

Alfred 代码模版库

目录

	录				强连通分量缩点 (SCC) 割边与割边缩点 (EBCC)	19 20
1	比赛配置 and 奇技淫巧	3			二分图最大权匹配 (MaxAssignment,基	20
	1.1 多组数据代码模板	3		0.0	于 KM)	22
	1.2 快读快写	3		5.6	一般图最大匹配 (Graph, 带花树算法).	24
	1.3 关闭流与 C 风格输入输出的同步	3			2-SAT	26
	1.4 .clang-format	4			最大流	26
	1.5 debug.h	4		5.9	最小费用可行流(或最大流)	28
	1.6 火车头	5		5.10	树链剖分	29
	1.7 c-cpp-properties.json	6			快速幂	31
	1.8 launch.json	7			欧拉筛	31
	1.9 settings.json	7			单点欧拉函数	32
	$1.10~tasks.json~\dots \dots \dots \dots \dots \dots$	7		5.14	exgcd	32
_	WL 10 / + 1 -			5.15	组合数	32
2	数据结构	8		5.16	树状数组	33
	2.1 珂朵莉树	8		5.17	Splay	34
		9			取模类, 按需写	37
		9 10		5.19	马拉车	40
	2.4 PBDS 大常数平衡树	10		5.20	Z 函数	41
	2.6 并查集	10		5.21	SA 后缀数组	41
	2.7 出现次数统计	11				
	2.8 01-Trie	12	6		ashi 代码库 (备用)	42
	2.9 滑动窗口	14			$O(n \log n) - O(1) \text{ RMQ} \dots \dots \dots$	42
	2.0 11773 [2] [2] [2] [2] [2] [2] [2] [2] [2] [2]				$O(n \log n) - O(\log n)$ LCA	43
3	数学(数论)算法	<b>15</b>			树状数组	44
	3.1 带模整数类	15		6.4	并查集	44
	3.2 计算几何	15		6.5	轻重权树剖分	45
	3.3 组合数学	16		6.6	强连通分量	46
	3.4 拉格朗日插值	17			双连通分量	47
					二分图匹配	49
4	字符串算法	18			最小费用最大流	50
	4.1 字符串哈希	18			AhoCorasick 自动机	52
۲	iionala 从列庆 (冬田 - 停切连担山 iggue)	10			后缀数组	53
5	<b>jiangly 代码库 (备用,侵权请提出 issue)</b> 5.1 int128 输出流自定义	18 18		6.12	LU 分解	54
	5.2 常用数学运算库函数及 gcd 重载		7	<b>7:1</b> —∶	类问题的处理方法	55
	0.4 印用	13	•	V.1 :	スロベルス性リム	99

# 1 比赛配置 and 奇技淫巧

## 1.1 多组数据代码模板

Listing 1: template.cpp

```
#include <bits/stdc++.h>
2
   using namespace std;
3 using i64 = long long;
   const i64 N = 1e5 + 10;
4
5
   int t = 1;
6
    inline void solve(int Case) {
7
        // your code here;
8
9
    inline void optimizeIO(void) {
10
        ios::sync with stdio(false);
11
        cin.tie(NULL), cout.tie(NULL);
12
13
    inline void init(void) {}
    int main(int argc, char const *argv[]) {
14
        optimizeIO(), init(), cin >> t;
15
        for (int i = 1; i <= t; i++) solve(i);</pre>
16
17
        return 0;
18
   }
```

## 1.2 快读快写

### Listing 2: fast-io.cpp

```
1
    namespace fastIO {
2
        char c, f, e = 0;
3
        namespace usr {
            template <class Tp>
4
            inline int read( Tp &x) {
5
                x = f = 0, c = getchar();
6
                while (!isdigit(c) && !e) f = c == '-', e \mid = c == EOF, c = getchar();
7
                while (isdigit(c) && !e) x = (x << 1) + (x << 3) + (c ^ 48), c = getchar();
8
                return (e |= c == EOF) ? 0 : ((f ? x = -x : 0), 1);
9
10
            template <class Tp>
11
12
            inline void write( Tp x) {
13
                if (x < 0) putchar('-'), x = -x;
14
                if (x > 9) write(x / 10);
15
                putchar((x % 10) ^ 48);
16
17
            template <typename \mathbb{T}, typename... \mathbb{V}>
            inline void read(T &t, V &...v) { read(t), read(v...); }
18
            template <typename T, typename... V>
19
20
            inline void write(T t, V... v) {
21
                write(t), putchar('_'), write(v...);
22
23
24
   using namespace fastIO::usr;
```

# 1.3 关闭流与 C 风格输入输出的同步

Listing 3: io-sync-off.cpp

```
1 inline void optimizeIO(void) {
2    ios::sync_with_stdio(false);
3    cin.tie(NULL), cout.tie(NULL);
4 }
```

## 1.4 .clang-format

### Listing 4: .clang-format

```
1 BasedOnStyle: LLVM
2 AlignAfterOpenBracket: BlockIndent
3  # AlignConsecutiveAssignments: Consecutive
4 AlignArrayOfStructures: Right
5 UseTab: Never
6 IndentWidth: 4
7 TabWidth: 4
8 BreakBeforeBraces: Attach
9 AllowShortIfStatementsOnASingleLine: AllIfsAndElse
10 AllowShortLoopsOnASingleLine: true
11 AllowShortBlocksOnASingleLine: true
12 IndentCaseLabels: true
13 ColumnLimit: 0
14 AccessModifierOffset: −4
15 NamespaceIndentation: All
16 FixNamespaceComments: false
17
   AllowShortCaseLabelsOnASingleLine: true
18 AlwaysBreakTemplateDeclarations: MultiLine
19 BinPackParameters: true
20 BraceWrapping:
     AfterCaseLabel: true
21
     AfterClass: true
22
23 AlignConsecutiveMacros: AcrossEmptyLinesAndComments
24 AlignTrailingComments: Always
```

## 1.5 debug.h

### Listing 5: debug.h

```
/**
 1
     * @file
 2
                        debug.h
 3
      * @author
                        Dr.Alfred (abonlinejudge@163.com)
      * @brief
 4
                        Local Debug Printer
      * @version
                        1.0
 5
      * @date
                        2023-12-30
 6
 7
 8
      * @copyright Copyright (c) 2019—now < Rhodes Island Inc.>
 9
      */
10
11
     #include <bits/stdc++.h>
12
13
14 using std::cerr;
15 using std::pair;
16 using std::string;
17
18
    const long long dbg_inf = 9e18 + 19260817;
19
20
    void __print(int x) { cerr << x; }</pre>
21
    \begin{tabular}{ll} \textbf{void} & \_\texttt{print}(\textbf{long} \ \texttt{x}) & \{ \ \texttt{cerr} << \ \texttt{x}; \ \} \\ \end{tabular}
            print(long long x) {
22
23
         if (x != dbg_inf) {
24
              cerr << x;
25
         } else {
              cerr << "inf";</pre>
26
27
    }
28
    void print(unsigned x) { cerr << x; }</pre>
29
    void print(unsigned long x) { cerr << x; }</pre>
30
31
    void print(unsigned long long x) { cerr << x; }</pre>
    void print(float x) { cerr << x; }</pre>
    void print(double x) { cerr << x; }</pre>
```

1 比赛配置 AND 奇技淫巧 1.6 火车头

```
void print(long double x) { cerr << x; }</pre>
    void __print(char x) { cerr << '\'' << x << '\''; }</pre>
    void __print(const char *x) { cerr << '\"' << x << '\"'; }</pre>
36
    \begin{tabular}{ll} \textbf{void} \begin{tabular}{ll} \textbf{print}(\textbf{const} \ \text{string \&x}) & \{ \ \text{cerr} << \ ' \ ''' << \ x << \ ' \ '''; \ \} \\ \end{tabular}
37
    void __print(bool x) { cerr << (x ? "true" : "false"); }</pre>
38
    void __print(__int128_t x) {
39
         if (x < 0) cerr << '-', x = -x;
40
41
         if (x > 9) __print(x / 10);
         cerr << char((x % 10) ^ 48);
42
43
    }
44
    void dbgEndl(void) { cerr << '\n'; }</pre>
45
46
    template <typename T, typename V>
    void print(const pair<T, V> &x) {
47
         cerr << '{', __print(x.first), cerr << ",_", __print(x.second), cerr << '}';</pre>
48
49
    template <typename T>
50
    void print(const T &x) {
51
52
         int f = 0;
         cerr << '{';
53
         for (auto i : x) cerr << (f++ ? ", " : ""),     print(i);</pre>
54
55
         cerr << "}";
56
    }
    void print() { cerr << "]\n"; }</pre>
57
58
    template <typename T, typename... V>
    void print(T t, V... v) {
59
          _print(t);
60
         if (sizeof...(v)) cerr << ", _";
61
62
         _print(v...);
63
64
    #ifdef DEBUG
65
    // To customize a struct/class to print, just define the print function.
66
67
    #ifndef NO DBG COLOR
68
    #define dbg(x...)
         cerr << "\e[91m" << __func__ << ":" << __LINE__ << "_[" << #x << "] _=_["; \
69
70
         _print(x);
         cerr << "\e[39m";
71
72
73
    #define short dbg(x...) \
        cerr << "\e[91m[["; \
74
         print(x);
75
76
         cerr << "\e[39m";
77
78
    #define dbg(x...)
         cerr << __func__ << ":" << __LINE__ << "_[" << #x << "] _=_["; \
79
80
         print(x);
    #define short_dbg(x...) \
81
        cerr << "[";
82
         print(x);
83
84
    #endif // !NO DBG COLOR
85
86
87
    #define dbg(x...)
88
    #endif
```

# 1.6 火车头

### Listing 6: optimize-header.h

```
1  #pragma GCC optimize(3)
2  #pragma GCC target("avx")
3  #pragma GCC optimize("Ofast")
4  #pragma GCC optimize("inline")
5  #pragma GCC optimize("-fgcse")
6  #pragma GCC optimize("-fgcse-lm")
```

```
#pragma GCC optimize("-fipa-sra")
    #pragma GCC optimize("-ftree-pre")
    #pragma GCC optimize("-ftree-vrp")
9
    #pragma GCC optimize("-fpeephole2")
10
    #pragma GCC optimize("-ffast-math")
11
    #pragma GCC optimize("-fsched-spec")
12
    #pragma GCC optimize("unroll-loops")
13
    #pragma GCC optimize("-falign-jumps")
14
    #pragma GCC optimize("-falign-loops")
15
    #pragma GCC optimize("-falign-labels")
16
    #pragma GCC optimize("-fdevirtualize")
17
    #pragma GCC optimize("-fcaller-saves")
18
    #pragma GCC optimize("-fcrossjumping")
19
    #pragma GCC optimize("-fthread-jumps")
20
    #pragma GCC optimize("-funroll-loops")
21
    #pragma GCC optimize("-fwhole-program")
22
    #pragma GCC optimize("-freorder-blocks")
23
    #pragma GCC optimize("-fschedule-insns")
24
    #pragma GCC optimize("inline-functions")
25
    #pragma GCC optimize("-ftree-tail-merge")
26
    #pragma GCC optimize("-fschedule-insns2")
27
    #pragma GCC optimize("-fstrict-aliasing")
28
    #pragma GCC optimize("-fstrict-overflow")
29
    #pragma GCC optimize("-falign-functions")
30
    #pragma GCC optimize("-fcse-skip-blocks")
31
    #pragma GCC optimize("-fcse-follow-jumps")
32
    #pragma GCC optimize("-fsched-interblock")
33
    #pragma GCC optimize("-fpartial-inlining")
34
35
    #pragma GCC optimize("no-stack-protector")
36
    #pragma GCC optimize("-freorder-functions")
37
    #pragma GCC optimize("-findirect-inlining")
38
    #pragma GCC optimize("-fhoist-adjacent-loads")
39
    #pragma GCC optimize("-frerun-cse-after-loop")
40
    #pragma GCC optimize("inline-small-functions")
    #pragma GCC optimize("-finline-small-functions")
41
    #pragma GCC optimize("-ftree-switch-conversion")
42
    #pragma GCC optimize("-foptimize-sibling-calls")
43
    #pragma GCC optimize("-fexpensive-optimizations")
44
    #pragma GCC optimize("-funsafe-loop-optimizations")
45
    #pragma GCC optimize("inline-functions-called-once")
46
    #pragma GCC optimize("-fdelete-null-pointer-checks")
```

# 1.7 c-cpp-properties.json

### Listing 7: c-cpp-properties.json

```
1
2
        "configurations": [
3
                 "name": "macos-gcc-arm64",
4
                 "includePath": [
5
                     "${workspaceFolder}/**",
6
                     "/usr/local/include/ac-library/"
7
8
                 "compilerPath": "/usr/local/bin/g++",
9
                 "cStandard": "c17",
10
                 "cppStandard": "c++20",
11
                 "intelliSenseMode": "macos-gcc-arm64",
12
13
                 "compilerArgs": [],
                 "configurationProvider": "ms-vscode.makefile-tools"
14
15
            }
16
        ],
        "version": 4
17
18
    }
```

## 1.8 launch.json

### Listing 8: launch.json

```
{
1
        "version": "0.2.0",
2
        "configurations": [
3
4
            {
                 "name": "(lldb) Launch",
5
                 "type": "cppdbg",
6
                "request": "launch",
7
8
                 "program": "${fileDirname}/compiled.out",
9
                 "args": [],
                "stopAtEntry": false,
10
                 "cwd": "dollar{fileDirname}",
11
                 "environment": [],
12
                 "externalConsole": true,
13
                 "internalConsoleOptions": "neverOpen",
14
                 "MIMode": "lldb",
15
                 "setupCommands": [
16
17
                     {
18
                         "description": "Enable_pretty-printing_for_lldb",
19
                         "text": "-enable-pretty-printing",
20
                         "ignoreFailures": false
21
22
                 "preLaunchTask": "Compile"
23
24
            }
25
        ],
    }
26
```

# 1.9 settings.json

#### Listing 9: settings.json

```
1
   {
2
       "files.defaultLanguage": "cpp",
3
       "editor.formatOnType": true,
       "editor.suggest.snippetsPreventQuickSuggestions": false,
4
       "editor.acceptSuggestionOnEnter": "off",
5
       "C Cpp.clang_format_sortIncludes": true,
6
       "C Cpp.errorSquiggles": "disabled",
7
       "C_Cpp.default.defines": ["LOCAL", "DEBUG"]
8
9
   }
```

