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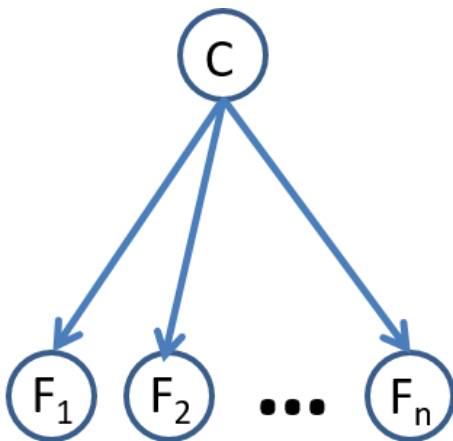
**Time taken** 1 hour 29 mins

**Grade** 44.00 out of 75.00 (58.67%)

#### Information

## Distributions and BNs

Consider the following Bayesian network:



#### Information

Which of the following expressions (if any) correctly describe the full joint distribution represented by this network?

### Question 1

Correct

Mark 1.00 out of 1.00

$$P(C) \sum_i P(F_i | C)$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 2**

Correct

Mark 1.00 out of 1.00

$$P(C) \prod_i P(F_i)$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 3**

Incorrect

Mark 0.00 out of 1.00

$$P(F_1, \dots, F_n, C)$$

- ☐ True
- ☒ False ✗

The correct answer is 'True'.

**Question 4**

Correct

Mark 1.00 out of 1.00

$$\prod_i P(F_i \mid F_{i-1})$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 5**

Correct

Mark 1.00 out of 1.00

$$P(C)P(C \mid F_1, \dots, F_n)$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 6**

Correct

Mark 1.00 out of 1.00

$$P(C) \prod_i P(F_i \mid F_{i-1})$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Information**

From now on, suppose all the variables in the above model are Boolean. Which of the following statements are correct?

**Question 7**

Correct

Mark 1.00 out of 1.00

If we reversed the direction of all arrows, the full joint distribution represented by the network would remain the same, if we keep the distribution tables unchanged

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 8**

Correct

Mark 1.00 out of 1.00

If we reversed the direction of all arrows, the number of non-redundant parameters would grow to a number of  $(n \cdot 2^n)$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 9**

Correct

Mark 1.00 out of 1.00

If we remove edge  $(C \rightarrow F_1)$ ,  $(C)$  and  $(F_2)$  become conditionally independent, given  $(F_1)$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 10**

Incorrect

Mark 0.00 out of 1.00

Adding an edge  $(F_1 \rightarrow F_2)$  would lead to an illegal (cyclic) graph structure

- ☒ True ✗
- ☐ False

The correct answer is 'False'.

**Question 11**

Incorrect

Mark 0.00 out of 1.00

If we remove the edge  $(C \rightarrow F_1)$ , then  $(C, F_2, \dots, F_n \sim F_1)$

- ☐ True
- ☒ False ✗

The correct answer is 'True'.

**Question 12**

Incorrect

Mark 0.00 out of 1.00

If we reversed the direction of all arrows, the number of non-redundant parameters would grow to a number of  $(2^{n+n})$

- ☐ True
- ☒ False ✗

The correct answer is 'True'.

**Question 13**

Correct

Mark 1.00 out of 1.00

If we removed the edge  $(C \rightarrow F_1)$ , the number of non-redundant parameters in the network would be halved

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 14**

Incorrect

Mark 0.00 out of 1.00

Adding an edge  $(F_1 \rightarrow F_2)$  would double the number of non-redundant parameters in  $(F_2)$ 's probability table

- ☐ True
- ☒ False ✗

The correct answer is 'True'.

**Information**

Now consider having to learn the parameters of this network from a set of training examples  $\mathcal{D} = \{\mathbf{x}_1, \dots, \mathbf{x}_M\}$ , in a maximum-likelihood (ML) way. Which of the following statements are true?

**Question 15**

Incorrect

Mark 0.00 out of 1.00

Each training example is a vector with  $(n)$  components

- ☒ True ✗
- ☐ False

The correct answer is 'False'.

**Question 16**

Correct

Mark 1.00 out of 1.00

When estimating  $P(F_i \mid C)$  for some variable  $F_i$ , the distributions of the other variables  $F_j, j \neq i$  in  $\mathcal{D}$  are irrelevant

- ☒ True ✓
- ☐ False

The correct answer is 'True'.

