Prove Max-Ergotropy

October 23, 2020

For now i'll work with direct Lanczos diagonalization.
We break the symmetry with a little field h and then we calculate on two sites in the middle of the chain:

- $E(\rho)/2|\epsilon_0|$
- $E(\rho^{pass})/2|\epsilon_0|$
- $E(\rho^{a-pass})/2|\epsilon_0|$
- Ergotropy = $E(\rho)/2|\epsilon_0| - E(\rho^{pass})/2|\epsilon_0|$
- Anti-Ergotropy $E(\rho^{a-pass})/2|\epsilon_0| - E(\rho)/2|\epsilon_0|$
- Max-Ergotropy $E(\rho^{a-pass})/2|\epsilon_0| - E(\rho^{pass})/2|\epsilon_0|$

It's important to normalize everything since the quantities depend strongly on the Hamiltonian eigenvalues

We also calculate some known quantities :

- Purity
- Entropy
- Concurrence

1 Experimental trials, broken symmetry

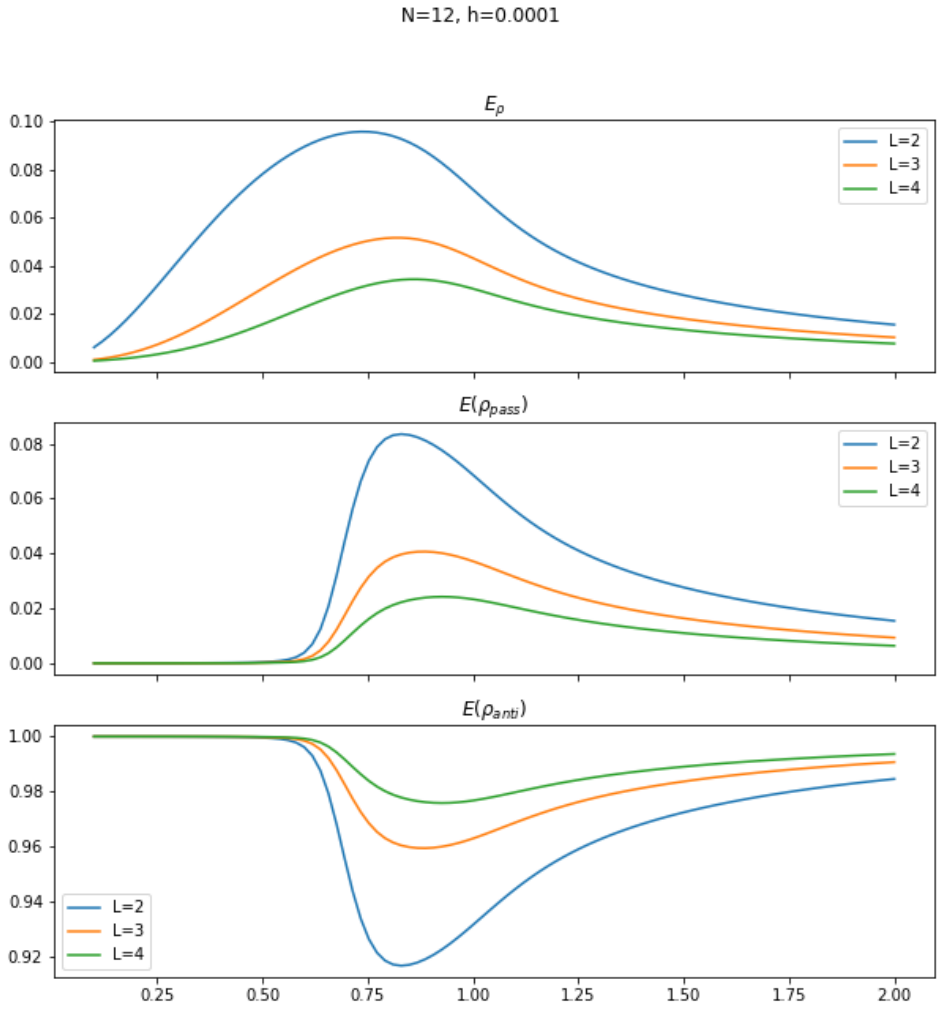

Figure 1

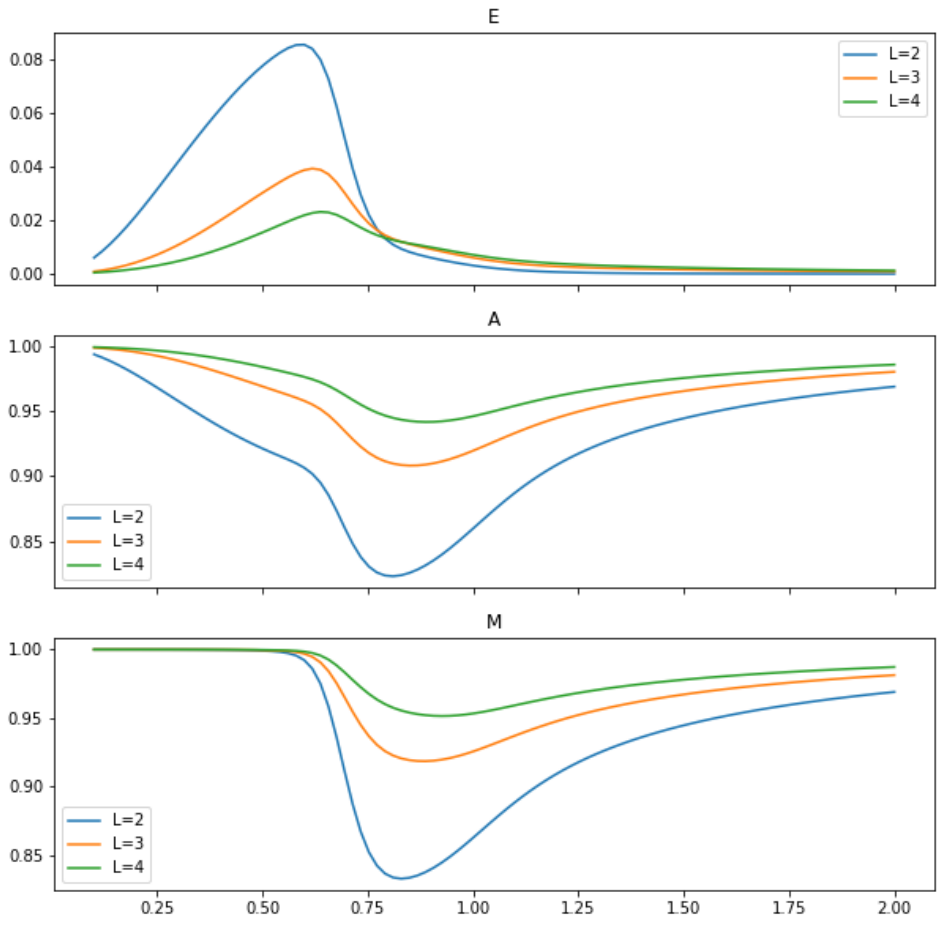
N=12, h=0.0001
E
A
M
L=2
L=3
L=4
0.08
0.06
0.04
0.02
0.00
1.00
0.95
0.90
0.85
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Figure 2

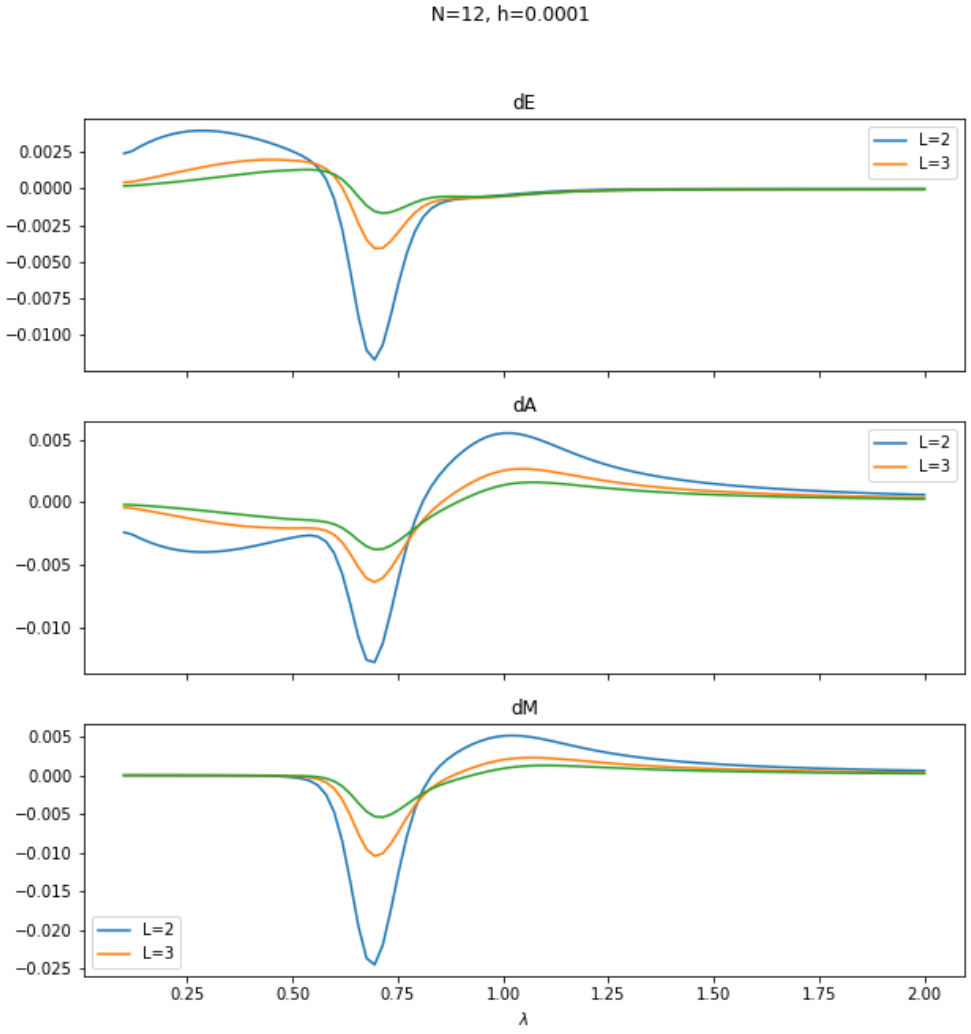
N=12, h=0.0001
dE
L=2
L=3
0.0025
0.0000
−0.0025
−0.0050
−0.0075
−0.0100
dA
L=2
L=3
0.005
0.000
−0.005
−0.010
dM
L=2
L=3
0.005
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
λ

