cos(\alpha)}{1-\left(\operatorname{sen}(\alpha)\cos(\theta)\right)^2}$$

CONVERTO I GRADI IN RADIANTI

→ LE FUNZ. TRIGONOMETRICHE USANO I RADIANTI

$$\theta_E = 100° = \frac{100\pi}{180} = \frac{5}{9}\pi = 1{,}7453$$

$$\alpha = 50° = \frac{50\pi}{180} = \frac{5}{18}\pi = 0{,}8727$$

$$\Rightarrow \tau(\theta) = \frac{\cos\left(\frac{5}{18}\pi\right)}{1-\left(\operatorname{sen}\left(\frac{5}{18}\pi\right)\cos\left(\frac{5}{9}\pi\right)\right)^2} = 0{,}6544$$

FORMA DI STATO

$x_1 = \theta \qquad x_2 = \omega \qquad u = C_m$

$$\Rightarrow \dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{\tau(x_1)}{J}u - \frac{\beta x_2}{J} - \frac{K}{J}x_1 \longrightarrow \dot{x}_2 = \frac{1}{J}\left(\tau(x_1)C_m - \beta x_2 - Kx_1\right)$$

$$\Rightarrow \dot{x} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}x_2\\ \frac{1}{J}\left(\tau(x_1)C_m - \beta x_2 - Kx_1\right)\end{bmatrix} \qquad y = x_1$$

$$\Rightarrow x = \begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad u = C_m \qquad y = x_1$$

PUNTO DI EQUILIBRIO

$\dot{x}_1 = 0$

$\dot{x}_2 = 0$

$$\Rightarrow \omega = 0$$

$$0 = \frac{\tau(\theta_E)}{J}C_m - \underbrace{\frac{\beta x_2}{J}}_{0} - \frac{K}{J}x_1$$

$$\frac{\tau(\theta_E)}{J}C_m = \frac{K}{J}x_1$$

$$C_m = \frac{K}{J}x_1 \cdot \frac{J}{\tau(\theta_E)} = \frac{K\theta_E}{\tau(\theta_E)} = 1333{,}6 \quad (1333{,}5340)$$

$$\left.\begin{aligned}x_{1E} &= \theta_E = 1{,}7453\\ x_{2E} &= 0\end{aligned}\right\} x_E = \begin{bmatrix}1{,}7453\\ 0\end{bmatrix} = \begin{bmatrix}\theta_E\\ \omega_E\end{bmatrix}$$

$$\left. u_E = C_m = 1333{,}6 \ \mathrm{Nm} \right\} u_E$$

LINEARIZZAZIONE

LINEARIZZO INTORNO ALL'EQUILIBRIO $\longrightarrow \begin{aligned}\delta\dot{x} &= A\delta x + B\delta u\\ \delta y &= C\delta x + D\delta u\end{aligned} \longrightarrow A = \left.\frac{\partial f}{\partial x}\right|_{x_E, u_E}, \quad B = \left.\frac{\partial f}{\partial u}\right|_{x_E, u_E}, \quad C = \left.\frac{\partial h}{\partial x}\right|_{x_E, u_E} = [1 \ \ 0], \quad D = \left.\frac{\partial h}{\partial u}\right|_{x_E, u_E} = [0]$

CALCOLO DERIVATA
$$\frac{\partial \tau}{\partial \theta} = \frac{-2\cos(\alpha)\left(\operatorname{sen}(\alpha)\cos(\theta)\right)\left(\operatorname{sen}(\alpha)\operatorname{sen}(\theta)\right)}{\left[1-\left(\operatorname{sen}(\alpha)\cos(\theta)\right)^2\right]^2} = 0{,}1337$$

EQUAZIONI LINEARIZZATE

$$\delta\dot{x}_1 = \delta x_2$$

$$\delta\dot{x}_2 = -\frac{K}{J}\delta x_1 - \frac{\beta}{J}\delta x_2 + \frac{\tau(\theta_E)}{J}\delta u$$

