Biostat 778: Homework 1

November 16, 2016

1 Improving consistent estimators

Let $\tilde{\theta}_n$ be an estimator of θ such that $\sqrt{n}(\tilde{\theta}_n - \theta) \to \mathcal{N}(0, \Sigma(\theta))$ and $\Sigma(\theta) < \infty$. Let $\hat{\theta}_n$ be the maximum likelihood estimator of θ. Let $\tilde{\theta}_n^{(1)}$ be single iteration of Newton's method applied to $\tilde{\theta}$, i.e.

$$\tilde{\theta}_n^{(1)} = \tilde{\theta}_n - \ell''(\tilde{\theta}_n)^{-1}\ell'(\tilde{\theta}_n)$$

Show that $\tilde{\theta}_n^{(1)}$ is asymptotically equivalent to the MLE.

2 Logistic Regression with Penalization

Write a function that can fit a logistic regression model while allowing for an L^2 penalty on the parameters. That is, if $\ell(\beta)$ is the log-likelihood for the parameter vector β, then you want to maximize the penalized log-likelihood

$$\ell^{\star}(\beta, \lambda) = \ell(\beta) - \lambda\beta'\beta$$

Write your own implementation of Newton's method to optimize the penalized likelihood. The output of the function should be the maximum (penalized) likelihood estimates of β and the asymptotic standard errors for each of the elements of $\hat{\beta}$.