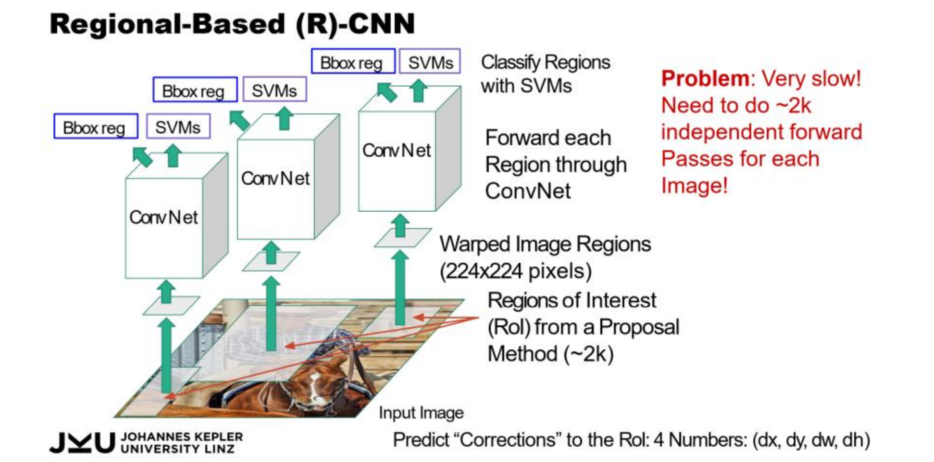
Computer Vision

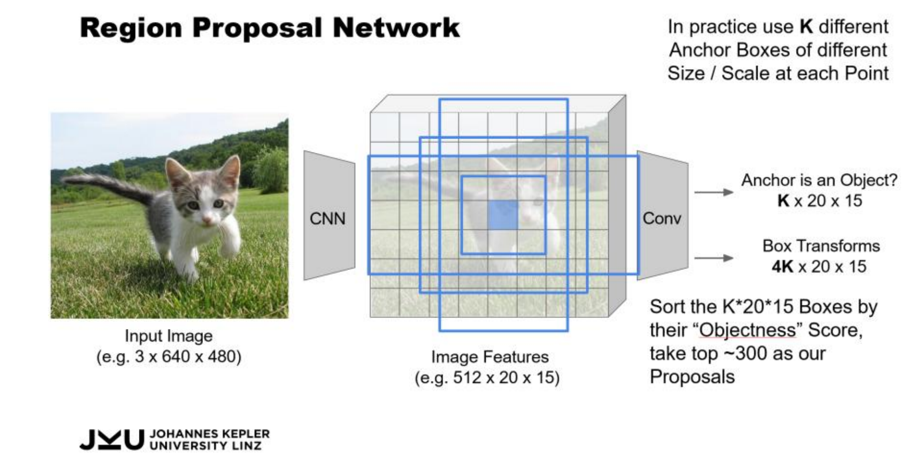
Lecture 9: Multi-View Geometry

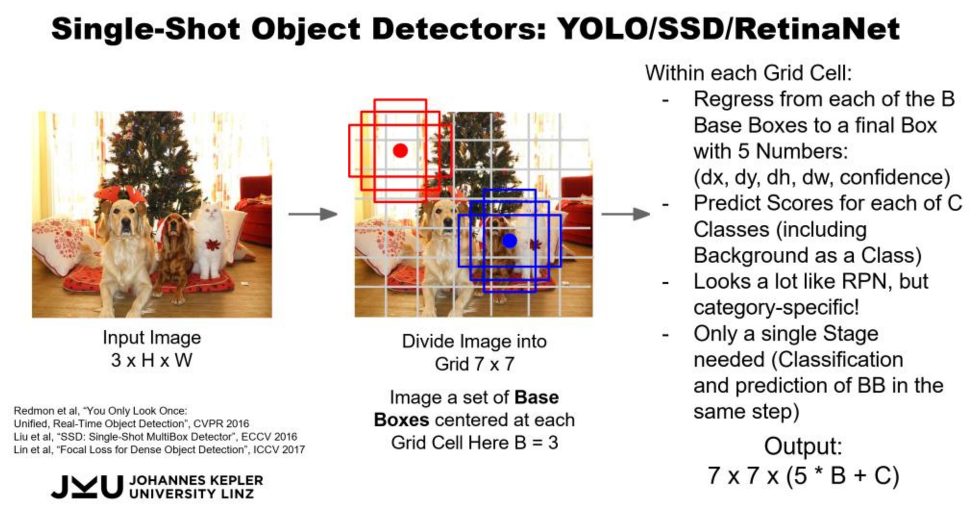
Oliver Bimber

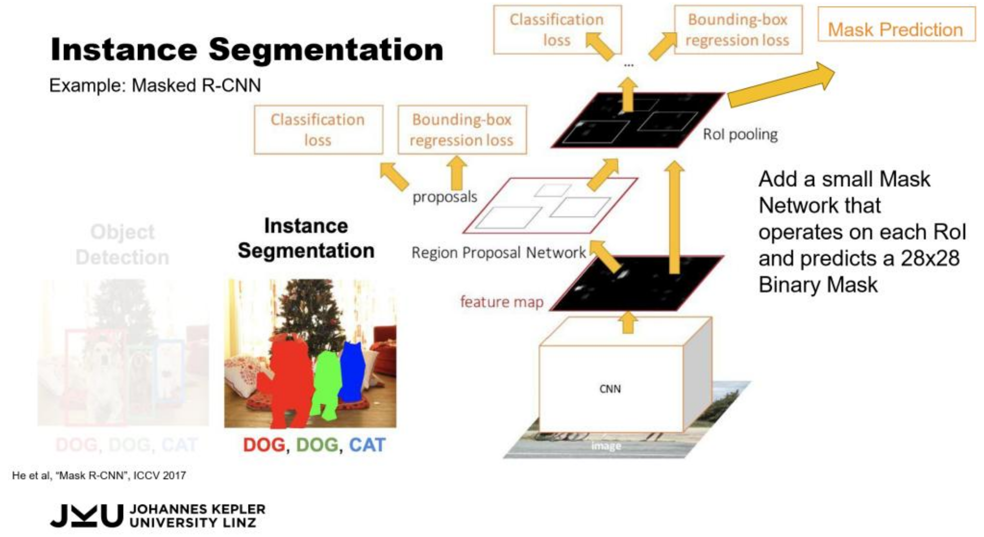
JOHANNES KEPLER UNIVERSITY LINZ

Last Week: Object Detection









Course Overview

CW	Topic	Date	Place	Lab
41	Introduction and Course Overview	07.10.2025	Zoom	Lab 1
42	Capturing Digital Images	14.10.2025	Zoom	Lab 2
43	Digital Image Processing	21.10.2025	Zoom	Assignment 1
44	Machine Learning	28.10.2025	Zoom	
45	Feature Extraction	04.11.2025	Zoom	Open Lab 1
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48	Object Detection	25.11.2025	Zoom	Assignment 3
49	Multi-View Geometry	02.12.2025	Zoom	Open Lab 3
50	3D Vision	10.12.2025	Zoom	Assignment 4
3	Trends in Computer Vision	13.01.2026	Zoom	
4	Q&A	20.01.2026	Zoom	Open Lab 4
5	Exam	27.01.2026	HS1 (Linz), S1/S3 (Vienna), S5 (Bregenz)	
9	Retry Exam	24.02.2026	tba	

Goal: Depth-Estimation

Depth from Stereo(-scopic Images) = two Views

Two Problems:

1.) Finding matching Features
2.) Computing Depth of corresponding Scenepoint (relative to Cameras)

Recap: Simplified Mathematical Camera Model

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \underbrace{\overset{[3x3]}{\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{Intrinsic Parameters (K)}} \cdot \underbrace{\overset{[3x3]}{\begin{bmatrix} \mathbf{R} \end{bmatrix}} \cdot \overset{[3x4]}{\begin{bmatrix} \mathbf{I} & \mathbf{-T} \end{bmatrix}}}_{\text{Extrinsic Parameters}} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

Note, that in Practice there are more intrinsic Parameters: e.g. Principle Point and Skew Angle describing Misalignment of Sensor on Optical Axis.

Recap: Simplified Mathematical Camera Model

$$p = MP \longrightarrow p = \frac{1}{z} MP \qquad M = K(\, R \quad t \,)$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \underbrace{\overset{[3x3]}{\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{\text{Intrinsic Parameters (K)}} \cdot \underbrace{\overset{[3x3]}{\begin{bmatrix} R \end{bmatrix}} \cdot \overset{[3x4]}{\begin{bmatrix} I & -T \end{bmatrix}}}_{\text{Extrinsic Parameters}} \cdot \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

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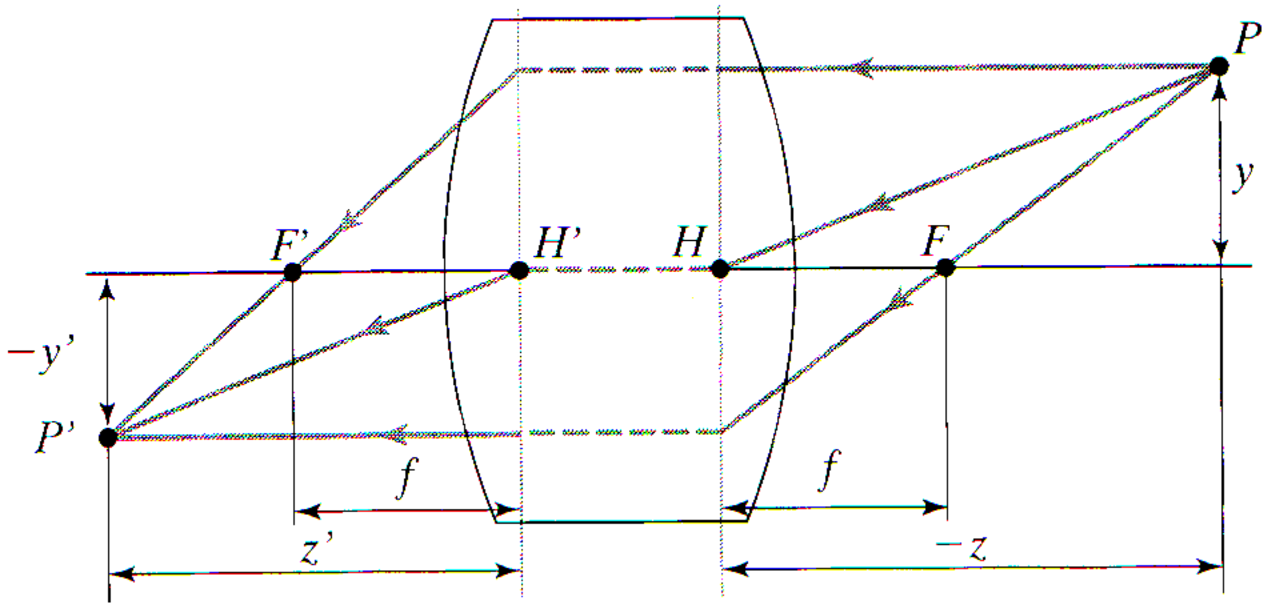
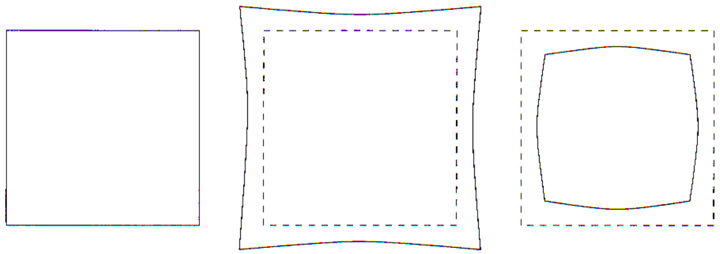
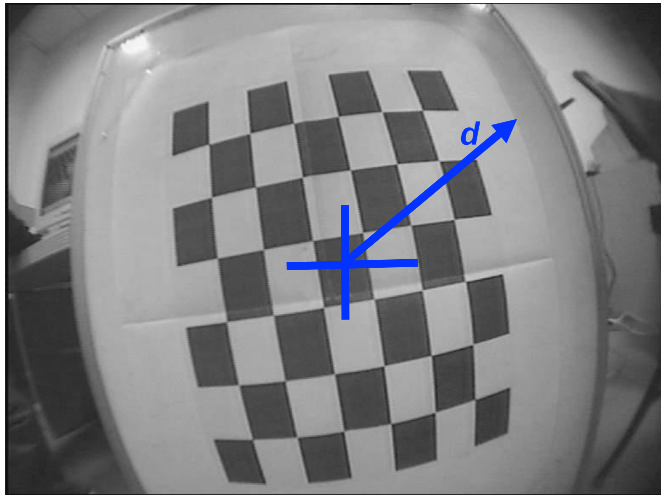
Recap: Radial Distortion



Thick Lens



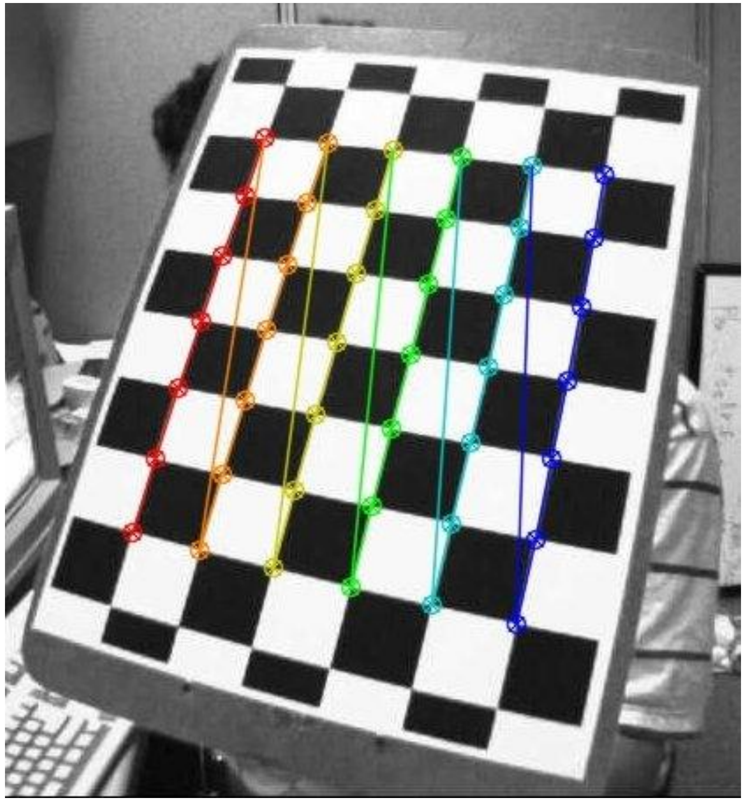
Pincushion Barrel



Radial Distortion

If Lens Distortion is considered, our Mathematical Model becomes non-linear (usually a Polynomial Function)!

