

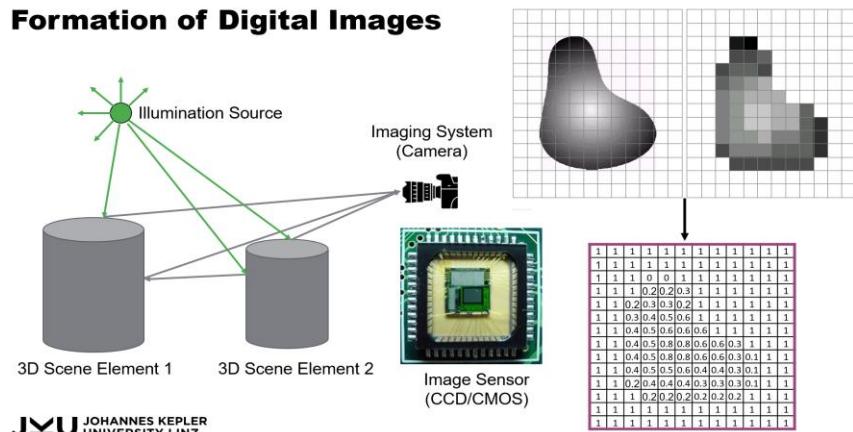
# Computer Vision



Lecture 3: Digital Image Processing  
Oliver Bimber

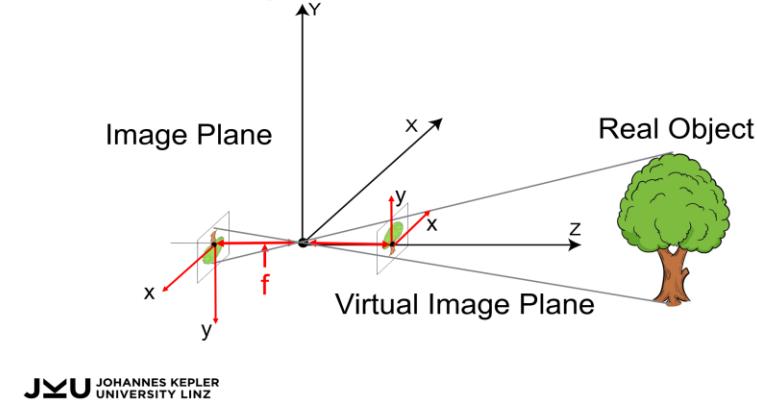
# Last Week: Capturing Digital Images

## Formation of Digital Images



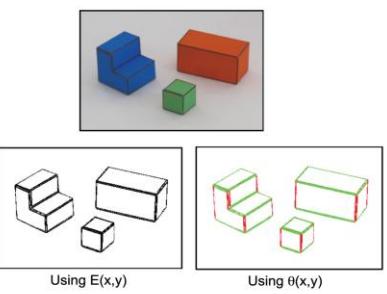
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## Perspective Projection



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## On a lower Level: Detect Features (e.g., Edges)



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Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

Approximation image derivative:

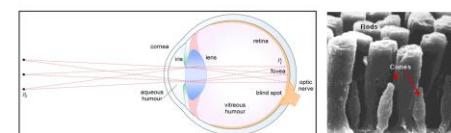
$$\frac{\partial I}{\partial x} \simeq I(x, y) - I(x - 1, y)$$

$$\text{Edge strength: } E(x, y) = |\nabla I(x, y)|$$

$$\text{Edge orientation: } \theta(x, y) = \angle \nabla I = \arctan \frac{\partial I / \partial y}{\partial I / \partial x}$$

$$\text{Edge normal: } n = \frac{\nabla I}{|\nabla I|}$$

## How Humans do it?



The Human Eye is a biological Camera (Lens, Aperture=Iris, Sensor=Retina) and is used for Measurement=Sensing.



The Perception of the Visual Cortex is really what we want to mimic with Computer Vision!

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# Course Overview

CW	Topic	Date	Place	Lab
41	Introduction and Course Overview	07.10.2025	Zoom	Lab 1
42	Capturing Digital Images	14.10.2025	Zoom	Lab 2
→ 43	Digital Image Processing	21.10.2025	Zoom	Assignment 1
44	Machine Learning	28.10.2025	Zoom	
45	Feature Extraction	04.11.2025	Zoom	Open Lab 1
46	Segmentation	11.11.2025	Zoom	Assignment 2
47	Optical Flow	18.11.2025	Zoom	Open Lab 2
48	Object Detection	25.11.2025	Zoom	Assignment 3
49	Multi-View Geometry	02.12.2025	Zoom	Open Lab 3
50	3D Vision	09.12.2025	Zoom	Assignment 4
3	Trends in Computer Vision	13.01.2026	Zoom	
4	Q&A	20.01.2026	Zoom	Open Lab 4
5	Exam	27.01.2026	HS1 (Linz), S1/S3 (Vienna), S5 (Bregenz)	
9	Retry Exam	24.02.2026	tba	

# Recap: Detect Features (e.g., Edges)

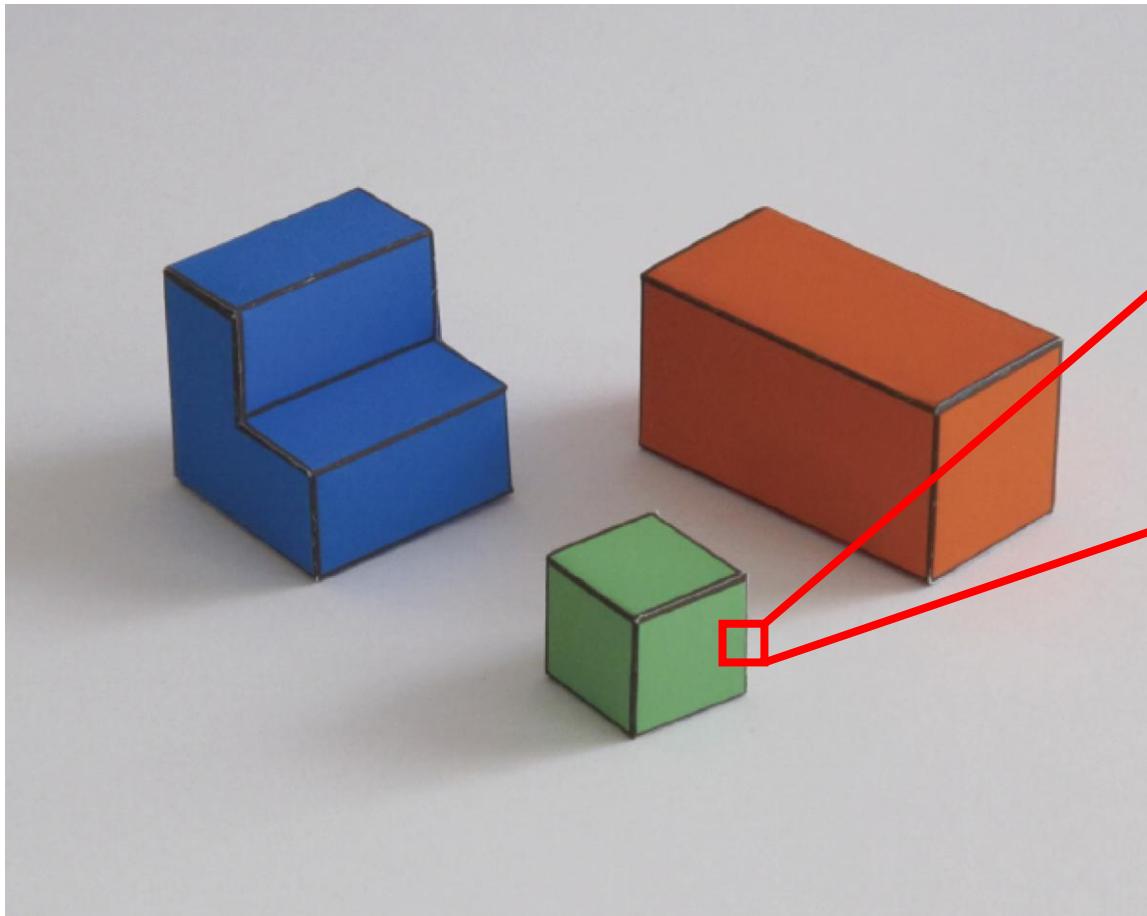


Image Patch

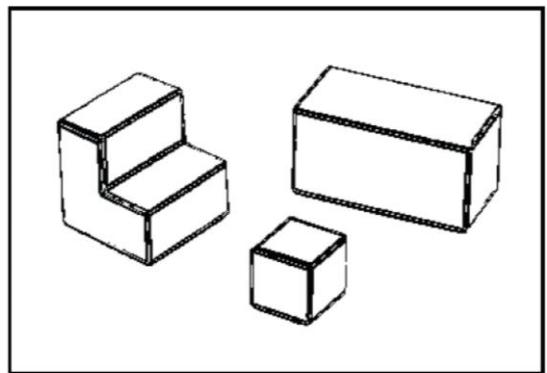
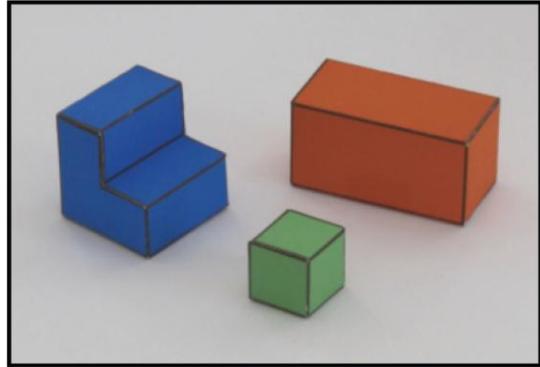
... 125, 126, 50, 10, 223, 223, ...  
..., 124, 126, 50, 10, 223, 224, ...  
..., 125, 127, 51, 9, 223, 224, ...  
...

What Measure can tell us that there is an Edge here?