### 1.10 tasks.json

### Listing 10: tasks.json

```
// https://code.visualstudio.com/docs/editor/tasks
1
2
       "version": "2.0.0",
3
      "tasks": [
4
5
          {
              "label": "Compile", // 任务名称,与launch.json的preLaunchTask相对应
6
             "command": "g++", // 要使用的编译器, C++用g++
7
              "args": [
8
                 "${file}",
9
                 "-o", // 指定输出文件名,不加该参数则默认输出a.exe, Linux下默认a.out
10
11
                 "${fileDirname}/compiled.out",
12
                 "-g", // 生成和调试有关的信息
13
                 // "-arch aarch64",
                 // "-m64", // 不知为何有时会生成16位程序而无法运行,此条可强制生成64位的
14
                 "-Wall", // 开启额外警告
15
                 "-std=c++20", // c++14
16
```

```
"-DLOCAL",
17
18
                "-DDEBUG",
                "-03",
19
                "-ld classic", // will be deprecated
20
                "-Wno-char-subscripts",
21
22
                "-I",
                "/usr/local/include/ac-library/"
23
                                         // 手动扩大栈空间
24
                // "-stack=268435456"
             ], // 编译的命令, 其实相当于VSC帮你在终端中输了这些东西
25
             "type": "process", // process是把预定义变量和转义解析后直接全部传给command: shell相当于先打开
26
              shell再输入命令,所以args还会经过shell再解析一遍
27
             "group": {
                "kind": "build",
28
                "isDefault": true // 不为true时ctrl shift B就要手动选择了
29
30
             "presentation": {
31
                "echo": true,
32
                "reveal": "always", // 执行任务时是否跳转到终端面板,可以为always, silent, never。具体参见
33
                 VSC的文档,即使设为never,手动点进去还是可以看到
                "focus": false, // 设为true后可以使执行task时焦点聚集在终端,但对编译C/C++来说,设为true没
34
                "panel": "shared" // 不同的文件的编译信息共享一个终端面板
35
36
             "problemMatcher": "$gcc" // 捕捉编译时终端里的报错信息到问题面板中,修改代码后需要重新编译才会
37
              再次触发
             // 本来有Lint,再开problemMatcher就有双重报错,但MinGw的Lint效果实在太差了;用Clange可以注释掉
38
39
         }
40
      1
41
   }
```

# 2 数据结构

## 2.1 珂朵莉树

支持区间推平,颜色段统计,在随机数据下期望复杂度为  $O(n \log n)$  的暴力数据结构。

Listing 11: chtholly.cpp

```
#include <set>
1
2
3
    struct ChthollyTree {
4
        typedef long long 11;
5
        struct Node {
6
            mutable 11 1, r, v;
7
            inline bool operator<(const Node &x) const { return 1 < x.1; }</pre>
8
9
        std::set<Node> tr;
10
        typedef std::set<Node>::iterator iterator;
        ChthollyTree(void) = default;
11
12
        ChthollyTree(int rng, int val) { init(rng, val); }
        inline void init(ll rng, ll val) noexcept {
13
14
            tr.insert({1, rng, val}), tr.insert({rng + 1, rng + 1, 0});
15
        inline iterator begin(void) const noexcept { return tr.begin(); }
16
17
        inline iterator end(void) const noexcept { return tr.end(); }
18
        inline iterator split(ll pos) {
19
            auto it = tr.lower_bound({pos, 0, 0});
20
            if (it != tr.end() && it->1 == pos) return it;
            11 1 = (--it) -> 1, r = it -> r, v = it -> v;
21
22
            tr.erase(it), tr.insert(\{1, pos - 1, v\});
23
            return tr.insert({pos, r, v}).first;
24
        inline void assign(ll l, ll r, ll v) {
25
26
            auto R = split(r + 1), L = split(l);
27
            tr.erase(L, R), tr.insert({1, r, v});
```

2 数据结构 2.2 树状数组

```
28
        template <class Functor> // func(iterator)
29
        inline void modify(ll 1, ll r, _Functor func) {
30
            auto R = split(r + 1), L = split(l);
31
            for (auto it = L; it != R; it++) func(it);
32
33
        template <class Functor> // func(ll &, iterator)
34
35
        inline 11 query(11 1, 11 r, _Functor func) {
36
            11 \text{ ans} = 0;
37
            auto R = split(r + 1);
38
            for (auto it = split(l); it != R; it++) func(ans, it);
39
            return ans;
40
41
    } ;
```

## 2.2 树状数组

维护满足结合律且可差分信息的,常数较小的数据结构。

Listing 12: fenwick.cpp

```
template <class T>
2
    struct Fenwick {
3
        std::vector<T> c;
4
        inline int lowbit(int x) { return x & -x; }
5
        inline void merge (T &x, T &y) { x = x + y; }
6
        inline T subtract(T x, T y) { return x - y; }
7
        inline void update(size_t pos, T x) {
8
            for (pos++; pos < c.size(); pos += lowbit(pos)) merge(c[pos], x);</pre>
9
        inline void clear(void) {
10
            for (auto &x : c) x = T();
11
12
13
        inline T query(size_t pos) {
14
            T ans = T();
            for (pos++; pos; pos ^= lowbit(pos)) merge(ans, c[pos]);
15
16
            return ans;
17
18
        inline T query(size_t l, size_t r) {
            return subtract(query(r), query(l - 1));
19
20
        Fenwick(size t len) : c(len + 2) {}
21
   } ;
22
```

## 2.3 静态可重区间信息(支持 ${ m RMQ}$ )

基于 ST 表,支持静态数组可重区间信息的数据结构。

Listing 13: sparse-table.cpp

```
1
    template <class T>
    struct MaxInfo {
2
        T val;
3
4
        MaxInfo() { val = T(); }
5
        template <class InitT>
        MaxInfo(InitT x) { val = x; }
6
7
        MaxInfo operator+ (MaxInfo &x)
8
            return {std::max(val, x.val)};
9
10
    } ;
    template <class T>
11
    struct MinInfo {
12
        T val:
13
        MinInfo() { val = T(); }
14
        template <class InitT>
15
```

```
16
        MinInfo(InitT x) { val = x; }
17
        MinInfo operator+ (MinInfo &x)
18
            return {std::min(val, x.val)};
19
20
    };
21
    template <class T>
    struct GcdInfo {
22
23
        T val;
24
        GcdInfo() { val = T(); }
25
        template <class InitT>
26
        GcdInfo(InitT x) { val = x; }
27
        GcdInfo operator+ (GcdInfo &x) {
28
            return {std::gcd(val, x.val)};
29
    };
30
    template <class T>
31
    struct SparseTable {
32
   private:
33
34
        int n;
35
        std::vector<std::vector<T>> ST;
36
37
   public:
38
        SparseTable() {}
39
        SparseTable(int N) : n(N), ST(N, std::vector<T>( lg(N) + 1)) {}
40
        template <class InitT>
        SparseTable(std::vector<InitT> init) : SparseTable(init.size()) {
41
            for (int i = 0; i < n; i++) ST[i][0] = T(init[i]);</pre>
42
            for (int i = 1; (1 << i) <= n; i++) {</pre>
43
44
                for (int j = 0; j + (1 << i) - 1 < n; j++) {
45
                     ST[j][i] = ST[j][i-1] + ST[j + (1 << (i-1))][i-1];
46
47
48
49
        inline T query(int 1, int r) { // 0 based
            int w = std::__lg(r - l + 1);
50
            return ST[1] [w] + ST[r - (1 << w) + 1] [w];
51
52
53
    } ;
```

## 2.4 PBDS 大常数平衡树

GNU PBDS 提供的大常数基于 rb-tree 的平衡树。

### Listing 14: pbds-balance-tree.cpp

```
#include <bits/extc++.h>
#include <bits/stdc++.h>

using namespace std;

using namespace __gnu_pbds;

// TreeTag can also be __gnu_pbds::splay_tree_tag

template <class T, class Cmp, class TreeTag = rb_tree_tag

using BalanceTree = tree<T, null_type, Cmp, TreeTag, tree_order_statistics_node_update>;
```

### 2.5 离散化容器

### Listing 15: discretization.cpp

```
1 template <class _Tp>
2 struct Mess {
3    std::vector<_Tp> v;
4    bool initialized = false;
5    inline _Tp origin(int idx) { return v[idx - 1]; }
6    inline void insert(_Tp x) { v.push_back(x); }
```

2 数据结构 2.6 并查集

```
7
        template <typename T, typename... V>
8
        inline void insert(T x, V... v) { insert(x), insert(v...); }
9
        inline void init(void) {
10
            sort(v.begin(), v.end()), initialized = true;
            v.erase(unique(v.begin(), v.end()), v.end());
11
12
13
        inline void clear(void) { v.clear(), initialized = false; }
        inline int query(_Tp x) {
14
15
            if (!initialized) init();
16
            return lower bound(v.begin(), v.end(), x) - v.begin() + 1;
17
18
        inline bool exist( Tp x) { return origin(query(x)) == x; }
19
   };
```

### 2.6 并查集

### Listing 16: dsu.cpp

```
struct DSU {
1
        std::vector<int> fa, siz;
2
3
        DSU(int n) : fa(n + 1), siz(n + 1, 1) {
            std::iota(fa.begin(), fa.end(), 0);
4
5
6
        inline int find(int x) {
            return fa[x] == x ? x : fa[x] = find(fa[x]);
8
        // true if x and y were not in the same set, false otherwise.
9
10
        inline bool merge(int x, int y) {
11
            int fx = find(x), fy = find(y);
            if (fx == fy) return false;
12
            if (siz[fx] < siz[fy]) swap(fx, fy);</pre>
13
            fa[fy] = fx, siz[fx] += siz[fy], siz[fy] = 0;
14
            return true;
15
16
        // x \rightarrow y, a.k.a let x be son of y (disable merge by rank).
17
        inline bool directed merge(int x, int y) {
18
19
            int fx = find(x), fy = find(y);
20
            if (fx == fy) return false;
21
            fa[fx] = fy, siz[fy] += siz[fx], siz[fx] = 0;
            return true;
22
23
24
   };
```

### 2.7 出现次数统计

O(nlogn) 预处理, O(logn) 查找的出现次数在线统计

Listing 17: appear-statistics.cpp

```
#include <bits/stdc++.h>
1
    template <class Tp>
3
    struct Mess {
4
5
        std::vector< Tp> v;
6
        bool initialized = false;
        inline _Tp origin(int idx) { return v[idx - 1]; }
7
        inline void insert(_Tp x) { v.push_back(x); }
8
        template <typename T, typename... V >
9
        inline void insert(T x, V... v) { insert(x), insert(v...); }
10
        inline void init(void) {
11
12
            sort(v.begin(), v.end()), initialized = true;
13
            v.erase(unique(v.begin(), v.end()), v.end());
14
15
        inline int query(_Tp x) {
16
            if (!initialized) init();
```

2 数据结构 2.8 01-Trie

```
return lower bound(v.begin(), v.end(), x) - v.begin() + 1;
17
18
19
        inline bool exist( Tp x) { return origin(query(x)) == x; }
20
    } ;
21
22
    template <class T>
    class AppearStats { // Appear Statistics.
23
24
    private:
25
        Mess<T> M;
26
        size t n;
27
        std::vector<std::vector<int>> pos;
28
29
    public:
30
        AppearStats(void) : n(0) {}
        AppearStats(std::vector<T> &init) : n(init.size()) { init(init); }
31
        inline void _init(std::vector<T> &init) {
32
             for (auto item : init) M.insert(item);
33
             n = init.size(), M.init(), pos.resize(M.v.size());
34
35
             for (size t i = 0; i < n; i++) {</pre>
36
                 pos[M.query(init[i]) - 1].push back(i);
37
38
39
        // Use \lceil base \rceil as the beginning of index, return -1 if x doesn't exist.
40
        inline int first(int 1, int r, T x, int base = 0) {
41
             1 -= base, r -= base;
             if (!M.exist(x)) return -1;
42
             std::vector\leqint\geq &P = pos[M.query(x) - 1];
43
             auto it = std::lower_bound(P.begin(), P.end(), 1);
44
45
             return it == P.end() || *it > r ? -1 : *it + base;
46
47
        // Use \begin{bmatrix} base \end{bmatrix} as the beginning of index, return -1 if x doesn't exist.
48
        inline int last(int 1, int r, T x, int base = 0) {
49
             1 -= base, r -= base;
50
             if (!M.exist(x)) return -1;
51
             std::vector\langle int \rangle &P = pos[M.query(x) - 1];
             auto it = std::upper bound(P.begin(), P.end(), r);
52
             return it == P.begin() || *std::prev(it) < 1 ? -1 : *std::prev(it) + base;</pre>
53
54
        inline int count(int 1, int r, T x, int base = 0) {
55
             1 -= base, r -= base;
56
57
             if (!M.exist(x)) return 0;
             std::vector(int) &P = pos[M.query(x) - 1];
58
             auto L = std::lower bound(P.begin(), P.end(), 1);
59
             auto R = std::upper bound(P.begin(), P.end(), r);
60
61
             if (L == P.end() || R == P.begin()) return 0;
62
             if (*L > r || *std::prev(R) < 1) return 0;</pre>
63
             return R - L;
64
65
    };
```

## 2.8 01-Trie

### Listing 18: binary-trie.cpp

```
// Thanks neal for this template.
   const int BITS = 30;
   const int INF = 1e9 + 7;
3
    struct BinaryTrie { // 01-Trie
5
        static const int ALPHABET = 2;
        struct Node {
6
7
            const int parent;
                                   // How many words EXACTLY here.
            int words here = 0;
8
            int starting with = 0; // How many words have the PREFIX of this node.
9
10
            int min index = INF; // The minimum index of words which have PREFIX of this node.
11
            int max index = -INF; // The maximum index of words which have PREFIX of this node.
12
            std::array<int, ALPHABET> child;
```