Recap: Camera Calibration

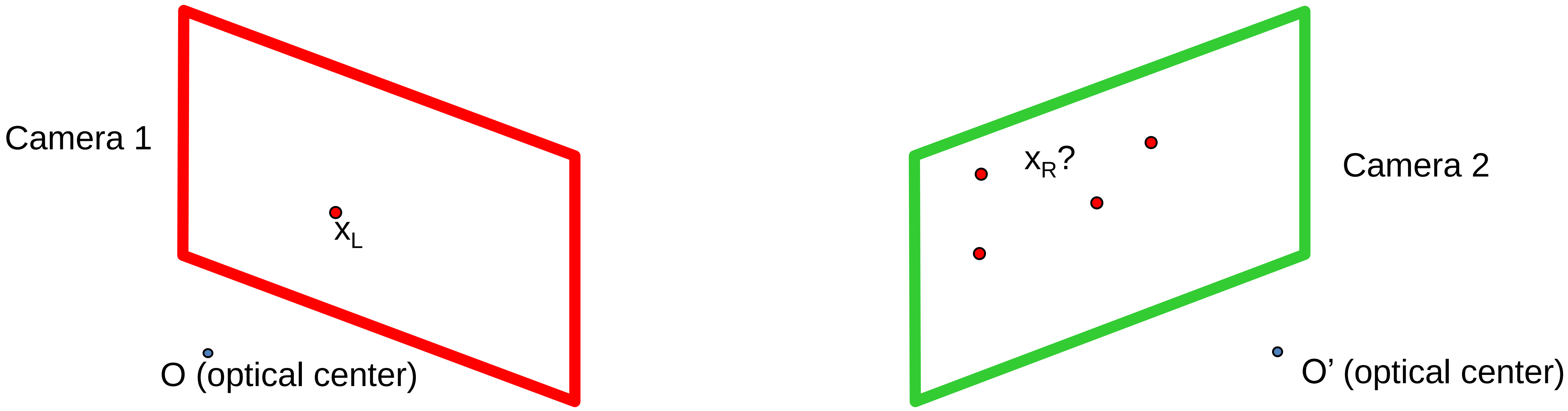


Images of known Calibration Pattern

cv.calibrateCamera() returns the camera matrix, distortion coefficients, rotation and translation vectors etc.

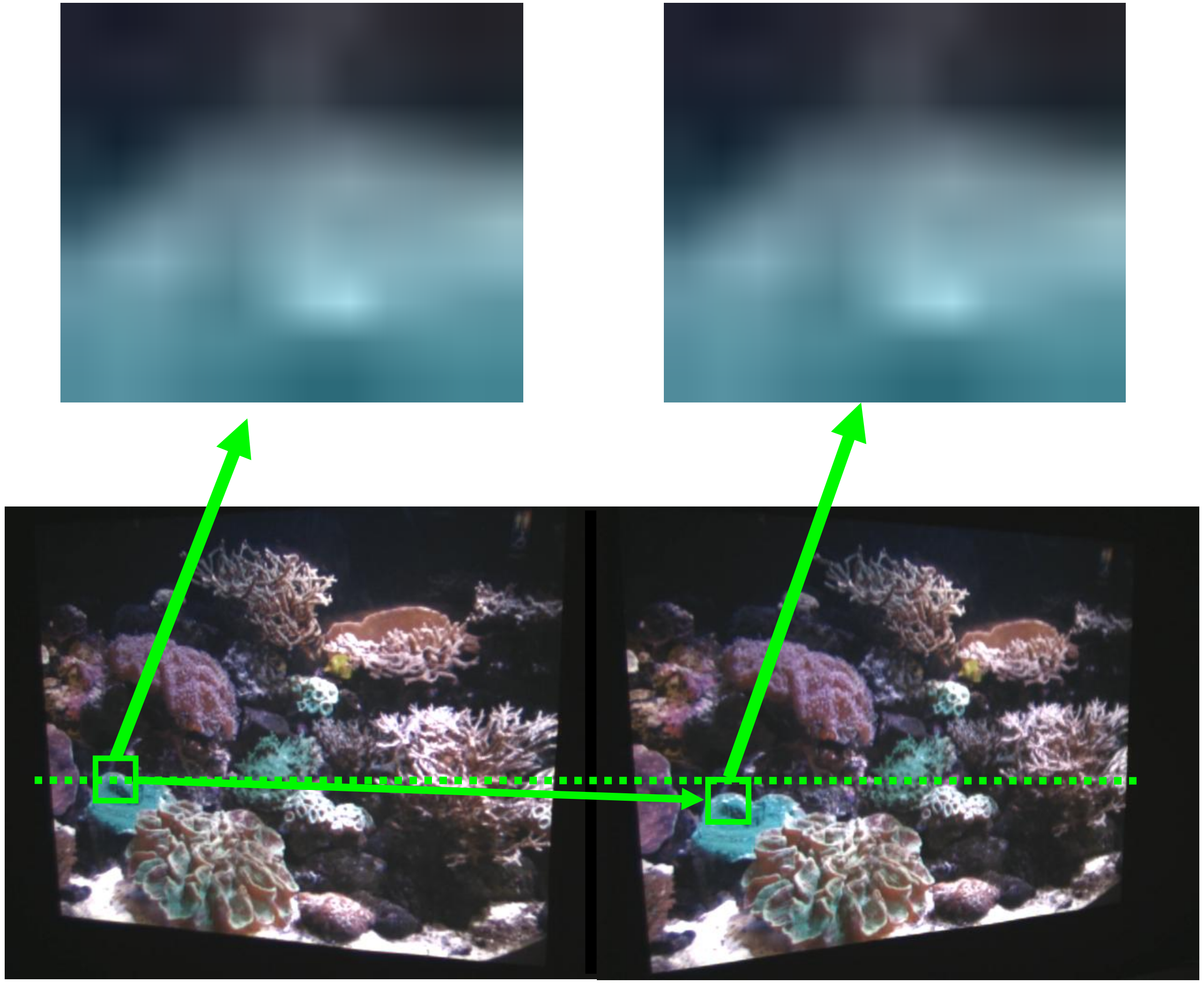
In OpenCV: https://docs.opencv.org/4.x/dc/dbb/tutorial_py_calibration.html

Two Calibrated Cameras



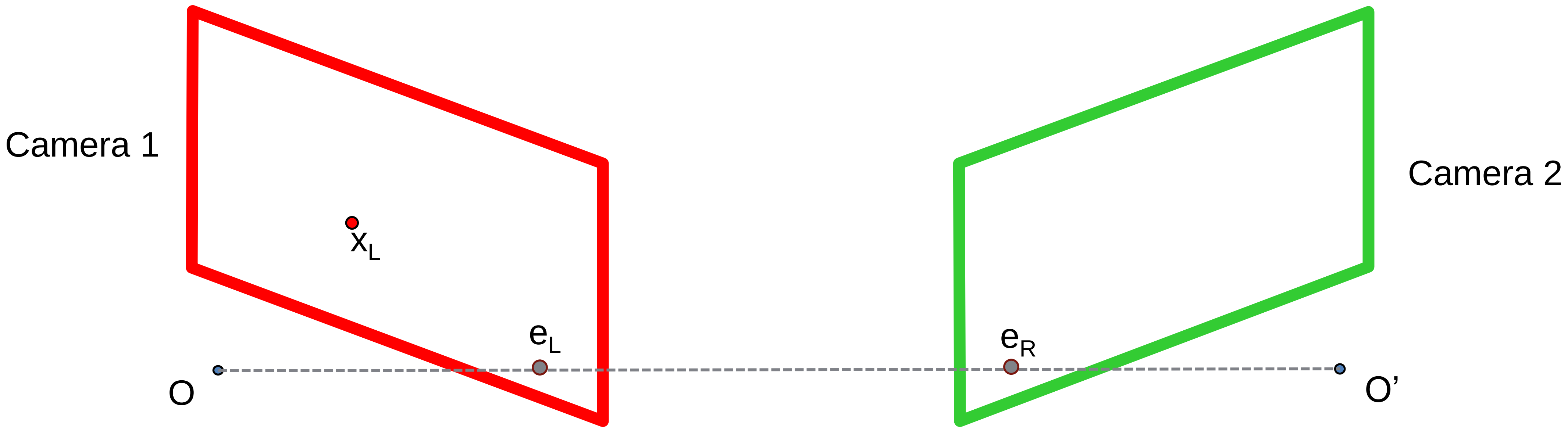
If we see a Point in Cam 1, are there any Constraints on where we will find it in Cam 2?

Feature Matching Problem



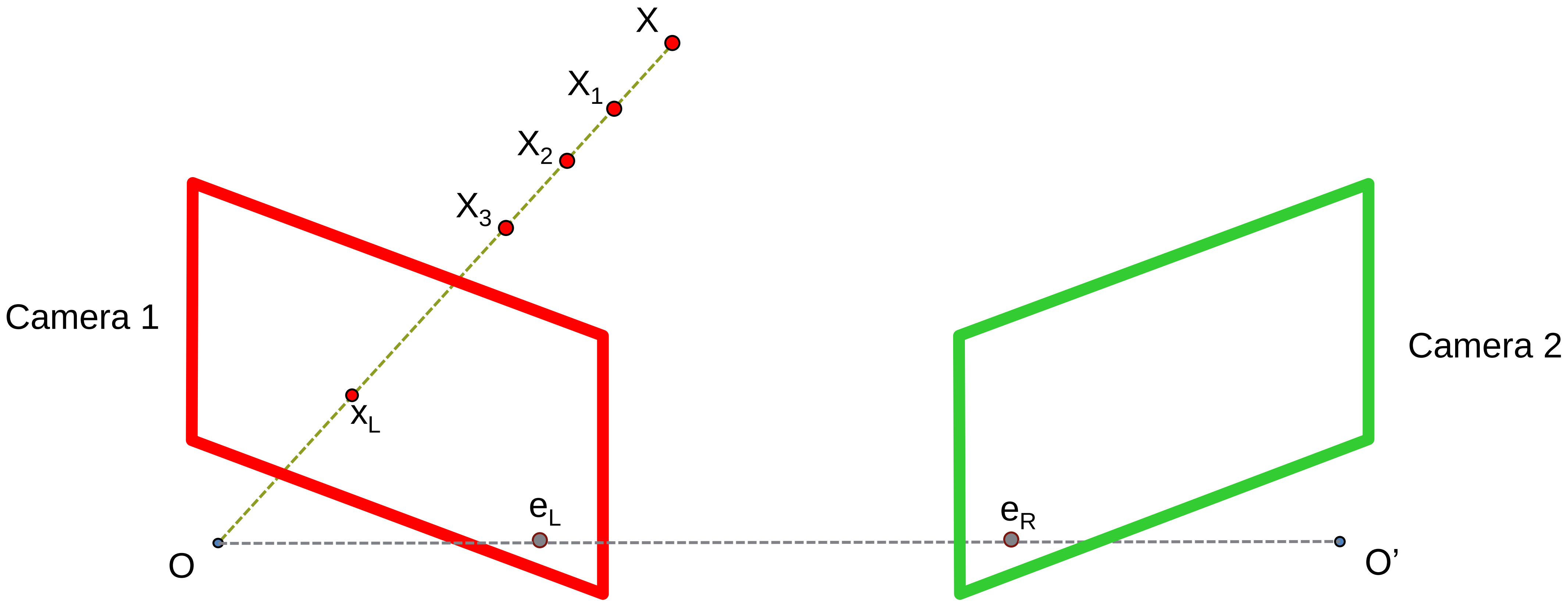
Is it really a 2D Search Problem?