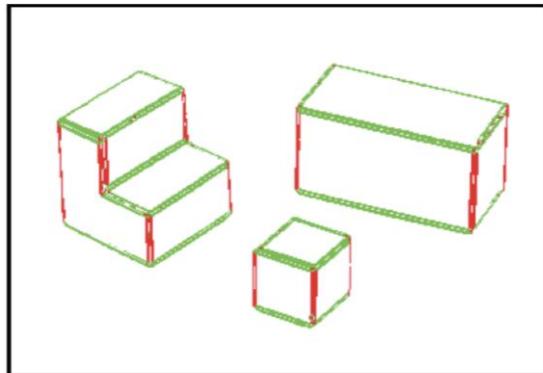
Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

# Recap: Detect Features (e.g., Edges)



Using  $E(x,y)$



Using  $\theta(x,y)$

Image gradient:

$$\nabla \mathbf{I} = \left( \frac{\partial \mathbf{I}}{\partial x}, \frac{\partial \mathbf{I}}{\partial y} \right)$$

Approximation image derivative:

$$\frac{\partial \mathbf{I}}{\partial x} \simeq \mathbf{I}(x, y) - \mathbf{I}(x - 1, y)$$

Edge strength

$$E(x, y) = |\nabla \mathbf{I}(x, y)|$$

Edge orientation:

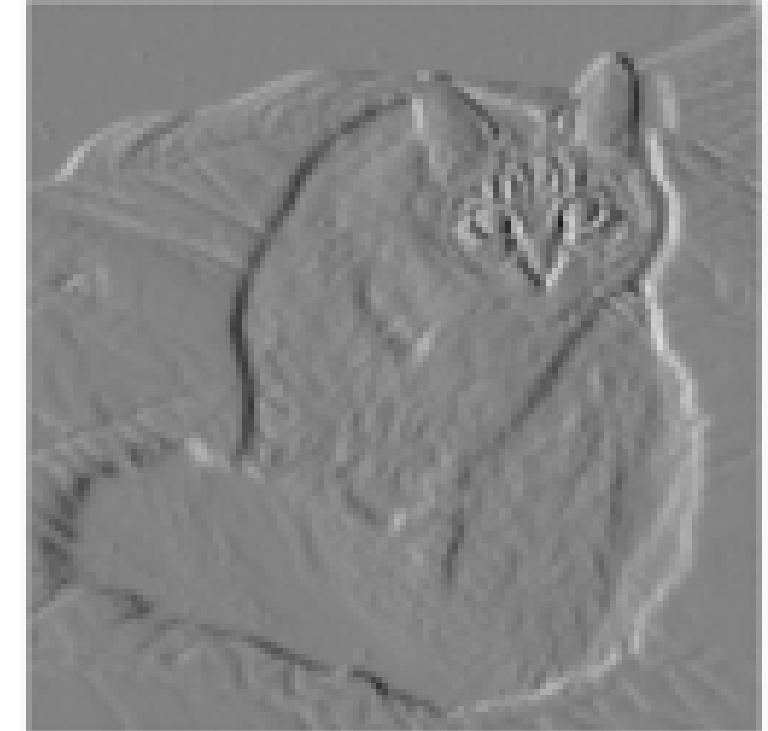
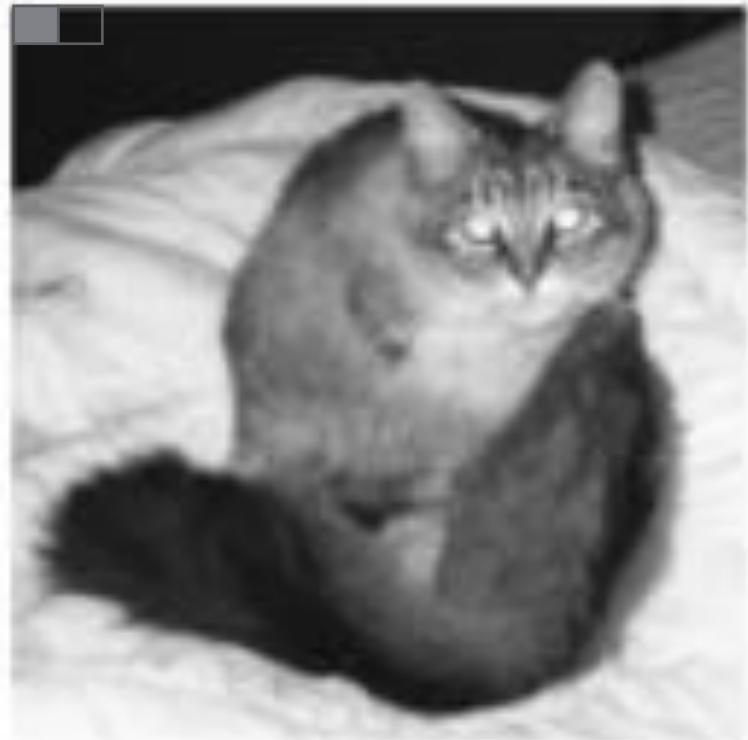
$$\theta(x, y) = \angle \nabla \mathbf{I} = \arctan \frac{\partial \mathbf{I} / \partial y}{\partial \mathbf{I} / \partial x}$$

Edge normal:

$$\mathbf{n} = \frac{\nabla \mathbf{I}}{|\nabla \mathbf{I}|}$$

# Image Gradients

What happens if we apply this to every Pixel?

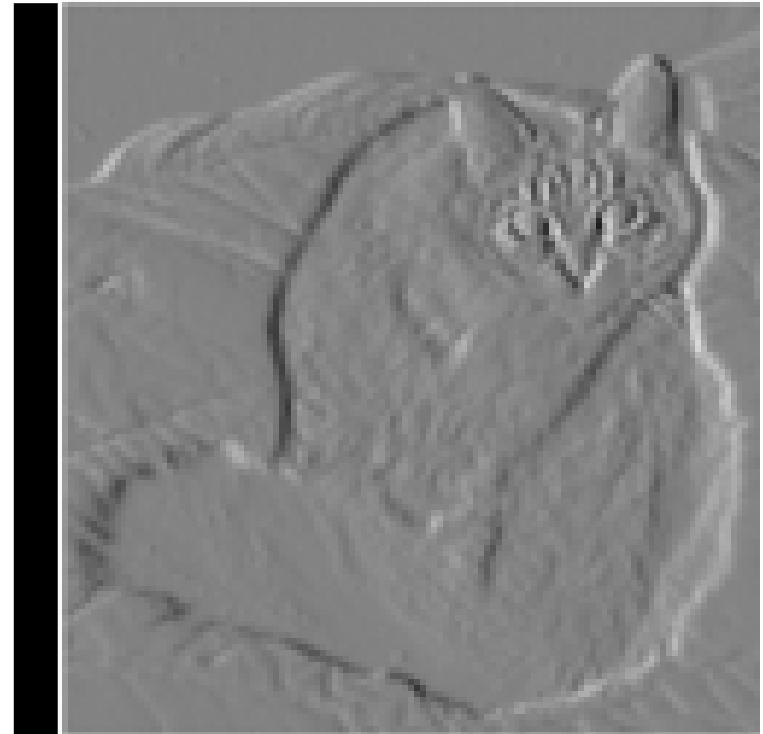


$$\mathbf{I}(x, y)$$

$$\frac{\partial \mathbf{I}}{\partial x} \simeq \mathbf{I}(x, y) - \mathbf{I}(x - 1, y)$$

# Image Gradients

And what happens if we shift the Image?



$$\mathbf{I}(x, y)$$

This is called Shift-Invariance.

$$\frac{\partial \mathbf{I}}{\partial x} \simeq \mathbf{I}(x, y) - \mathbf{I}(x - 1, y)$$

# Convolution

x

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0

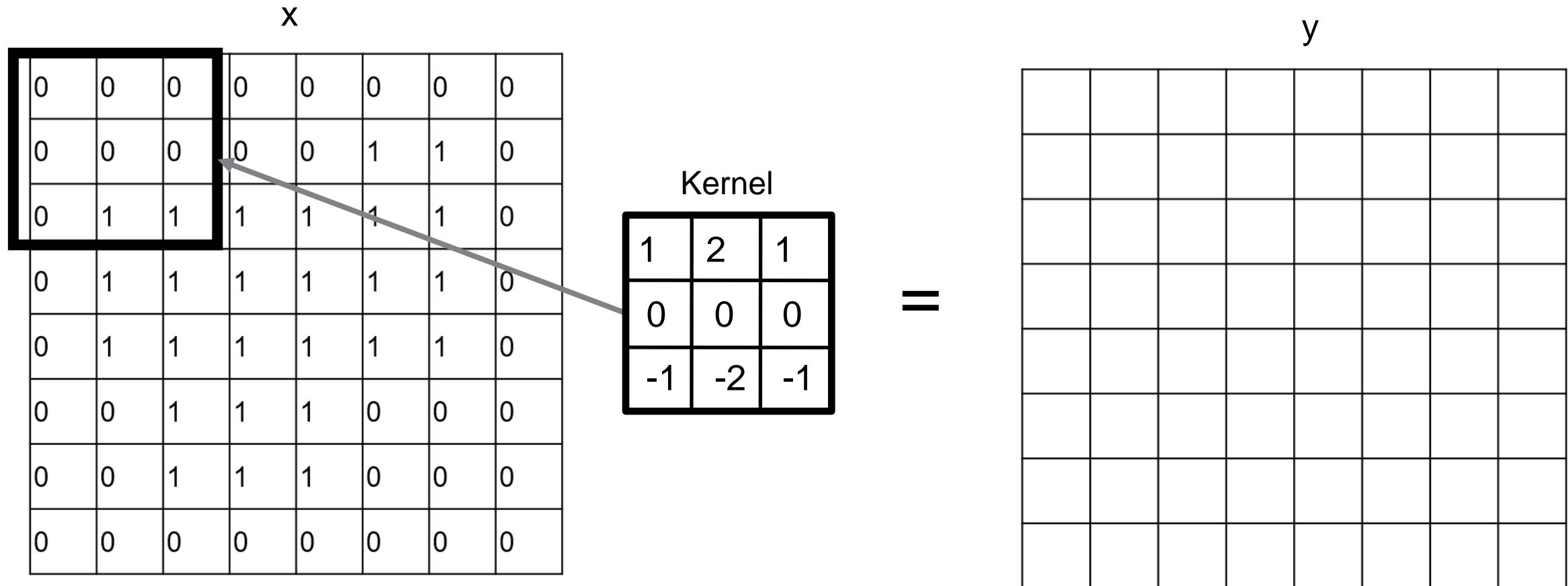
Kernel

1	2	1
0	0	0
-1	-2	-1

=

y


# Convolution



# Convolution

X

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y


# Convolution

x

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	
0	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	0	
0	0	1	1	1	0	0	0	
0	0	1	1	1	0	0	0	
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y


# Convolution

x

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	
0	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	0	
0	0	1	1	1	0	0	0	
0	0	1	1	1	0	0	0	
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y

	-3	-4						

# Convolution

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y


# Convolution

$x$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	
0	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0	
0	0	1	1	1	0	0	0	
0	0	0	0	0	0	0	0	

Kernel

1	2	1
0	0	0
-1	-2	-1

=

$y$

	-3	-4	-4	-4				

# Convolution

x

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	
0	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0	
0	0	1	1	1	0	0	0	
0	0	0	0	0	0	0	0	

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y

	-3	-4	-4	-4	-4			

# Convolution

x

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y

	-3	-4	-4	-4	-4	-3		

# Convolution

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	1	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y

	-3	-4	-4	-4	-4	-3		
-3								

# Convolution

x

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	1	0	0	0
0	0	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0

Kernel

1	2	1
0	0	0
-1	-2	-1

=

y

-3	-4	-4	-4	-4	-4	-3		
-3	-4	-4	-3	-1	0			
0	0	0	0	0	0	0		
2	1	0	1	3	3			
2	1	0	1	3	3			
1	3	4	3	1	0			

