JUNK

For $\lambda \to 0$:

$$\begin{array}{l} g_l = -\left(\frac{1+\gamma}{2} L_{l+1} + \frac{1-\gamma}{2} L_{l-1}\right) \quad l \in \text{ odd} \\ g_l = 0 \quad l \in \text{ even} \end{array} \tag{1}$$

where:

$$L_l = \frac{2}{\pi} \int_0^{\pi/2} d\phi \frac{\cos \phi l}{\sqrt{\cos^2 \phi + \gamma^2 \sin^2 \phi}} \tag{2}$$

For $E(\rho)$:

$$\begin{aligned} E(\rho) = & r_1[e_1(r_1|e_1) + e_4(r_1|e_4)] + \\ & r_2 e_3 + \\ & r_3 e_2 + \\ & r_4[e_1(r_4|e_1) + e_4(r_4|e_4)] \end{aligned} \tag{3}$$

$$\begin{aligned} E(\rho) = & r_1[e_1(r_1|e_1) + e_4(r_1|e_4)] + \\ & 1/8(1-\nu_-)(1+\nu_+) + \\ & -1/8(1+\nu_-)(1-\nu_+) + \\ & r_4[e_1(r_4|e_1) + e_4(r_4|e_4)] \end{aligned} \tag{4}$$

We should express the eigenvalues of H in terms of the nus maybe. Or better we can do the trace multiplication in some smart way.

Without Identity:

$$\mathrm{Tr}\left(\sigma_a \sigma_b \sigma_c \sigma_d\right) = 2\left(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}\right) \tag{5}$$

$$\mathrm{Tr}\left(\sigma_a \sigma_b \sigma_c\right) = 2i\varepsilon_{abc} \tag{6}$$

$$\mathrm{Tr}\left(\sigma_a \sigma_b\right) = 2\delta_{ab} \tag{7}$$

$E(\rho) = \mathrm{Tr}[H\rho]$

$$\rho_2 = \frac{1}{4}[(\sigma_1^0\sigma_2^0) - g_{-1}\left(\sigma_1^x\sigma_2^x\right) - g_1\left(\sigma_1^y\sigma_2^y\right) + (g_0^2 - g_1 g_{-1})\left(\sigma_1^z\sigma_2^z\right) + g_0\left(\sigma_1^0\sigma_2^z + \sigma_1^z\sigma_2^0\right)] \tag{8}$$

$$H = -\frac{1}{2}\left(\sigma^x\sigma^x + \lambda\sigma^z\sigma^0 + \lambda\sigma^0\sigma^z\right) \tag{9}$$

4 terms: xxxx , yyxx, zzxx 2 for everyone

3 terms: 0zxx, z0xx, xxz0, xx0z, yyz0, yy0z, zzz0, zz0z 0 for everyone

2 terms 0zz0 0z0z z0z0 z00z 00xx 2 for everyone

1 term 00z0 000z =0 for everyone

Result, not zero:
xxxx , yyxx, zzxx, 0zz0 0z0z z0z0 z00z 00xx
$-1/4(-g_{-1} - g_1 + g_0^2 - g_1 g_{-1} + 4g_0\lambda + 1)$

$$\left.\begin{pmatrix} \sqrt{4\lambda^2+1}+2\lambda & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 2\lambda-\sqrt{4\lambda^2+1} & 0 & 0 & 1 \end{pmatrix}\right\} \tag{10}$$

$$\begin{pmatrix} \sqrt{4g_0^2+(g_1-g_{-1})^2}+2g_0 & 0 & 0 & g_1-g_{-1} \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \sqrt{4g_0^2+(g_1-g_{-1})^2}-2g_0 & 0 & 0 & g_1-g_{-1} \end{pmatrix} \tag{11}$$

Calling $\alpha = \frac{g_{-1}-g_1}{2g_0}$:

$$\begin{pmatrix} -\sqrt{1+\alpha^2}-1 & 0 & 0 & \alpha \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ \sqrt{1+\alpha^2}-1 & 0 & 0 & \alpha \end{pmatrix} \tag{12}$$

Eigenstates in the same order:

$$\begin{pmatrix} -\frac{\sqrt{4g_0^2+(g_1-g_{-1})^2}+2g_0}{g_{-1}-g_1} & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -\frac{2g_0-\sqrt{4g_0^2+(g_1-g_{-1})^2}}{g_{-1}-g_1} & 0 & 0 & 1 \end{pmatrix} \tag{13}$$