1D-Discrete Dynamical Systems (Assignment Sheet 1)

Introduction to Chaos Applied to Systems, Processes, and Products (ETSIDI, UPM)

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Contents

Exercise 1

Consider the map:

$$x_{n+1} = \cos(x_n)$$

- Plot several orbits (x_n vs n) starting from the interval $[-\pi, \pi]$.
- Create a **cobweb** plot (x_{n+1} vs x_n) and identify the fixed points.

Exercise 2

For each of the following maps, generate a **cobweb plot** starting from an appropriate initial condition x_1:

- $x_{n+1} = x_n^2$ for x_1 in $[-1, 1]$.
- $x_{n+1} = 3x_n - x_n^3$ for x_1 in $[-2, 2]$ and also for $x_1 = 2.1$.
- $x_{n+1} = e^{-x_n}$ for x_1 in $[0, 1]$.

Exercise 3

Consider the simple linear map:

$$x_{n+1} = \mu \cdot x_n$$

- Find the **fixed points** and analyze their **nature** based on the value of μ.
- Extend this analysis to study the fixed points of the generic nonlinear map:

$$x_{n+1} = g(x_n, \mu)$$

Exercise 4

Analyze the stability of the fixed points for the maps given in **Exercise 2**.

Exercise 5

The logistic map:

$$x_{n+1} = \mu \cdot x_n(1 - x_n)$$

is a **paradigm of chaos** in the domain $[0, 1]$.

- Identify the fixed points.
- Generate cobweb plots for the following values of $\mu : 2.0, 2.5, 3.0, 3.5, 3.7, 4.0$.

Exercise 6

Plot the **bifurcation diagrams** for the following maps:

1. $x_{n+1} = \mu \sin(x_n)$.
2. $x_{n+1} = \mu x_n - x_n^3$.
3. $x_{n+1} = e^{-\mu(1-x_n)}$.
4. $x_{n+1} = \mu \cdot x_n(1 - x_n)$ for μ in $[2, 4]$.
5. $x_{n+1} = \mu \frac{x_n^2}{1+x_n^2}$.
6. $x_{n+1} = \mu \cdot x_n + x_n^3 - x_n^5$ for μ in $[-2, 2]$.

Note: Ensure that your plots use sufficient iterations to observe long-term behavior.