2 数据结构 2.8 01-Trie

```
Node (int p = -1): parent(p) { child.fill(-1); }
13
14
        };
15
        static const int ROOT = 0;
16
        std::vector<Node> tr = {Node()};
        BinaryTrie(int total length = -1) { // Sum of |s|, leave -1 if don't know.
17
18
            if (total_length >= 0) tr.reserve(total_length + 1);
19
20
        // Returns the Node reference of word.
21
        // NOTICE: this function creates a new Node if word isn't in the trie.
22
        Node & operator[] (uint64 t word) {
23
            return tr[build(word, 0)];
24
25
        // Get or create c-th (c = 0, 1) child of node
26
        // Returns BinaryTrie node.
        int get or create child(int node, int c) {
27
            if (tr[node].child[c] == -1) {
28
                tr[node].child[c] = (int)tr.size();
29
                tr.push back(Node(node));
30
31
32
            return tr[node].child[c];
33
        // Build rootpath of word, insert delta (个) words
34
35
        // Returns BinaryTrie node.
36
        int build(uint64 t word, int delta) {
            int node = ROOT;
37
            for (int i = BITS - 1; i >= 0; i—) {
38
                tr[node].starting_with += delta;
39
                node = get or create child(node, word >> i & 1);
40
41
42
            tr[node].starting with += delta;
43
            return node;
44
45
        // Insert a word with the index of index, INF if index is unknown.
46
        // Returns BinaryTrie node.
47
        int insert(uint64_t word, int index = INF) {
48
            int node = build(word, 1);
            tr[node].words_here += 1;
49
            for (int x = node; x != -1; x = tr[x].parent) {
50
                if (index != INF) {
51
                    tr[x].min index = std::min(tr[x].min index, index);
52
53
                    tr[x].max index = std::max(tr[x].max index, index);
54
55
            }
56
            return node;
57
58
        // Find such an x inserted in the trie that word ^ x is minimized.
59
        // Returns such x (x is certain).
        uint64_t query_min(uint64_t word) {
60
            int node = ROOT;
61
            uint64 t val = 0;
62
            for (int i = BITS - 1; i >= 0; i---) {
63
64
                int go bit = word >> i & 1;
                if (tr[node].child[go bit] == -1) {
65
66
                    go bit ^= 1;
67
68
                val |= 1ull << go bit;
69
                node = tr[node].child[go bit];
70
71
            return val;
72
73
        // Find such an x inserted in the trie that word ^ x is maximized.
74
        // Returns such x (x is certain).
        uint64 t query max(uint64 t word) {
75
            int node = ROOT;
76
77
            uint64 t val = 0;
78
            for (int i = BITS - 1; i >= 0; i—) {
```

2 数据结构 2.9 滑动窗口

```
79
                 int go bit = (word >> i & 1) ^ 1;
80
                 if (tr[node].child[go bit] == -1) {
81
                     go bit ^= 1;
82
                 }
                 val |= 1ull << go bit;
83
84
                 node = tr[node].child[go bit];
85
86
             return val;
87
         // CF1983F: Find such an x inserted in the trie that word ^ x < upper bound
88
89
         // Returns a pair {min index, max index} of x.
90
         std::pair<int, int> query ub(uint64 t word, uint64 t upper bound) {
91
             int mn = INF, mx = -INF, node = ROOT;
             for (int i = BITS - 1; i >= 0; i---) {
92
                 int word bit = word >> i & 1;
                                                     // digit i of word
93
                 int ub_bit = upper_bound >> i & 1; // digit i of ub
94
                 if (ub bit == 1 && tr[node].child[word_bit] != -1) {
95
                     // if digit i of ub is 1, then we can choose either
96
97
                     // the subtree of word bit or word bit ^ 1.
98
                     mn = std::min(mn, tr[tr[node].child[word bit]].min index);
99
                     mx = std::max(mx, tr[tr[node].child[word bit]].max index);
100
101
                 // else if digit i of ub is 0, then we can only choose
102
                 // the subtree of word bit. (otherwise, we will violate the range)
                 node = tr[node].child[word bit ^ ub bit];
103
                 if (node == -1) break;
104
105
106
             return {mn, mx};
107
         }
108
    };
```

### 2.9 滑动窗口

# Listing 19: sliding-window.cpp

```
template <class T> // default max.
2
    std::vector<T> sliding_window(std::vector<T> A, size_t k) {
3
         std::vector<T> res;
         std::deque<size_t> Q;
4
         for (size t i = 0; i < A.size(); i++) {</pre>
5
             if (!Q.empty() && Q[0] + k == i) {
6
7
                  Q.pop_front();
8
             while (!Q.empty() && A[Q.back()] <= A[i]) {</pre>
9
10
                  Q.pop back();
11
12
             Q.push back(i);
13
             if (i >= k - 1) { // warning: assert k >= 1
14
                  res.push_back(A[Q[0]]);
15
16
         }
17
         return res;
18
    }
19
    template <class T>
20
    std::vector<std::vector<T>> grid sliding window(
21
         std::vector<std::vector<T>> &A, size_t x, size_t y
22
23
         const size t n = A.size(), m = A[0].size();
24
         std::vector<std::vector<T>> cols(m - y + 1);
         std::vector\langle \text{std}: \text{vector} \langle \text{T} \rangle \rangle ans (n - x + 1, \text{std}: \text{vector} \langle \text{T} \rangle (m - y + 1));
25
         for (size t i = 0; i < n; i++) {</pre>
26
             std::vector<T> res = sliding window(A[i], y);
27
             for (size t j = 0; j <= m - y; j++) {
28
29
                  cols[j].push back(res[j]);
30
31
         }
```

```
for (size_t j = 0; j <= m - y; j++) {
    std::vector<T> res = sliding_window(cols[j], x);

for (size_t i = 0; i <= n - x; i++) {
    ans[i][j] = res[i];

    }

return ans;

}</pre>
```

# 3 数学(数论)算法

## 3.1 带模整数类

Listing 20: mod-int.cpp

```
template <int mod>
   inline int64_t down(int64_t x) { return x >= mod ? x - mod : x; }
   template <int mod>
3
   struct ModInt {
        int64 t x;
5
6
        ModInt() = default;
7
        ModInt(int64_t x) : x((x % mod + mod) % mod) {}
8
        friend istream &operator>>(istream &in, ModInt &a) { return in >> a.x; }
9
        friend ostream &operator<<(ostream &out, ModInt a) { return out << a.x; }</pre>
10
        friend ModInt operator+(ModInt a, ModInt b) { return down(mod)(a.x + b.x); }
        friend ModInt operator-(ModInt a, ModInt b) { return down<mod> (a.x - b.x + mod); }
11
        friend ModInt operator*(ModInt a, ModInt b) { return (_int128)a.x * b.x % mod; }
12
        friend ModInt operator/ (ModInt a, ModInt b) { return a * ~b; }
13
14
        friend ModInt operator^ (ModInt a, long long b) {
15
            ModInt ans = 1;
            for (; b; b >>= 1, a *= a)
16
17
                if (b & 1) ans *= a;
18
            return ans;
19
        friend ModInt operator\sim (ModInt a) { return a ^ \land (mod - 2); }
20
        friend ModInt operator—(ModInt a) { return down (mod - a.x); }
21
22
        friend ModInt &operator+= (ModInt &a, ModInt b) { return a = a + b; }
        friend ModInt &operator—= ModInt \&a, ModInt b) { return a = a - b; }
23
        friend ModInt &operator*=(ModInt &a, ModInt b) { return a = a * b; }
24
        friend ModInt &operator/=(ModInt &a, ModInt b) { return a = a / b; }
25
        friend ModInt &operator^= (ModInt &a, long long b) { return a = a ^ b; }
26
27
        friend ModInt &operator++ (ModInt &a) { return a += 1; }
28
        friend ModInt operator++ (ModInt &a, int) {
29
            ModInt x = a;
30
            a += 1;
            return x;
31
32
33
        friend ModInt &operator— (ModInt &a) { return a -= 1; }
34
        friend ModInt operator— (ModInt &a, int) {
35
            ModInt x = a;
            a -= 1;
36
37
            return x;
38
        friend bool operator== (ModInt a, ModInt b) { return a.x == b.x; }
39
        friend bool operator!=(ModInt a, ModInt b) { return ! (a == b); }
40
41
   };
   using mint = ModInt<>;
42
```

### 3.2 计算几何

Listing 21: computation-geometry.cpp

```
1 template <class T>
2 struct Point {
```

3 数学(数论)算法 3.3 组合数学

```
Тх, у;
 3
 4
        Point (void) = default;
 5
        Point(T X, T Y) : x(X), y(Y) {}
 6
        inline bool operator==(const Point B) {
 7
            return x == B.x && y == B.y;
 8
 9
        friend std::ostream &operator<<(std::ostream &out, Point P) {</pre>
            return out << "(" << P.x << ", _" << P.y << ")";
10
11
        friend std::istream &operator>>(std::istream &in, Point &P) {
12
13
            return in >> P.x >> P.y;
14
15
    template <class T>
16
17
    struct Line {
        T A, B, C; // Ax + By + C = 0
18
        Line(void) = default;
19
20
        Line(T a, T b, T c) : A(a), B(b), C(c) \{\} // Ax + By + C = 0
21
        Line(T k, T b) : A(k), B(-1), C(b) {}
                                                  // y = kx + b
22
   };
    template <class T>
23
    inline int sign(T x) {
25
        return x == 0 ? 0 : (x < 0 ? -1 : 1);
26
    template <class T>
27
    inline bool parallel(Line<T> P, Line<T> Q) {
28
        return P.A * Q.B == P.B * Q.A;
29
30
31
    template <class T>
32
    inline Point<T> intersect(Line<T> P, Line<T> Q) {
33
        assert(!parallel(P, Q));
34
        return Point<T>{
35
            (P.C * Q.B - Q.C * P.B) / (Q.A * P.B - P.A * Q.B),
36
             (P.C * Q.A - Q.C * P.A) / (P.A * Q.B - Q.A * P.B)
37
        };
38
    template <class T>
39
    inline Line<T> get_line(Point<T> P, Point<T> Q) {
40
        assert(!(P == Q));
41
42
        if (P.x == Q.x) {
43
            return Line\langle T \rangle (-1, 0, P.x);
        } else if (P.y == Q.y) {
44
45
            return Line<T>(0, -1, P.y);
46
        } else {
47
            return Line<T>(
48
                Q.y - P.y, P.x - Q.x, P.y * Q.x - P.x * Q.y
49
            );
50
        }
51
52
    template <class T>
53
    inline bool point on line(Point<T> P, Line<T> L) {
        return L.A * P.x + L.B * P.y + L.C == 0;
54
55
56
    template <class T>
57
    inline T dis square(Point<T> P, Point<T> Q) {
        return (P.x - Q.x) * (P.x - Q.x) + (P.y - Q.y) * (P.y - Q.y);
58
59
    }
```

### 3.3 组合数学

Listing 22: comb.cpp

```
1 // require: math/mod-int.cpp
2 template <class mint>
3 struct Comb {
4 int n;
```

3 数学(数论)算法 3.4 拉格朗日插值

```
5
        std::vector<mint> _fac, _invfac, _inv;
6
        Comb() : n{0}, fac{1}, invfac{1}, inv{0} {}
7
        Comb(int n) : Comb() { init(n); }
8
        inline void init(int m) {
             _fac.resize(m + 1), _inv.resize(m + 1), _invfac.resize(m + 1);
9
            for (int i = n + 1; i <= m; i++) {</pre>
10
                fac[i] = fac[i - 1] * i;
11
12
13
             invfac[m] = \sim fac[m];
            for (int i = m; i > n; i—) {
14
                _{invfac[i-1] = _{invfac[i]} * i;}
15
16
                 inv[i] = invfac[i] * fac[i-1];
17
18
            n = m;
19
20
        inline mint fac(int m) {
            if (m > n) init(m);
21
            return _fac[m];
22
23
24
        inline mint invfac(int m) {
25
            if (m > n) init(m);
26
            return invfac[m];
27
28
        inline mint inv(int m) {
29
            if (m > n) init(m);
            return _inv[m];
30
31
        inline mint binom(int n, int m) {
32
33
            if (n < m | | m < 0) return 0;</pre>
34
            return fac(n) * invfac(m) * invfac(n - m);
35
36
37
    Comb<mint> comb;
```

### 3.4 拉格朗日插值

#### Listing 23: lagrange.cpp

```
// require: math/mod-int.cpp, math/comb.cpp
    inline mint lagrange(std::vector@mint> &x, std::vector@mint> &y, mint k) {
3
        mint ans = 0, cur;
4
        const int n = x.size();
        for (int i = 0; i < n; i++) {</pre>
5
6
            cur = y[i];
            for (int j = 0; j < n; j++) {
7
8
                if (j == i) continue;
9
                cur *= (k - x[j]) / (x[i] - x[j]);
10
11
            ans += cur;
        }
12
13
        return ans;
14
    }
15
    // y[0] is placeholder.
16
    // If for all integer x_i in [1, n], we have f(x_i) = y_i (mod p), find f(k) mod p.
17
    inline mint cont lagrange(std::vector<mint> &y, mint k) {
18
        mint ans = 0;
19
        const int n = y.size() - 1;
20
        std::vectormint> pre(n + 1, 1), suf(n + 2, 1);
        for (int i = 1; i \le n; i ++) pre[i] = pre[i - 1] * (k - i);
21
        for (int i = n; i \ge 1; i—) suf[i] = suf[i + 1] * (k - i);
22
        for (int i = 1; i <= n; i++) {</pre>
23
            mint A = pre[i - 1] * suf[i + 1];
24
            mint B = \text{comb.fac}(i - 1) * \text{comb.fac}(n - i);
25
26
            ans += ((n - i) \& 1 ? -1 : 1) * y[i] * A / B;
27
28
        return ans;
```

```
29
30
   // find 1^k + 2^k + ... + n^k. in O(k) of time complexity.
31
   inline mint sum_of_kth_powers(mint n, int k) {
32
        mint sum = 0;
        std::vector<mint> Y{0};
33
34
        for (int i = 1; i <= k + 2; i++) {</pre>
            Y.push_back(sum += (mint)i ^ k);
35
36
37
        return cont lagrange(Y, n);
38
```

# 4 字符串算法

# 4.1 字符串哈希

Listing 24: hashed-string.cpp

```
template <int mod, int seed>
   struct SingleHash {
3
        int n;
4
        std::vector<int> pow, h;
5
       SingleHash(void) = default;
6
       SingleHash(std::string &s) { init(s); }
7
        inline void init(std::string &s) {
8
            n = s.size(), h.assign(n + 2, 0), pow.assign(n + 2, 1);
9
            for (int i = 1; i <= n; i++) {</pre>
                pow[i] = 111 * pow[i - 1] * seed % mod;
10
                h[i] = (111 * h[i-1] * seed + s[i-1]) % mod;
11
12
13
        inline int get hash(int 1, int r) {
14
15
            return (h[r + 1] - 111 * h[1] * pow[r - 1 + 1] % mod + mod) % mod;
16
        inline bool check_same(int 11, int r1, int 12, int r2) {
17
            return get_hash(11, r1) == get_hash(12, r2);
18
19
20
   };
   struct HashedString {
21
        SingleHash<998244353, 477> H1;
22
        SingleHash<1000000007, 233> H2;
23
       HashedString(void) = default;
24
25
       HashedString(std::string &s) : H1(s), H2(s) {}
26
        inline void init(std::string &s) {
27
            H1.init(s), H2.init(s);
28
        std::pair<int, int> get_hash(int 1, int r) { // not recommended.
29
            return {H1.get_hash(l, r), H2.get_hash(l, r)};
30
31
32
        inline bool check_same(int 11, int r1, int 12, int r2) {
33
            return H1.check_same(11, r1, 12, r2) && H2.check_same(11, r1, 12, r2);
34
        inline bool check period(int 1, int r, int p) {
35
36
            return check_same(1, r - p, 1 + p, r);
37
   } ;
38
```