MATRICI SISTEMA LINEARIZZATO

$$A = \begin{bmatrix}0 & 1\\ -\frac{K}{J} & -\frac{\beta}{J}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{5}{4} & -\frac{1}{800}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1{,}25 & -1{,}25\cdot 10^{-3}\end{bmatrix}$$

$$B = \begin{bmatrix}0\\ \frac{\tau(\theta_E)}{J}\end{bmatrix} = \begin{bmatrix}0\\ 0{,}001636\end{bmatrix} = \begin{bmatrix}0\\ 1{,}636\cdot 10^{-3}\end{bmatrix}$$

$$C = [1 \quad 0]$$

$$D = 0$$

FUNZIONE DI TRASFERIMENTO

$$G(s) = C(sI-A)^{-1}B$$

$$(sI-A) = \begin{bmatrix}s & 1\\ -1{,}25 & s+1{,}25\cdot 10^{-3}\end{bmatrix}$$

$$\det(sI-A) = s(s+1{,}25\cdot 10^{-3}) + 1{,}25 = s^2 + 1{,}25\cdot 10^{-3}s + 1{,}25$$

$$(-1)^{i+j}\cdot C_{ij} \Rightarrow \begin{aligned}&(-1)^{1+1}\cdot(s+1{,}25\cdot 10^{-3})\\ &(-1)^{1+2}\ (-1{,}25)\\ &(-1)^{2+1}\ (1)\\ &(-1)^{2+2}\ (s)\end{aligned} \Rightarrow \begin{bmatrix}s+1{,}25\cdot 10^{-3} & -1\\ 1{,}25 & s\end{bmatrix}$$

$$A^{-1} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}^{-1} = \frac{1}{\det(A)}\begin{pmatrix}C_{11} & C_{21}\\ C_{12} & C_{22}\end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$$

$$(sI-A)^{-1} = \frac{\operatorname{ADJ}(sI-A)}{\det(sI-A)}$$

$$\operatorname{ADJ}\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}s+1{,}25\cdot 10^{-3} & -1\\ 1{,}25 & s\end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{s^2+1{,}25\cdot 10^{-3}s+1{,}25}\begin{bmatrix}s+1{,}25\cdot 10^{-3} & -1\\ 1{,}25 & s\end{bmatrix} =$$

$$= \begin{bmatrix}\frac{s+1{,}25\cdot 10^{-3}}{s^2+1{,}25\cdot 10^{-3}+1{,}25} & \frac{-1}{s^2+1{,}25\cdot 10^{-3}+1{,}25}\\ \frac{1{,}25}{s^2+1{,}25\cdot 10^{-3}+1{,}25} & \frac{s}{s^2+1{,}25\cdot 10^{-3}+1{,}25}\end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}B = \frac{1{,}636\cdot 10^{-3}}{s^2+1{,}25\cdot 10^{-3}s+1{,}25}$$

$$C\cdot(sI-A)^{-1} = [1 \ \ 0]\begin{bmatrix}\frac{s+1{,}25\cdot 10^{-3}}{s^2+1{,}25\cdot 10^{-3}s+1{,}25} & \frac{-1}{s^2+1{,}25\cdot 10^{-3}s+1{,}25}\\ \frac{1{,}25}{s^2+1{,}25\cdot 10^{-3}s+1{,}25} & \frac{s}{s^2+1{,}25\cdot 10^{-3}s+1{,}25}\end{bmatrix} =$$

$$= \begin{bmatrix}\frac{s+1{,}25\cdot 10^{-3}}{s^2+1{,}25\cdot 10^{-3}s+1{,}25} & \frac{-1}{s^2+1{,}25\cdot 10^{-3}s+1{,}25}\end{bmatrix}$$

$$C\cdot(sI-A)^{-1}\cdot B = \begin{bmatrix}\frac{s+1{,}25\cdot 10^{-3}}{s^2+1{,}25\cdot 10^{-3}s+1{,}25} & \frac{-1}{s^2+1{,}25\cdot 10^{-3}s+1{,}25}\end{bmatrix}\cdot\begin{bmatrix}0\\ 1{,}636\cdot 10^{-3}\end{bmatrix} =$$

$$= \frac{1{,}636\cdot 10^{-3}}{s^2+1{,}25\cdot 10^{-3}s+1{,}25}$$

PROGETTO REGOLATORE

ERRORE A REGIME $|e_\infty| \leq e^* = 0{,}01$

MARGINE DI FASE $M_\varphi \geq 33°$

SOVRAELONGAZIONE $S\% \leq 16\%$

TEMPO DI ASSESTAMENTO $T_{a,\varepsilon} = 0{,}003\ s$

ATTENUAZIONE DISTURBI SULL'USCITA → ALMENO 50 dB PER $\omega \in [0\,;\ 0{,}8]$

ATTENUAZIONE RUMORE DI MISURA → ALMENO 72 dB PER $\omega \in [1{,}2\cdot 10^5\,;\ 5\cdot 10^6]$

⇒ PER SODDISFARE QUESTE SPECIFICHE USO UN REGOLATORE PID CON FUNZ. DI TRASF.

$$R(s) = K_p + \frac{K_i}{s} + K_d s$$

① PER ELIMINARE L'ERRORE A REGIME DEVO AGGIUNGERE