# Convolution

			x							
			0	0	0	0	0	0	0	0
			0	0	0	0	0	1	1	0
0	1	1	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	1	1	0	
0	1	1	1	1	1	1	1	1	0	
0	0	1	1	1	1	0	0	0	0	
0	0	1	1	1	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	

Kernel

1	2	1
0	0	0
-1	-2	-1

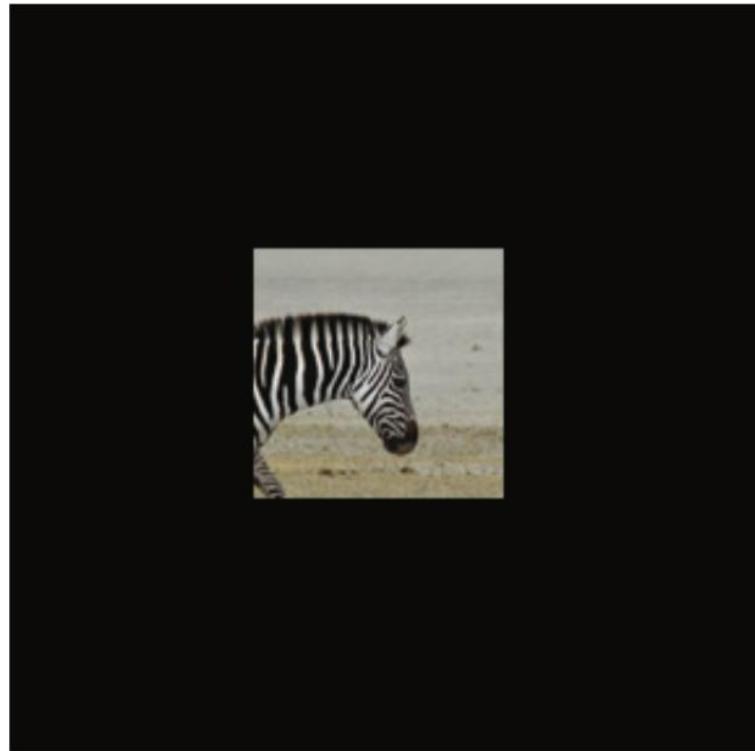
=

			y							
			?							
			-3	-4	-4	-4	-4	-3		
			-3	-4	-4	-3	-1	0		
			0	0	0	0	0	0	0	
			2	1	0	1	3	3		
			2	1	0	1	3	3		
			1	3	4	3	1	0		

You have to handle the Border Cases: e.g. Zero-Padding.

# Example: Zero Padding

Zero padding



$$\odot \begin{matrix} * \\ \square \\ \uparrow \end{matrix} =$$

11x11 ones



# Convolution

$$y[m, n] = x * h = \sum_{k, l=-N}^N x[m+k, n+l]h[-k, -l]$$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0

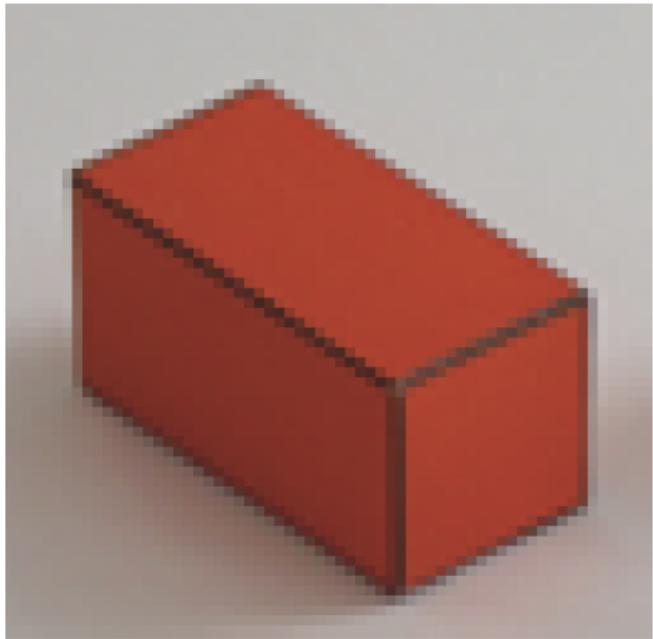
1	-1	-2	-1
0	0	0	0
-1	1	2	1

$\uparrow$   
I  $k \rightarrow -1 \quad 0 \quad 1$

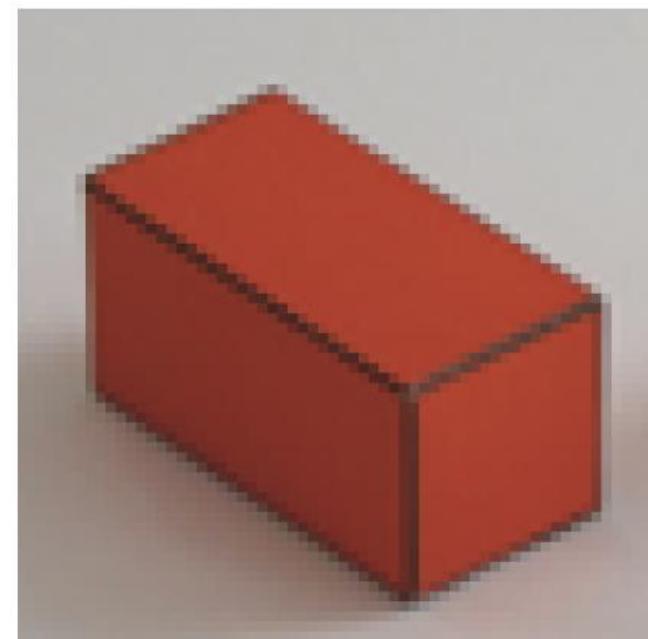
In this case,  $N = 1$ .

Convolution is a shift-invariant linear Operation.

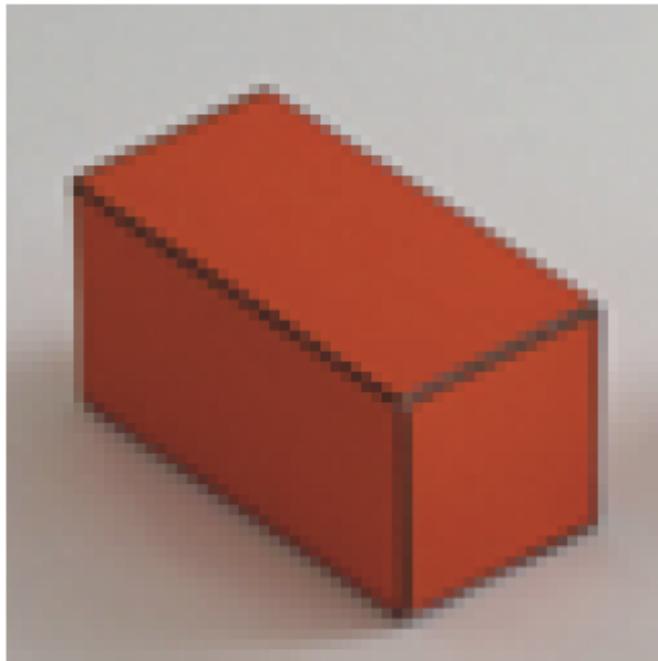
# Example: Impulse

 $\circledast$ 

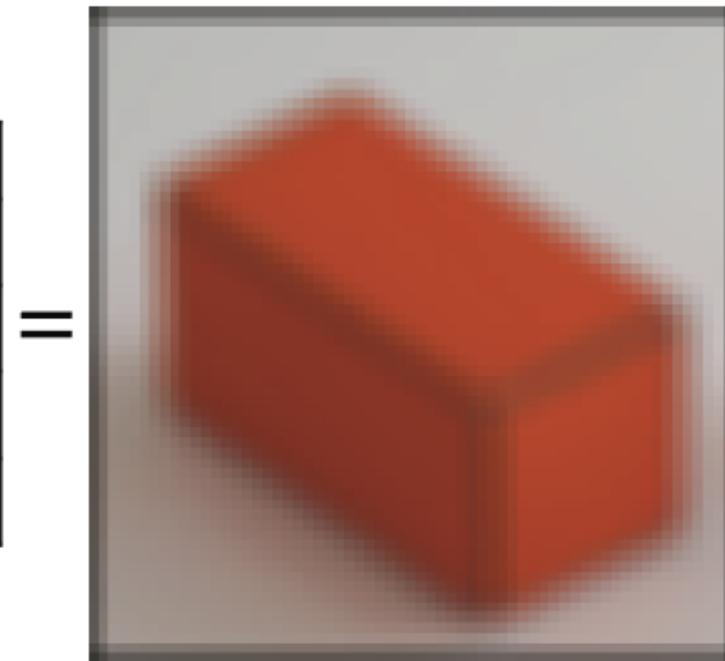
0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

 $=$ 

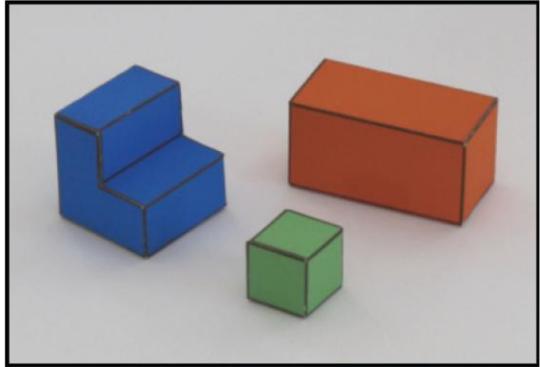
# Example: Blurring (Box Filter)

 $\circledast$ 

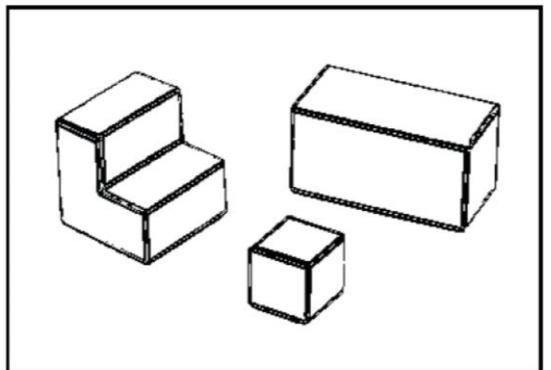
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

 $/25$ 

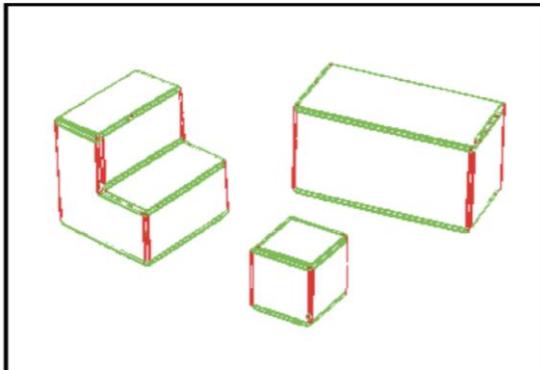
# Recap: Gradients



$$K_{Gx} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$
$$K_{Gy} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$