Epipolar Constraints

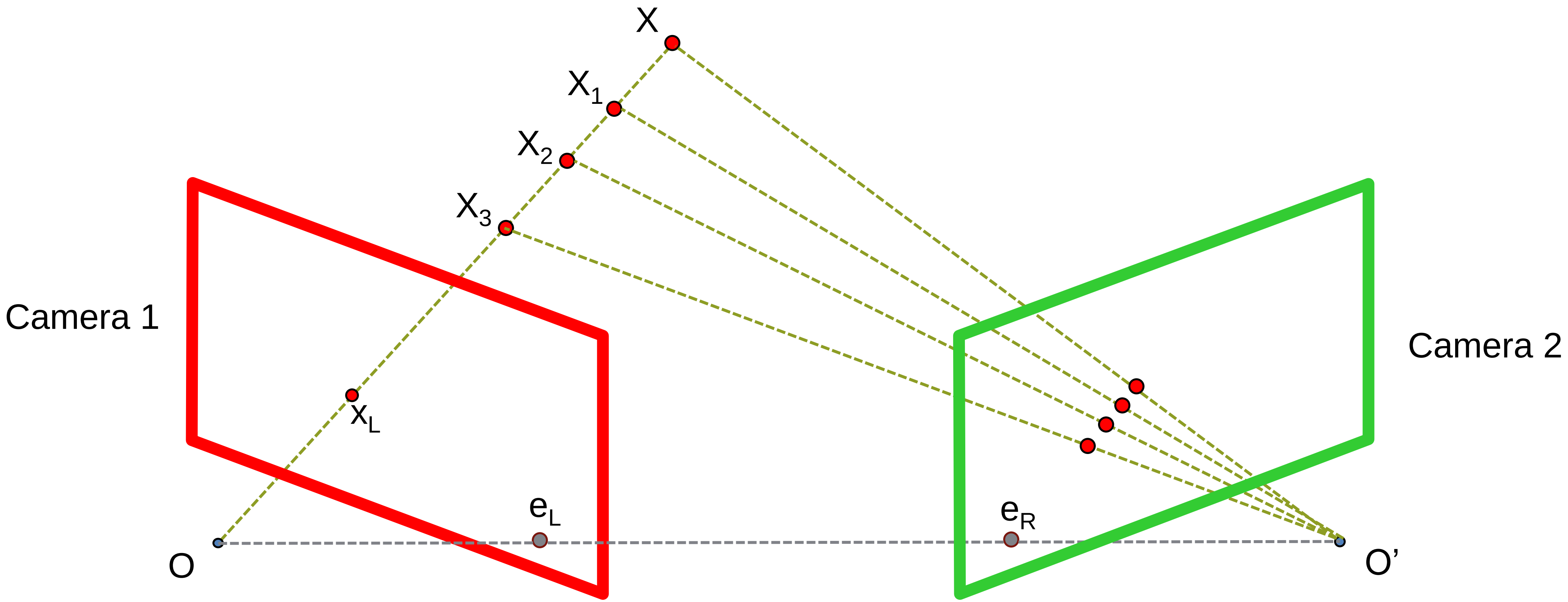


Epipoles: project each Optical Center onto other Image Plane
(Distance between Optical Centers is called Baseline)

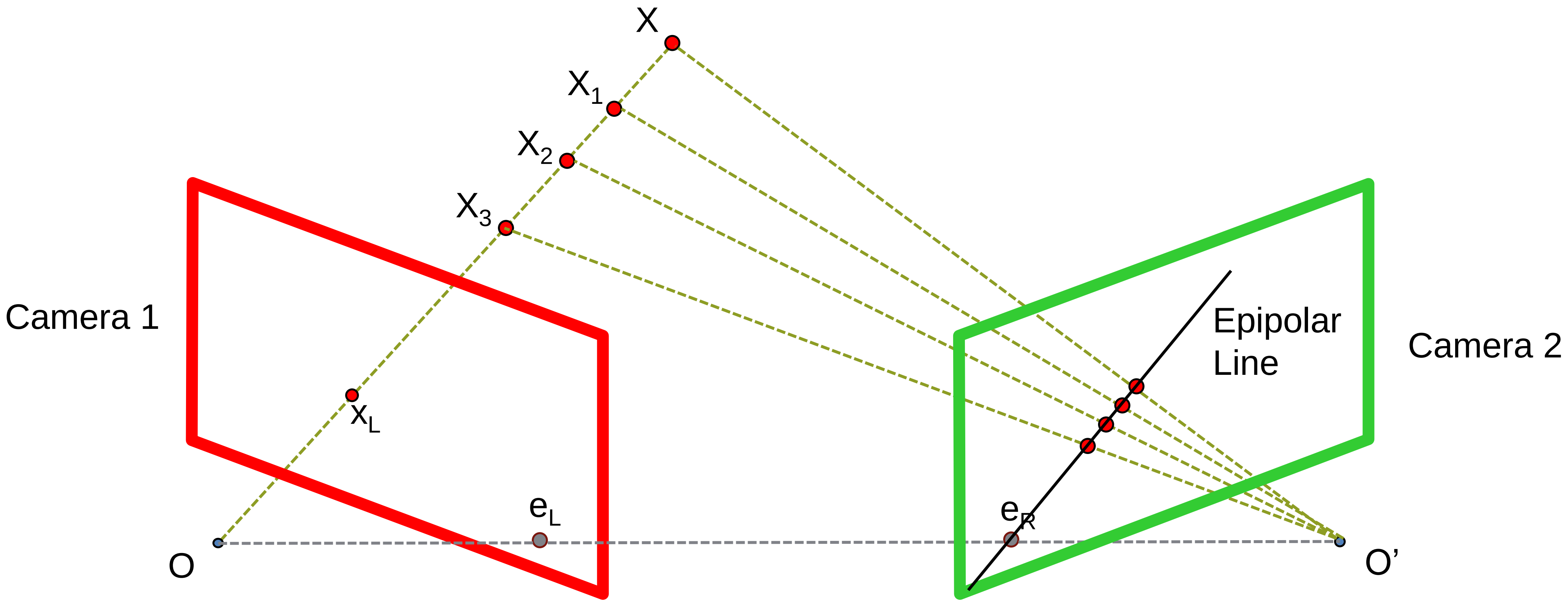
Epipolar Constraints



Epipolar Constraints



Epipolar Constraints



It's a 1D Search Problem!
But how do we get the Epipolar Line for a given Point?

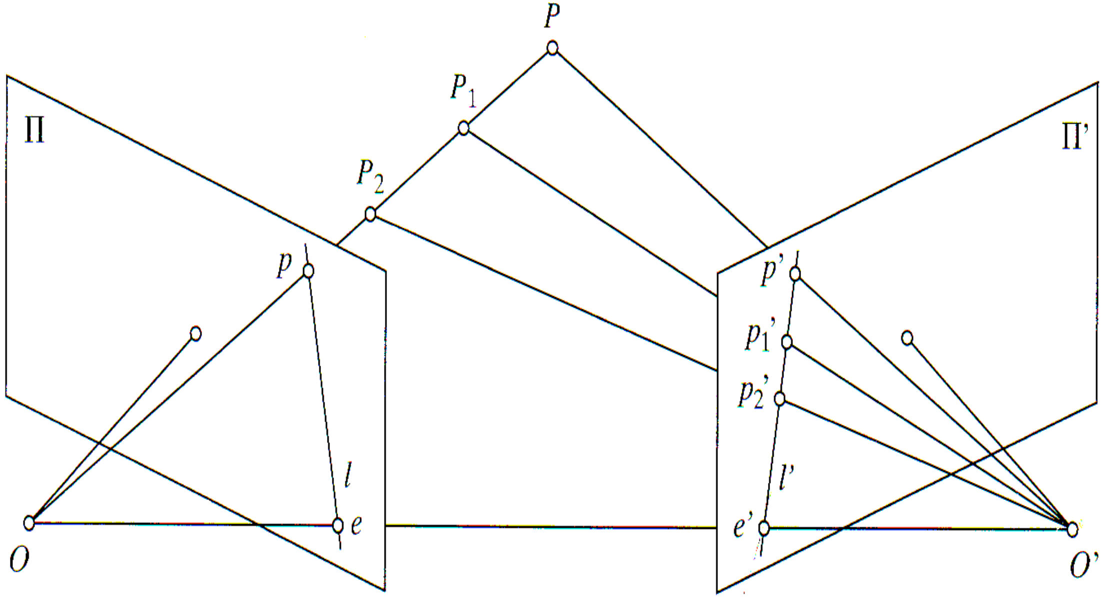
Epipolar Constraints

Calibrated Case
p,p’ on normalized Image Plane, Intrinsic K known:

$$p^T \varepsilon p' = 0$$

$l = \varepsilon p'$, $l' = \varepsilon^T p$

Line Parameters of Epipolar Line on normalized Image Plane, ***ε*** → 3x3 Essential Matrix



Uncalibrated Case
p,p’ on physical Image Plane, Intrinsic K unknown:

$$p^T F p' = 0$$

$l = Fp'$, $l' = F^T p$

Line Parameters of Epipolar Line on physical Image Plane, ***F*** → 3x3 Fundamental Matrix

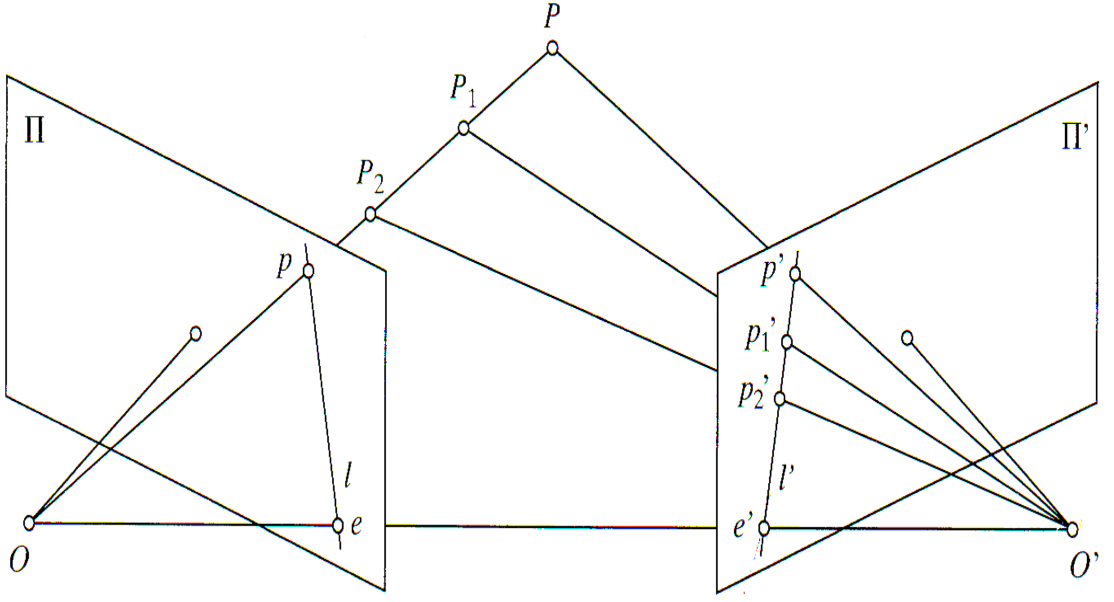
Epipolar Constraints

Calibrated Case
p,p' on normalized Image Plane, Intrinsic K known:

$$p^T \varepsilon p' = 0$$

$l = \varepsilon p'$, $l' = \varepsilon^T p$

Line Parameters of Epipolar Line on normalized Image Plane, $\boldsymbol{\varepsilon}$ → 3x3 Essential Matrix



How do we get the Matrices?