Figure 3

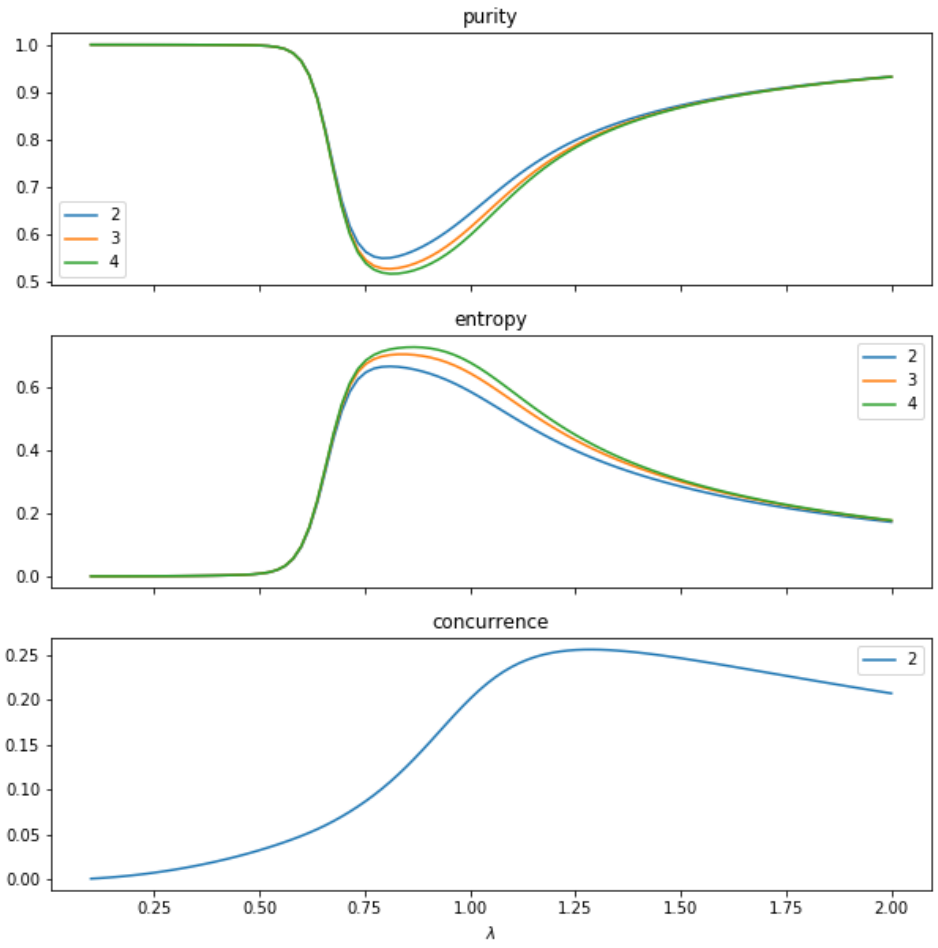
N=12, h=0.0001
purity
1.0
0.9
0.8
0.7
0.6
0.5
2
3
4
entropy
0.6
0.4
0.2
0.0
2
3
4
concurrence
0.25
0.20
0.15
0.10
0.05
0.00
2
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
λ