Using  $E(x,y)$



Using  $\theta(x,y)$

Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

Approximation image derivative:

$$\frac{\partial I}{\partial x} \simeq I(x, y) - I(x - 1, y)$$

Edge strength

$$E(x, y) = |\nabla I(x, y)|$$

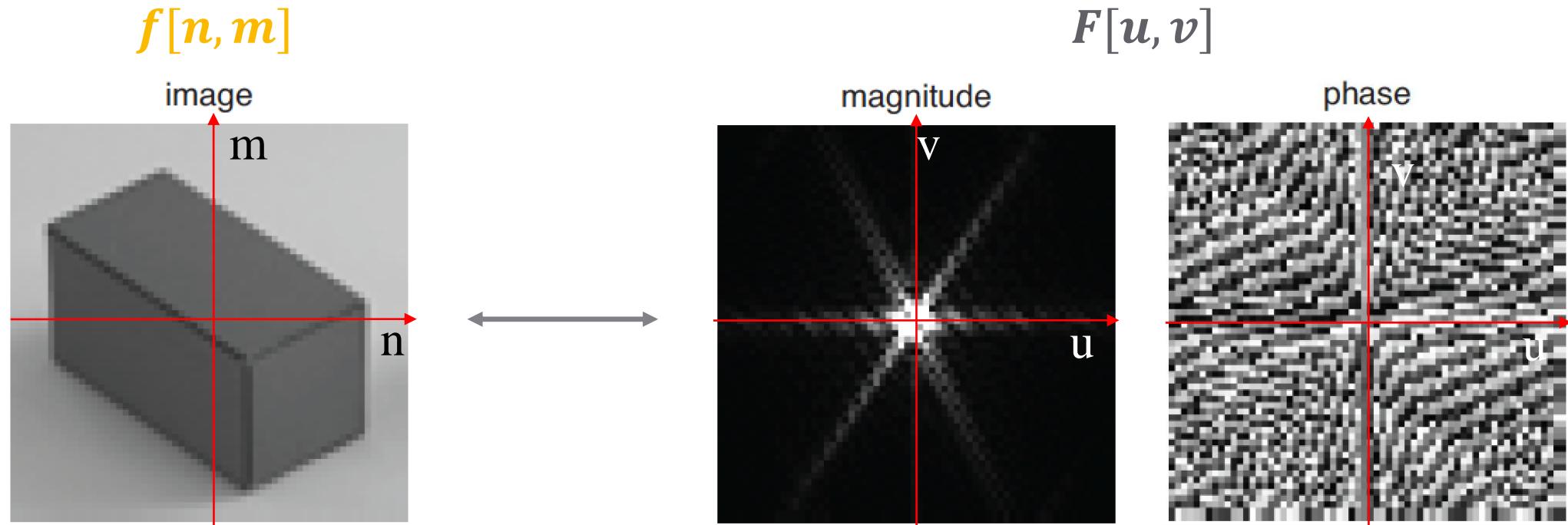
Edge orientation:

$$\theta(x, y) = \angle \nabla I = \arctan \frac{\partial I / \partial y}{\partial I / \partial x}$$

Edge normal:

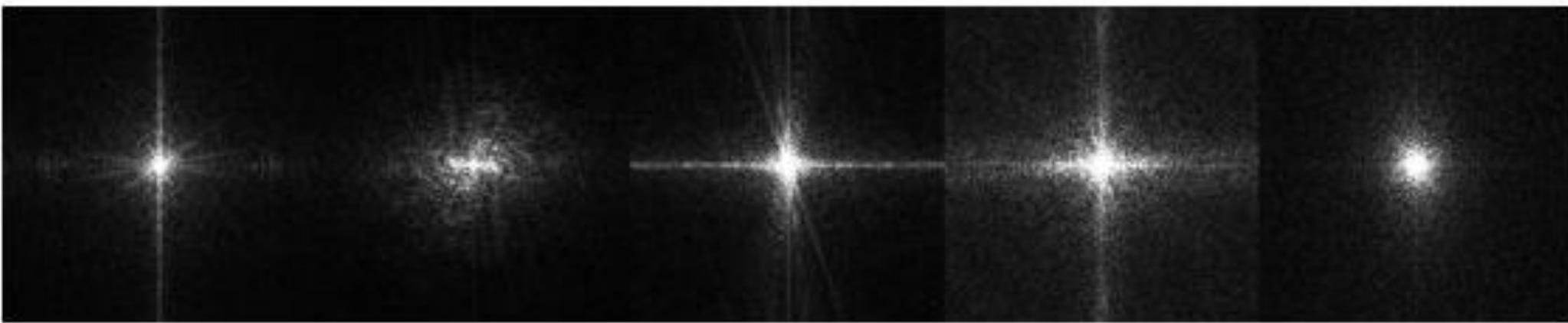
$$\mathbf{n} = \frac{\nabla I}{|\nabla I|}$$

# Spatial vs. Frequency Domain



Fourier Transformation /  
Inverse Fourier Transformation

# Spectra of Natural Images



# Convolution Theorem

The Fourier transform of the convolution is the product of Fourier transforms

$$f[n, m] = h * g \quad \leftrightarrow \quad F[u, v] = G[u, v]H[u, v]$$

The Fourier transform of the product is the convolution of Fourier transforms

$$f[n, m] = g[n, m]h[n, m] \quad \leftrightarrow \quad F[u, v] = \frac{1}{NM}G[u, v]*H[u, v]$$

# Convolution Properties

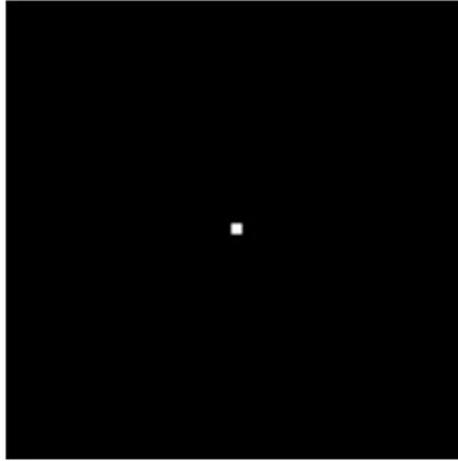
	Spatial Domain	Fourier Domain
Commutative	$f * g = g * f$	$FG = GF$
Associative	$(f * g) * h = f * (g * h)$	$(FG)H = F(GH)$
Distributive	$(f + g) * h = f * h + g * h$	$(F + G)H = FH + GH$
Linear	$(af + bg) * h = af * h + bg * h$	$(aF + bG)H = aFH + bGH$
Shift Invariant	$f(x + t) * h = (f * h)(x + t)$	$(e^{i\omega t} F)H = e^{i\omega t}(FH)$

# Example: Box Filter

$f$



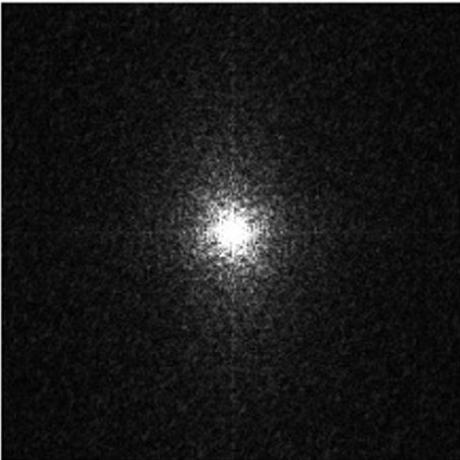
$g$



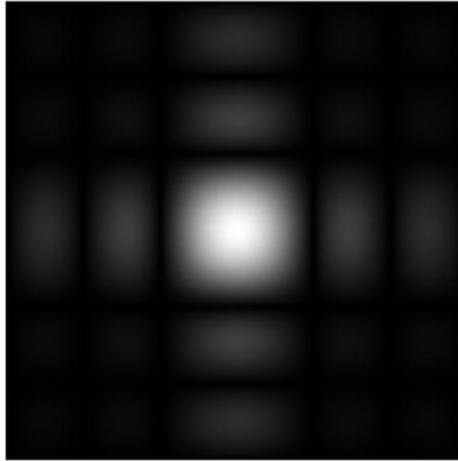
$f * g$



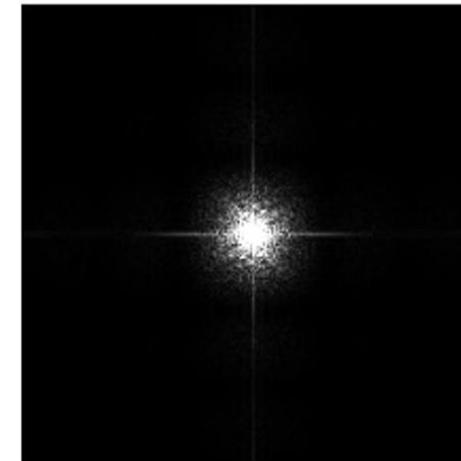
$|F|$



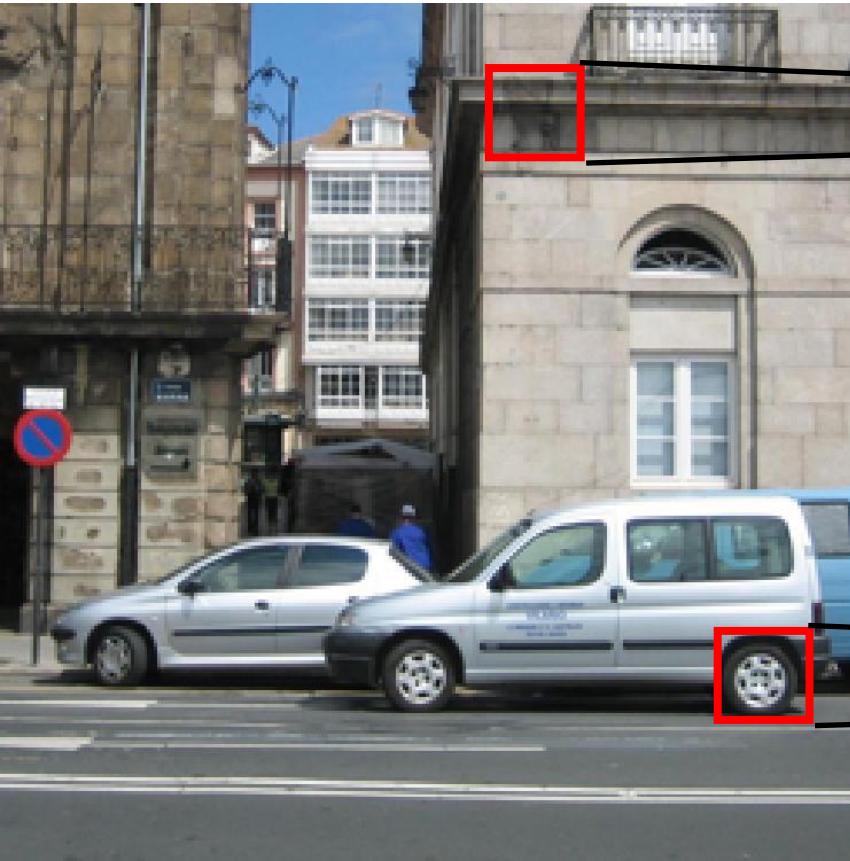
$|G|$



$|FG|$



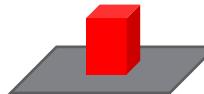
# Low-Pass Filter (Box Filter)