Uncalibrated Case
p,p' on physical Image Plane, Intrinsic K unknown:

$$p^T F p' = 0$$

$l = Fp'$, $l' = F^T p$

Line Parameters of Epipolar Line on physical Image Plane, ***F*** → 3x3 Fundamental Matrix

Example: The uncalibrated Case

$$p^T F p' = 0$$

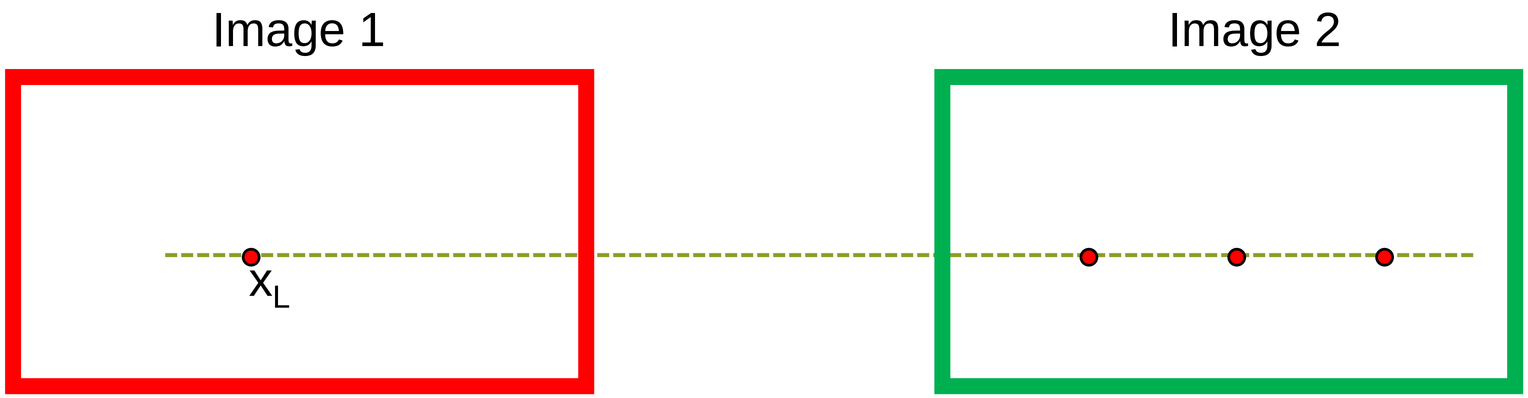
$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix}^T \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$E = \sum_{i=1}^{n} (p_i^T F p'_i)^2$$

To get the Matrix-Coefficients, you need to have sufficient amount of known corresponding point-pairs (>8)

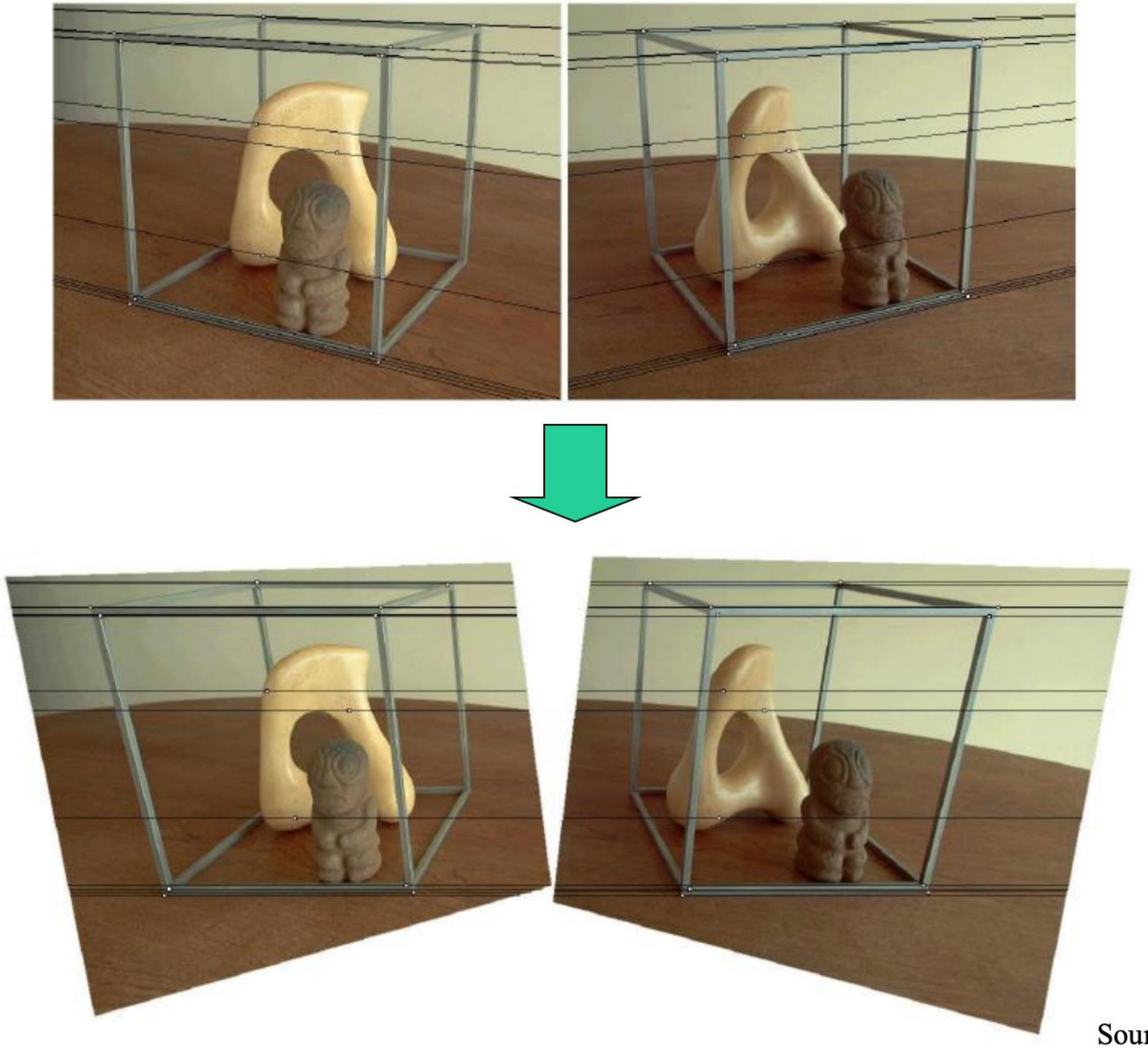
Image Rectification

If cameras are only displaced horizontally, Epipolar Lines are horizontal!



Matching Point must lie on this Line!

Image Rectification

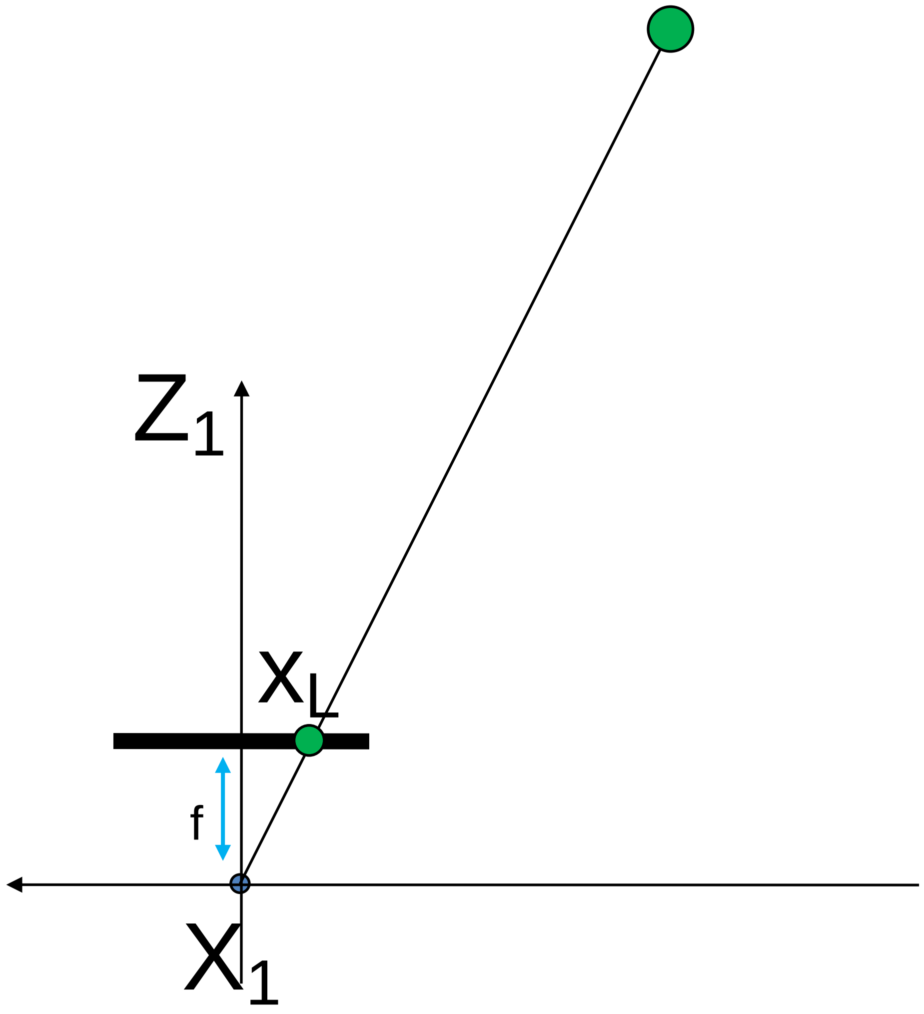


Source: Alyosha Efros

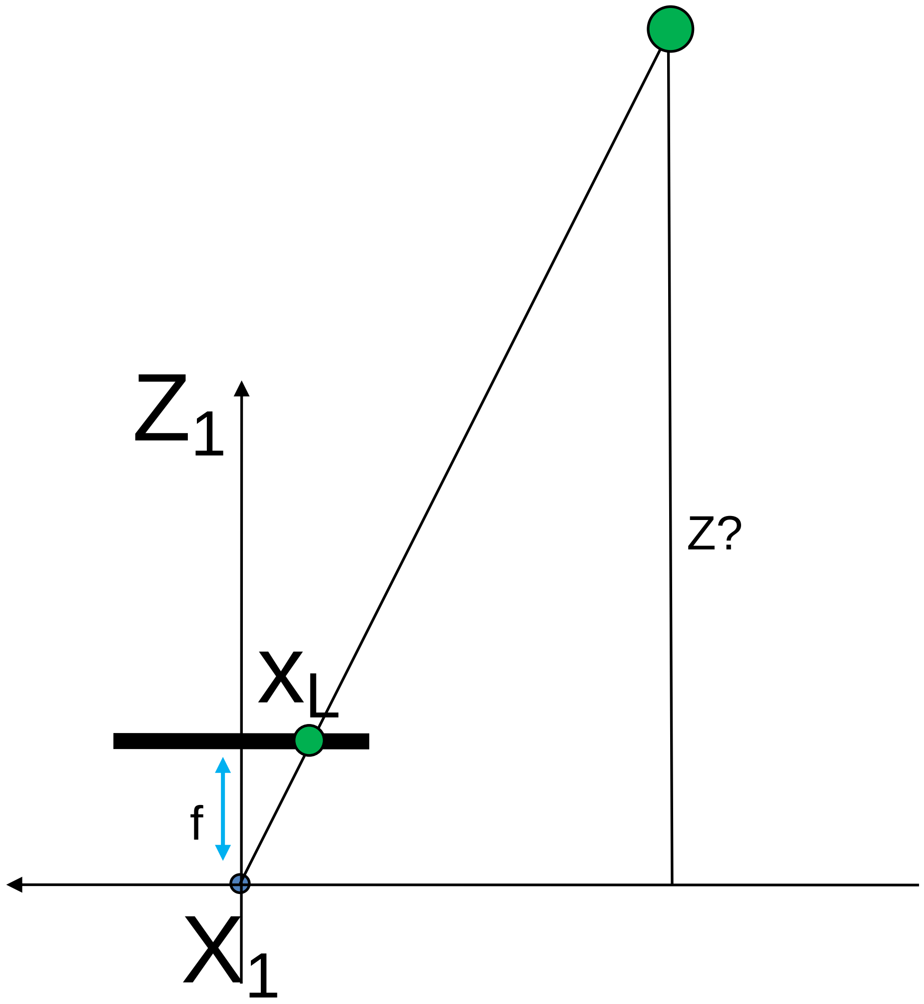
Correspondence Search

Assuming Images are rectified, we only need to search for Matches along horizontal Scanlines.

