1D-Continuous Dynamical Systems (Assignment Sheet 2)

Introduction To Chaos Applied To Systems, Processes And Products (ETSIDI, UPM)

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Exercise 1

Consider the system

$$\dot{x} = x^2 - 1$$

Solve and plot $\dot{x}$ versus t for:

- **Case a**: $x(0) < -1$
- **Case b**: $-1 < x(0) < 1$
- **Case c**: $x(0) > 1$

Question: What can be concluded about the points $x = 1$ and $x = -1$?

Exercise 2

Given the system

$$\dot{x} = 3x - x^3$$

- Find all the **fixed points**.
- Analyze their **stability**.
- Solve **numerically** with $x(0) = 5$ and plot for large t.
- Animate the solution.

Exercise 3

The Logistic Flow is defined as:

$$\dot{x} = \mu x(1 - x)$$

for $\mu > 0$.

- Find **fixed points** as a function of μ.
- Perform a **stability analysis**.
- Solve **numerically** with $x(0) = 0.5$ and animate the trajectory.

Exercise 4

Solve analytically the **Logistic Flow** (Exercise 3) and compare it with the numerical solution using `ode45` for:

- $x(0) = 0.1$, with $\mu = 2$.

Exercise 5

For the following systems, **plot** $\dot{x}$ vs x, find **fixed points**, and analyze **stability**:

- $\dot{x} = \sin(x)e^{-x}$
- $\dot{x} = e^{-x} - 10\cos(x)$
- $\dot{x} = 1 + \frac{\cos(x)}{2}$

Exercise 6

Consider an **electrical circuit** with a **linear resistor** R in series with **capacitor** C and **battery** with constant voltage V. Let $Q(t)$ denote the charge of the capacitor at time $t \geq 0$.

- Plot $Q(t)$ when the capacitor starts with no charge.

Question: What changes if the resistor is nonlinear, that is, if the voltage across the resistor depends on the electric current? Try the relation:

$$V(I) = R\tanh(I)$$