256X256

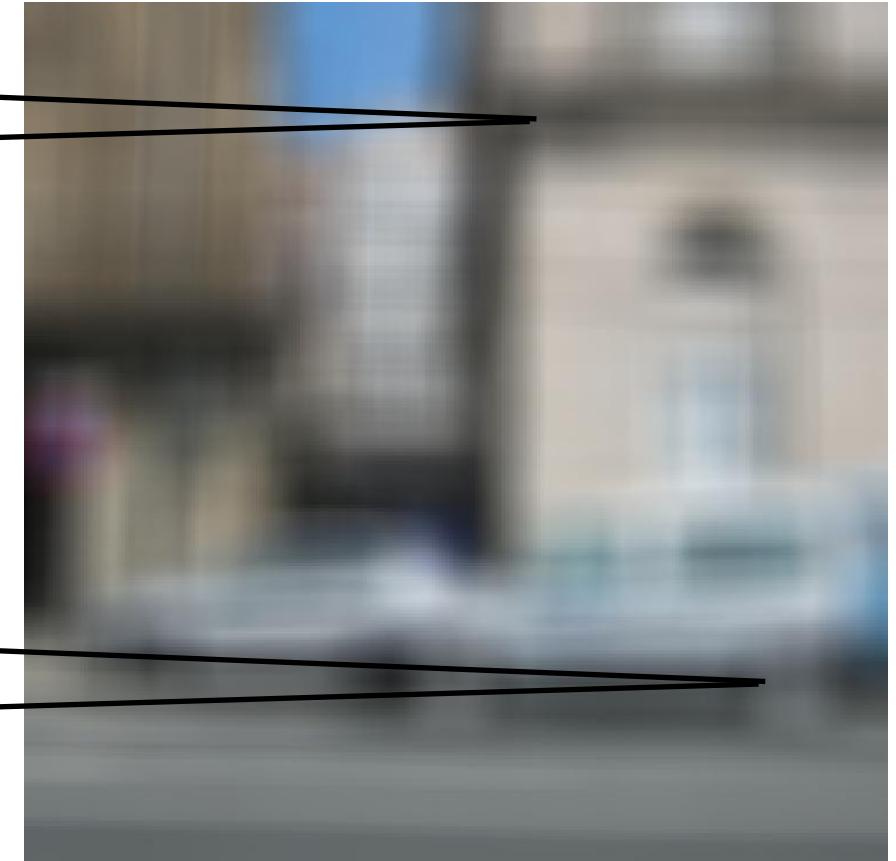
- Replaces each pixel with an Average of its Neighborhood.
- Achieves smoothing Effect (remove sharp Features, but also Noise).

mean



=

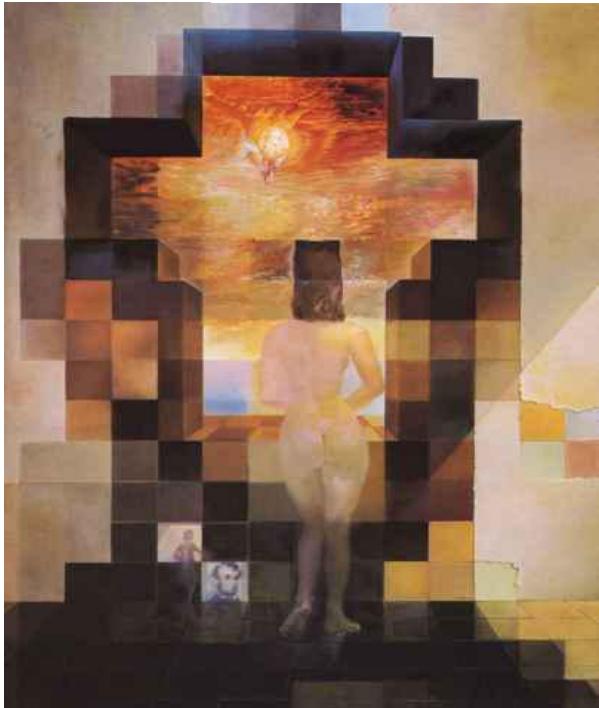
mean



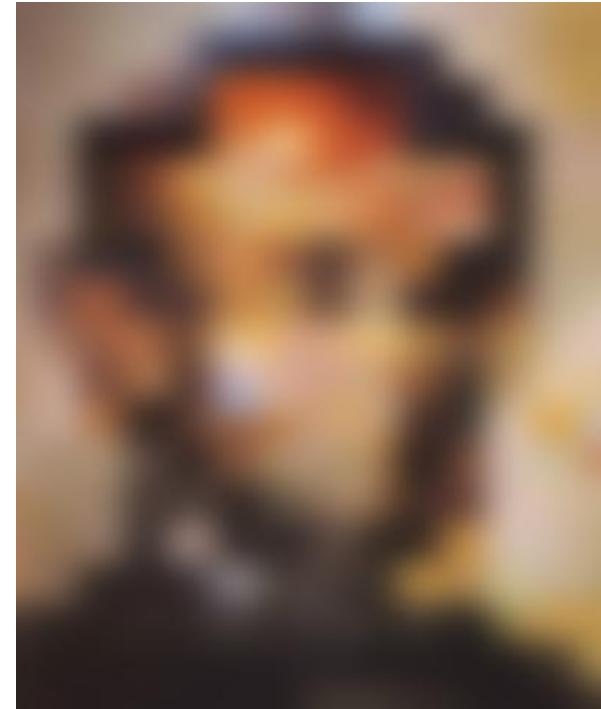
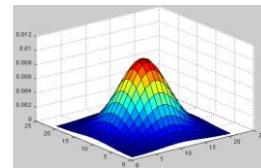
256X256

# Low-Pass Filter (Gaussian Filter)

$$g(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp -\frac{x^2 + y^2}{2\sigma^2}$$



\*

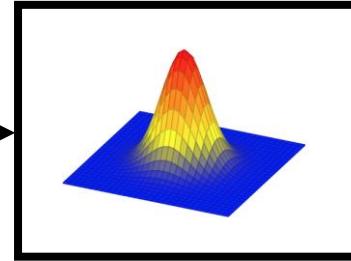


- Convolution of two Gaussians is a Gaussian.
- The (continuous) Fourier Transform of a Gaussian is another Gaussian.
- Gaussians are separable (2x1D Convolution instead of 1x2D).

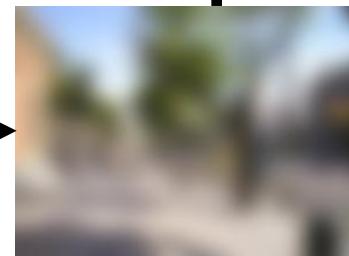
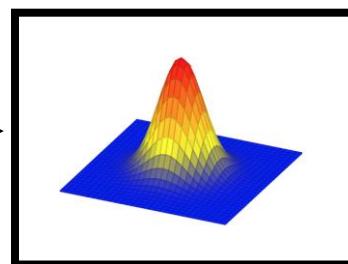
# High-Pass Filter (Laplacian Filter)



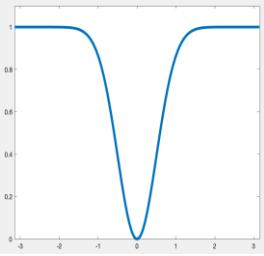
Gaussian Filter



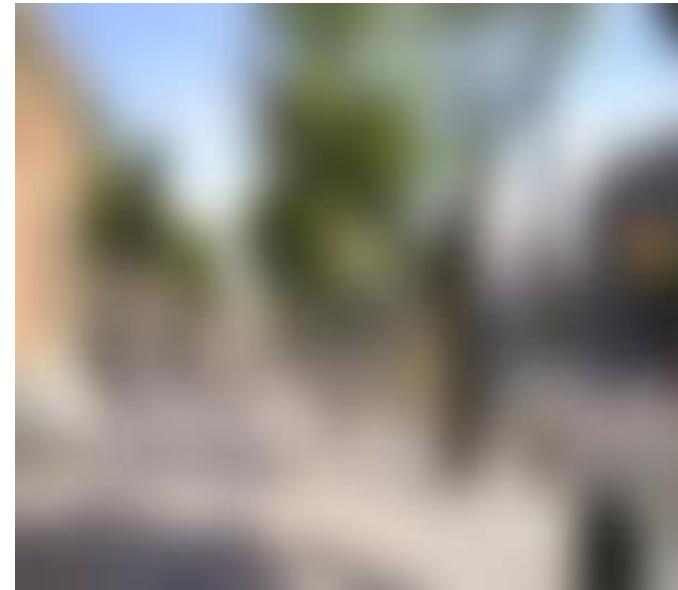
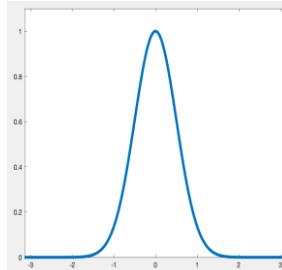
Laplacian Filter



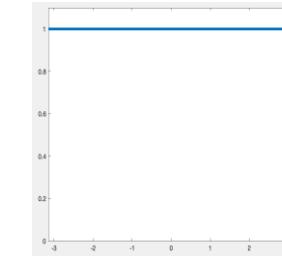
# High-Pass Filter (Laplacian Filter)



+



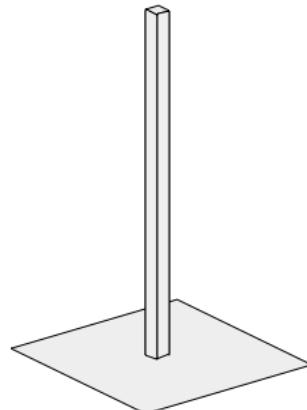
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# High-Pass Filter (Laplacian Filter)

