Book #1 of 2

Name: ____________________

ID# (last 4 digits): ____________ Section: ____________

Show all work clearly using proper notation and explain your reasoning in English where appropriate. Answers must be justified using techniques that have been taught in this course, and answers without such justification may receive less than full credit – or no credit at all – even if the answer is correct.

This exam is closed book. Calculators, electronic devices, notes, books, formula sheets, and other outside materials are not allowed. Phones must be turned off and put away.

Unless otherwise stated, give exact answers: e.g., write π and $\sqrt{2}$ instead of 3.14 and 1.41. However, when an expression simplifies to a well-known value, you must use that value. For example, you must write 1 instead of e^0, and you must write $\pi/3$ instead of $\arccos(1/2)$.

This exam has 8 questions, printed in 2 booklets, for a total of 100 points.

Question	Points	Score
1.	12	
2.	12	
3.	12	
4.	14	
5.	10	
6.	10	
7.	12	
8.	18	
Total:	100	

1. Let $\boldsymbol{v} = \boldsymbol{i} - \lambda \boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{w} = \lambda \boldsymbol{i} - 4\boldsymbol{j} + \mu \boldsymbol{k}$, where λ and μ are real numbers.

2 pts (a) Calculate $\|\boldsymbol{v}\|$ in terms of λ and μ.

$\|\boldsymbol{v}\| =$ ____________________

5 pts (b) Calculate $\boldsymbol{v} \cdot \boldsymbol{w}$ and $\boldsymbol{v} \times \boldsymbol{w}$ in terms of λ and μ.

$\boldsymbol{v} \cdot \boldsymbol{w} =$ ____________________

$\boldsymbol{v} \times \boldsymbol{w} =$ ____________________

5 pts (c) Use your answer to part (b) to find the values of λ and μ such that $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel. (Assume λ and μ are ***positive*** real numbers.)

$\lambda =$ ____________________

$\mu =$ ____________________

2. The lines ℓ_1 and ℓ_2 are given by the following parametrizations.

$$\begin{aligned} \ell_1 : \quad \boldsymbol{r}_1(t) &= \langle -2, -1, 4 \rangle + t\,\langle -5, 5, 1 \rangle \\ \ell_2 : \quad \boldsymbol{r}_2(t) &= \langle 0, -10, 10 \rangle + t\,\langle 3, 4, -7 \rangle \end{aligned}$$

6 pts (a) Show that ℓ_1 and ℓ_2 intersect and find the point of intersection. Is this point also a collision point? Explain.

point of intersection: ____________________

collision point? (yes/no): ____________________

6 pts (b) Find an equation of the plane $\mathcal{P}$ that contains both ℓ_1 and ℓ_2.

equation of $\mathcal{P}$: ______________________________

3. Consider the curve $\mathcal{C}$ with parametrization

$$\boldsymbol{r}(t) = (t^2 - 3)\boldsymbol{i} + (3t^2 + 5)\boldsymbol{j} + \tfrac{2}{3}t^3\boldsymbol{k} \quad , \quad t \geq 0$$

6 pts (a) Find the length of $\mathcal{C}$ over the interval $0 \leq t \leq 1$.

length: ____________________

6 pts (b) Find the curvature of $\mathcal{C}$ at the point $\boldsymbol{r}(1)$. You may use the formula

$$\kappa(t) = \frac{\|\boldsymbol{r}''(t) \times \boldsymbol{r}'(t)\|}{\|\boldsymbol{r}'(t)\|^3}$$

curvature: ____________________

4. *Assume that the positive x-axis points East and the positive y-axis points North.*
Suppose you are hiking on a terrain modeled by the equation $z = \sqrt{3}xy - 2x^2 - 1$ and you are standing at the point $(1, \sqrt{3}, 0)$.

5 pts (a) Determine the angle of inclination you would encounter if you headed due West.

angle of inclination: ____________________

5 pts (b) Determine the steepest slope you could encounter from your position and the compass direction measured in degrees anticlockwise from East that you would head to realize this steepest slope.

steepest slope: ____________________

compass direction: ____________________

4 pts (c) In what direction should you head to encounter no change in elevation? Give your answer as an angle measured in degrees anticlockwise from East.

compass direction: ____________________

This page is for scratch work. Do not detach this sheet.

Book #2 of 2

Name: ________________________________

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10 pts **5.** Calculate the following limit or show that it does not exist.

$$\lim_{(x,y)\to(0,0)} \left(\frac{xy}{3x^2+2y^2} \right)$$

value of limit: ____________________

10 pts **6.** Find all points on the graph of $z = xy^3 + 10y^{-1} + 12$ where the vector $\boldsymbol{n} = \langle 16, -7, 2 \rangle$ is normal to the tangent plane.

point(s) on graph: ______________________________

12 pts **7.** Let r, s, and t be independent parameters and suppose x, y, and z are given by

$$\begin{aligned} x &= 2r - 3s + t \\ y &= 5r + 2s - 6t \\ z &= -r + s \end{aligned}$$

Let $w = f(x, y, z)$ where f is an arbitrary differentiable function. Calculate the sum

$$A(r,s,t) = \frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} + \frac{\partial w}{\partial t}$$

Write your answer as a function of r, s, and t. Simplify as much as possible.
(Since f is arbitrary, your answer may still contain the symbol f or related symbols. But you must write your answer as a function of r, s, and t.)

$A(r, s, t) =$ ______________________________

8. *Note: This problem continues onto the next page.*

Let $f(x, y) = x^2 + y^2 - xy - 6x$

9 pts (a) Find the critical point of f and the corresponding critical value. Then classify it as a local minimum, local maximum, or neither (saddle).

critical point: ____________________

critical value: ____________________

classification: ____________________

Note: This is a continuation of the problem on the previous page.

Recall $f(x, y) = x^2 + y^2 - xy - 6x$. Let S be the square $\{(x, y) : 0 \leq x \leq 6, 0 \leq y \leq 6\}$.

9 pts

(b) Find the minimum and maximum values of f on each of the four edges of S. Then determine the global extreme values of f on S. *Fill in the table below as you work.*

edge of S	bottom edge	right edge	top edge	left edge
minimum value of f				
maximum value of f				

global minimum value: ____________________

global maximum value: ____________________

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