Figure 4

2 Experimental trials, preserved symmetry

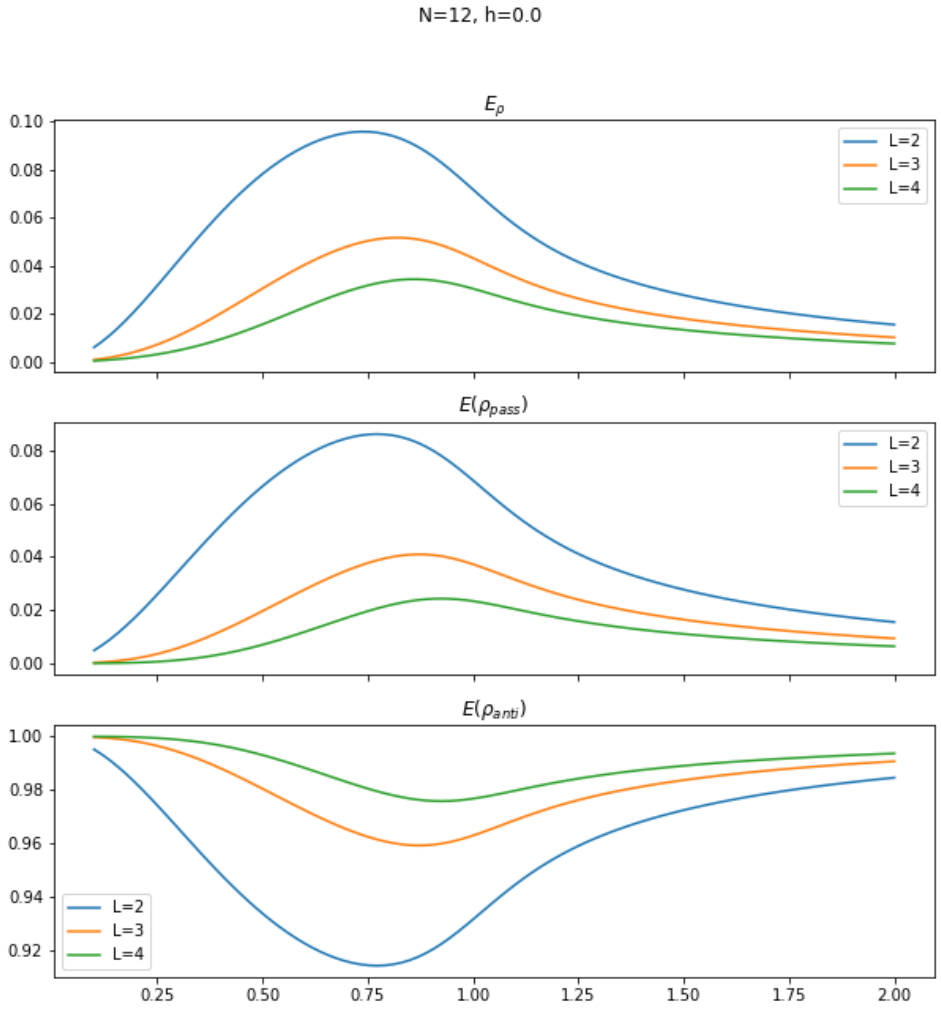

Figure 5

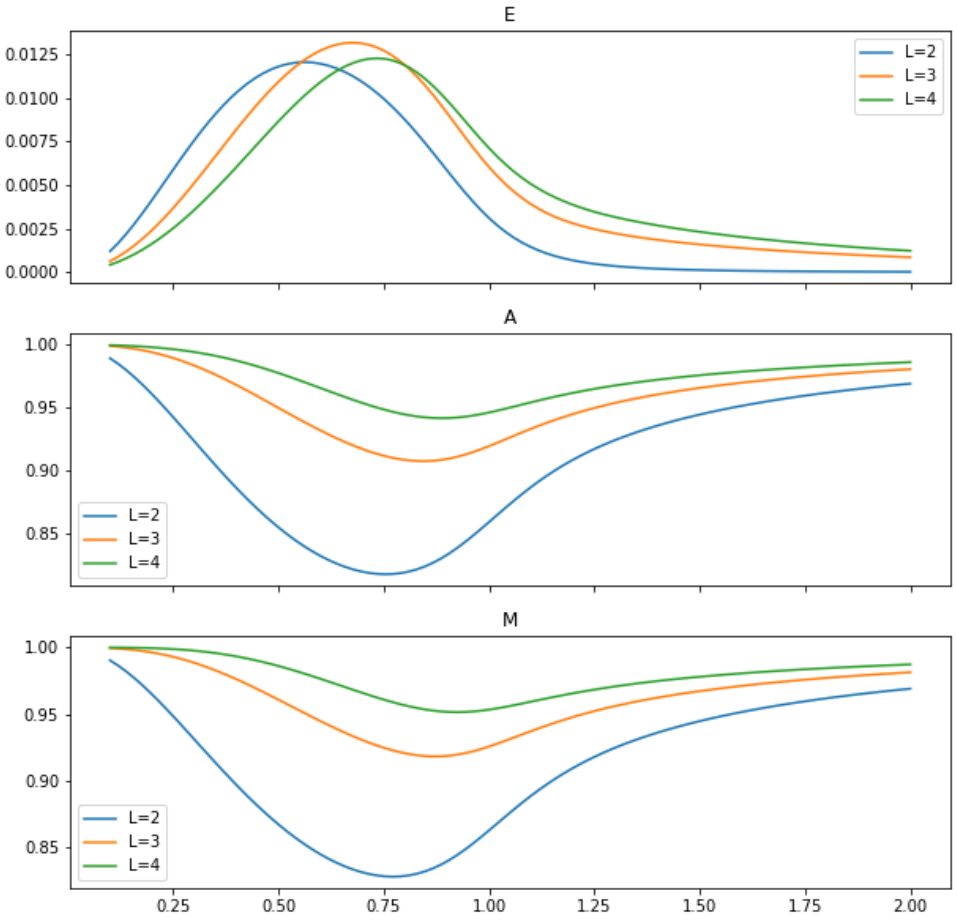
N=12, h=0.0
E
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
L=2
L=3
L=4
A
1.00
0.95
0.90
0.85
L=2
L=3
L=4
M
1.00
0.95
0.90
0.85
L=2
L=3
L=4