**Question 17**

Incorrect

Mark 0.00 out of 1.00

If all the training examples in  $\mathcal{D}$  were identical and \*only\* contained  $\text{false}$  values for all variables, all parameters estimated for the model will be zero

- ☒ True ✗
- ☐ False

The correct answer is 'False'.

**Question 18**

Correct

Mark 1.00 out of 1.00

The estimate  $\hat{P}(C = \text{true})$  for the parentless variable  $C$  will always be 1

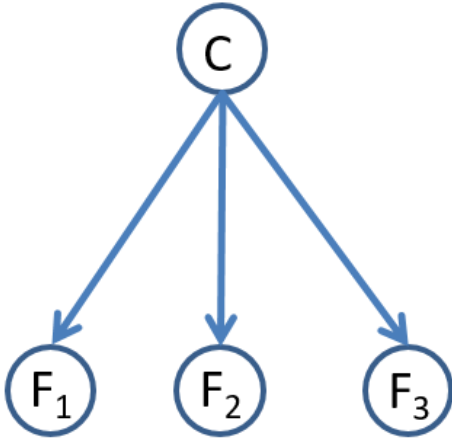
- ☐ True
- ☒ False ✓

The correct answer is 'False'.

### Information

## Inference BNs

Consider the following simple Bayesian network model:



with all variables being binary with two values  $t$  and  $f$ , and the following (Conditional) Probability Tables (where the three variables  $F_1 \dots F_3$  have identical tables):

$P(C):$	t	f
	0.5	0.5

$P(F_i   C):$	t	f
t	0.5	0.5
f	0.0	1.0

### Information

Calculate the following probabilities (where  $c$  stands for  $C = t$ ,  $\neg f_i$  for  $F_i = f$  etc). **Enter all decimal places.**

### Question 19

Correct

Mark 1.00 out of 1.00

$$P(\neg c, \neg f_1, \neg f_2, \neg f_3)$$

Answer:



The correct answer is: 0.5

**Question 20**

Incorrect

Mark 0.00 out of 1.00

$$P(\neg c, f_1, f_2, f_3)$$

Answer: 

The correct answer is: 0

**Question 21**

Incorrect

Mark 0.00 out of 1.00

$$P(\neg f_1, \neg f_2, \neg f_3)$$

Answer: 

The correct answer is: 0.5625

**Question 22**

Correct

Mark 1.00 out of 1.00

$$P(c, \neg f_1, \neg f_2, \neg f_3)$$

Answer: 

The correct answer is: 0.0625



**Question 23**

Correct

Mark 1.00 out of 1.00

How many topological orderings are there for this graph structure?

- ☐ 1
- ☐ 2
- ☐ 3
- ☒ 6 ✓
- ☐ 7
- ☐ 12

The correct answer is:

6

**Information**

Suppose we have drawn the following set of samples from the Bayesian network above:

C	F1	F2	F3
t	t	f	f
t	t	f	f
t	t	t	f
t	t	t	t
t	t	t	t
f	f	f	f
f	f	f	f
t	f	f	f
t	f	f	f
t	f	f	f

What estimates will be get by Rejection Sampling for the following queries:

**Question 24**

Incorrect

Mark 0.00 out of 1.00

$$P(c \mid \neg f_1, \neg f_2, \neg f_3):$$

0.33 ✖

$$P(\neg c \mid \neg f_1, \neg f_2, \neg f_3):$$

0.66 ✖

**Question 25**

Incorrect

Mark 0.00 out of 1.00

$P(c \mid \neg f_1):$   ✖

$P(\neg c \mid \neg f_1):$   ✖

**Question 26**

Correct

Mark 1.00 out of 1.00

$P(c \mid f_1):$   ✔

$P(\neg c \mid f_1):$   ✔

**Information**

Consider the query  $P(C \mid f_1)$ . What is the correct weight for the following samples, when we do likelihood weighting?

**Question 27**

Incorrect

Mark 0.00 out of 1.00

$\neg c, f_1, \neg f_2, \neg f_3$

Answer:  ✖

The correct answer is: 0

**Question 28**

Correct

Mark 1.00 out of 1.00

$c, f_1, f_2, f_3$

Answer:  ✔

The correct answer is: 0.5

**Question 29**

Correct

Mark 1.00 out of 1.00

 $c, f_1, f_2, \neg f_3$ 

Answer: 0.5



The correct answer is: 0.5

**Information**

Consider the query  $P(C \mid f_1)$ . Which of the following (if any) is a correct Gibbs Resampling Distribution for variable  $C$ ?