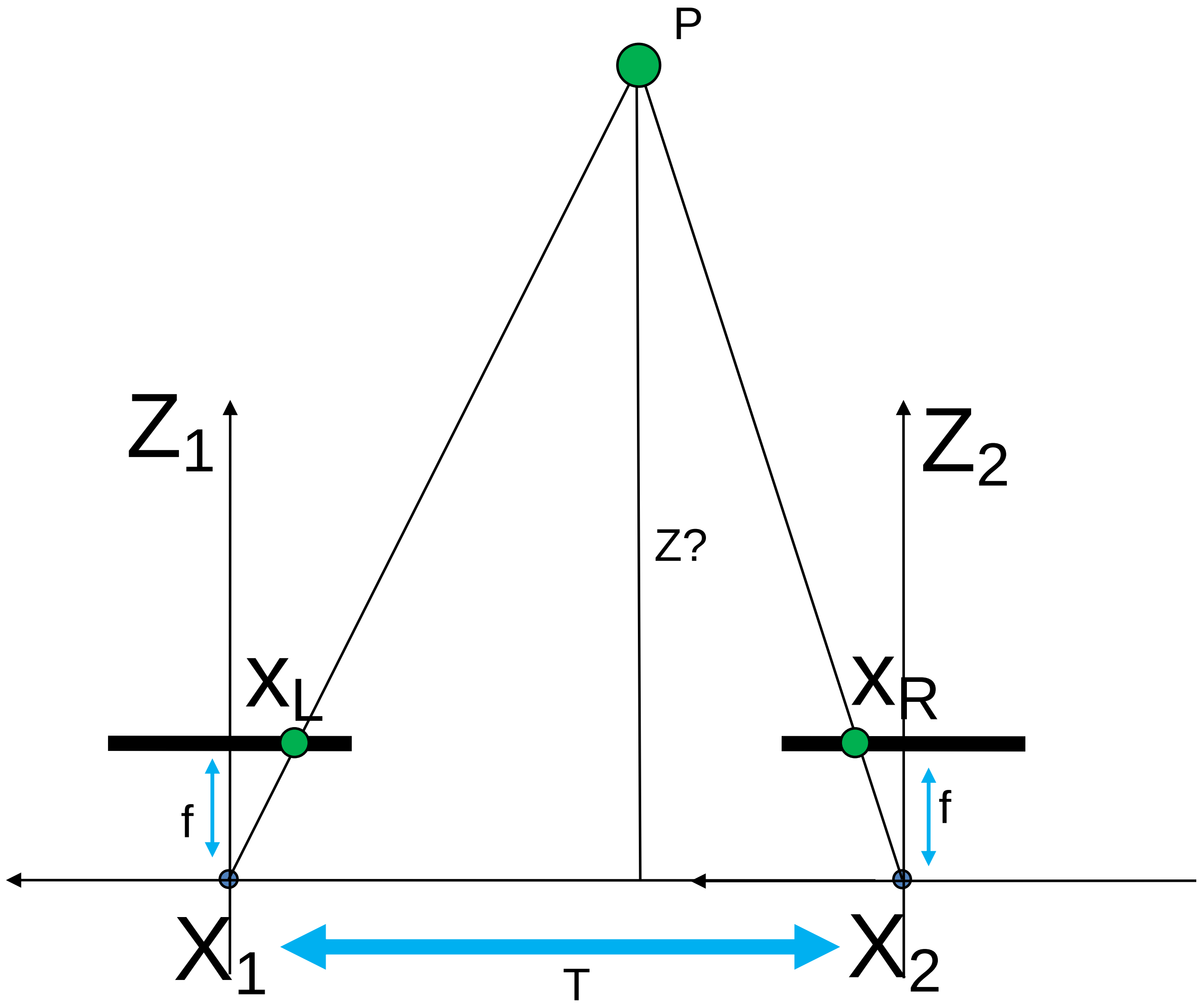
Stereoscopic Depth-Reconstruction



Stereoscopic Depth-Reconstruction



Stereoscopic Depth-Reconstruction



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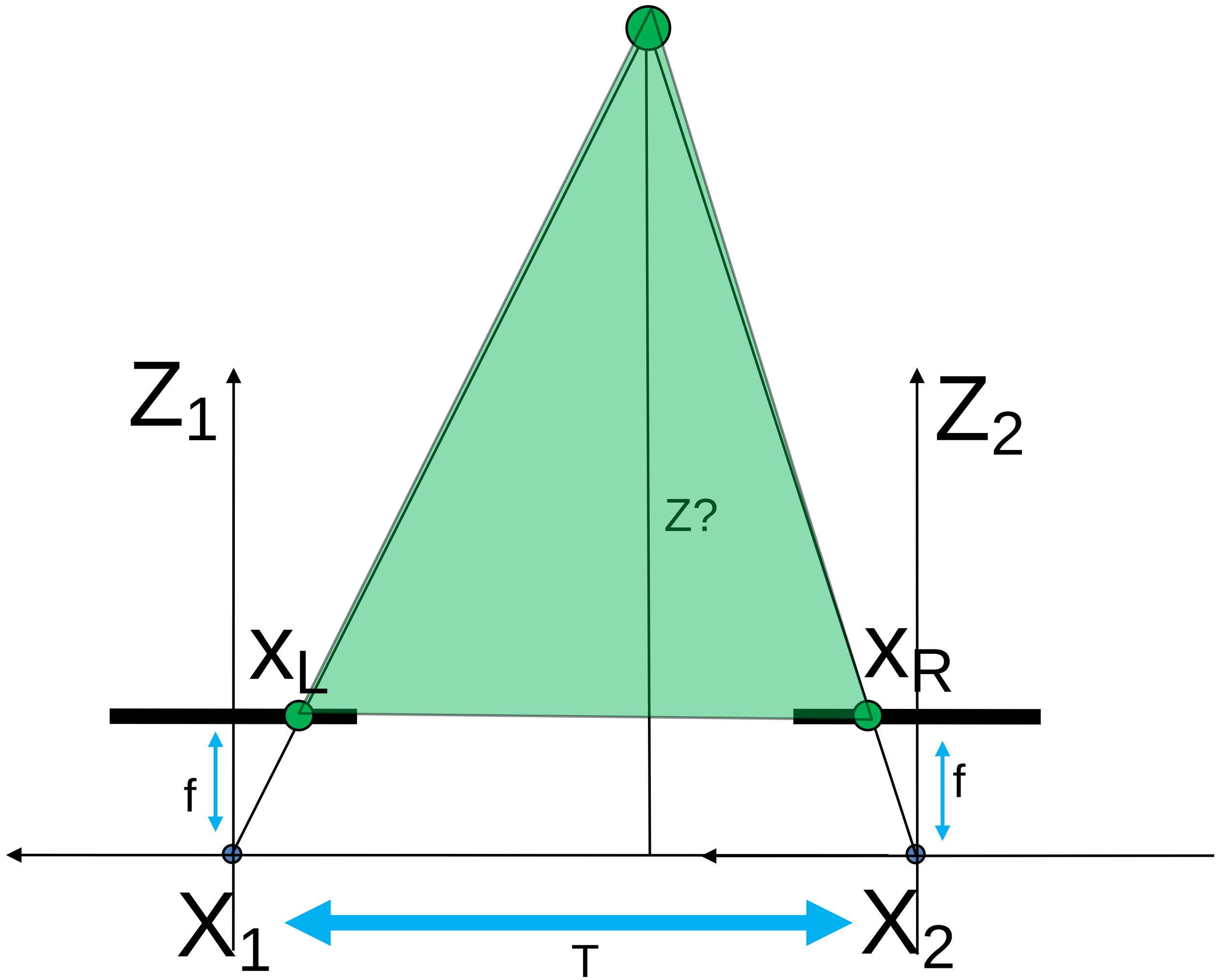

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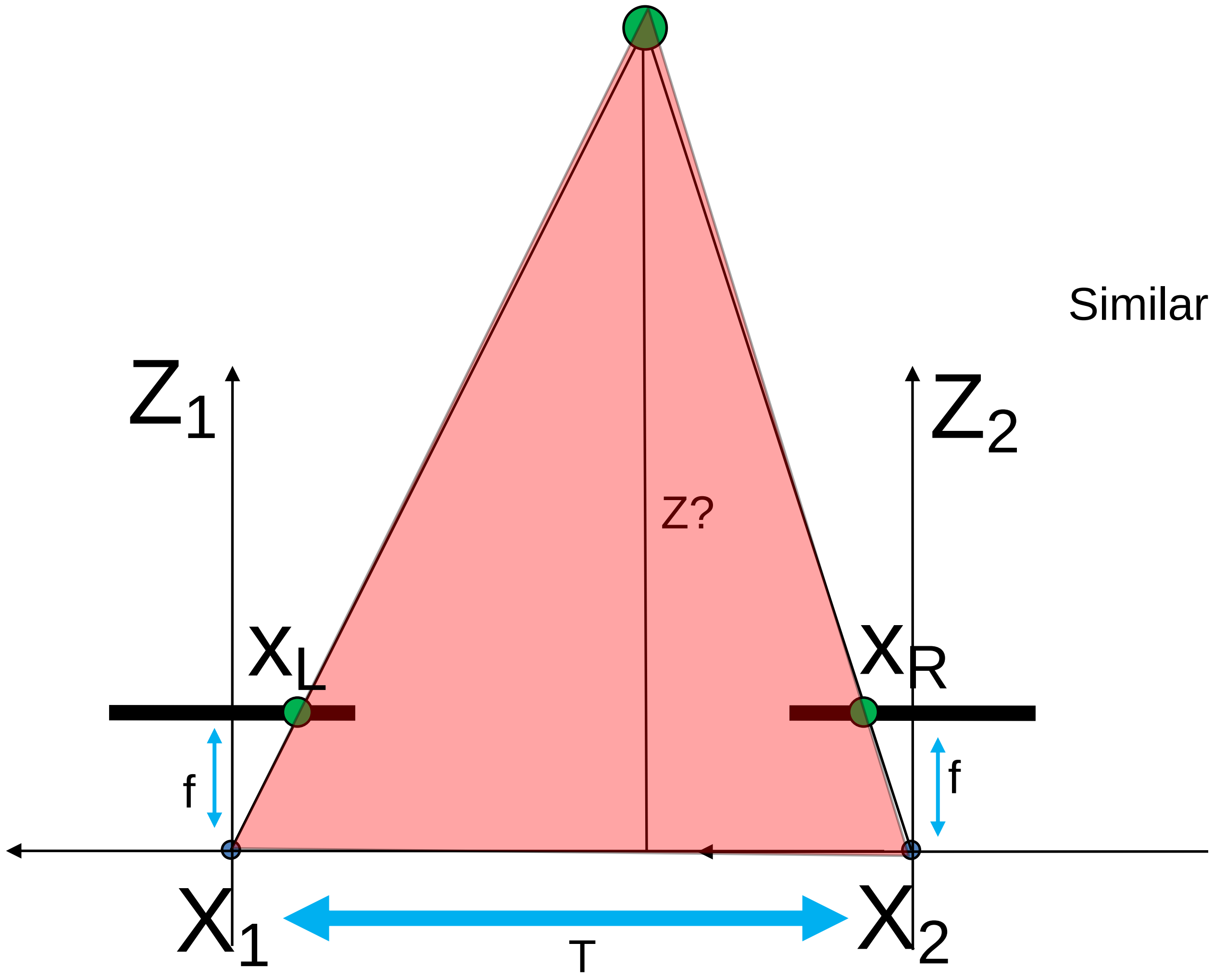
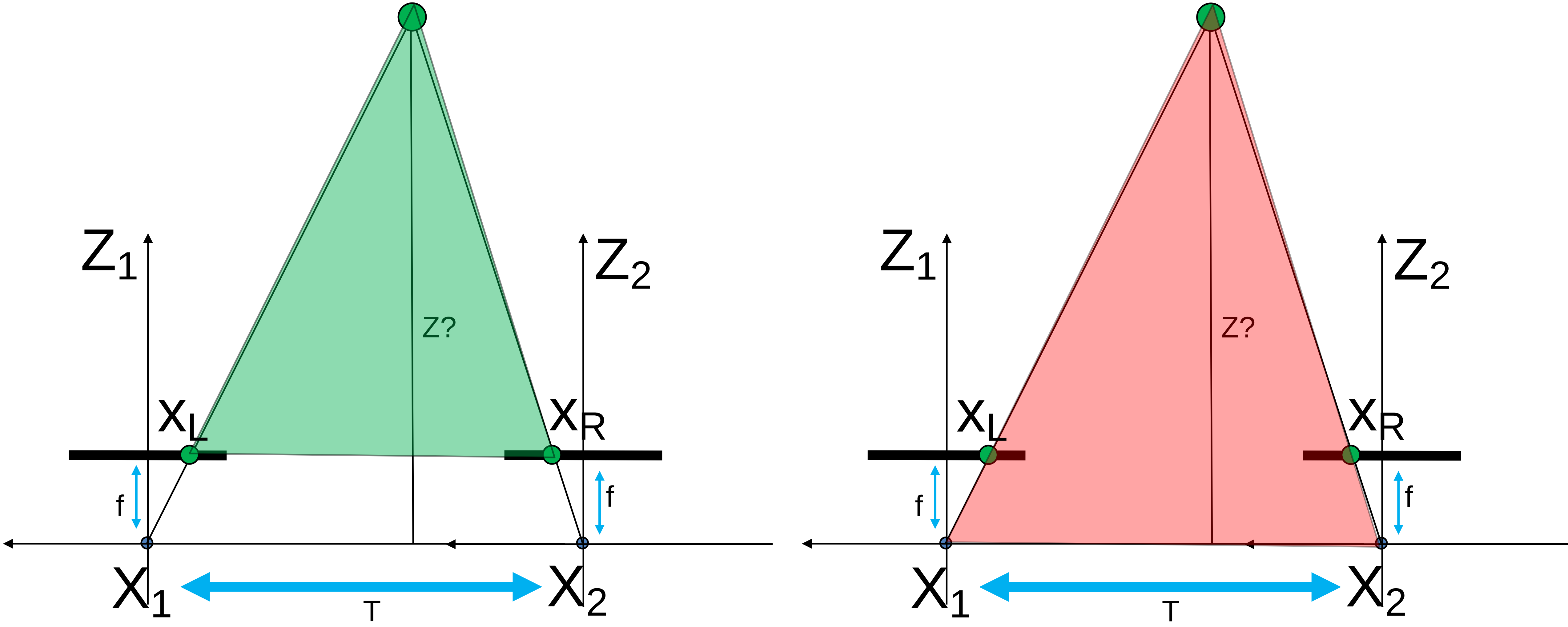