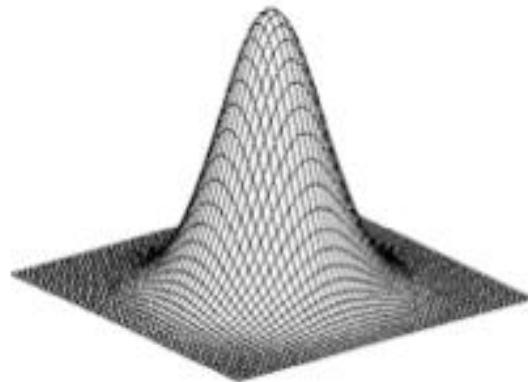
$$(\nabla^2 f)(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$K_{\nabla^2} = -1 \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



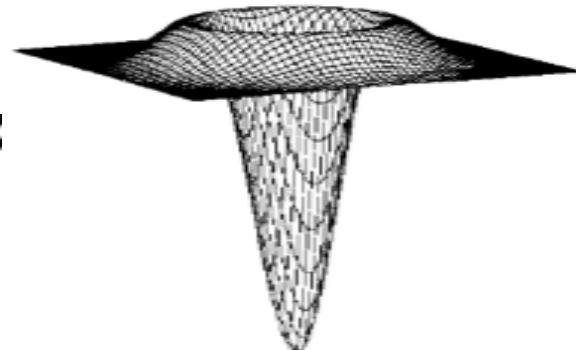
**unit**

-



**Gaussian**

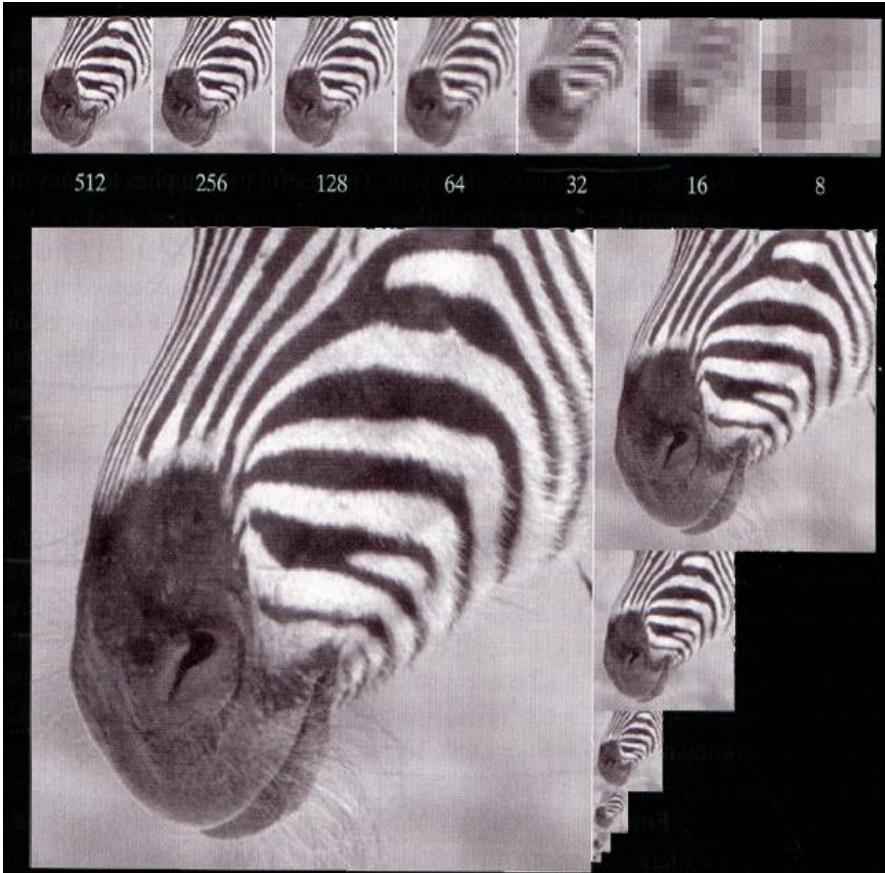
$\approx$



**Laplacian**

# Image Pyramids (Scale Invariance)

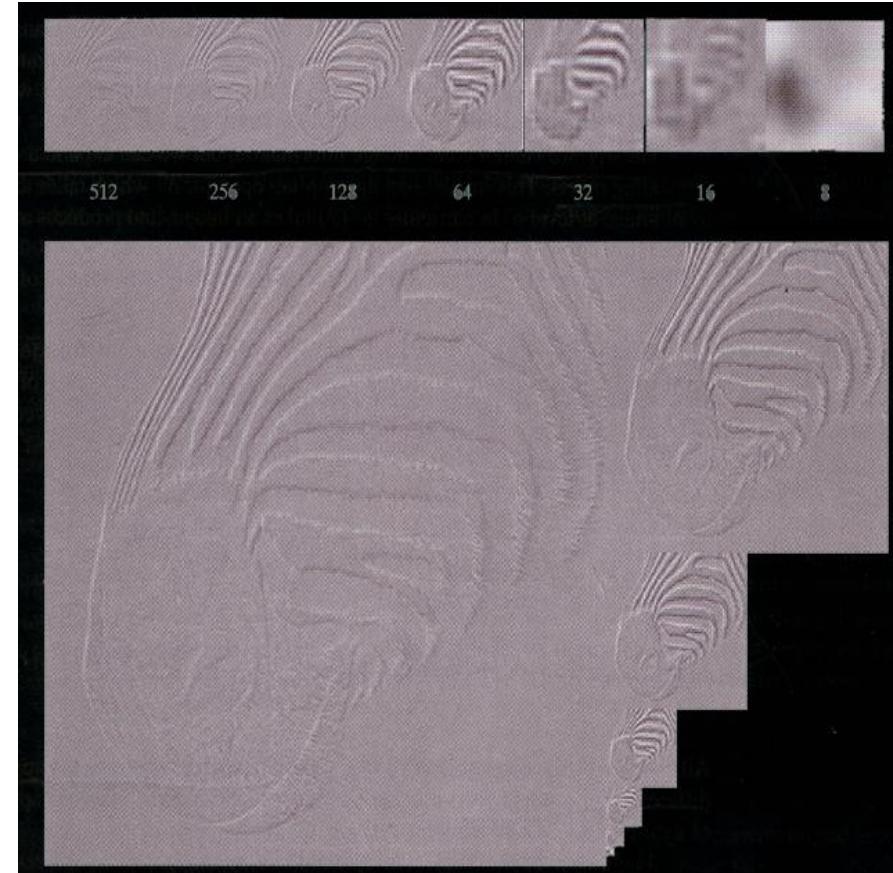
Gaussian Pyramid



$$P_G^{n+1}(I) = \downarrow (G_s \otimes P_G^n(I))$$

$$P_G^1(I) = I$$

Laplacian Pyramid



$$P_L^n(I) = P_G^n(I) - \uparrow P_G^{n+1}(I)$$

$$P_L^n(I) = P_L^n(I) + \uparrow P_L^{n+1}(I) \rightarrow I$$

# Deconvolution / Inverse Filtering

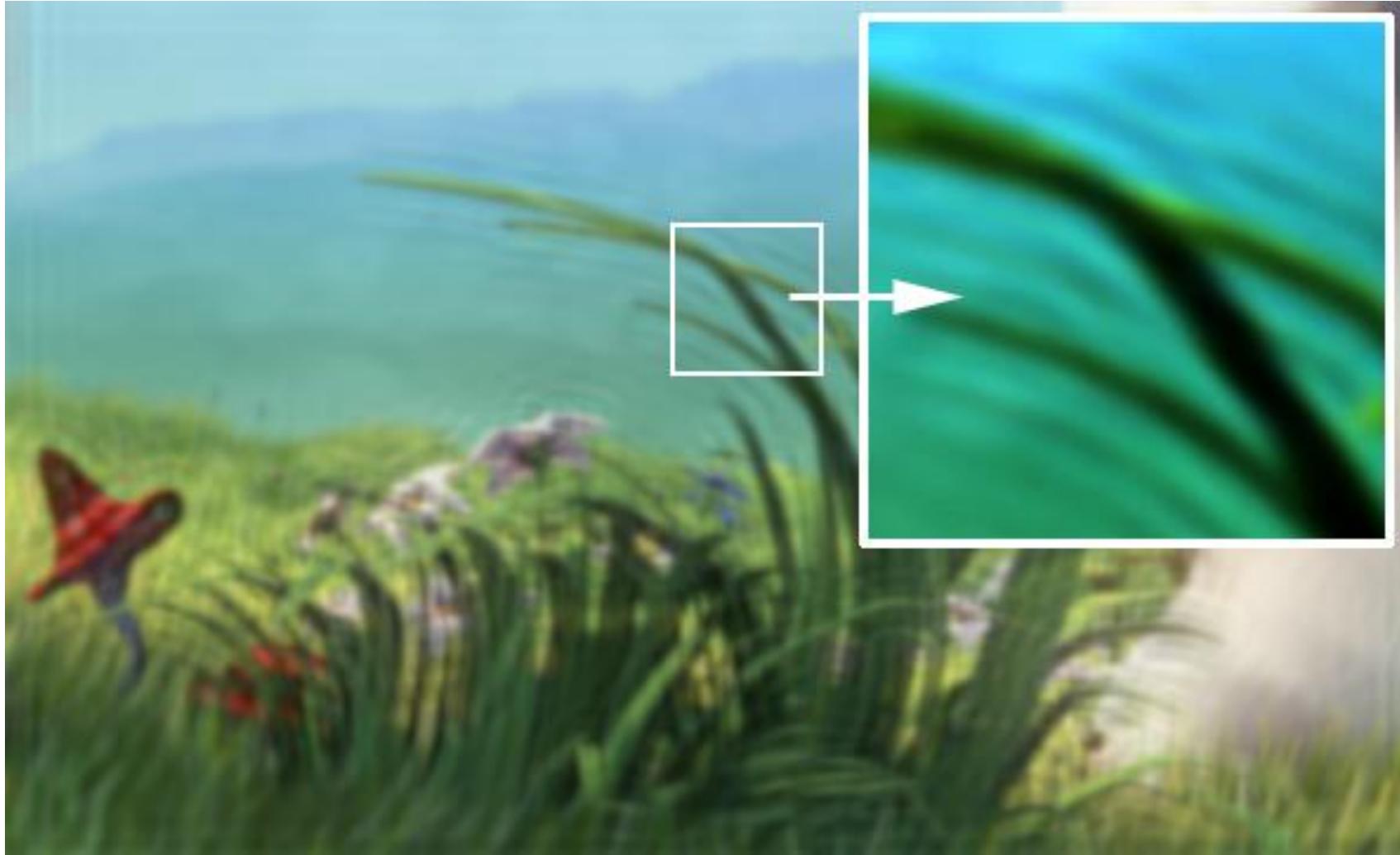
$$I(x, y) * K_s(x, y) = I'(x, y) \quad \hat{I}(f_x, f_y) \cdot \hat{K}_s(f_x, f_y) = \hat{I}'(f_x, f_y)$$

Convolution in Spatial Domain.....is.....Multiplication in Frequency Domain

Does that mean that  
Deconvolution in Spatial Domain  
is Division in Frequency Domain?

$$\hat{I}(f_x, f_y) = \frac{\hat{I}'(f_x, f_y)}{\hat{K}_s(f_x, f_y)}$$

# Deconvolution / Inverse Filtering



This is simplified and does not consider Noise.