**Question 30**

Incorrect

Mark 0.00 out of 1.00

 $1/Z \times P(C \mid F_1, F_2, F_3)$ 

- ☐ True
- ☒ False ✖

The correct answer is 'True'.

**Question 31**

Correct

Mark 1.00 out of 1.00

 $1/Z \times P(C)P(F_1 \mid C)P(F_2 \mid C)P(F_3 \mid C)$ 

- ☒ True ✔
- ☐ False

The correct answer is 'True'.

**Question 32**

Correct

Mark 1.00 out of 1.00

$$1/Z \times P(C)P(F_1 | C)$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 33**

Correct

Mark 1.00 out of 1.00

$$1/Z \times P(F_1 | C)$$

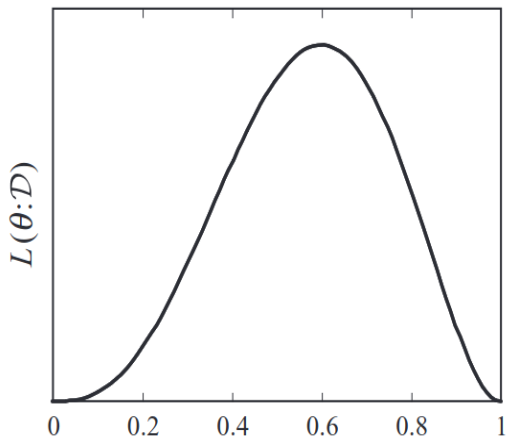
- ☐ True
- ☒ False ✓

The correct answer is 'False'.

#### Information

### Parameter Learning

Consider the example, from the lecture slides, of learning the parameter  $\theta = P(heads)$  for our thumbtack, where we had the following likelihood function  $L(\theta, \mathcal{D})$  based on a set  $\mathcal{D}$  of  $N = 5$  observations, of which  $k = 3$  were heads, and  $(N - k) = 2$  were tails:



#### Information

Which of the following statements are true (about **this specific likelihood function**, relating to **this specific dataset**):

#### Question 34

Incorrect

Mark 0.00 out of 1.00

$$L(0 : \mathcal{D}) = 0$$

- ☐ True
- ☒ False ✖

The correct answer is 'True'.

**Question 35**

Correct

Mark 1.00 out of 1.00

$$L(0.4 : \mathcal{D}) = L(0.6 : \mathcal{D}) - 2$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 36**

Correct

Mark 1.00 out of 1.00

$$L(0.6 : \mathcal{D}) = 3/5$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 37**

Correct

Mark 1.00 out of 1.00

$$L(0.6 : \mathcal{D}) = P(\mathcal{D} \mid \theta = 0.6)$$

- ☒ True ✓
- ☐ False

The correct answer is 'True'.

**Information**

Which of the following statements are true **in general** (i.e., about any possible likelihood function over one parameter  $\theta$ , and any dataset):

**Question 38**

Correct

Mark 1.00 out of 1.00

The likelihood for any  $\theta$  cannot be  $> 1.0$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 39**

Incorrect

Mark 0.00 out of 1.00

The likelihood for  $\theta = 0$  and  $\theta = 1$  is always 0

- ☒ True ✗
- ☐ False

The correct answer is 'False'.

**Question 40**

Correct

Mark 1.00 out of 1.00

The likelihood function is a probability distribution over  $\theta$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 41**

Correct

Mark 1.00 out of 1.00

A likelihood function can only have one maximum

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

## Information

Assume we want to do Bayesian parameter estimation for some parameter  $\theta$ , and assume we use a uniform distribution over  $\theta$  as the prior, which gives the same probability to all values of  $\theta$ . Which of the following statements are correct:

### Question 42

Correct

Mark 1.00 out of 1.00

$$P(\theta \mid \mathcal{D}) = P(\mathcal{D} \mid \theta)$$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

### Question 43

Correct

Mark 1.00 out of 1.00

The posterior will be proportional to the likelihood

- ☒ True ✓
- ☐ False

The correct answer is 'True'.

### Question 44

Correct

Mark 1.00 out of 1.00

The posterior mean estimate can be different from the MAP estimate

- ☒ True ✓
- ☐ False

The correct answer is 'True'.



