

2D-Discrete Dynamical Systems (Assignment Sheet 3)

Introduction To Chaos Applied To Systems, Processes And Products (ETSIDI, UPM)

Alfonso Allen-Perkins, Juan Carlos Bueno and Eduardo Faleiro

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Introduction: Plotting 2D-orbits in R

Before solving the exercises, let's go through an example of how to plot the orbits of a 2D map using R. We will use the Gingerbreadman map, which is a simple example of a chaotic dynamical system that generates intricate, non-repeating patterns from a piecewise linear transformation.

The Gingerbreadman map is given by the equations:

$$\begin{aligned}x_{n+1} &= 1 - y_n + |x_n| \\ y_{n+1} &= x_n\end{aligned}$$

To visualize orbits of this system in R, follow these steps:

1. Load ggplot2 for visualization:

```
library(ggplot2)
```

2. Define the function to generate the Gingerbreadman map:

```
gingerbreadman_map <- function(x0, y0, n) {  
  x <- numeric(n)  
  y <- numeric(n)  
  x[1] <- x0  
  y[1] <- y0  
  
  for (i in 2:n) {  
    x[i] <- 1 - y[i-1] + abs(x[i-1])  
    y[i] <- x[i-1]  
  }  
}
```

```

  y[i] <- x[i-1]
}

data.frame(x, y)
}

```

2. Generate orbit with initial conditions:

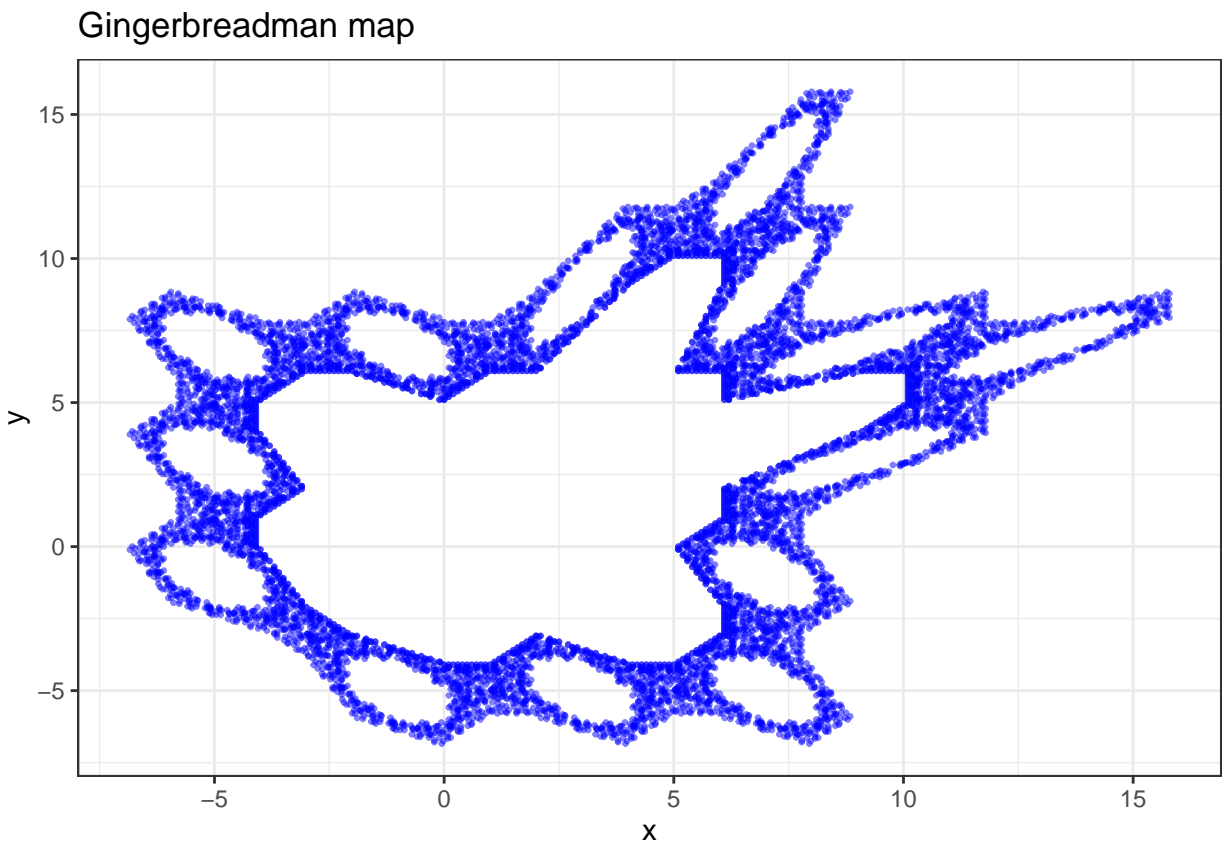
```
data_gbm <- gingerbreadman_map(1.1, 6.1, 10000)
```