# 5 jiangly 代码库 (备用,侵权请提出 issue)

## 5.1 int128 输出流自定义

Listing 25: others/i128-stream.cpp

1 #include <iostream>

```
2
 3
   using i128 = int128;
 4
    std::istream & operator>>(std::istream is, i128 &n) {
 5
 6
        std::string s;
 7
        is >> s;
 8
        for (auto c : s) {
            n = n * 10 + (c - '0');
 9
10
        return is;
11
12
13
14
    std::ostream &operator<<(std::ostream &os, i128 n) {
15
        std::string s;
        while (n) {
16
            s += '0' + n % 10;
17
            n /= 10;
18
19
20
        std::reverse(s.begin(), s.end());
        return os << s;
21
22
   }
```

# 5.2 常用数学运算库函数及 gcd 重载

Listing 26: others/clf.cpp

```
using i64 = long long;
   using i128 = int128;
   inline i64 ceilDiv(i64 n, i64 m) {
 3
 4
        if (n >= 0) {
            return (n + m - 1) / m;
 5
 6
        } else {
 7
            return n / m;
 8
 9
    }
    inline i64 floorDiv(i64 n, i64 m) {
10
        if (n >= 0) {
11
            return n / m;
12
        } else {
13
            return (n - m + 1) / m;
14
15
16
17
    template <class T>
18
    inline void chmax(T &a, T b) {
19
        if (a < b) a = b;
20
    template <class T>
21
    inline void chmin(T &a, T b) {
22
        if (!(a < b)) a = b;
23
24
   inline i128 gcd(i128 a, i128 b) {
25
        return b ? gcd(b, a % b) : a;
26
27
```

## 5.3 强连通分量缩点 (SCC)

Listing 27: graph/scc.cpp

```
1 #include <vector>
2
3 struct SCC {
4    int n;
5    std::vector<std::vector<int>> adj;
6    std::vector<int>> stk;
7    std::vector<int>> dfn, low, bel;
```

```
8
        int cur, cnt;
 9
        SCC() {}
10
        SCC(int n) {
11
            init(n);
12
13
14
        void init(int n) {
15
             this\rightarrown = n;
16
17
            adj.assign(n, {});
18
            dfn.assign(n, -1);
19
            low.resize(n);
20
            bel.assign(n, -1);
21
            stk.clear();
            cur = cnt = 0;
22
23
24
25
        void addEdge(int u, int v) {
26
            adj[u].push_back(v);
27
28
29
        void dfs(int x) {
30
            dfn[x] = low[x] = cur++;
31
            stk.push_back(x);
32
            for (auto y : adj[x]) {
33
                 if (dfn[y] == -1) {
34
                     dfs(y);
35
36
                     low[x] = std::min(low[x], low[y]);
37
                 } else if (bel[y] == -1) {
38
                     low[x] = std::min(low[x], dfn[y]);
39
40
             }
41
            if (dfn[x] == low[x]) {
42
                 int y;
43
                 do {
44
                     y = stk.back();
45
                     bel[y] = cnt;
46
                     stk.pop back();
47
                 } while (y != x);
48
49
                 cnt++;
50
             }
51
52
53
        std::vector<int> work() {
            for (int i = 0; i < n; i++) {</pre>
54
                 if (dfn[i] == -1) {
55
                     dfs(i);
56
57
58
59
            return bel;
60
61
    };
```

# 5.4 割边与割边缩点 (EBCC)

Listing 28: graph/ebcc.cpp

```
#include <set>
#include <vector>

std::set<std::pair<int, int>> E;

struct EBCC {
   int n;
```

```
8
         std::vector<std::vector<int>> adj;
 9
         std::vector<int> stk;
        std::vector<int> dfn, low, bel;
10
11
        int cur, cnt;
12
        EBCC() {}
13
        EBCC(int n) {
14
             init(n);
15
16
17
18
         void init(int n) {
19
             this\rightarrown = n;
20
             adj.assign(n, {});
21
             dfn.assign(n, -1);
             low.resize(n);
22
23
             bel.assign(n, -1);
24
             stk.clear();
25
             cur = cnt = 0;
26
27
28
         void addEdge(int u, int v) {
29
             adj[u].push back(v);
30
             adj[v].push back(u);
31
32
        \textbf{void} \ \texttt{dfs}(\textbf{int} \ \texttt{x, int} \ \texttt{p}) \quad \{
33
             dfn[x] = low[x] = cur++;
34
             stk.push_back(x);
35
36
37
             for (auto y : adj[x]) {
38
                 if (y == p) {
39
                      continue;
40
                 if (dfn[y] == -1) {
41
42
                      E.emplace(x, y);
43
                      dfs(y, x);
                      low[x] = std::min(low[x], low[y]);
44
                  } else if (bel[y] == -1 \&\& dfn[y] < dfn[x]) {
45
46
                      E.emplace(x, y);
47
                      low[x] = std::min(low[x], dfn[y]);
48
49
50
51
             if (dfn[x] == low[x]) {
52
                 int y;
53
                 do {
54
                      y = stk.back();
55
                      bel[y] = cnt;
56
                      stk.pop_back();
57
                  } while (y != x);
                 cnt++;
58
59
             }
60
61
62
         std::vector<int> work() {
             dfs(0, -1);
63
             return bel;
64
65
66
         struct Graph {
67
68
             int n;
             std::vector<std::pair<int, int>> edges;
69
70
             std::vector<int> siz;
71
             std::vector<int> cnte;
72
73
        Graph compress() {
```

```
74
             Graph g;
75
             g.n = cnt;
76
             g.siz.resize(cnt);
77
             g.cnte.resize(cnt);
             for (int i = 0; i < n; i++) {</pre>
78
                 g.siz[bel[i]]++;
79
                 for (auto j : adj[i]) {
80
81
                     if (bel[i] < bel[j]) {
82
                          g.edges.emplace back(bel[i], bel[j]);
                      } else if (i < j) {
83
84
                          g.cnte[bel[i]]++;
85
86
                 }
87
88
             return g;
89
    };
90
```

## 5.5 二分图最大权匹配 (MaxAssignment, 基于 KM)

Listing 29: graph/bigraph-weight-match.cpp

```
#include <queue>
 2
    #include <vector>
 3
 4
    template <class T>
 5
    struct MaxAssignment {
 6
    public:
 7
        T solve(int nx, int ny, std::vector<std::vector<T>> a) {
            assert(0 <= nx && nx <= ny);
 8
 9
            assert(int(a.size()) == nx);
            for (int i = 0; i < nx; ++i) {</pre>
10
11
                 assert(int(a[i].size()) == ny);
12
                 for (auto x : a[i])
13
                     assert(x >= 0);
14
15
            auto update = [&] (int x) {
16
                 for (int y = 0; y < ny; ++y) {
17
                     if (lx[x] + ly[y] - a[x][y] < slack[y]) {
18
                         slack[y] = lx[x] + ly[y] - a[x][y];
19
20
                         slackx[y] = x;
21
22
                 }
23
            };
24
25
            costs.resize(nx + 1);
26
            costs[0] = 0;
            lx.assign(nx, std::numeric_limits<T>::max());
27
            ly.assign(ny, 0);
28
            xy.assign(nx, -1);
29
30
            yx.assign(ny, -1);
31
            slackx.resize(ny);
            for (int cur = 0; cur < nx; ++cur) {</pre>
32
33
                 std::queue<int> que;
34
                 visx.assign(nx, false);
35
                 visy.assign(ny, false);
36
                 slack.assign(ny, std::numeric limits<T>::max());
37
                 p.assign(nx, -1);
38
                 for (int x = 0; x < nx; ++x) {
39
                     if (xy[x] == -1) {
40
41
                         que.push(x);
42
                         visx[x] = true;
43
                         update(x);
44
                     }
```

```
45
 46
47
                 int ex, ey;
                 bool found = false;
48
                 while (!found) {
49
50
                     while (!que.empty() && !found) {
                          auto x = que.front();
51
52
                          que.pop();
53
                          for (int y = 0; y < ny; ++y) {
54
                              if (a[x][y] == lx[x] + ly[y] && !visy[y]) {
55
                                  if (yx[y] == -1) {
56
                                      ex = x;
57
                                      ey = y;
58
                                      found = true;
59
                                      break;
                                  }
60
61
                                  que.push(yx[y]);
62
                                  p[yx[y]] = x;
                                  visy[y] = visx[yx[y]] = true;
63
64
                                  update(yx[y]);
65
                              }
 66
                          }
67
                     if (found)
68
                          break;
69
70
                     T delta = std::numeric_limits<T>::max();
71
                     for (int y = 0; y < ny; ++y)
 72
 73
                          if (!visy[y])
 74
                              delta = std::min(delta, slack[y]);
 75
                     for (int x = 0; x < nx; ++x)
 76
                          if (visx[x])
 77
                              lx[x] = delta;
 78
                     for (int y = 0; y < ny; ++y) {
79
                          if (visy[y]) {
                              ly[y] += delta;
80
                          } else {
81
                              slack[y] -= delta;
82
                          }
83
84
                     for (int y = 0; y < ny; ++y) {
85
                          if (!visy[y] && slack[y] == 0) {
86
87
                              if (yx[y] == -1) {
88
                                  ex = slackx[y];
89
                                  ey = y;
90
                                  found = true;
91
                                  break;
92
                              }
93
                              que.push(yx[y]);
                              p[yx[y]] = slackx[y];
94
95
                              visy[y] = visx[yx[y]] = true;
96
                              update(yx[y]);
97
                          }
98
                      }
99
                 }
100
                 costs[cur + 1] = costs[cur];
101
                 for (int x = ex, y = ey, ty; x != -1; x = p[x], y = ty) {
102
                     costs[cur + 1] += a[x][y];
103
                     if (xy[x] != -1)
104
                         costs[cur + 1] -= a[x][xy[x]];
105
                     ty = xy[x];
106
107
                     xy[x] = y;
108
                     yx[y] = x;
109
                 }
110
             }
```

```
111
             return costs[nx];
112
         }
113
         std::vector<int> assignment() {
114
             return xy;
         }
115
116
         std::pair<std::vector<T>, std::vector<T>> labels() {
             return std::make pair(lx, ly);
117
118
         }
119
         std::vector<T> weights() {
             return costs;
120
121
122
     private:
123
         std::vector<T> lx, ly, slack, costs;
124
125
         std::vector<int> xy, yx, p, slackx;
         std::vector<bool> visx, visy;
126
    };
127
```

# 5.6 一般图最大匹配 (Graph, 带花树算法)

Listing 30: graph/general-match.cpp

```
#include <queue>
    #include <vector>
 3
 4
    struct Graph {
 5
        int n;
 6
        std::vector<std::vector<int>> e;
 7
        Graph(int n) : n(n), e(n) {}
        void addEdge(int u, int v) {
 8
 9
            e[u].push back(v);
10
            e[v].push_back(u);
11
        }
12
        std::vector<int> findMatching() {
13
            std::vector\leqint\geq match(n, -1), vis(n), link(n), f(n), dep(n);
14
15
            // disjoint set union
            auto find = [&] (int u) {
16
                while (f[u] != u)
17
                     u = f[u] = f[f[u]];
18
19
                return u;
20
            };
21
            auto lca = [&] (int u, int v) {
                u = find(u);
24
                v = find(v);
25
                while (u != v) {
26
                     if (dep[u] < dep[v])
27
                         std::swap(u, v);
                     u = find(link[match[u]]);
28
29
                 }
30
                return u;
31
            };
32
33
            std::queue<int> que;
34
            auto blossom = [&] (int u, int v, int p) {
35
                while (find(u) != p) {
36
                     link[u] = v;
                     v = match[u];
37
                     if (vis[v] == 0) {
38
                         vis[v] = 1;
39
                         que.push(v);
40
41
42
                     f[u] = f[v] = p;
43
                     u = link[v];
44
```

```
45
 46
             // find an augmenting path starting from u and augment (if exist)
47
             auto augment = [&] (int u) {
 48
                  while (!que.empty())
 49
50
                      que.pop();
51
                  std::iota(f.begin(), f.end(), 0);
52
53
54
                  // vis = 0 corresponds to inner vertices, vis = 1 corresponds to outer vertices
55
                  std::fill(vis.begin(), vis.end(), -1);
56
57
                  que.push(u);
58
                  vis[u] = 1;
                  dep[u] = 0;
59
60
61
                  while (!que.empty()) {
                      int u = que.front();
62
63
                      que.pop();
                      for (auto v : e[u]) {
64
                          if (vis[v] == -1) {
 65
 66
67
                              vis[v] = 0;
68
                              link[v] = u;
                              dep[v] = dep[u] + 1;
69
 70
                               // found an augmenting path
71
                               if (match[v] == -1) {
 72
 73
                                   for (int x = v, y = u, temp; y != -1; x = temp, y = x == -1 ? -1 : link[x]) {
 74
                                       temp = match[y];
                                       match[x] = y;
 75
 76
                                       match[y] = x;
 77
 78
                                   return;
79
80
                              vis[match[v]] = 1;
81
                              dep[match[v]] = dep[u] + 2;
82
                              que.push(match[v]);
83
84
                           } else if (vis[v] == 1 && find(v) != find(u)) {
85
                               // found a blossom
86
87
                              int p = lca(u, v);
88
                              blossom(u, v, p);
89
                              blossom(v, u, p);
90
                           }
                      }
91
92
                  }
             };
93
94
95
             // find a maximal matching greedily (decrease constant)
             auto greedy = [&]() {
96
97
                  for (int u = 0; u < n; ++u) {
98
                      if (match[u] != -1)
99
                          continue;
100
                      for (auto v : e[u]) {
                           \textbf{if} \ (\texttt{match[v]} == -1) \ \{ \\
101
                              match[u] = v;
102
103
                              match[v] = u;
                              break;
104
105
                          }
                      }
106
107
                  }
108
             };
109
110
             greedy();
```