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Figure 6

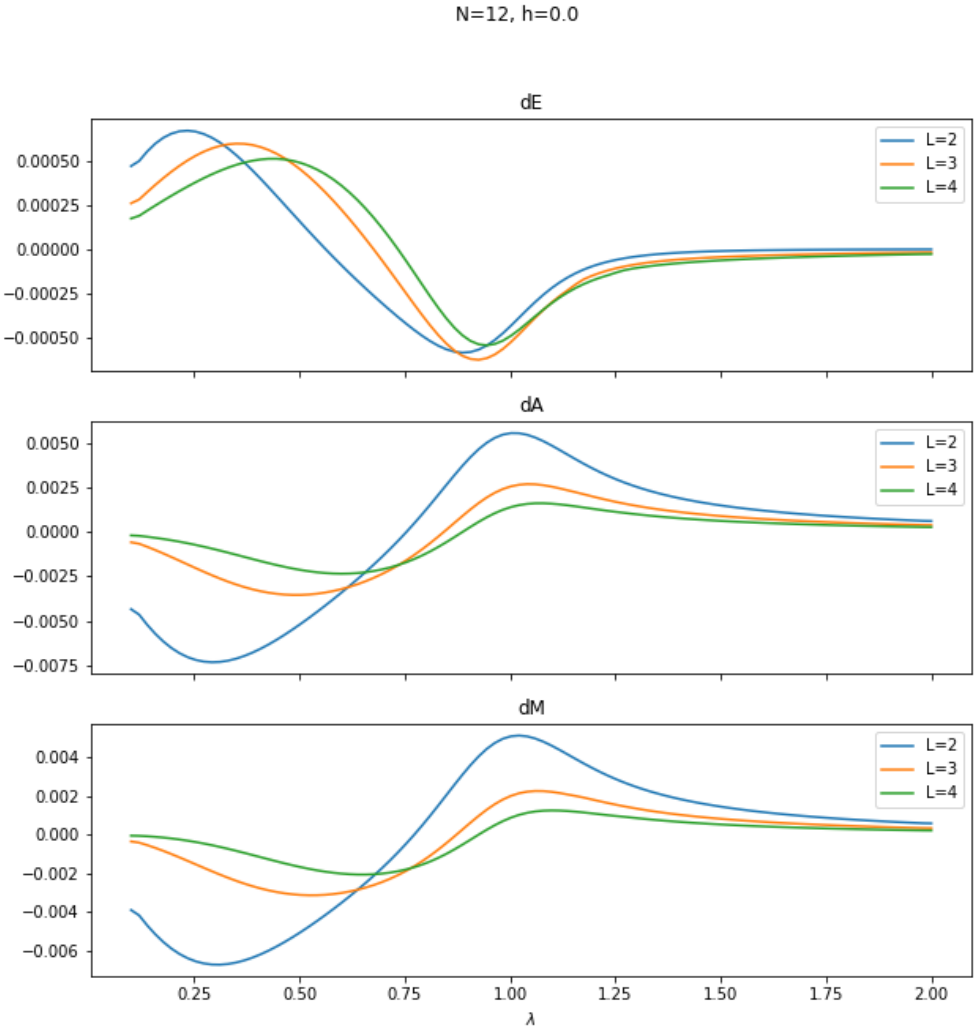
N=12, h=0.0
dE
L=2
L=3
L=4
0.00050
0.00025
0.00000
−0.00025
−0.00050
dA
L=2
L=3
L=4
0.0050
0.0025
0.0000
−0.0025
−0.0050
−0.0075
dM
L=2
L=3
L=4
0.004
0.002
0.000
−0.002
−0.004
−0.006
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
λ

