2D-Discrete Dynamical Systems (Assignment Sheet 3)

Introduction To Chaos Applied To Systems, Processes And Products (ETSIDI, UPM)

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Introduction: Plotting 2D-orbits in R

Before solving the exercises, let's go through an example of how to plot the orbits of a 2D map using R. We will use the Gingerbreadman map, which is a simple example of a chaotic dynamical system that generates intricate, non-repeating patterns from a piecewise linear transformation.

The Gingerbreadman map is given by the equations:

$$x_{n+1} = 1 - y_n + |x_n|$$
$$y_{n+1} = x_n$$

To visualize orbits of this system in R, follow these steps:

1. **Load ggplot2 for visualization:**

```
library(ggplot2)
```

2. **Define the function to generate the Gingerbreadman map:**

```
gingerbreadman_map <- function(x0, y0, n) {
x <- numeric(n)
y <- numeric(n)
x[1] <- x0
y[1] <- y0

for (i in 2:n) {
  x[i] <- 1 - y[i-1] + abs(x[i-1])
```

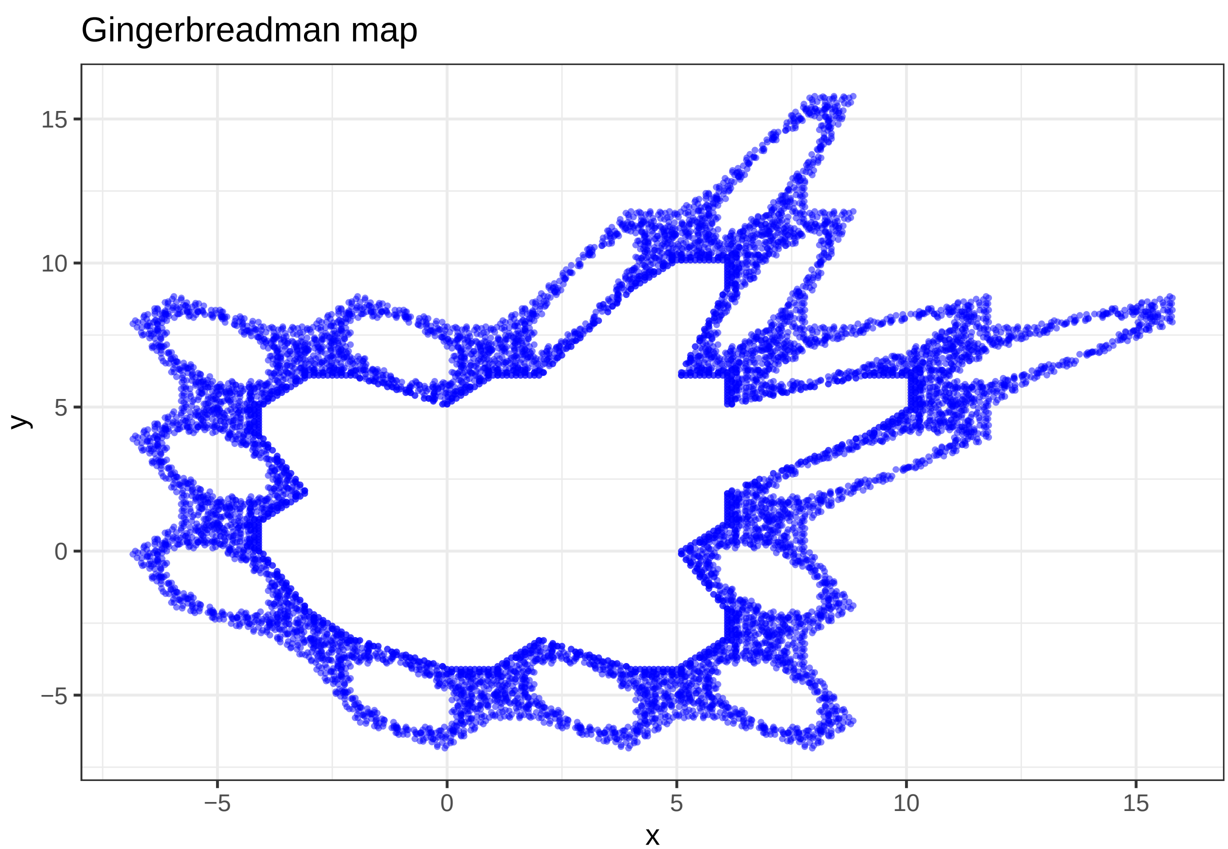
```
  y[i] <- x[i-1]
}

data.frame(x, y)
}
```