### 5.7 2-SAT

Listing 31: graph/2-sat.cpp

```
1
    #include <vector>
 2
    struct TwoSat {
 3
 4
        int n;
        std::vector<std::vector<int>> e;
 5
        std::vector(bool) ans;
 6
        TwoSat(int n) : n(n), e(2 * n), ans(n) {}
 8
        void addClause(int u, bool f, int v, bool g) {
 9
            e[2 * u + !f].push back(2 * v + g);
10
            e[2 * v + !g].push back(2 * u + f);
11
12
        bool satisfiable() {
            std::vector<int> id(2 * n, −1), dfn(2 * n, −1), low(2 * n, −1);
13
            std::vector<int> stk;
14
            int now = 0, cnt = 0;
15
            std::function<void(int)> tarjan = [&] (int u) {
16
17
                stk.push back(u);
                dfn[u] = low[u] = now++;
18
                for (auto v : e[u]) {
19
20
                     if (dfn[v] == -1) {
21
                         tarjan(v);
22
                         low[u] = std::min(low[u], low[v]);
23
                     } else if (id[v] == -1) {
24
                         low[u] = std::min(low[u], dfn[v]);
25
                     }
26
27
                if (dfn[u] == low[u]) {
                     int v;
28
29
                     do {
30
                         v = stk.back();
31
                         stk.pop back();
32
                         id[v] = cnt;
                     } while (v != u);
33
34
                     ++cnt;
                }
35
36
            };
            for (int i = 0; i < 2 * n; ++i)</pre>
37
                if (dfn[i] == -1) tarjan(i);
38
            for (int i = 0; i < n; ++i) {</pre>
39
                if (id[2 * i] = id[2 * i + 1]) return false;
40
41
                ans[i] = id[2 * i] > id[2 * i + 1];
42
43
            return true;
44
        std::vector<bool> answer() { return ans; }
45
46
    };
```

### 5.8 最大流

Listing 32: graph/max-flow.cpp

```
1 constexpr int inf = 1E9;
```

```
template <class T>
 3
    struct MaxFlow {
        struct _Edge {
 4
 5
            int to;
 6
            T cap;
 7
             _Edge(int to, T cap) : to(to), cap(cap) {}
 8
        };
 9
10
        int n;
        std::vector< Edge> e;
11
12
        std::vector<std::vector<int>> g;
13
        std::vector<int> cur, h;
14
        MaxFlow() {}
15
        MaxFlow(int n) {
16
17
             init(n);
18
19
        void init(int n) {
20
21
            this\rightarrown = n;
            e.clear();
22
23
             g.assign(n, {});
24
            cur.resize(n);
            h.resize(n);
25
        }
26
27
        bool bfs(int s, int t) {
28
29
            h.assign(n, -1);
30
            std::queue<int> que;
31
            h[s] = 0;
32
            que.push(s);
33
            while (!que.empty()) {
34
                 const int u = que.front();
35
                 que.pop();
                 for (int i : g[u]) {
36
                     auto [v, c] = e[i];
37
                     if (c > 0 \&\& h[v] == -1) {
38
                         h[v] = h[u] + 1;
39
                         if (v == t) {
40
41
                              return true;
                         }
42
43
                         que.push(v);
44
45
                 }
46
47
            return false;
48
        }
49
        T dfs(int u, int t, T f) {
50
             if (u == t) {
51
52
                 return f;
53
54
             auto r = f;
55
             for (int &i = cur[u]; i < int(g[u].size()); ++i) {</pre>
56
                 const int j = g[u][i];
57
                 auto [v, c] = e[j];
                 if (c > 0 \&\& h[v] == h[u] + 1) {
58
59
                     auto a = dfs(v, t, std::min(r, c));
60
                     e[j].cap = a;
                     e[j ^ 1].cap += a;
61
                     r -= a;
62
                     if (r == 0) {
63
64
                         return f;
65
                     }
66
                 }
             }
67
```

```
68
              return f - r;
 69
70
         void addEdge(int u, int v, T c) {
71
             g[u].push_back(e.size());
              e.emplace_back(v, c);
 72
73
              g[v].push_back(e.size());
 74
              e.emplace back(u, 0);
 75
 76
         T flow(int s, int t) {
 77
              T ans = 0;
 78
              while (bfs(s, t)) {
 79
                  cur.assign(n, 0);
 80
                  ans += dfs(s, t, std::numeric limits<T>::max());
81
82
              return ans;
         }
83
84
         std::vector bool minCut() {
85
86
             std::vector<bool> c(n);
              for (int i = 0; i < n; i++) {</pre>
87
                  c[i] = (h[i] != -1);
88
 89
90
              return c;
91
         }
92
         struct Edge {
93
             int from;
94
              int to;
95
 96
              T cap;
97
              T flow;
98
         };
 99
         std::vector Edge> edges() {
100
              std::vector<Edge> a;
101
              for (int i = 0; i < e.size(); i += 2) {</pre>
102
                  Edge x;
                  x.from = e[i + 1].to;
103
                  x.to = e[i].to;
104
                  x.cap = e[i].cap + e[i + 1].cap;
105
                  x.flow = e[i + 1].cap;
106
107
                  a.push back(x);
108
109
             return a;
110
111
     };
```

## 5.9 最小费用可行流(或最大流)

Listing 33: graph/max-cost-flow-graph.cpp

```
struct MCFGraph {
1
2
        struct Edge {
3
            int v, c, f;
4
            Edge(int v, int c, int f) : v(v), c(c), f(f) {}
5
        };
6
        const int n;
7
        std::vector<Edge> e;
8
        std::vector<std::vector<int>> g;
9
        std::vector<i64> h, dis;
10
        std::vector<int> pre;
       bool dijkstra(int s, int t) {
11
            dis.assign(n, std::numeric limits<i64>::max());
12
            pre.assign(n, -1);
13
            std::priority queue<std::pair<i64, int>, std::vector<std::pair<i64, int>>, std::greater<std::pair<
14
              i64, int>>> que;
15
            dis[s] = 0;
16
            que.emplace(0, s);
```

```
17
            while (!que.empty()) {
18
                i64 d = que.top().first;
19
                int u = que.top().second;
20
                que.pop();
                if (dis[u] < d) continue;</pre>
21
22
                for (int i : g[u]) {
                    int v = e[i].v;
23
24
                    int c = e[i].c;
25
                    int f = e[i].f;
                    if (c > 0 \&\& dis[v] > d + h[u] - h[v] + f) {
26
27
                         dis[v] = d + h[u] - h[v] + f;
28
                         pre[v] = i;
29
                         que.emplace(dis[v], v);
30
                }
31
32
            return dis[t] != std::numeric_limits<i64>::max();
33
34
35
        MCFGraph(int n) : n(n), g(n) {}
        void addEdge(int u, int v, int c, int f) { // 可行流
36
37
            if (f < 0) {
38
                g[u].push back(e.size());
39
                e.emplace back(v, 0, f);
40
                g[v].push_back(e.size());
41
                e.emplace back(u, c, -f);
            } else {
42
                g[u].push_back(e.size());
43
                e.emplace_back(v, c, f);
44
45
                g[v].push back(e.size());
46
                e.emplace back(u, 0, -f);
47
            }
48
49
        // void addEdge(int u, int v, int c, int f) { // 最大流
50
               g[u].push back(e.size());
51
        //
               e.emplace_back(v, c, f);
        //
               g[v].push back(e.size());
52
        //
53
               e.emplace_back(u, 0, -f);
        // }
54
        std::pair<int, i64> flow(int s, int t) {
55
56
            int flow = 0;
            i64 cost = 0;
57
58
            h.assign(n, 0);
            while (dijkstra(s, t)) {
59
60
                for (int i = 0; i < n; ++i) h[i] += dis[i];</pre>
61
                int aug = std::numeric_limits<int>::max();
62
                for (int i = t; i != s; i = e[pre[i] ^ 1].v) aug = std::min(aug, e[pre[i]].c);
63
                for (int i = t; i != s; i = e[pre[i] ^ 1].v) {
                    e[pre[i]].c = aug;
64
                    e[pre[i] ^ 1].c += aug;
65
66
67
                flow += aug;
68
                cost += i64(aug) * h[t];
69
70
            return std::make pair(flow, cost);
71
72
    };
```

### 5.10 树链剖分

Listing 34: graph/hld.cpp

```
1 struct HLD {
2    int n;
3    std::vector<int> siz, top, dep, parent, in, out, seq;
4    std::vector<std::vector<int>> adj;
5    int cur;
```

```
6
 7
        HLD() {}
        HLD(int n) {
 8
            init(n);
 9
10
        void init(int n) {
11
            this\rightarrown = n;
12
13
            siz.resize(n);
14
            top.resize(n);
15
            dep.resize(n);
16
            parent.resize(n);
17
            in.resize(n);
18
            out.resize(n);
19
            seq.resize(n);
            cur = 0;
20
21
            adj.assign(n, {});
22
23
        void addEdge(int u, int v) {
24
            adj[u].push back(v);
            adj[v].push back(u);
25
26
27
        void work(int root = 0) {
28
            top[root] = root;
29
            dep[root] = 0;
30
            parent[root] = -1;
            dfs1(root);
31
            dfs2(root);
32
33
34
        void dfs1(int u) {
35
            if (parent[u] !=-1) {
36
                 adj[u].erase(std::find(adj[u].begin(), adj[u].end(), parent[u]));
37
38
39
            siz[u] = 1;
40
            for (auto &v : adj[u]) {
41
                 parent[v] = u;
                 dep[v] = dep[u] + 1;
42
43
                 dfs1(v);
                 siz[u] += siz[v];
44
45
                 if (siz[v] > siz[adj[u][0]]) {
                     std::swap(v, adj[u][0]);
46
47
48
            }
49
        void dfs2(int u) {
50
51
            in[u] = cur++;
52
            seq[in[u]] = u;
53
            for (auto v : adj[u]) {
                 top[v] = v == adj[u][0] ? top[u] : v;
54
55
                 dfs2(v);
56
57
            out[u] = cur;
58
59
        int lca(int u, int v) {
60
            while (top[u] != top[v]) {
61
                 if (dep[top[u]] > dep[top[v]]) {
62
                     u = parent[top[u]];
63
                 } else {
64
                     v = parent[top[v]];
65
66
67
            return dep[u] < dep[v] ? u : v;
68
69
70
        int dist(int u, int v) {
            return dep[u] + dep[v] - 2 * dep[lca(u, v)];
71
```

```
72
73
         int jump(int u, int k) {
74
             \textbf{if} \ (dep[u] < k) \ \{
75
                  return -1;
76
77
78
79
             int d = dep[u] - k;
80
81
             while (dep[top[u]] > d) {
82
                  u = parent[top[u]];
83
84
             return seq[in[u] - dep[u] + d];
85
         }
86
87
88
         bool isAncestor(int u, int v) {
89
             return in[u] <= in[v] && in[v] < out[u];</pre>
90
91
92
         int rootedParent(int u, int v) {
93
             std::swap(u, v);
94
             if (u == v) {
95
                  return u;
96
             if (!isAncestor(u, v)) {
97
98
                  return parent[u];
99
100
             auto it = std::upper bound(adj[u].begin(), adj[u].end(), v, [&] (int x, int y) {
101
                 return in[x] < in[y];</pre>
102
             \}) - 1;
103
              return *it;
104
105
         int rootedSize(int u, int v) {
106
             if (u == v) {
107
                  return n;
108
109
             if (!isAncestor(v, u)) {
110
                  return siz[v];
111
112
             return n - siz[rootedParent(u, v)];
113
114
115
116
         int rootedLca(int a, int b, int c) {
117
             return lca(a, b) ^ lca(b, c) ^ lca(c, a);
118
119
     } ;
```

# 5.11 快速幂

Listing 35: math/fast-pow.cpp

```
int power(int a, i64 b, int p) {
1
2
       int res = 1;
       for (; b; b /= 2, a = 1LL * a * a % p) {
3
           if (b % 2) {
4
5
               res = 1LL * res * a % p;
6
7
8
       return res;
9
   }
```

## 5.12 欧拉筛

Listing 36: math/euler-sieve.cpp

```
1
    std::vector<int> minp, primes;
 2
 3
    void sieve(int n) {
        minp.assign(n + 1, 0);
 4
        primes.clear();
 5
 6
        for (int i = 2; i <= n; i++) {</pre>
 7
            if (minp[i] == 0) {
 8
                minp[i] = i;
 9
10
                 primes.push_back(i);
11
            }
12
            for (auto p : primes) {
13
                 if (i * p > n) {
14
15
                    break;
16
                 }
17
                minp[i * p] = p;
18
                 if (p == minp[i]) {
19
                     break;
20
21
22
23
```

# 5.13 单点欧拉函数

# Listing 37: math/phi.cpp

```
1
    int phi(int n) {
 2
        int res = n;
        for (int i = 2; i * i <= n; i++) {</pre>
 3
            if (n % i == 0) {
 4
                 while (n % i == 0) {
 5
 6
                     n /= i;
 7
                 res = res / i * (i - 1);
 8
 9
            }
10
        if (n > 1) {
11
            res = res / n * (n-1);
12
13
14
        return res;
15
    }
```

### 5.14 exgcd

## Listing 38: math/exgcd.cpp

```
int exgcd(int a, int b, int &x, int &y) {
1
2
       if (!b) {
3
           x = 1, y = 0;
4
           return a;
5
       int g = exgcd(b, a % b, y, x);
6
7
       y = a / b * x;
       return g;
8
9
```

## 5.15 组合数

Listing 39: math/comb.cpp

```
1 struct Comb {
```

```
2
 3
        std::vector<Z> fac;
        std::vector<Z> _invfac;
 4
        std::vector<Z> _inv;
 5
 6
 7
        Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}
        Comb(int n) : Comb() {
 8
 9
             init(n);
10
11
12
        void init(int m) {
13
             m = std::min(m, Z::getMod() - 1);
14
             if (m <= n) return;</pre>
15
             _{\text{fac.resize(m + 1)}}
16
              invfac.resize(m + 1);
17
             _{inv.resize(m + 1);}
18
             for (int i = n + 1; i <= m; i++) {</pre>
19
20
                 _{fac[i]} = _{fac[i-1]} * i;
21
              invfac[m] = fac[m].inv();
22
23
             for (int i = m; i > n; i—) {
                 _{invfac[i-1] = _{invfac[i]} * i;}
24
                 _{inv[i]} = _{invfac[i]} * _{fac[i-1]};
25
26
27
             n = m;
        }
28
29
30
        Z fac(int m) {
31
             if (m > n) init(2 * m);
32
             return _fac[m];
33
34
        Z invfac(int m) {
35
             if (m > n) init(2 * m);
             return _invfac[m];
36
37
        Z inv(int m) {
38
             if (m > n) init(2 * m);
39
             return _inv[m];
40
41
        Z binom(int n, int m) {
42
             if (n < m | | m < 0) return 0;</pre>
43
44
             return fac(n) * invfac(m) * invfac(n - m);
45
        }
46
    } comb;
```