Figure 7

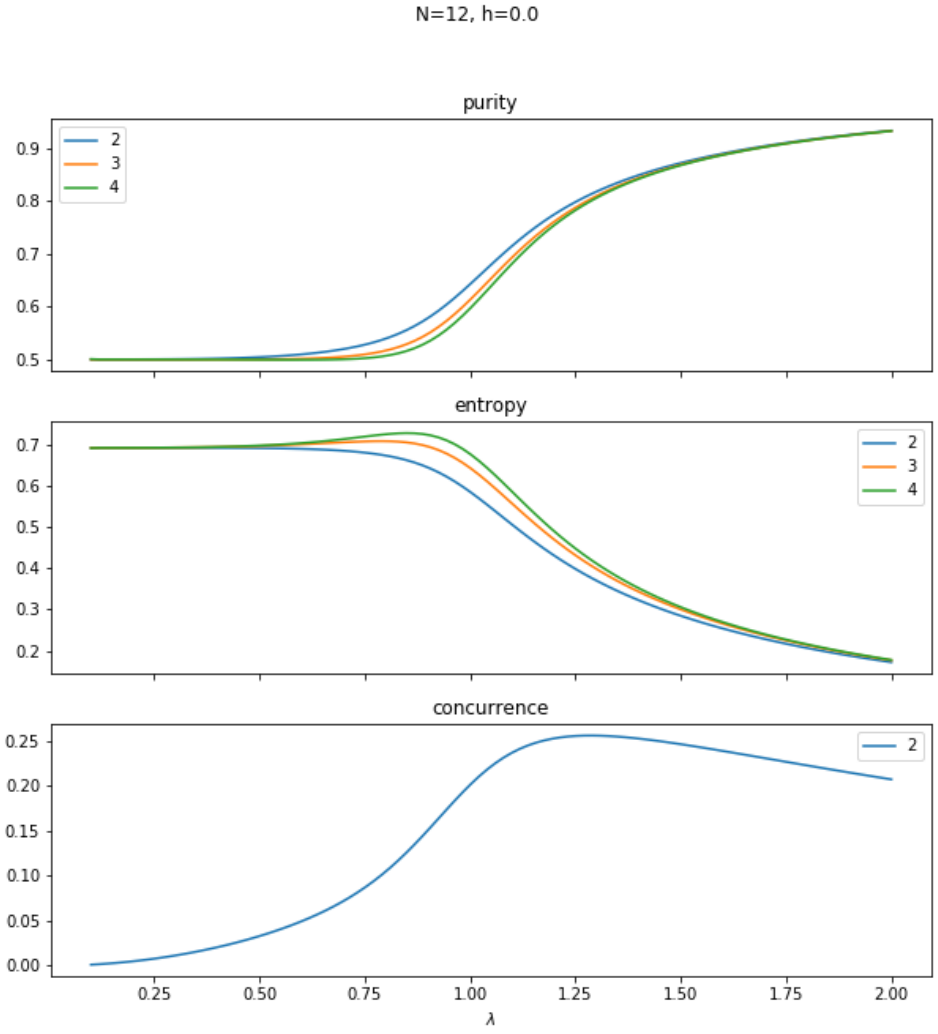
N=12, h=0.0
purity
2
3
4
0.9
0.8
0.7
0.6
0.5
entropy
2
3
4
0.7
0.6
0.5
0.4
0.3
0.2
concurrence
2
0.25
0.20
0.15
0.10
0.05
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
λ