3. Plot the orbit using ggplot2:

```

ggplot(data_gbm, aes(x = x, y = y)) +
  geom_point(color = 'blue', alpha = 0.5, size = .5) +
  labs(title = "Gingerbreadman map", x = "x", y = "y") +
  theme_bw()

```



You can modify the parameters to observe different behaviors of the above map using `ggplot2`.

Exercise 1: The Lotka-Volterra discrete map

The Lotka-Volterra discrete map models predator-prey interactions in a discrete-time framework, capturing oscillatory population dynamics and potential chaotic behavior. The long-term behavior of the system can converge to fixed points, periodic cycles, or chaotic attractors, depending on the parameter values.

The discrete 2D Lotka-Volterra map is given by:

$$x_n = \frac{\alpha x_{n-1} - \beta x_{n-1} y_{n-1}}{1 + \gamma x_{n-1}}$$

$$y_n = \frac{\delta y_{n-1} - \epsilon x_{n-1} y_{n-1}}{1 + \eta y_{n-1}}$$

where:

- Case 1: $\alpha = 170, \beta = 11, \gamma = 2.7, \delta = 50, \epsilon = 1.7, \eta = 7, x_1 = 7, y_1 = 5$
- Case 2: $\alpha = 1.001, \beta = 0.03, \gamma = 0.6, \delta = 1.002, \epsilon = 1.7, \eta = 0.9, x_1 = 0.0002, y_1 = 0.0006$

Plot the solution (orbit) of the discrete dynamical system for both cases.

Exercise 2: The Henon map (1976)

The Henon map is a simplified model of chaotic dynamics in two dimensions, originally derived from a model of stellar motion in galaxies. It exhibits a well-known strange attractor, which demonstrates sensitive dependence on initial conditions, a key feature of chaotic systems.

The Henon map is given by the equations:

$$x_{n+1} = y_n + 1 - ax_n^2$$

$$y_{n+1} = bx_n$$

where a and b are adjustable parameters. Set $a = 1.4$ and $b = 0.3$, then plot x_n vs y_n .

Exercise 3: The standard map (Chirikov 1971)

The Standard Map, also known as the Chirikov-Taylor Map, is a key model in Hamiltonian chaos, describing the transition from regular to chaotic motion in conservative dynamical systems. Depending on the parameter b , the system can display stable islands, chaotic seas, or a mixture of both.

The Standard map is defined as follows:

$$x_{n+1} = y_{n+1} + x_n$$

$$y_{n+1} = y_n + b \sin(x_n)$$

- (a) Plot various orbits for $b = 0$.
- (b) Plot the phase portrait for $b = 0.5$.
- (c) Show the phase portrait for $b = 1$ and $b = 2$.

Exercise 4: The Lozi map (1978)

The Lozi map is a piecewise linear version of the Henon map that retains its chaotic properties while being computationally simpler to analyze. It generates a strange attractor with a fractal structure, making it an important example in the study of deterministic chaos.

The Lozi map is defined by:

$$\begin{aligned}x_{n+1} &= y_n + 1 - a|x_n| \\ y_{n+1} &= bx_n\end{aligned}$$

where $-1 < b < 1$. Plot orbits when $a = 1.7$ and $b = 0.5$.