### 5.16 树状数组

Listing 40: ds/fenwick.cpp

```
template <typename T>
 1
    struct Fenwick {
 2
 3
        int n;
        std::vector<T> a;
 5
 6
        Fenwick(int n_ = 0) {
 7
            init(n );
 8
 9
        void init(int n ) {
10
            n = n_{,}
11
             a.assign(n, T{});
12
13
14
15
        void add(int x, const T &v) {
16
             for (int i = x + 1; i <= n; i += i & -i) {</pre>
```

```
17
                a[i-1] = a[i-1] + v;
            }
18
        }
19
20
21
        T sum(int x) {
22
            T ans{};
            for (int i = x; i > 0; i -= i & -i) {
23
                ans = ans + a[i-1];
24
25
26
            return ans;
27
28
29
        T rangeSum(int 1, int r) {
30
            return sum(r) - sum(1);
31
32
33
        int select(const T &k) {
34
            int x = 0;
            T cur{};
35
            for (int i = 1 << std:: lq(n); i; i /= 2) {
36
37
                if (x + i \le n \&\& cur + a[x + i - 1] \le k) {
38
                    x += i;
39
                    cur = cur + a[x - 1];
40
            }
41
42
            return x;
43
        }
44
   } ;
```

# 5.17 Splay

## Listing 41: ds/splay.cpp

```
struct Node {
 1
 2
        Node *1 = nullptr;
 3
        Node *r = nullptr;
        int cnt = 0;
 4
 5
        i64 sum = 0;
 6
    };
 7
    Node *add(Node *t, int 1, int r, int p, int v) {
 8
        Node *x = new Node;
 9
10
        if (t) {
             *x = *t;
11
12
13
        x->cnt += 1;
14
        x->sum += v;
        if (r - 1 == 1) {
15
16
             return x;
17
18
        int m = (1 + r) / 2;
19
         if (p < m) {
20
             x\rightarrow 1 = add(x\rightarrow 1, 1, m, p, v);
21
         } else {
22
             x \rightarrow r = add(x \rightarrow r, m, r, p, v);
23
24
        return x;
25
    }
26
    int find(Node *tl, Node *tr, int l, int r, int x) {
27
        if (r <= x) {
28
             return -1;
29
30
31
        if (1 >= x) {
32
             int cnt = (tr ? tr->cnt : 0) - (tl ? tl->cnt : 0);
             if (cnt == 0) {
```

```
34
                    return -1;
35
               if (r - 1 == 1) {
36
37
                    return 1;
38
39
40
          int m = (1 + r) / 2;
          int res = find(tl ? tl->l : tl, tr ? tr->l : tr, l, m, x);
41
42
          if (res == -1) {
43
               res = find(tl ? tl\rightarrowr : tl, tr ? tr\rightarrowr : tr, m, r, x);
44
45
          return res;
46
47
     std::pair<int, i64> get(Node *t, int 1, int r, int x, int y) {
48
          if (1 >= y || r <= x || !t) {</pre>
49
               return {0, OLL};
50
51
          if (1 >= x && r <= y) {</pre>
52
               return {t->cnt, t->sum};
53
54
55
          int m = (1 + r) / 2;
56
          auto [cl, sl] = get(t->1, l, m, x, y);
57
          auto [cr, sr] = get(t->r, m, r, x, y);
          return {cl + cr, sl + sr};
58
59
     }
60
     struct Tree {
61
62
          int add = 0;
63
          int val = 0;
64
          int id = 0;
65
          Tree *ch[2] = {};
66
          Tree *p = nullptr;
67
     };
68
     int pos(Tree *t) {
69
70
         return t\rightarrow p\rightarrow ch[1] == t;
71
72
     void add(Tree *t, int v) {
73
74
          t->val += v;
75
          t->add += v;
76
     }
77
78
     void push(Tree *t) {
79
          if (t->ch[0]) {
80
               add(t\rightarrow ch[0], t\rightarrow add);
81
          if (t->ch[1]) {
82
83
               add(t\rightarrow ch[1], t\rightarrow add);
84
85
          t\rightarrow add = 0;
86
87
88
     void rotate(Tree *t) {
89
          Tree *q = t\rightarrow p;
          int x = !pos(t);
90
91
          q \rightarrow ch[!x] = t \rightarrow ch[x];
          if (t\rightarrow ch[x]) t\rightarrow ch[x]\rightarrow p = q;
92
          t\rightarrow p = q\rightarrow p;
93
94
          if (q\rightarrow p) q\rightarrow p\rightarrow ch[pos(q)] = t;
          t\rightarrow ch[x] = q;
95
96
          q \rightarrow p = t;
97
98
99
    void splay(Tree *t) {
```

```
100
          std::vector<Tree *> s;
101
          for (Tree *i = t; i\rightarrow p; i = i\rightarrow p) s.push back(i\rightarrow p);
          while (!s.empty()) {
102
103
              push(s.back());
              s.pop_back();
104
105
          }
          push(t);
106
107
          while (t->p) {
108
              if (t->p->p) {
109
                   if (pos(t) == pos(t \rightarrow p)) rotate(t \rightarrow p);
110
                   else rotate(t);
111
112
              rotate(t);
          }
113
     }
114
115
     void insert(Tree *&t, Tree *x, Tree *p = nullptr) {
116
117
          if (!t) {
              t = x;
118
              x \rightarrow p = p;
119
120
              return;
121
          }
122
          push(t);
123
          124
              insert(t\rightarrow ch[0], x, t);
125
126
          } else {
              insert(t\rightarrow ch[1], x, t);
127
128
          }
129
     }
130
131
     void dfs(Tree *t) {
132
          if (!t) {
133
              return;
134
135
          push(t);
          dfs(t->ch[0]);
136
          std::cerr << t->val << "_";
137
          dfs(t\rightarrow ch[1]);
138
139
     }
140
     std::pair<Tree *, Tree *> split(Tree *t, int x) {
141
142
          if (!t) {
143
              return {t, t};
144
          }
          Tree *v = nullptr;
145
          Tree *j = t;
146
          for (Tree *i = t; i;) {
147
              push(i);
148
149
               j = i;
              if (i->val >= x) {
150
151
                   v = i;
152
                   i = i \rightarrow ch[0];
153
               } else {
154
                   i = i \rightarrow ch[1];
155
          }
156
157
158
          splay(j);
          if (!v) {
159
160
              return {j, nullptr};
161
162
163
          splay(v);
164
          Tree *u = v \rightarrow ch[0];
165
```

```
166
          if (u) {
167
               v\rightarrow ch[0] = u\rightarrow p = nullptr;
168
          // std::cerr << "split " << x << "\n";
169
          // dfs(u);
170
          // std::cerr << "\n";
171
          // dfs(v);
172
          // std::cerr << "\n";
173
174
          return {u, v};
175
176
177
     Tree *merge(Tree *1, Tree *r) {
178
          if (!1) {
179
               return r;
180
          if (!r) {
181
               return 1;
182
183
          Tree *i = 1;
184
          while (i->ch[1]) {
185
               i = i \rightarrow ch[1];
186
187
188
          splay(i);
189
          i\rightarrow ch[1] = r;
          r \rightarrow p = i;
190
          return i;
191
192
     }
```

# 5.18 取模类, 按需写

### Listing 42: ds/mod.cpp

```
template <class T>
 1
 2
    constexpr T power(T a, i64 b) {
 3
        T res = 1;
        for (; b; b /= 2, a *= a) {
 4
            if (b % 2) {
 5
                res *= a;
 6
 7
 8
        }
 9
        return res;
10
    }
11
    constexpr i64 mul(i64 a, i64 b, i64 p) {
12
13
        i64 \text{ res} = a * b - i64(1.L * a * b / p) * p;
14
        res %= p;
        if (res < 0) {
15
16
            res += p;
17
        return res;
18
19
    }
    template <i64 P>
20
    struct MLong {
21
22
        i64 x;
23
        constexpr MLong() : x{} {}
24
        constexpr MLong(i64 x) : x{norm(x % getMod())} {}
25
        static i64 Mod;
26
        constexpr static i64 getMod() {
27
            if (P > 0) {
28
29
                return P;
            } else {
30
31
                return Mod;
32
33
        constexpr static void setMod(i64 Mod ) {
```

```
35
             Mod = Mod ;
36
         }
         constexpr i64 norm(i64 x) const {
37
38
             if (x < 0) {
                 x += getMod();
39
40
             if (x \ge getMod()) {
41
42
                 x = getMod();
43
44
             return x;
45
46
         constexpr i64 val() const {
47
             return x;
48
         explicit constexpr operator i64() const {
49
50
             return x;
51
         constexpr MLong operator-() const {
52
53
             MLong res;
             res.x = norm(getMod() - x);
54
55
             return res;
56
57
         constexpr MLong inv() const {
58
             assert(x != 0);
             return power(*this, getMod() -2);
59
60
         constexpr MLong & operator* = (MLong rhs) & {
61
             x = mul(x, rhs.x, getMod());
62
63
             return *this;
64
65
         constexpr MLong & operator += (MLong rhs) & {
66
             x = norm(x + rhs.x);
             return *this;
67
68
69
         constexpr MLong & operator -= (MLong rhs) & {
             x = norm(x - rhs.x);
70
             return *this;
71
72
         constexpr MLong & operator/= (MLong rhs) & {
73
             return *this *= rhs.inv();
74
75
         friend constexpr MLong operator* (MLong lhs, MLong rhs) {
76
77
             MLong res = lhs;
78
             res *= rhs;
79
             return res;
80
81
         friend constexpr MLong operator+ (MLong lhs, MLong rhs) {
             MLong res = lhs;
82
             res += rhs;
83
84
             return res;
85
         friend constexpr MLong operator-(MLong lhs, MLong rhs) {
86
87
             MLong res = lhs;
88
             res -= rhs;
89
             return res;
90
         friend constexpr MLong operator/ (MLong lhs, MLong rhs) {
91
             MLong res = lhs;
92
             res /= rhs;
93
94
             return res;
95
         friend constexpr std::istream &operator>>(std::istream &is, MLong &a) {
96
97
             i64 v;
98
             is >> v;
99
             a = MLong(v);
100
             return is;
```

```
101
102
         friend constexpr std::ostream &operator (std::ostream &os, const MLong &a) {
103
             return os << a.val();</pre>
104
         friend constexpr bool operator (MLong lhs, MLong rhs) {
105
106
             return lhs.val() == rhs.val();
107
         friend constexpr bool operator!=(MLong lhs, MLong rhs) {
108
             return lhs.val() != rhs.val();
109
110
111
     };
112
113
     template <>
     i64 MLong<0LL>::Mod = i64(1E18) + 9;
114
115
    template <int P>
116
     struct MInt {
117
         int x:
118
119
         constexpr MInt() : x{} {}
         constexpr MInt(i64 x) : x{norm(x % getMod())} {}
120
121
122
         static int Mod;
123
         constexpr static int getMod() {
            if (P > 0) {
124
125
                 return P;
             } else {
126
127
                 return Mod;
128
             }
129
         }
130
         constexpr static void setMod(int Mod ) {
131
             Mod = Mod ;
132
133
         constexpr int norm(int x) const {
134
             if (x < 0) {
135
                 x += getMod();
136
             if (x \ge getMod()) {
137
138
                 x = getMod();
139
             return x;
140
141
         }
         constexpr int val() const {
142
143
             return x;
144
145
         explicit constexpr operator int() const {
146
             return x;
147
         }
148
         constexpr MInt operator-() const {
            MInt res;
149
150
             res.x = norm(qetMod() - x);
151
             return res;
152
         }
153
         constexpr MInt inv() const {
154
             assert(x != 0);
155
             return power(*this, getMod() -2);
156
         constexpr MInt & operator* = (MInt rhs) & {
157
             x = 1LL * x * rhs.x % getMod();
158
             return *this;
159
160
         constexpr MInt & operator += (MInt rhs) & {
161
             x = norm(x + rhs.x);
162
163
             return *this;
164
165
         constexpr MInt & operator -= (MInt rhs) & {
166
             x = norm(x - rhs.x);
```

```
167
             return *this;
168
         }
         constexpr MInt & operator /= (MInt rhs) & {
169
             return *this *= rhs.inv();
170
171
172
         friend constexpr MInt operator* (MInt lhs, MInt rhs) {
             MInt res = lhs;
173
             res *= rhs;
174
175
             return res;
176
177
         friend constexpr MInt operator+ (MInt lhs, MInt rhs) {
178
             MInt res = lhs;
179
             res += rhs;
180
             return res;
181
         friend constexpr MInt operator-(MInt lhs, MInt rhs) {
182
             MInt res = lhs;
183
             res -= rhs;
184
185
             return res;
186
         friend constexpr MInt operator/ (MInt lhs, MInt rhs) {
187
188
             MInt res = lhs;
189
             res /= rhs;
190
             return res;
191
         friend constexpr std::istream &operator>>(std::istream &is, MInt &a) {
192
             i64 v;
193
             is >> v;
194
195
             a = MInt(v);
196
             return is;
197
198
         friend constexpr std::ostream &operator<< (std::ostream &os, const MInt &a) {
199
             return os << a.val();</pre>
200
         friend constexpr bool operator== (MInt lhs, MInt rhs) {
201
202
             return lhs.val() == rhs.val();
203
         friend constexpr bool operator!=(MInt lhs, MInt rhs) {
204
             return lhs.val() != rhs.val();
205
206
207
     };
208
209
     template <>
210
     int MInt<0>::Mod = 998244353;
211
212
     template <int V, int P>
213
    constexpr MInt<P> CInv = MInt<P> (V) .inv();
214
    constexpr int P = 1000000007;
215
216
    using Z = MInt<P>;
```

#### 5.19 马拉车

Listing 43: string/manacher.cpp

```
std::vector<int> manacher(std::vector<int> s) {
1
2
        std::vector<int> t{0};
3
        for (auto c : s) {
4
            t.push back(c);
            t.push back(0);
5
6
7
        int n = t.size();
        std::vector<int> r(n);
8
9
        for (int i = 0, j = 0; i < n; i++) {</pre>
10
            if (2 * j - i) = 0 \&\& j + r[j] > i) {
                 r[i] = std::min(r[2 * j - i], j + r[j] - i);
11
```