Division by small Values in Frequency Domain leads to Ringing Artefacts in Spatial Domain.

Better is to apply regularized Techniques: Wiener Filter, Regularized Filter, Lucy-Richardson Algorithm, Blind Deconvolution

# Inverse Filtering



Original Image

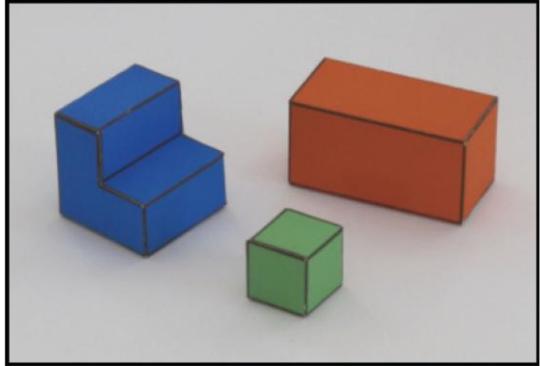


After Convolution  
(Motion Blur)

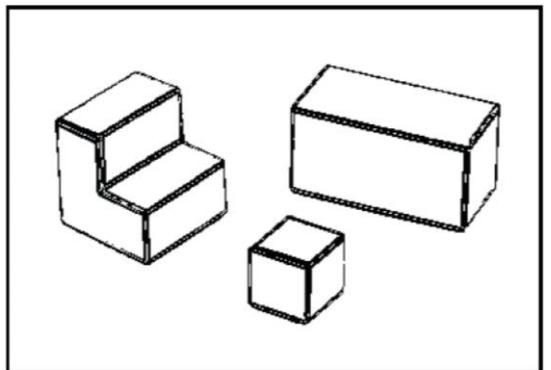


After Inverse Filtering

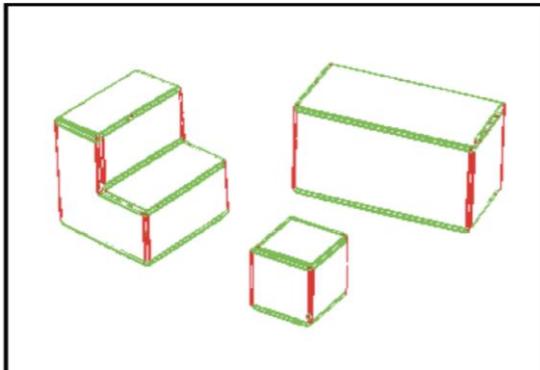
# Recap: Gradients



$$K_{Gx} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$
$$K_{Gy} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$



Using  $E(x,y)$



Using  $\theta(x,y)$

Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

Approximation image derivative:

$$\frac{\partial I}{\partial x} \simeq I(x, y) - I(x - 1, y)$$

Edge strength

$$E(x, y) = |\nabla I(x, y)|$$

Edge orientation:

$$\theta(x, y) = \angle \nabla I = \arctan \frac{\partial I / \partial y}{\partial I / \partial x}$$

Edge normal:

$$\mathbf{n} = \frac{\nabla I}{|\nabla I|}$$

# Spatial vs. Gradient Domain

Gradient Domain

Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

$\uparrow$        $\uparrow$   
 $I_x$        $I_y$

$$\text{div}(\nabla I) = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} = I_{xx} + I_{yy} = \nabla^2 I$$

Laplacian!

$$\text{curl}(\nabla I) = \frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y} = I_{yx} - I_{xy}$$

We can consider this as Coefficients of a Vectorfield.

# Spatial vs. Gradient Domain

Laplacian!

$$\text{div}(\nabla I) = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} = I_{xx} + I_{yy} = \nabla^2 I$$

Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

$$I_x$$

$$I_y$$

$$\text{curl}(\nabla I) = \frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y} = I_{yx} - I_{xy}$$

Gradient Domain

Spatial Domain

Derivative / Integral

$$I = \int \nabla I$$

We can consider this as Coefficients of a Vectorfield.

Integrating this Vectorfield results in the original Image as long as the Vectorfield has zero Curl.

# Spatial vs. Gradient Domain

Laplacian!

$$\text{div}(\nabla I) = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} = I_{xx} + I_{yy} = \nabla^2 I$$

Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

$$I_x$$

$$I_y$$

We can consider this as Coefficients of a Vectorfield.

Gradient Domain

Spatial Domain

Derivative / Integral

$$I = \int \nabla I$$

**Processing the Vectorfield is called  
Gradient Domain Processing.**

Integrating this Vectorfield results in the original Image as long as the Vectorfield has zero Curl. **In Practice: that does not work as the Vectorfields is no longer conservative anymore after processing it (it has non-zero Curl).**

# Spatial vs. Gradient Domain

Laplacian!

$$\text{div}(\nabla I) = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} = I_{xx} + I_{yy} = \nabla^2 I$$

Image gradient:

$$\nabla I = \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

$$I_x$$

$$I_y$$

We can consider this as Coefficients of a Vectorfield.

Gradient Domain

Spatial Domain

Derivative / Integral

$$I = \int \nabla I$$

**Processing the Vectorfield is called  
Gradient Domain Processing.**

Integrating this Vectorfield results in the original Image as long as the Vectorfield has zero Curl. **In Practice: that does not work as the Vectorfields is no longer conservative anymore after processing it (it has non-zero Curl).**

Instead: solve 2D Poisson Equation:  $\nabla^2 I = \text{div}(G)$

# Examples

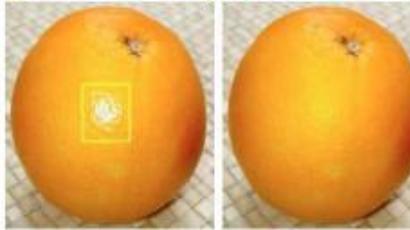
Image Fusion



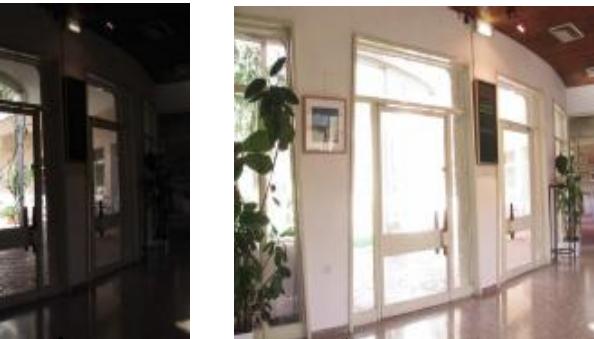
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Removing Reflections