#### Information

## Model Fitting and Structure Learning

Consider a world with three Boolean variables  $A, B, C$  with full joint distribution  $P(A, B, C)$ , and four potential Bayesian network graph structures over these variables:

- (1)  $A \quad B \quad C$
- (2)  $A \longrightarrow B \longrightarrow C$
- (3)  $A \quad B \longleftarrow C$
- (4)  $A \longrightarrow B \longleftarrow C$

Also, suppose we have a training set  $\mathcal{D}$  of 100 examples sampled i.i.d. from the above joint distribution. Which of the following statements are true (always assuming that a network graph is parametrised with its ML parameters on  $\mathcal{D}$ )?

### Question 45

Incorrect

Mark 0.00 out of 1.00

The description length of a model depends not on the number of parent relations, but on the size of its parameters (probabilities in the CPDs)

- ☒ True ✗
- ☐ False

The correct answer is 'False'.

### Question 46

Correct

Mark 1.00 out of 1.00

If we replaced the model dimension term in the BIC score with any constant value  $> 0$ , a learning algorithm using this score would always learn the model (1), regardless of the data in  $\mathcal{D}$

- ☐ True
- ☒ False ✓

The correct answer is 'False'.

**Question 47**

Incorrect

Mark 0.00 out of 1.00

The BIC score will always prefer structure (1) to model (4), because it requires strictly fewer parameters

- ☒ True ✖
- ☐ False

The correct answer is 'False'.

**Question 48**

Correct

Mark 1.00 out of 1.00

Models (3) and (4) will always have the same likelihood on  $\mathcal{D}$

- ☐ True
- ☒ False ✔

The correct answer is 'False'.

**Question 49**

Incorrect

Mark 0.00 out of 1.00

Model (1) can never have a higher likelihood than Model (2) on  $\mathcal{D}$ .

- ☐ True
- ☒ False ✖

The correct answer is 'True'.

**Question 50**

Incorrect

Mark 0.00 out of 1.00

Model structure (2) is preferable compared to (4) because it models in the direction of causality.

- ☒ True ✖
- ☐ False

The correct answer is 'False'.

**Question 51**

Correct

Mark 1.00 out of 1.00

If all four models had the same likelihood relative to  $\mathcal{D}$ , the BIC score would prefer model

- ☐ 2
- ☒ 1 ✓
- ☐ 4
- ☐ 3

The correct answer is:

1

#### Information

## Temporal Models / HMMs

Consider a HMM defined over a discrete state variable  $S = \{s_1, \dots, s_N\}$  and a discrete observation variable  $O = \{o_1, \dots, o_M\}$ . Let matrix **A** be the state transition matrix, matrix **B** the observation model, and  $\Pi$  the initial state distribution model, as defined in the lecture slides.

#### Information

Give the following quantities:

#### Question 52

Incorrect

Mark 0.00 out of 1.00

Give the dimension of **B**

rows:  ✖ , columns:  ✖

#### Question 53

Incorrect

Mark 0.00 out of 1.00

Give the dimension of  $P(o_2 \mid S)$

numbers:  ✖

#### Question 54

Correct

Mark 1.00 out of 1.00

Give the dimension of  $P(S \mid o_1)$

numbers:  ✔

#### Question 55

Correct

Mark 1.00 out of 1.00

Give the dimension of **A**

rows:  ✔ , columns:  ✔

**Question 56**

Correct

Mark 1.00 out of 1.00

Give the dimension of  $P(O \mid s_N)$ 

numbers:

**Question 57**

Correct

Mark 1.00 out of 1.00

Give the dimension of  $\Pi$ 

rows:



, columns:

**Information**

Assume the above HMM is unrolled to a length  $T = 2$  (call the resulting network HMM2). Which of the following are valid topological orderings w.r.t. HMM2?

**Question 58**

Incorrect

Mark 0.00 out of 1.00

 $s^{(0)}, s^{(1)}, o^{(1)}, s^{(2)}, o^{(2)}$ ☐ True☒ False 

The correct answer is 'True'.

**Question 59**

Incorrect

Mark 0.00 out of 1.00

 $\mathbf{S}^{(0:2)}, \mathbf{O}^{(1:2)}$ ☐ True☒ False 

The correct answer is 'True'.