#### 5.20 Z 函数

#### Listing 44: string/z-func.cpp

```
std::vector<int> zFunction(std::string s) {
 1
        int n = s.size();
 2
        std::vector<int> z(n + 1);
 3
 4
        z[0] = n;
 5
        for (int i = 1, j = 1; i < n; i++) {</pre>
 6
             z[i] = std::max(0, std::min(j + z[j] - i, z[i - j]));
 7
            while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]])  {
 8
                 z[i]++;
 9
            if (i + z[i] > j + z[j]) {
10
11
                 j = i;
12
13
        return z;
14
    }
15
```

#### 5.21 SA 后缀数组

### Listing 45: string/suffix-array.cpp

```
struct SuffixArray {
 2
        int n;
        std::vector<int> sa, rk, lc;
 3
        SuffixArray(const std::string &s) {
 4
 5
            n = s.length();
            sa.resize(n);
 6
 7
            lc.resize(n-1);
 8
            rk.resize(n);
 9
            std::iota(sa.begin(), sa.end(), 0);
10
            std::sort(sa.begin(), sa.end(), [&](int a, int b) { return s[a] < s[b]; });
11
            rk[sa[0]] = 0;
12
            for (int i = 1; i < n; ++i)</pre>
                rk[sa[i]] = rk[sa[i-1]] + (s[sa[i]] != s[sa[i-1]]);
13
            int k = 1;
14
            std::vector<int> tmp, cnt(n);
15
16
            tmp.reserve(n);
            while (rk[sa[n-1]] < n-1) {
17
18
                tmp.clear();
                for (int i = 0; i < k; ++i)
19
20
                    tmp.push_back(n - k + i);
21
                for (auto i : sa)
22
                    if (i >= k)
                         tmp.push back(i - k);
23
                std::fill(cnt.begin(), cnt.end(), 0);
24
                for (int i = 0; i < n; ++i)</pre>
25
26
                    ++cnt[rk[i]];
27
                for (int i = 1; i < n; ++i)</pre>
28
                    cnt[i] += cnt[i-1];
29
                for (int i = n - 1; i >= 0; —i)
30
                    sa[--cnt[rk[tmp[i]]]] = tmp[i];
```

```
31
                                                                                             std::swap(rk, tmp);
32
                                                                                             rk[sa[0]] = 0;
                                                                                             for (int i = 1; i < n; ++i)</pre>
33
                                                                                                                     {\tt rk[sa[i]] = rk[sa[i-1]] + (tmp[sa[i-1]] < tmp[sa[i]] \mid\mid sa[i-1] + k == n \mid\mid tmp[sa[i-1]] < tmp[sa[i-1]] = rk[sa[i-1]] = rk[
34
                                                                                                                                 -1] + k] < tmp[sa[i] + k]);
                                                                                             k *= 2;
35
36
                                                                       for (int i = 0, j = 0; i < n; ++i) {</pre>
37
38
                                                                                             if (rk[i] == 0) {
                                                                                                                      j = 0;
39
40
                                                                                               } else {
                                                                                                                     for (j -= j > 0; i + j < n \& sa[rk[i] - 1] + j < n \& s[i + j] == s[sa[rk[i] - 1] + j];)
41
42
                                                                                                                     lc[rk[i] - 1] = j;
43
                                                                                              }
44
45
                                                                       }
46
47
                       };
```

# 6 Watashi 代码库 (备用)

# **6.1** $O(n \log n) - O(1)$ **RMQ**

#### Listing 46: rmq.cpp

```
#include <algorithm> // copy
 1
 2
    #include <climits> // CHAR BIT
 3
    using namespace std;
 5
 6
    template <typename T>
 7
    struct RMQ {
 8
        int n;
 9
        vector<T> e;
10
        vector<vector<int>> rmq;
11
        static const int INT BIT = sizeof(4) * CHAR BIT;
12
        static inline int LG2(int i) { return INT_BIT - 1 - _builtin_clz(i); }
13
        static inline int BIN(int i) { return 1 << i; }</pre>
14
15
16
        int cmp(int 1, int r) const {
17
            return e[1] <= e[r] ? 1 : r;
18
19
        void init(int n, const T e[]) {
20
            this\rightarrown = n;
21
22
            vector<T>(e, e + n).swap(this->e);
23
24
             int m = 1;
25
            while (BIN(m) \leq n) {
26
                 ++m;
27
28
             vector<vector<int>>(m, vector<int>(n)).swap(rmq);
29
             for (int i = 0; i < n; ++i) {</pre>
30
                 rmq[0][i] = i;
31
32
            for (int i = 0; BIN(i + 1) <= n; ++i) {</pre>
33
                 for (int j = 0; j + BIN(i + 1) <= n; ++j) {</pre>
34
35
                     rmq[i + 1][j] = cmp(rmq[i][j], rmq[i][j + BIN(i)]);
36
37
             }
38
39
40
        int index(int 1, int r) const {
```

```
41
            int b = LG2(r - 1);
42
            return cmp(rmq[b][1], rmq[b][r - (1 << b)]);
43
44
        T value(int 1, int r) const {
45
            return e[index(l, r)];
46
47
48
    };
```

# **6.2** $O(n \log n) - O(\log n)$ **LCA**

#### Listing 47: lca.cpp

```
#include <algorithm>
    #include <cstdio>
    #include <vector>
 3
 4
    using namespace std;
 5
 6
 7
    const int MAXM = 16;
    const int MAXN = 1 << MAXM;</pre>
 8
10
    // LCA
    struct LCA {
11
12
        vector \le int = [MAXN];
13
        int d[MAXN], p[MAXN] [MAXM];
14
        void dfs_(int v, int f) {
15
            p[v][0] = f;
16
17
             for (int i = 1; i < MAXM; ++i) {</pre>
18
                 p[v][i] = p[p[v][i-1]][i-1];
19
20
             for (int i = 0; i < (int)e[v].size(); ++i) {</pre>
21
                 int w = e[v][i];
22
                 if (w != f) {
23
                     d[w] = d[v] + 1;
24
                     dfs_(w, v);
25
26
             }
        }
27
28
29
        int up (int v, int m) {
            for (int i = 0; i < MAXM; ++i) {</pre>
30
                 if (m & (1 << i)) {
31
32
                     v = p[v][i];
33
34
35
            return v;
        }
36
37
38
        int lca(int a, int b) {
            if (d[a] > d[b]) {
39
40
                 swap(a, b);
41
42
             b = up (b, d[b] - d[a]);
43
            if (a == b) {
44
                 return a;
45
             } else {
                 for (int i = MAXM - 1; i \ge 0; —i) {
46
                     if (p[a][i] != p[b][i]) {
47
                         a = p[a][i];
48
                         b = p[b][i];
49
50
                     }
51
52
                 return p[a][0];
```

```
54
55
        void init(int n) {
56
             for (int i = 0; i < n; ++i) {</pre>
57
                 e[i].clear();
58
59
60
        }
61
        void add(int a, int b) {
62
63
             e[a].push back(b);
64
             e[b].push_back(a);
65
66
        void build() {
67
             d[0] = 0;
68
69
             dfs_(0, 0);
70
71
    } lca;
```

#### 6.3 树状数组

Listing 48: bit.cpp

```
#include <vector>
 2
    using namespace std;
 3
 4
    template<typename T = int>
 5
 6
    struct BIT {
      vector<T> a;
 7
 8
 9
      void init(int n) {
10
        vector<T>(n + 1).swap(a);
11
12
13
      void add(int i, T v) {
        for (int j = i + 1; j < (int)a.size(); j = (j | (j - 1)) + 1) {
14
          a[j] += v;
15
16
        }
17
      }
18
      // [0, i)
19
20
      T sum(int i) const {
21
        T ret = T();
        for (int j = i; j > 0; j = j & (j - 1)) {
22
23
          ret += a[j];
24
25
        return ret;
26
27
28
      T get(int i) const {
29
        return sum(i + 1) - sum(i);
30
31
      void set(int i, T v) {
32
33
        add(i, v - get(i));
34
35
   } ;
```

#### 6.4 并查集

Listing 49: union-find.cpp

```
1 #include <vector>
2
```

```
using namespace std;
 4
    struct DisjointSet {
 5
        vector<int> p;
 6
 7
 8
        void init(int n) {
 9
            p.resize(n);
             for (int i = 0; i < n; ++i) {</pre>
10
11
                 p[i] = i;
12
13
14
15
        int getp(int i) {
            return i == p[i] ? i : (p[i] = getp(p[i]));
16
17
18
        bool setp(int i, int j) {
19
            i = getp(i);
20
            j = getp(j);
21
            p[i] = j;
22
            return i != j;
23
24
25
   } ;
```

# 6.5 轻重权树剖分

#### Listing 50: chain-decomp.cpp

```
#include <cstdio>
 1
    #include <vector>
    #include <algorithm>
 3
 4
 5
    using namespace std;
 7
    const int MAXM = 16;
 8
    const int MAXN = 1 << MAXM;</pre>
 9
   // Heavy-Light Decomposition
10
   struct TreeDecomposition {
11
      vector<int> e[MAXN], c[MAXN];
12
      int s[MAXN];
                    // subtree size
13
                      // parent id
      int p[MAXN];
14
      int r[MAXN];
                      // chain root id
15
      int t[MAXN];
                      // timestamp, index used in segtree
16
17
      int ts;
18
      void dfs (int v, int f) {
19
       p[v] = f;
20
        s[v] = 1;
21
        for (int i = 0; i < (int)e[v].size(); ++i) {</pre>
22
23
          int w = e[v][i];
24
          if (w != f) {
25
            dfs (w, v);
26
            s[v] += s[w];
27
28
        }
29
30
      void decomp_(int v, int f, int k) {
31
        t[v] = ts++;
32
        c[k].push back(v);
33
        r[v] = k;
34
35
36
        int x = 0, y = -1;
37
        for (int i = 0; i < (int)e[v].size(); ++i) {</pre>
          int w = e[v][i];
```

```
39
          if (w != f) {
40
            if (s[w] > x) {
41
              x = s[w];
42
               y = w;
43
             }
44
          }
45
        if (y != -1) {
46
47
          decomp_(y, v, k);
48
49
        for (int i = 0; i < (int)e[v].size(); ++i) {</pre>
50
51
          int w = e[v][i];
          if (w != f && w != y) {
52
            decomp_(w, v, w);
53
54
55
        }
      }
56
57
58
      void init(int n) {
59
        for (int i = 0; i < n; ++i) {</pre>
60
          e[i].clear();
61
        }
62
      }
63
      void add(int a, int b) {
64
65
        e[a].push_back(b);
66
        e[b].push_back(a);
67
68
69
      void build() { // !!
70
        ts = 0;
        dfs(0, 0);
71
72
        decomp (0, 0, 0);
73
   } hld;
74
```

### 6.6 强连通分量

Listing 51: scc.cpp

```
#include <algorithm>
    #include <stack>
    #include <vector>
 5
   using namespace std;
 6
    struct SCCTarjan {
 7
 8
        int n;
        vector<vector<int>> e;
 9
10
        vector<int> id;
11
        vector<vector<int>> scc;
12
13
14
        void init(int n) {
15
            this\rightarrown = n;
16
            vector<vector<int>> (n) .swap(e);
17
             id.resize(n);
            dfn.resize(n);
18
            low.resize(n);
19
        }
20
21
22
        void add(int a, int b) {
23
             e[a].push_back(b);
24
```

```
26
        vector<int> dfn, low;
27
        int timestamp;
        stack<int> s;
28
29
        void dfs(int v) {
30
            dfn[v] = timestamp++;
31
            low[v] = dfn[v];
32
33
            s.push(v);
            for (vector<int>::const iterator w = e[v].begin(); w != e[v].end(); ++w) {
34
35
                if (dfn[*w] == -1) {
36
                     dfs(*w);
37
                     low[v] = min(low[v], low[*w]);
38
                 } else if (dfn[*w] != -2) {
                     low[v] = min(low[v], dfn[*w]);
39
40
41
            }
42
            if (low[v] == dfn[v]) {
43
                vector<int> t;
44
                do {
45
                     int w = s.top();
46
47
                     s.pop();
48
                     id[w] = (int)scc.size();
                     t.push_back(w);
49
                     dfn[w] = -2;
50
                 } while (t.back() != v);
51
                scc.push_back(t);
52
            }
53
54
        }
55
56
        int gao() {
57
            scc.clear();
58
            stack<int>().swap(s);
59
            timestamp = 0;
60
            fill(dfn.begin(), dfn.end(), -1);
61
            for (int i = 0; i < n; ++i) {</pre>
62
                if (dfn[i] == -1) {
63
                     dfs(i);
64
65
66
67
            return (int)scc.size();
68
69
    } ;
```

# 6.7 双连通分量

Listing 52: bcc.cpp

```
#include <algorithm>
 1
    #include <stack>
    #include <utility>
 3
    #include <vector>
 4
 5
 6
    using namespace std;
 7
 8
    // TODO: cannot handle duplicate edges
 9
    struct Tarjan {
        int n;
10
        vector<vector<int>> e;
11
12
        vector<int> cut;
13
        vector<pair<int, int>> bridge;
14
15
        vector<vector<pair<int, int>>> bcc;
16
17
        void init(int n) {
```

```
18
             this\rightarrown = n;
19
             e.clear();
20
             e.resize(n);
21
             dfn.resize(n);
22
             low.resize(n);
23
         }
24
25
         void add(int a, int b) {
26
             // assert(find(e[a].begin(), e[a].end(), b) == e[a].end());
27
             e[a].push back(b);
28
             e[b].push back(a);
29
30
31
         vector<int> dfn, low;
32
         int timestamp;
33
         stack<pair<int, int>> s;
34
35
         void dfs(int v, int p) {
             int part = p == -1 ? 0 : 1;
36
37
             dfn[v] = low[v] = timestamp++;
             for (vector<int>::const iterator w = e[v].beqin(); w != e[v].end(); ++w) {
38
39
                 pair<int, int> f = make pair(min(v, *w), max(v, *w));
40
                 if (dfn[*w] == -1) {
41
                      s.push(f);
                      dfs(*w, v);
42
                      low[v] = min(low[v], low[*w]);
43
                      \textbf{if} \ (\texttt{dfn[v]} \ \mathop{<=}\ \texttt{low[*w])} \ \{
44
                          // articulation point
45
46
                          if (++part == 2) {
47
                               cut.push back(v);
48
49
                          // articulation edge
50
                          if (dfn[v] < low[*w]) {
51
                              bridge.push back(f);
52
                          // biconnected component (2-vertex-connected)
53
                          vector<pair<int, int>> t;
54
55
                          do {
                               t.push back(s.top());
56
57
                               s.pop();
                          } while (t.back() != f);
58
59
                          bcc.push back(t);
60
61
                  } else if (*w != p && dfn[*w] < dfn[v]) {</pre>
62
                      s.push(f);
63
                      low[v] = min(low[v], dfn[*w]);
64
65
             }
         }
66
67
68
         void gao() {
69
             cut.clear();
70
             bridge.clear();
71
             bcc.clear();
72
73
             timestamp = 0;
             stack<pair<int, int>>().swap(s);
74
75
             fill(dfn.begin(), dfn.end(), -1);
76
             for (int i = 0; i < n; ++i) {</pre>
77
                 if (dfn[i] == -1) {
78
                      dfs(i, -1);
79
80
81
             }
82
         }
83
    };
```

```
84
     struct BridgeBlockTree {
85
         Tarjan<MAXN> bcc;
86
         DisjointSet<MAXN▷ ds;
87
         vector<int> e[MAXN];
88
89
90
         void init(int n) {
             bcc.init(n);
91
             ds.init(n);
92
93
94
         void add(int a, int b) {
95
96
             bcc.add(a, b);
97
98
         void gao() {
99
             bcc.gao();
100
101
             for (const auto &i : bcc.bcc) {
102
                 if (i.size() > 1) {
                      for (const auto &j : i) {
103
                          ds.setp(j.first, j.second);
104
105
106
107
             for (const auto &i : bcc.bridge) {
108
                 int a = ds.getp(i.first);
109
                 int b = ds.getp(i.second);
110
                 e[a].push_back(b);
111
112
                 e[b].push_back(a);
113
             }
114
         }
115
116
         int id(int v) {
117
             return ds.getp(v);
118
     };
119
```