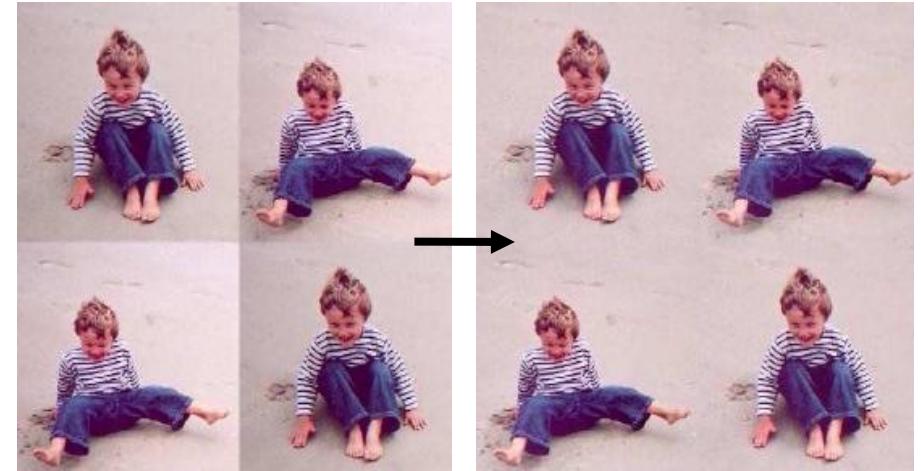
HDR Compression



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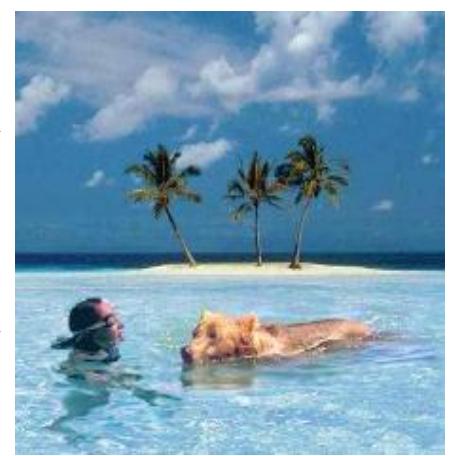
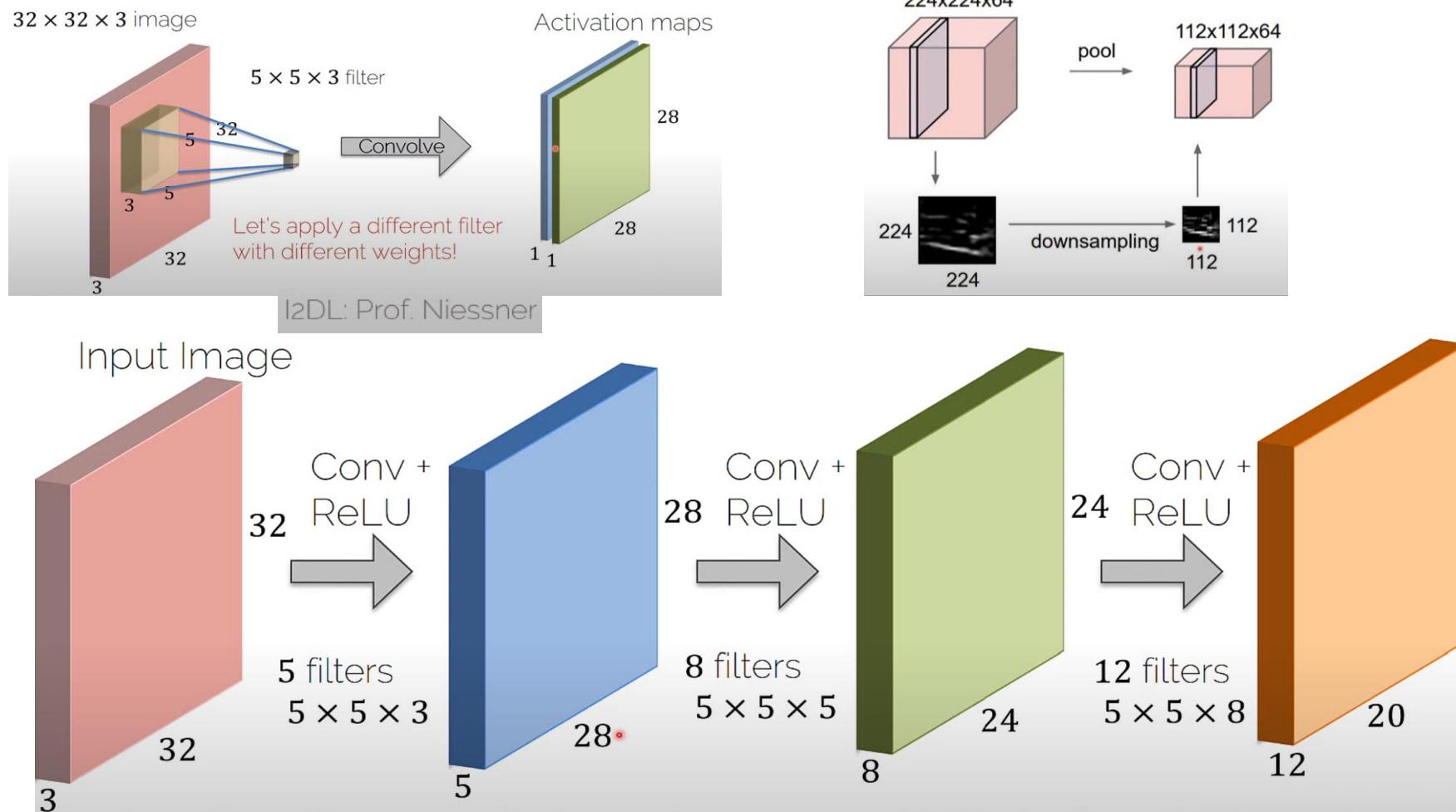
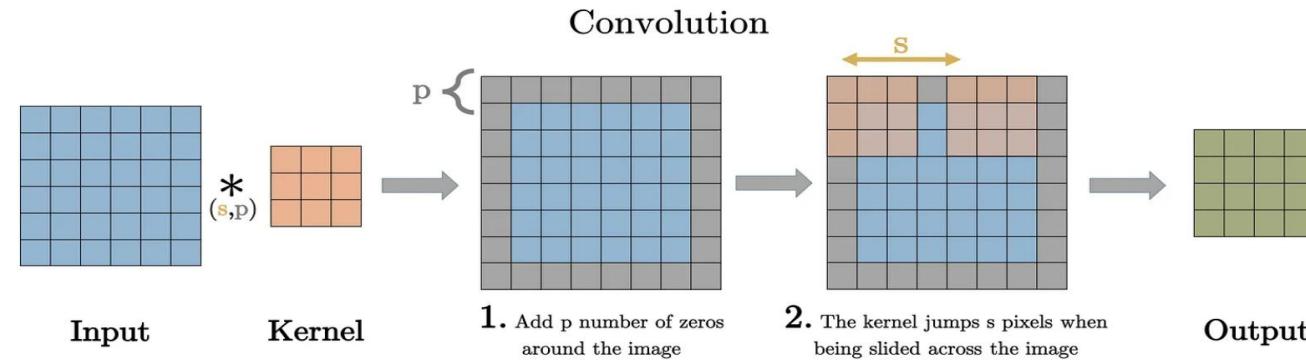
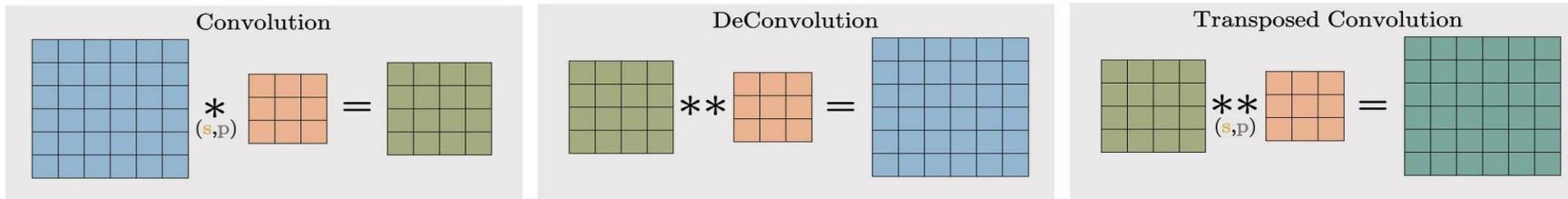


Image Stitching / Editing

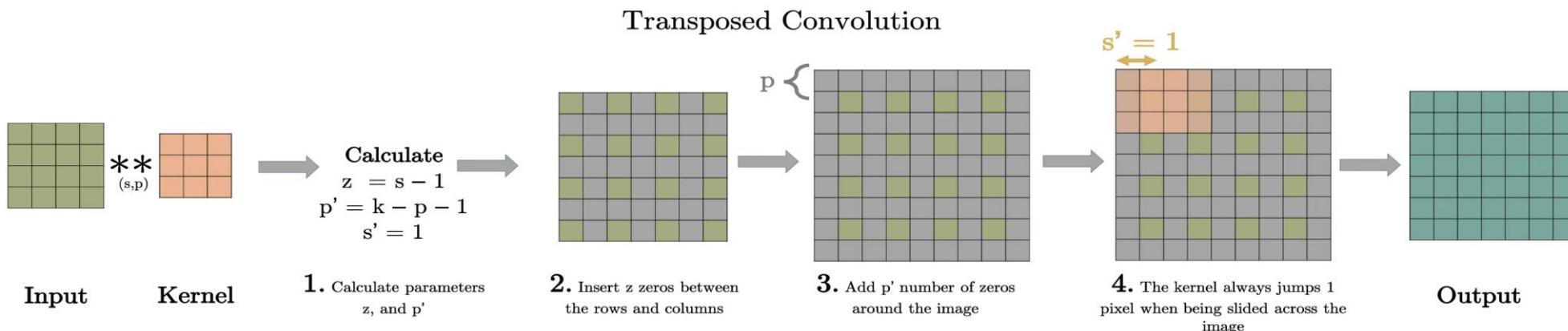
# Convolutional Neural Networks (CNNs)



# Transposed Convolution



Transposed  
Convolution used  
for Upsampling in  
CCNs

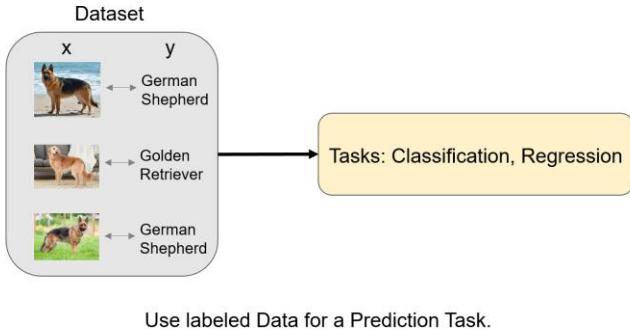


# Course Overview

CW	Topic	Date	Place	Lab
41	Introduction and Course Overview	07.10.2025	Zoom	Lab 1
42	Capturing Digital Images	14.10.2025	Zoom	Lab 2
43	Digital Image Processing	21.10.2025	Zoom	Assignment 1
→ 44	Machine Learning	28.10.2025	Zoom	
45	Feature Extraction	04.11.2025	Zoom	Open Lab 1
46	Segmentation	11.11.2025	Zoom	Assignment 2
47	Optical Flow	18.11.2025	Zoom	Open Lab 2
48	Object Detection	25.11.2025	Zoom	Assignment 3
49	Multi-View Geometry	02.12.2025	Zoom	Open Lab 3
50	3D Vision	09.12.2025	Zoom	Assignment 4
3	Trends in Computer Vision	13.01.2026	Zoom	
4	Q&A	20.01.2026	Zoom	Open Lab 4
5	Exam	27.01.2026	HS1 (Linz), S1/S3 (Vienna), S5 (Bregenz)	
9	Retry Exam	24.02.2026	tba	

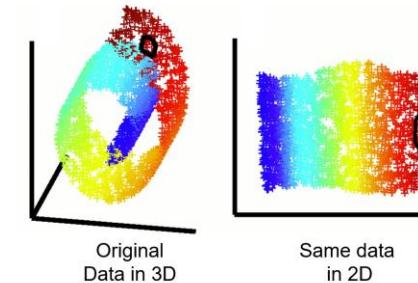
# Next Week: Machine Learning

## Supervised Learning

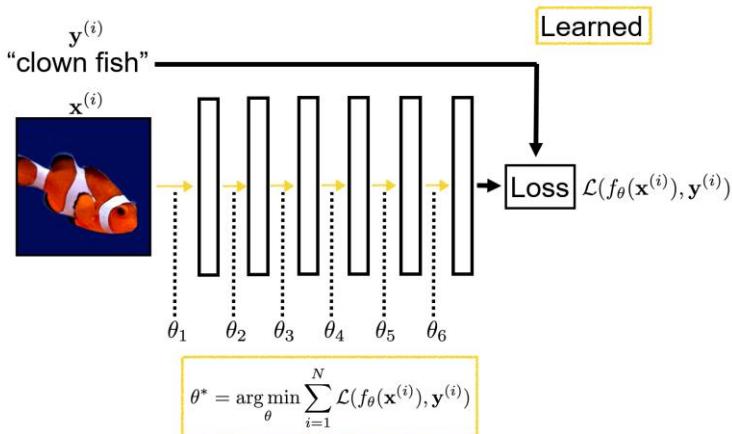


## Dimensionality Reduction

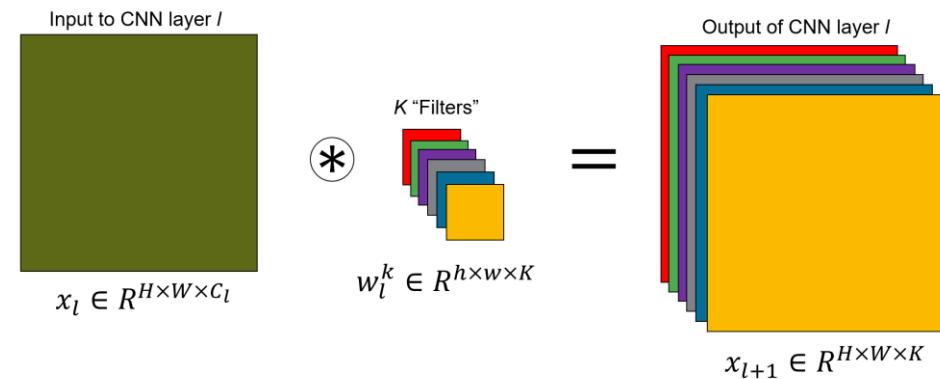
Given Data Points in  $d$  Dimensions, convert them to Data Points in  $k < d$  Dimensions with minimal loss of Information.



## Deep Learning



## Convolutional Neural Networks (CNNs)



# Thank You