**Question 60**

Incorrect

Mark 0.00 out of 1.00

$$S^{(0)}, O^{(1)}, S^{(1)}, O^{(2)}, S^{(2)}$$

☒ True ✖☐ False

The correct answer is 'False'.

**Information**

Which of the following statements are correct for any HMM?

**Question 61**

Correct

Mark 1.00 out of 1.00

Given an observation sequence  $\mathbf{O} = \mathbf{o}^{(1:t)}$ ,  $P(S^{(t+1)} | \mathbf{O})$  can be predicted by running the Filtering Algorithm, followed by the Prediction Algorithm.

☒ True ✔☐ False

The correct answer is 'True'.

**Question 62**

Correct

Mark 1.00 out of 1.00

Given an observation sequence  $\mathbf{O} = \mathbf{o}^{(1:t)}$ , one can also ignore the observations completely and still make a prediction for  $P(S^{(t)})$

☒ True ✔☐ False

The correct answer is 'True'.

**Question 63**

Correct

Mark 1.00 out of 1.00

The message  $\sum p$  in the prediction algorithm always sums to 1

- ☒ True ✓
- ☐ False

The correct answer is 'True'.

**Question 64**

Incorrect

Mark 0.00 out of 1.00

Smoothing could also be done by unrolling the HMM to the observation sequence length  $T$  and then calculating all distributions  $P(S^{(t)})$  via Inference by Enumeration

- ☐ True
- ☒ False ✗

The correct answer is 'True'.

#### Information

## Linear Gaussian Models

Consider the Kalman Filter model of a bicycle riding along a one-dimensional road, as introduced in the lecture slides, with random variables  $P$  (position),  $V$  (velocity),  $Z$  (GPS values).

#### Information

Which of the following distributions (if any) does the Kalman Filtering Algorithm calculate?

#### Question 65

Incorrect

Mark 0.00 out of 1.00

$$P(P^{(t)}, V^{(t)} \mid \mathbf{z}^{(1:t)})$$

- ☐ True
- ☒ False ❌

The correct answer is 'True'.

#### Question 66

Correct

Mark 1.00 out of 1.00

$$P(P^{(t)}, V^{(t)} \mid z^{(t)}, p^{(t-1)}, v^{(t-1)})$$

- ☐ True
- ☒ False ✅

The correct answer is 'False'.



**Question 67**

Incorrect

Mark 0.00 out of 1.00

$$P(P^{(t)}, V^{(t)} \mid P^{(t-1)}, V^{(t-1)})$$

☒ True ✖☐ False

The correct answer is 'False'.

**Question 68**

Correct

Mark 1.00 out of 1.00

$$P(P^{(t)}, V^{(t)} \mid \mathbf{z}^{(1:t)}, p^{(t-1)}, v^{(t-1)})$$

☐ True☒ False ✔

The correct answer is 'False'.

**Question 69**

Incorrect

Mark 0.00 out of 1.00

$$P(P^{(t)}, V^{(t)} \mid z^{(t)})$$

☒ True ✖☐ False

The correct answer is 'False'.

**Information**

Which of the following assumptions about the bicycle's movement are encoded in the specific transition matrix **A** and observation model **B** that we specified in the lecture slides:

**Question 70**

Incorrect

Mark 0.00 out of 1.00

that GPS reading  $Z^{(t)}$  and velocity  $V^{(t)}$  at the same time point are independent

- ☒ True ✖
- ☐ False

The correct answer is 'False'.

**Question 71**

Correct

Mark 1.00 out of 1.00

that the probability of slowing down by a certain amount is independent of the current velocity

- ☒ True ✔
- ☐ False

The correct answer is 'True'.

**Question 72**

Correct

Mark 1.00 out of 1.00

that the GPS sensor has the same inaccuracy everywhere

- ☒ True ✔
- ☐ False

The correct answer is 'True'.

**Question 73**

Incorrect

Mark 0.00 out of 1.00

that the distribution over possible GPS values at time  $t$  depends only on  $p^{(t)}$

- ☐ True
- ☒ False ✖

The correct answer is 'True'.

**Question 74**

Incorrect

Mark 0.00 out of 1.00

that the bicycle's velocity will tend to stay the same from moment to moment

- ☐ True
- ☒ False ✖

The correct answer is 'True'.

**Question 75**

Correct

Mark 1.00 out of 1.00

that the GPS sensor readings give us no information about velocity  $\vec{v}$

- ☐ True
- ☒ False ✔

The correct answer is 'False'.