Figure 8

3 Theoretical calculations, preserved symmetry

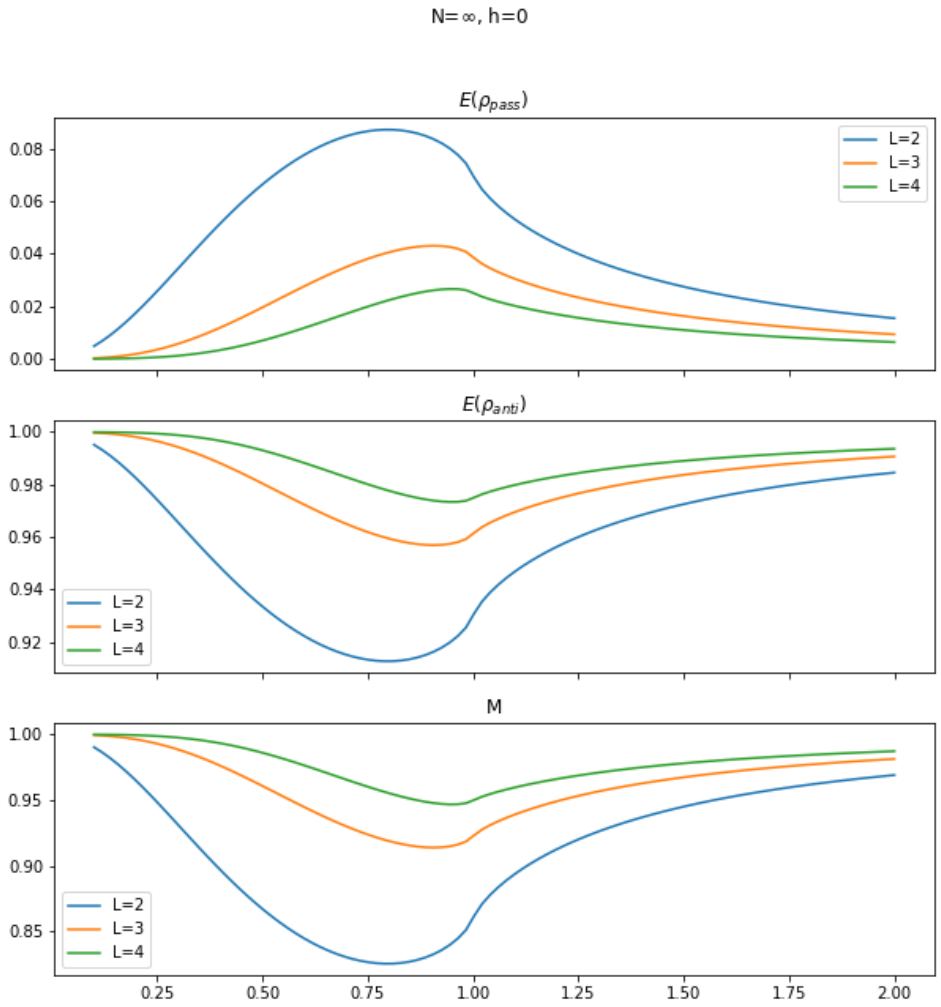

Figure 9

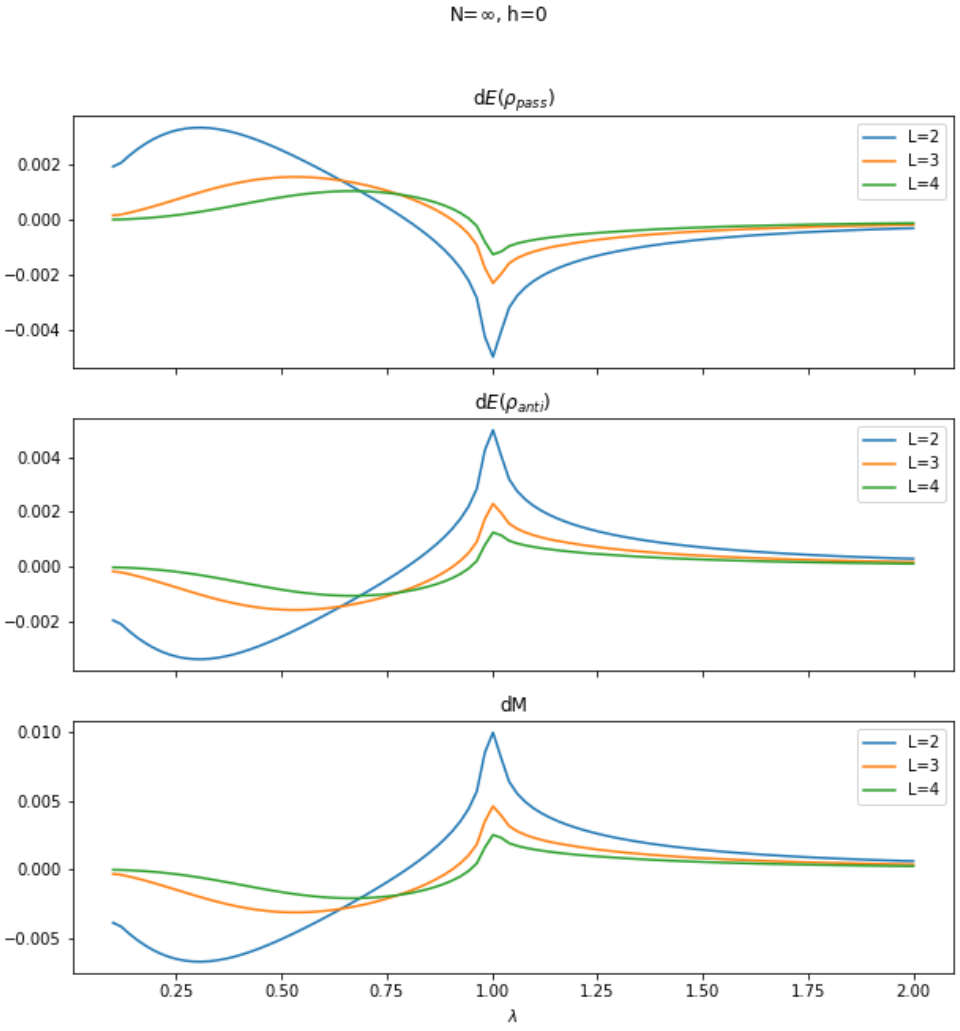
N=∞, h=0
dE(ρpass)
L=2
L=3
L=4
0.002
0.000
−0.002
−0.004
dE(ρanti)
0.004
0.002
0.000
−0.002
dM
0.010
0.005
0.000
−0.005
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
λ