Similar Triangles

Stereoscopic Depth-Reconstruction



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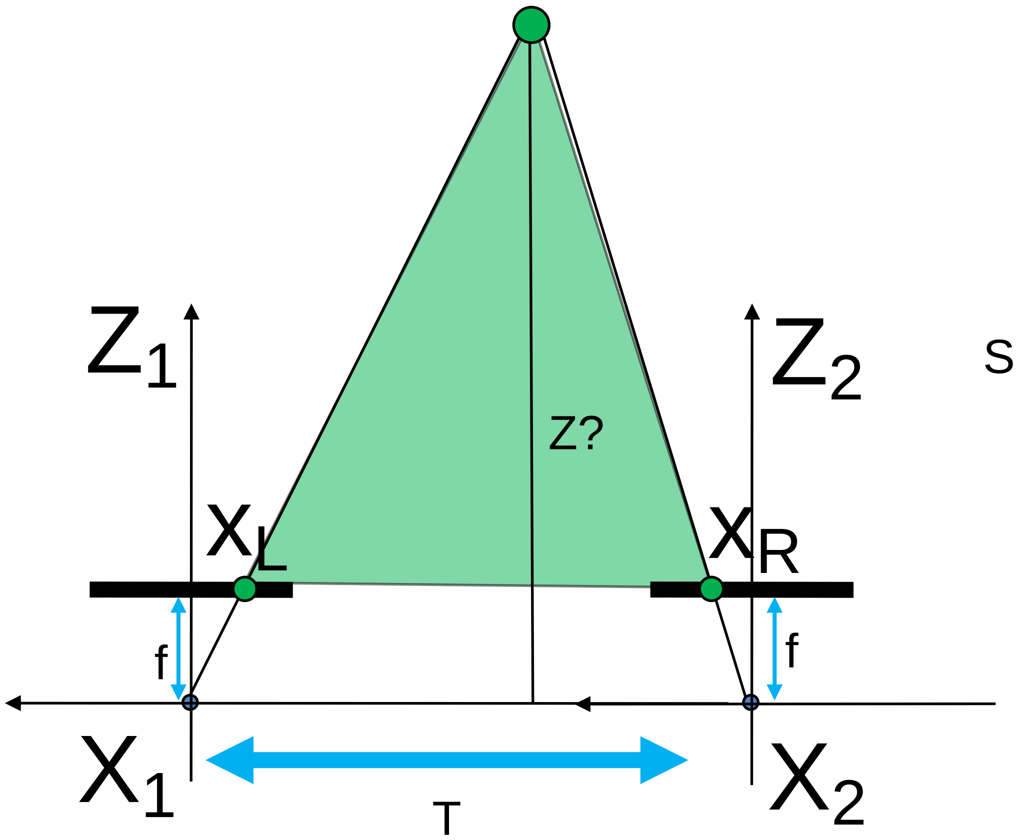
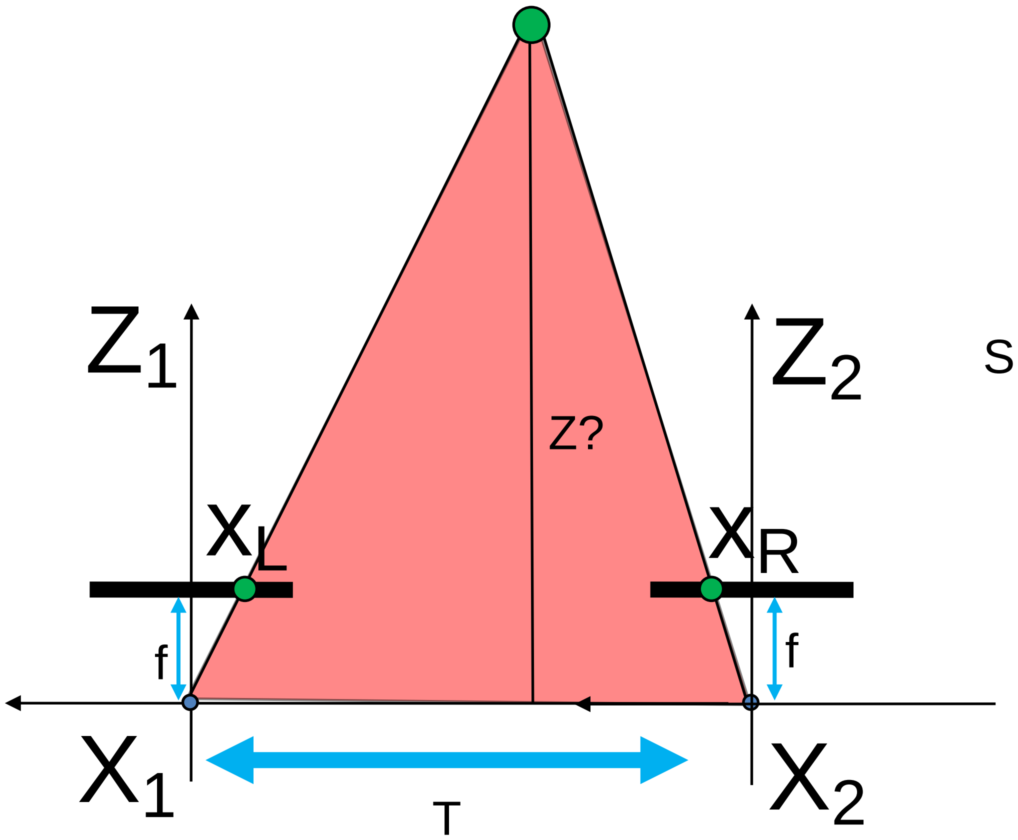


Similar Triangles:

$$\frac{T+X_R-X_L}{Z-f} =$$

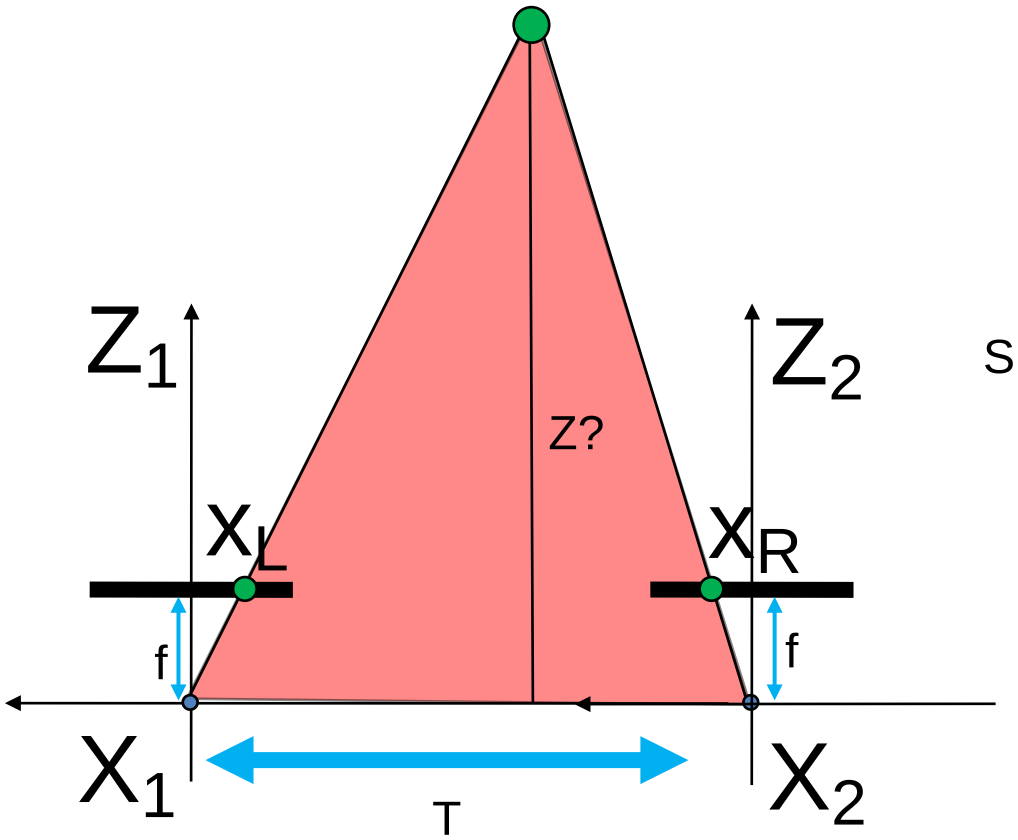
Stereoscopic Depth-Reconstruction



Similar Triangles:

$$\frac{T + X_R - X_L}{Z - f} = \frac{T}{Z}$$

Stereoscopic Depth-Reconstruction



Similar Triangles:

$$\frac{T+X_R-X_L}{Z-f} = \frac{T}{Z}$$

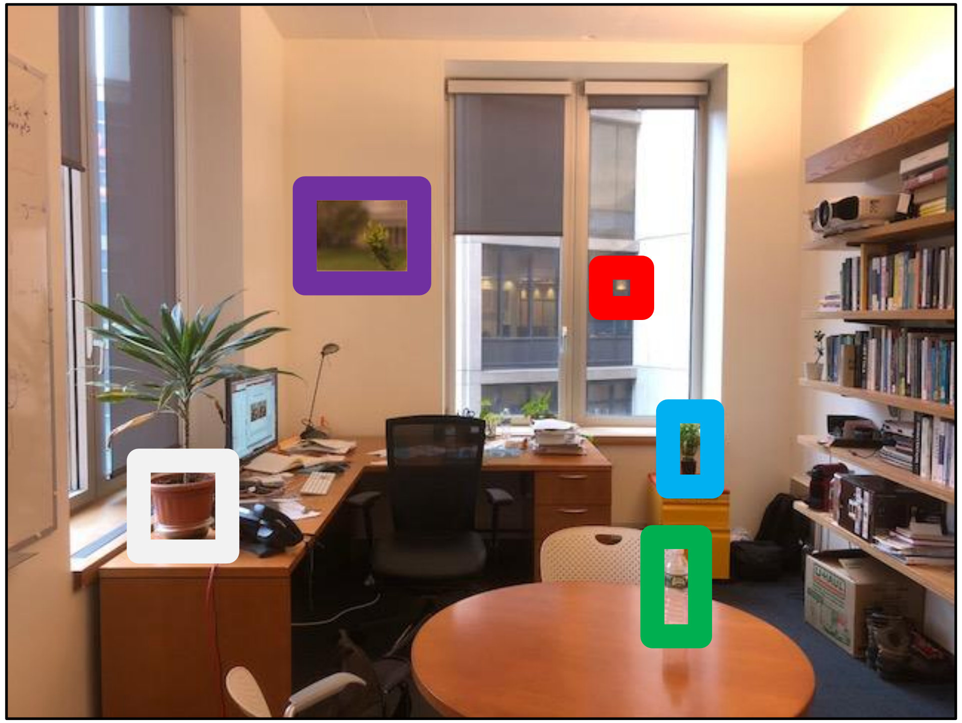
Solving for Z:

$$Z = f\,\frac{T}{X_L - X_R}$$

Disparity

So Z (depth) is proportional to Disparity.