2. **Generate orbit with initial conditions:**

```
data_gbm <- gingerbreadman_map(1.1, 6.1, 10000)
```

3. **Plot the orbit using `ggplot2`:**

```
ggplot(data_gbm, aes(x = x, y = y)) +
 geom_point(color = 'blue', alpha = 0.5, size = .5) +
 labs(title = "Gingerbreadman map", x = "x", y = "y") +
 theme_bw()
```



You can modify the parameters to observe different behaviors of the above map using `ggplot2`.

Exercise 1: The Lotka-Volterra discrete map

The Lotka-Volterra discrete map models predator-prey interactions in a discrete-time framework, capturing oscillatory population dynamics and potential chaotic behavior. The long-term behavior of the system can converge to fixed points, periodic cycles, or chaotic attractors, depending on the parameter values.

The discrete 2D Lotka-Volterra map is given by:

$$x_n = \frac{\alpha x_{n-1} - \beta x_{n-1} y_{n-1}}{1 + \gamma x_{n-1}}$$

$$y_n = \frac{\delta y_{n-1} - \epsilon x_{n-1} y_{n-1}}{1 + \eta y_{n-1}}$$

where:

- Case 1: $\alpha = 170, \beta = 11, \gamma = 2.7, \delta = 50, \epsilon = 1.7, \eta = 7, x_1 = 7, y_1 = 5$
- Case 2: $\alpha = 1.001, \beta = 0.03, \gamma = 0.6, \delta = 1.002, \epsilon = 1.7, \eta = 0.9, x_1 = 0.0002, y_1 = 0.0006$

Plot the solution (orbit) of the discrete dynamical system for both cases.

Exercise 2: The Henon map (1976)

The Henon map is a simplified model of chaotic dynamics in two dimensions, originally derived from a model of stellar motion in galaxies. It exhibits a well-known strange attractor, which demonstrates sensitive dependence on initial conditions, a key feature of chaotic systems.

The Henon map is given by the equations:

$$x_{n+1} = y_n + 1 - a x_n^2$$

$$y_{n+1} = b x_n$$

where a and b are adjustable parameters. Set $a = 1.4$ and $b = 0.3$, then plot x_n vs y_n.

Exercise 3: The standard map (Chirikov 1971)

The Standard Map, also known as the Chirikov-Taylor Map, is a key model in Hamiltonian chaos, describing the transition from regular to chaotic motion in conservative dynamical systems. Depending on the parameter b, the system can display stable islands, chaotic seas, or a mixture of both.

The Standard map is defined as follows:

$$x_{n+1} = y_{n+1} + x_n$$

$$y_{n+1} = y_n + b \sin(x_n)$$

- (a) Plot various orbits for $b = 0$.
- (b) Plot the phase portrait for $b = 0.5$.
- (c) Show the phase portrait for $b = 1$ and $b = 2$.

Exercise 4: The Lozi map (1978)

The Lozi map is a piecewise linear version of the Henon map that retains its chaotic properties while being computationally simpler to analyze. It generates a strange attractor with a fractal structure, making it an important example in the study of deterministic chaos.

The Lozi map is defined by:

$$x_{n+1} = y_n + 1 - a|x_n|$$
$$y_{n+1} = bx_n$$

where $-1 < b < 1$. Plot orbits when $a = 1.7$ and $b = 0.5$.