Figure 10

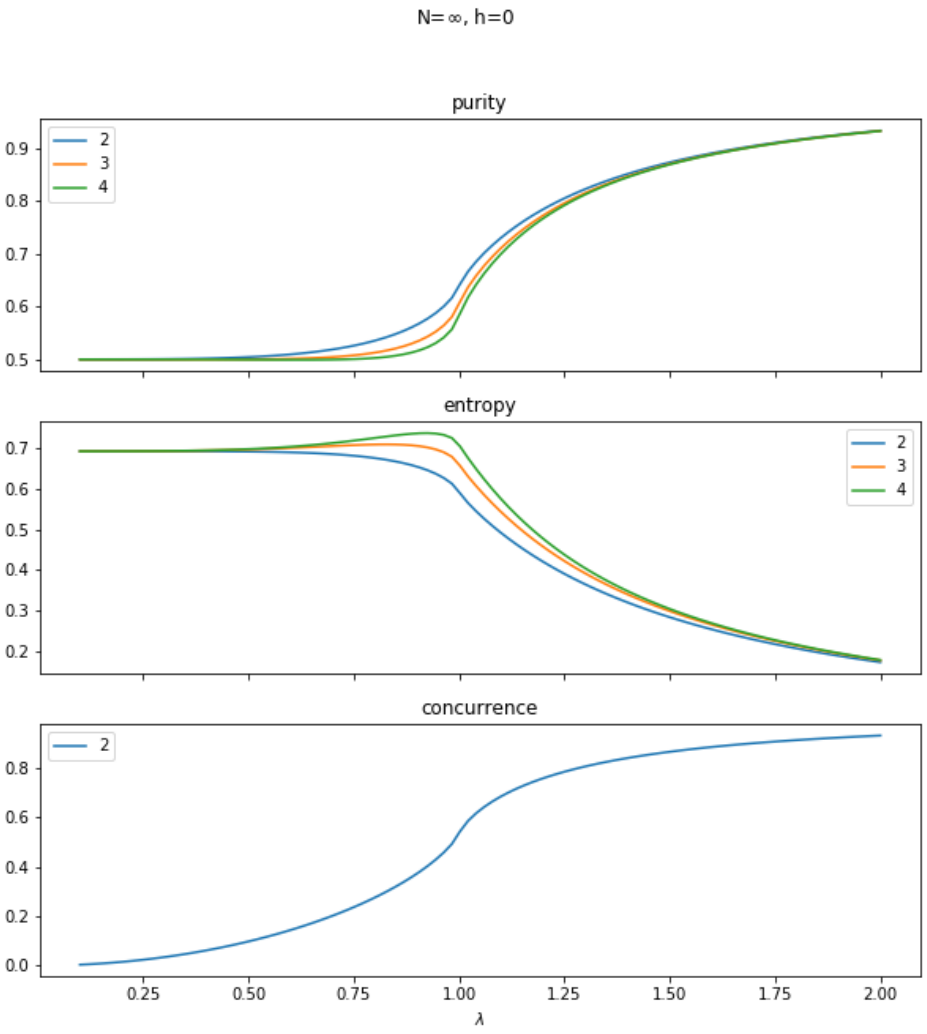
N=∞, h=0
purity
2
3
4
0.9
0.8
0.7
0.6
0.5
entropy
2
3
4
0.7
0.6
0.5
0.4
0.3
0.2
concurrence
2
0.8
0.6
0.4
0.2
0.0
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
λ

Figure 11