Measuring Disparity

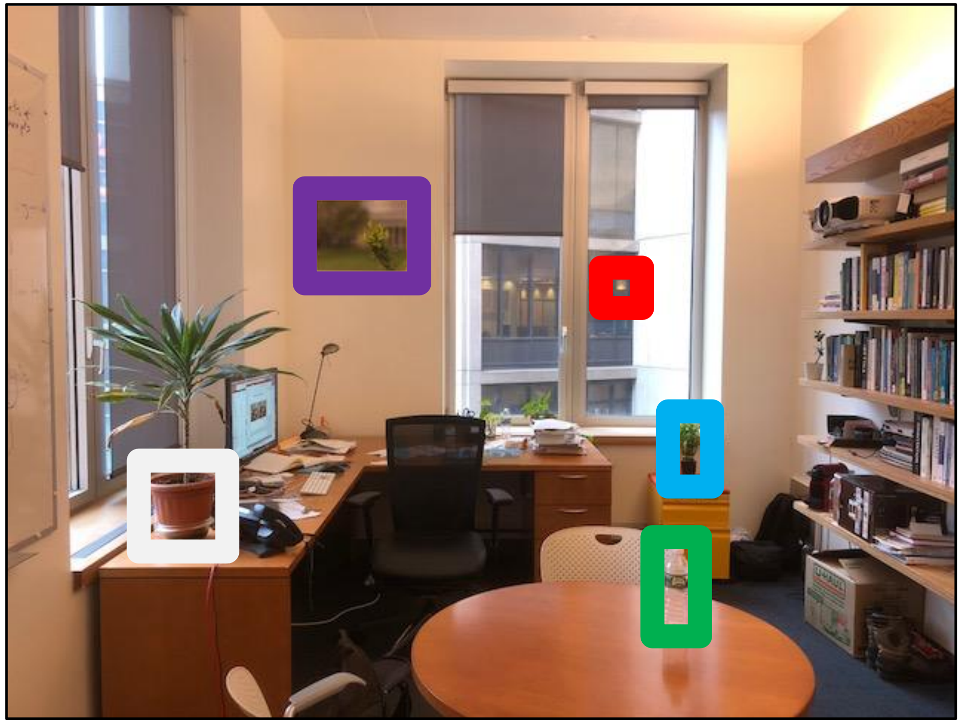


Left Image

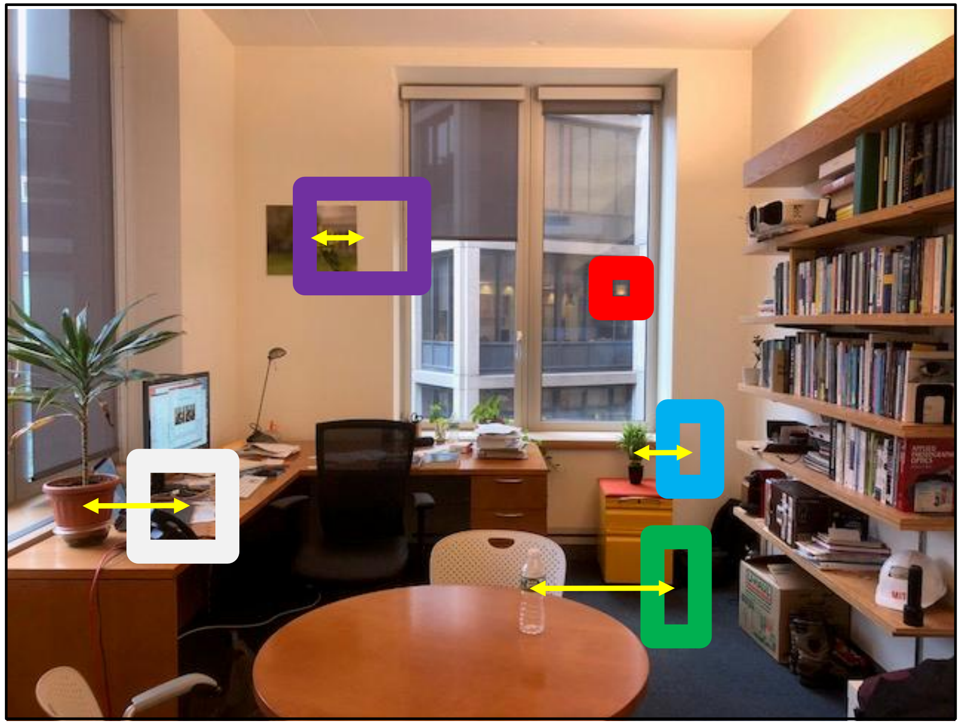
Right Image

(Taken by moving ~1m horizontally to the right)

Measuring Disparity



Left Image



Right Image

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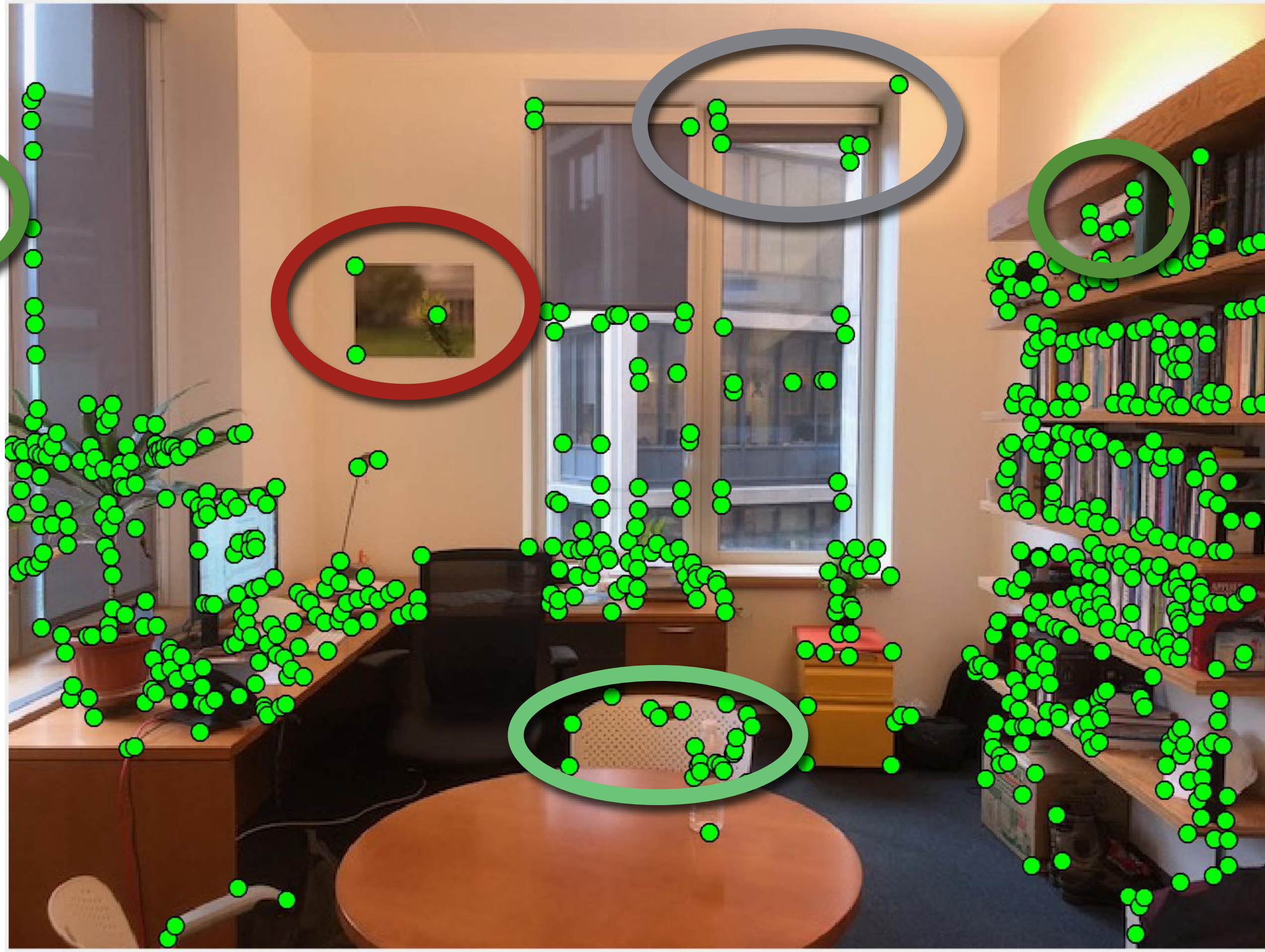
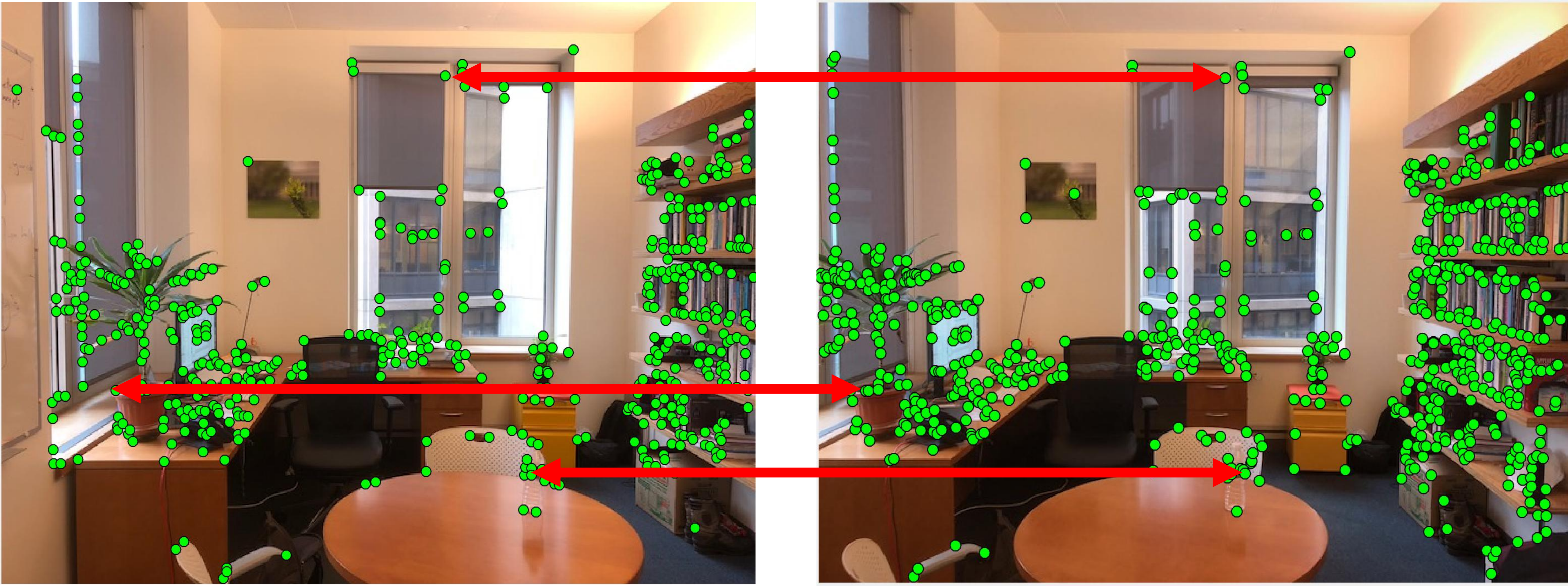
Example: SIFT Features



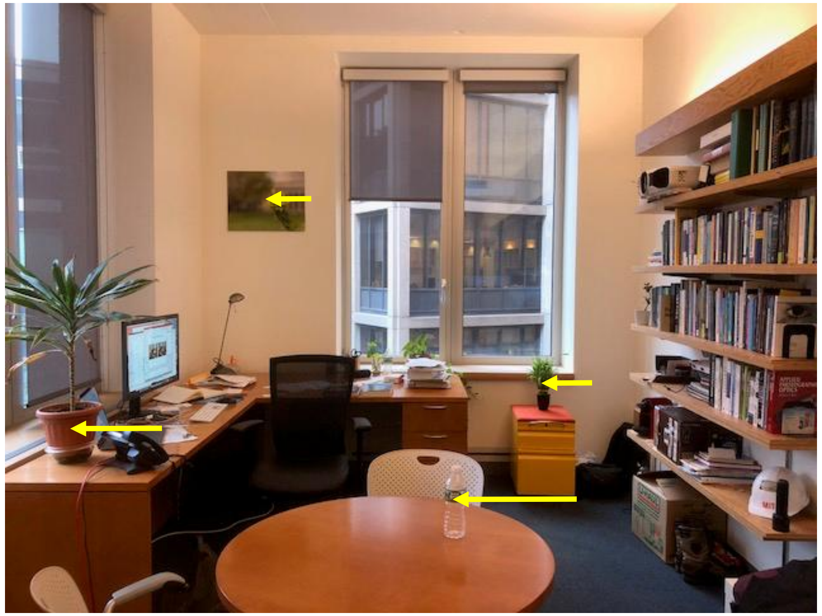
Example: SIFT Features



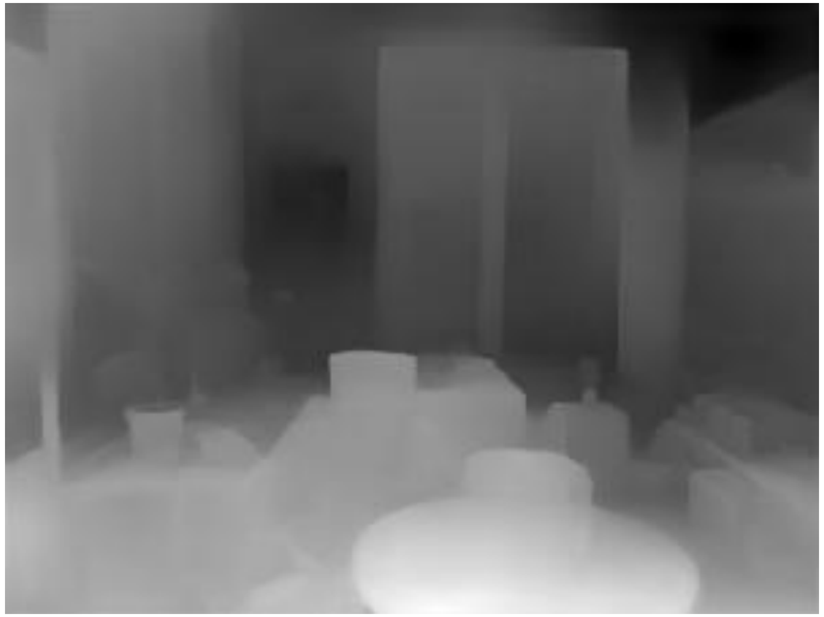
Disparity Maps

$I(x,y)$

$I'(x,y) = I(x+D(x,y), y)$



$D(x,y)$



$$Z(x,y) \propto \frac{1}{D(x,y)}$$

Structure-from-Motion

Can we estimate Depth (= Structure) and Camera Pose (= Motion) simultaneously from multiple Images?