### 6.8 二分图匹配

### Listing 53: bimatch.cpp

```
// maximum matchings in bipartite graphs
    // maximum cardinality bipartite matching
    // O(|V||E|), generally fast
 5
    #include <algorithm>
    #include <string>
 6
    #include <vector>
 7
 8
 9
    using namespace std;
10
11
    struct Hungarian {
12
        int nx, ny;
        vector<int> mx, my;
13
14
        vector<vector<int>> e;
15
16
        void init(int nx, int ny) {
17
            this\rightarrownx = nx;
            this->ny = ny;
18
            mx.resize(nx);
19
20
            my.resize(ny);
21
            e.clear();
            e.resize(nx);
22
23
            mark.resize(nx);
24
        }
```

```
26
        void add(int a, int b) {
27
             e[a].push back(b);
28
29
        // vector<bool> is evil!!!
30
31
        basic_string<bool> mark;
32
        bool augment(int i) {
33
34
             if (!mark[i]) {
35
                 mark[i] = true;
36
                 for (vectorint)::const iterator j = e[i].begin(); j != e[i].end(); ++j) {
37
                     if (my[*j] == -1 \mid | augment(my[*j])) {
                         mx[i] = *j;
38
                         my[*j] = i;
39
                         return true;
40
                     }
41
42
                 }
43
44
            return false;
45
        }
46
47
        int gao() {
48
            int ret = 0;
             fill(mx.begin(), mx.end(), -1);
49
             fill(my.begin(), my.end(), -1);
50
             for (int i = 0; i < nx; ++i) {</pre>
51
                 fill(mark.begin(), mark.end(), false);
52
                 if (augment(i)) {
53
54
                     ++ret;
55
56
57
            return ret;
58
59
    };
```

## 6.9 最小费用最大流

#### Listing 54: flow.cpp

```
#include <algorithm>
    #include <cstdio>
    #include <limits>
    #include <queue>
    #include <vector>
 7
    using namespace std;
 8
 9
    template <int MAXN, typename T = int, typename S = T>
    struct MinCostMaxFlow {
10
        struct NegativeCostCircuitExistsException {
11
12
        };
13
14
        struct Edge {
            int v;
15
16
            T c;
17
            S w;
18
            int b;
            Edge(int v, T c, S w, int b) : v(v), c(c), w(w), b(b) {}
19
20
21
        int n, source, sink;
22
        vector<Edge> e[MAXN];
23
24
25
        void init(int n, int source, int sink) {
26
            this\rightarrown = n;
            this->source = source;
```

```
28
             this->sink = sink;
29
             for (int i = 0; i < n; ++i) {</pre>
30
                 e[i].clear();
31
32
        }
33
        void addEdge(int a, int b, T c, S w) {
34
35
             e[a].push_back(Edge(b, c, w, e[b].size()));
36
             e[b].push back(Edge(a, 0, -w, e[a].size() - 1)); // TODO
37
38
39
        bool mark[MAXN];
40
        T maxc[MAXN];
41
        S minw[MAXN];
        int dist[MAXN];
42
        Edge *prev[MAXN];
43
44
45
        bool spfa() {
46
            queue<int> q;
47
             fill(mark, mark + n, false);
48
             fill(maxc, maxc + n, 0);
49
             fill(minw, minw + n, numeric limits<S>::max());
50
             fill(dist, dist + n, 0);
51
             fill(prev, prev + n, (Edge *)NULL);
            mark[source] = true;
52
            maxc[source] = numeric limits<S>::max();
53
            minw[source] = 0;
54
55
56
            q.push(source);
57
             while (!q.empty()) {
58
                 int cur = q.front();
59
                 mark[cur] = false;
60
                 q.pop();
61
                 for (typename vector Edge>::iterator it = e[cur].begin(); it != e[cur].end(); ++it) {
62
                     T c = min(maxc[cur], it->c);
                     if (c == 0) {
63
                         continue;
64
65
                     }
66
                     int v = it->v;
67
68
                     S w = minw[cur] + it \rightarrow w;
                     if (\min w[v] > w \mid | (\min w[v] == w \&\& \max c[v] < c)) { // TODO}
69
70
                         maxc[v] = c;
71
                         minw[v] = w;
72
                         dist[v] = dist[cur] + 1;
73
                         if (dist[v] >= n) {
74
                              return false;
75
                         }
                         prev[v] = &*it;
76
77
                         if (!mark[v]) {
78
                              mark[v] = true;
79
                              q.push(v);
80
                          }
81
                     }
82
                 }
83
84
             return true;
85
86
        pair<T, S> gao() {
87
            T sumc = 0;
88
             S sumw = 0;
89
90
            while (true) {
91
                 if (! spfa()) {
92
                     throw NegativeCostCircuitExistsException();
93
                 } else if (maxc[sink] == 0) {
```

```
94
                       break;
95
                   } else {
                       T c = maxc[sink];
96
97
                       sumc += c;
                       sumw += c * minw[sink];
98
99
                       int cur = sink;
100
                       while (cur != source) {
101
                           Edge *e1 = prev[cur];
102
103
                           e1->c -= c;
104
                           Edge *e2 = \&e[e1->v][e1->b];
105
                           e2->c += c;
106
                           cur = e2 \rightarrow v;
107
                  }
108
109
              return make_pair(sumc, sumw);
110
111
112
     };
```

#### 6.10 AhoCorasick 自动机

#### Listing 55: ac-automata.cpp

```
#include <algorithm>
 2
    #include <queue>
 3
 4
    using namespace std;
 5
 6
    struct AhoCorasick {
 7
        static const int NONE = 0;
        static const int MAXN = 1024;
 8
 9
        static const int CHARSET = 26;
10
11
        int end;
12
        int tag[MAXN];
13
        int fail[MAXN];
        int trie[MAXN] [CHARSET];
14
15
        void init() {
16
            tag[0] = NONE;
17
            fill(trie[0], trie[0] + CHARSET, -1);
18
            end = 1;
19
20
21
        int add(int m, const int *s) {
22
23
            int p = 0;
            for (int i = 0; i < m; ++i) {</pre>
24
                 if (trie[p][*s] == -1) {
25
                     tag[end] = NONE;
26
27
                     fill(trie[end], trie[end] + CHARSET, -1);
                     trie[p][*s] = end++;
28
29
                 }
30
                 p = trie[p][*s];
31
                 ++s;
32
33
            return p;
34
35
        void build(void) { // !!
36
            queue<int> bfs;
37
            fail[0] = 0;
38
            for (int i = 0; i < CHARSET; ++i) {</pre>
39
40
                 if (trie[0][i] != −1) {
41
                     fail[trie[0][i]] = 0;
42
                     bfs.push(trie[0][i]);
```

```
43
                 } else {
44
                     trie[0][i] = 0;
45
46
            while (!bfs.empty()) {
47
48
                 int p = bfs.front();
                 tag[p] |= tag[fail[p]];
49
50
                 bfs.pop();
51
                 for (int i = 0; i < CHARSET; ++i) {</pre>
                     if (trie[p][i] != −1) {
52
53
                          fail[trie[p][i]] = trie[fail[p]][i];
54
                         bfs.push(trie[p][i]);
55
                     } else {
56
                         trie[p][i] = trie[fail[p]][i];
57
58
                 }
             }
59
60
61
    } ac;
```

#### 6.11 后缀数组

Listing 56: sa.cpp

```
#include <algorithm>
 2
    #include <utility>
 3
    #include <vector>
 4
    using namespace std;
 5
 6
    struct SuffixArray {
 7
        vector<int> sa, rank, height;
 8
 9
        template <typename T>
10
        void init(int n, const T a[]) {
11
             sa.resize(n);
12
             rank.resize(n);
13
             vector<pair<T, int>> assoc(n);
14
             for (int i = 0; i < n; ++i) {</pre>
15
                 assoc[i] = make pair(a[i], i);
16
17
             sort(assoc.begin(), assoc.end());
18
             for (int i = 0; i < n; ++i) {</pre>
19
                 sa[i] = assoc[i].second;
20
21
                 if (i == 0 \mid | assoc[i].first != assoc[i - 1].first) {
22
                     rank[sa[i]] = i;
23
                 } else {
24
                     rank[sa[i]] = rank[sa[i-1]];
25
                 }
             }
26
27
28
             vector<int> tmp(n), cnt(n);
29
             vector<pair<int, int>> suffix(n);
             for (int m = 1; m < n; m <<= 1) {</pre>
30
31
                 // snd
32
                 for (int i = 0; i < m; ++i) {</pre>
33
                     tmp[i] = n - m + i;
34
                 for (int i = 0, j = m; i < n; ++i) {</pre>
35
                     if (sa[i] >= m) {
36
                          tmp[j++] = sa[i] - m;
37
38
                 }
39
40
                 // fst
41
                 fill(cnt.begin(), cnt.end(), 0);
42
                 for (int i = 0; i < n; ++i) {</pre>
```

```
43
                     ++cnt[rank[i]];
44
                 }
45
                 partial_sum(cnt.begin(), cnt.end(), cnt.begin());
                 for (int i = n - 1; i >= 0; —i) {
46
                     sa[--cnt[rank[tmp[i]]] = tmp[i];
47
48
                 }
                 //
49
                 for (int i = 0; i < n; ++i) {</pre>
50
51
                     suffix[i] = make pair(rank[i], i + m < n ? rank[i + m] : numeric_limits<int>::min());
52
53
                 for (int i = 0; i < n; ++i) {</pre>
                     if (i == 0 \mid \mid suffix[sa[i]] != suffix[sa[i-1]]) {
54
55
                         rank[sa[i]] = i;
56
                     } else {
                         rank[sa[i]] = rank[sa[i-1]];
57
58
59
                 }
60
             }
61
            height.resize(n);
62
             for (int i = 0, z = 0; i < n; ++i) {
63
64
                 if (rank[i] == 0) {
65
                     height[0] = z = 0;
66
                 } else {
                     int x = i, y = sa[rank[i] - 1];
67
                     z = \max(0, z - 1);
68
                     while (x + z < n \&\& y + z < n \&\& a[x + z] == a[y + z])  {
69
70
                         ++z:
71
72
                     height[rank[i]] = z;
73
                 }
74
             }
75
        }
76
   };
```

## 6.12 LU 分解

#### Listing 57: lu.cpp

```
const int MAXN = 128;
    const double EPS = 1e-10;
 3
    void LU(int n, double a[MAXN] [MAXN], int r[MAXN], int c[MAXN]) {
        for (int i = 0; i < n; ++i) {</pre>
 6
             r[i] = c[i] = i;
 7
        for (int k = 0; k < n; ++k) {</pre>
 8
             int ii = k, jj = k;
 9
             for (int i = k; i < n; ++i) {</pre>
10
                 for (int j = k; j < n; ++j) {
11
                     if (fabs(a[i][j]) > fabs(a[ii][jj])) {
12
13
                          ii = i;
14
                          jj = j;
15
                      }
16
                 }
17
18
             swap(r[k], r[ii]);
19
             swap(c[k], c[jj]);
             for (int i = 0; i < n; ++i) {</pre>
20
                 swap(a[i][k], a[i][jj]);
21
22
             for (int j = 0; j < n; ++j) {
23
                 swap(a[k][j], a[ii][j]);
24
25
26
             if (fabs(a[k][k]) < EPS) {
                 continue;
```

```
28
               for (int i = k + 1; i < n; ++i) {</pre>
29
                    a[i][k] = a[i][k] / a[k][k];
30
31
                    for (int j = k + 1; j < n; ++j) {
32
                         a[i][j] = a[i][k] * a[k][j];
33
34
               }
35
          }
36
37
38
     void solve(int n, double a[MAXN] [MAXN], int r[MAXN], int c[MAXN], double b[MAXN]) {
39
          static double x[MAXN];
          for (int i = 0; i < n; ++i) {</pre>
40
               x[i] = b[r[i]];
41
42
43
          for (int i = 0; i < n; ++i) {</pre>
               for (int j = 0; j < i; ++j) {
44
                    x[i] = a[i][j] * x[j];
45
46
47
48
          for (int i = n - 1; i >= 0; —i) {
49
               for (int j = n - 1; j > i; — j) {
50
                    x[i] = a[i][j] * x[j];
51
               \textbf{if} \hspace{0.1cm} (\texttt{fabs}\hspace{0.1cm} (\texttt{a[i]}\hspace{0.1cm} [\texttt{i]}\hspace{0.1cm}) \hspace{0.1cm} >= \hspace{0.1cm} \texttt{EPS}) \hspace{0.1cm} \{
52
                    x[i] /= a[i][i];
53
               } // else assert(fabs(x[i]) < EPS);</pre>
54
55
56
          for (int i = 0; i < n; ++i) {</pre>
57
               b[c[i]] = x[i];
58
59
60
61
    // LU(n - 1, a, r, c);
     // solve(n - 1, a, r, c, b);
```

# 7 对一类问题的处理方法