Structure-from-Motion in a Nutshell (and Simplified)

$P_j(j = 1,...,n)$ 3D Points

$i = 1,..,m$ Images (Camera View)

$j = 1,...,n$ Points

This is what is unkown

$$p_{ij} = M_i \begin{pmatrix} P_j \\ 1 \end{pmatrix}$$

This is what is known

Structure-from-Motion in a Nutshell (and Simplified)

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This is what is known

This is an Equation-System: How many Unkowns vs. how many Equations, and when can it be solved?

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Unknowns: 12m Coefficients in all m M + 3n Coefficients in all n P , **Equations**: 2 Equations for all nm p

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This is what is known

This is an Equation-System: How many Unkowns vs. how many Equations, and when can it be solved?

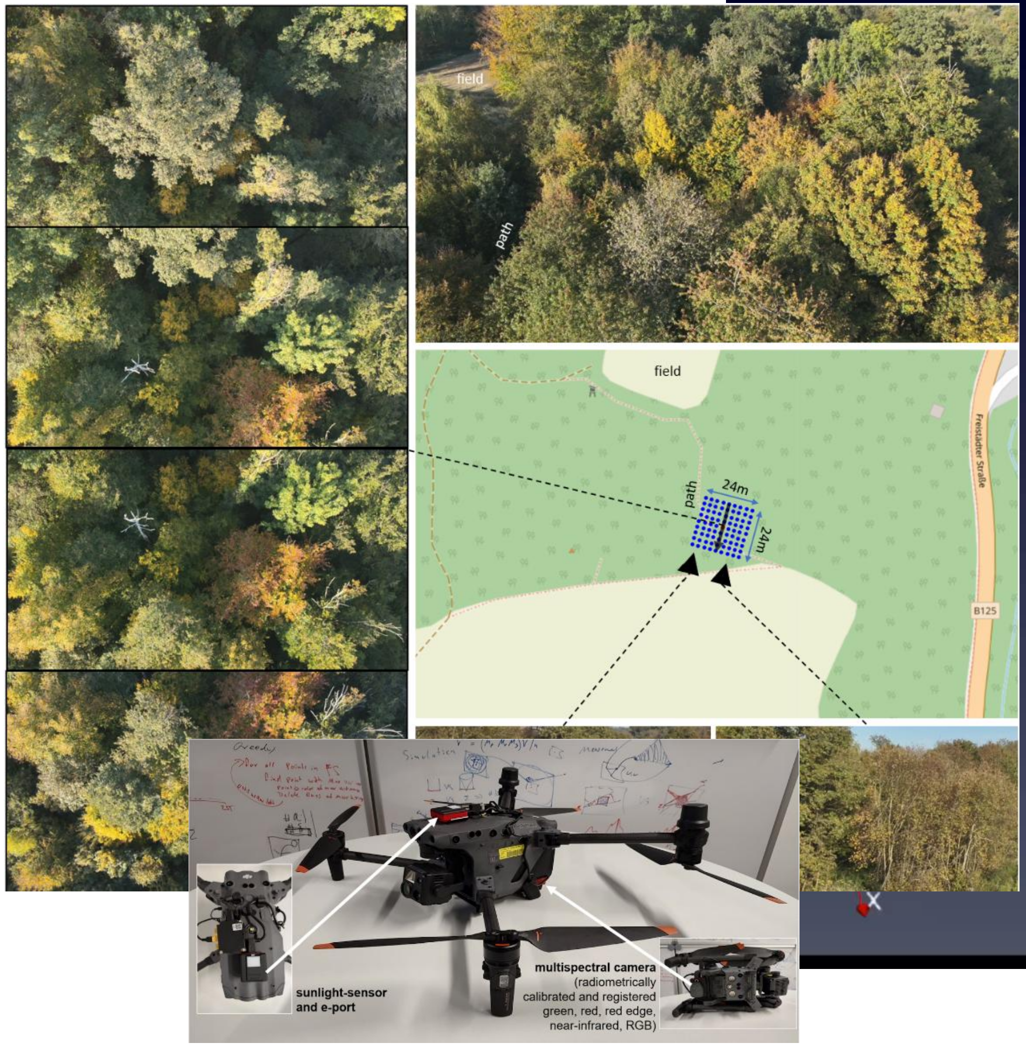
Unknowns: 12m Coefficients in all m M + 3n Coefficients in all n P , **Equations**: 2 Equations for all nm p

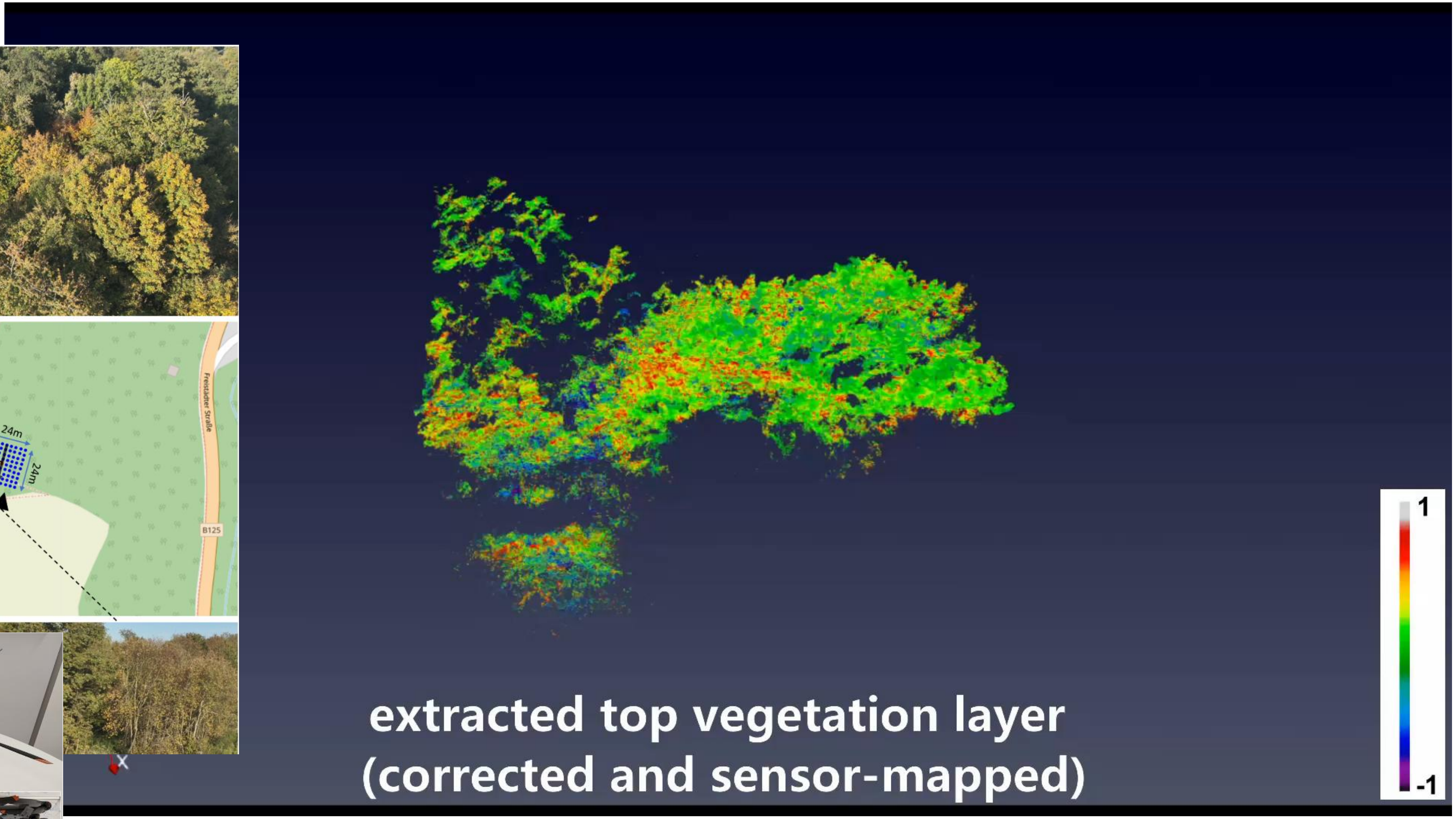
Solvable (simplified)**:** 2nm > 12m+3n (e.g., m=2 and n=25)

Structure-from-Motion

https://colmap.github.io/

Research Example: Reconstructing 3D Forest Health





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Journal of Remote Sensing
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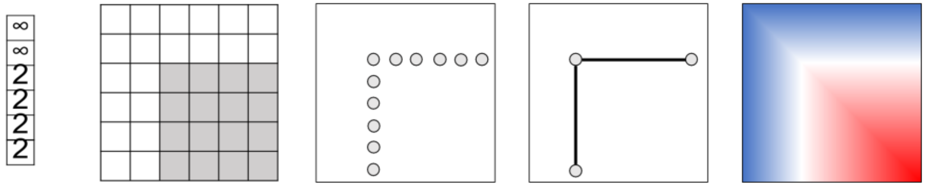
Next Week: 3D Vision

How to represent Depth

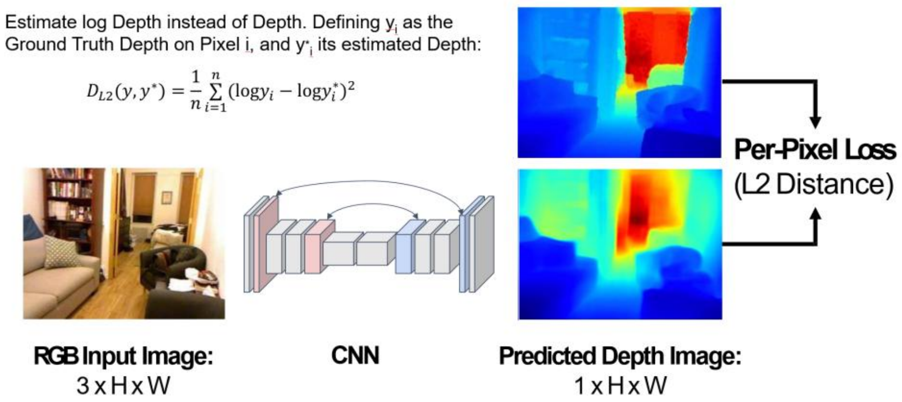


Depth Map | Voxel Grid | Point Cloud | Mesh | Implicit Surface

Predicting Depth Maps

Estimate log Depth instead of Depth. Defining y_i as the Ground Truth Depth on Pixel i, and y^*_i its estimated Depth:

$$D_{L2}(y, y^*) = \frac{1}{n}\sum_{i=1}^{n}(\log y_i - \log y_i^*)^2$$



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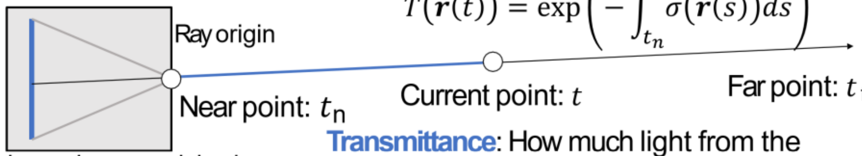
Volume Rendering

Color observed by the camera given by volume rendering equation:

$$C(\boldsymbol{r}) = \int_{t_n}^{t_f} T(\boldsymbol{r}(t))\sigma(\boldsymbol{r}(t))\boldsymbol{c}(\boldsymbol{r}(t), \boldsymbol{d})dt$$

$$T(\boldsymbol{r}(t)) = \exp\left(-\int_{t_n}^{t}\sigma(\boldsymbol{r}(s))ds\right)$$



Parameterize each ray as origin plus direction: $\boldsymbol{r}(t) = \boldsymbol{o} + t\boldsymbol{d}$
Volume Density is $\sigma(\boldsymbol{p}) \in [0,1]$
Color that a point **p** emits in direction **d** is $c(\boldsymbol{p}, \boldsymbol{d}) \in [0,1]^3$

Transmittance: How much light from the current point will reach the camera?

Compute transmittance by accumulating volume density up to current point

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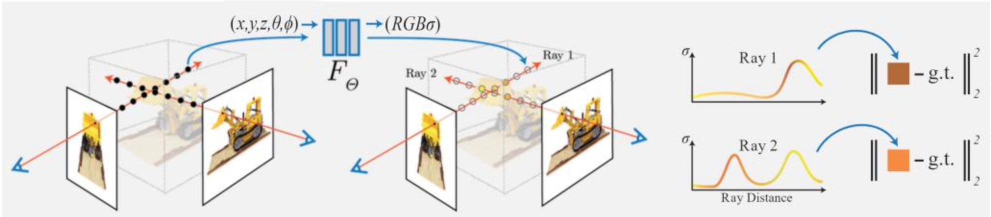
Neural Radiance Fields (NeRFs)

Fully-connected Network: Input Position p=x,y,z and Direction d=θ,Φ, and output Volume Density (σ) and RGB color



Mildenhall et al, "Representing Scenes as Neural Radiance Fields for View Synthesis", ECCV 2020

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Thank You