A report on the exam project number 14 - cubic sub-spline for data with derivitives

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This report details the implimentation of a cubic sub-spline for data with derivitives. It will explain how the sub-spline is made and the conditions and coefficients used to make it. The results of this will then be demonstrated using a simple cos(x) function as then the cubic sub-spline can be easily compared to this. After the derivatie and integral of the sub-spline was found and also plotted.

1 Introduction to sub-splines

Sub-splines are peicewise polynomials but unlike splines do not demand maximal differentiability of the spline. They can therefore do things such as minimize the wiggles and lumps that can be present with other splines. An exampkle of a sub-spline is the Akima sub-spline. For this project I have built a cubic sub-spline using the data x_i, y_i, y'_i where y'_i is the derivitive of y_i at point x_i. The sub-spline formula is given in equation (1).

$$S_i(x) = y_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3 \tag{1}$$

2 The coefficients b_i, c_i, d_i

As the cubic subspline is in the form of equation (1), the coefficients are determined by three conditions given in equations (2), (3), (4).

$$S_i(x_i + 1) = y_{i+1} \ . \tag{2}$$

$$S'_i(x_i) = y'_i \ . \tag{3}$$

$$S'_i(x_{i+1}) = y'_{i+1} \ . \tag{4}$$

Solving these equations gives the following for each of the coefficients,

$$b_i = y'_i \ . \tag{5}$$

$$c_i = \frac{3(p_i - b_i)}{h_i} - dp_i \ . \tag{6}$$

$$d_i = \frac{dp_i - 2c_i}{3h_i} \ . \tag{7}$$

where $h_i = x_{i+1} - x_i$, $p_i = (y_{i+1} - y_i)/h_i$ and $dp_i = (y'_{i+1} - y'_i)/h_i$. These can then be used to build a cubic sub-spline.

3 Testing the cubic sub-spline

Once the coefficients had been calculated and the cubic sub-spline had been made, it had to be tested to make sure it works. To do this a simple function of cos(x) was chosen as its derivitive is -sin(x). This would be an easy example to show if the sub-splione worked or not as it would be obvious upon viewing. For comparison the Akima sub-spline was also plotted as this would give me a sense of how well my sub-spline compared to one from D.V. Fedorov (2021) [1]. This example can be seen in figure (3).

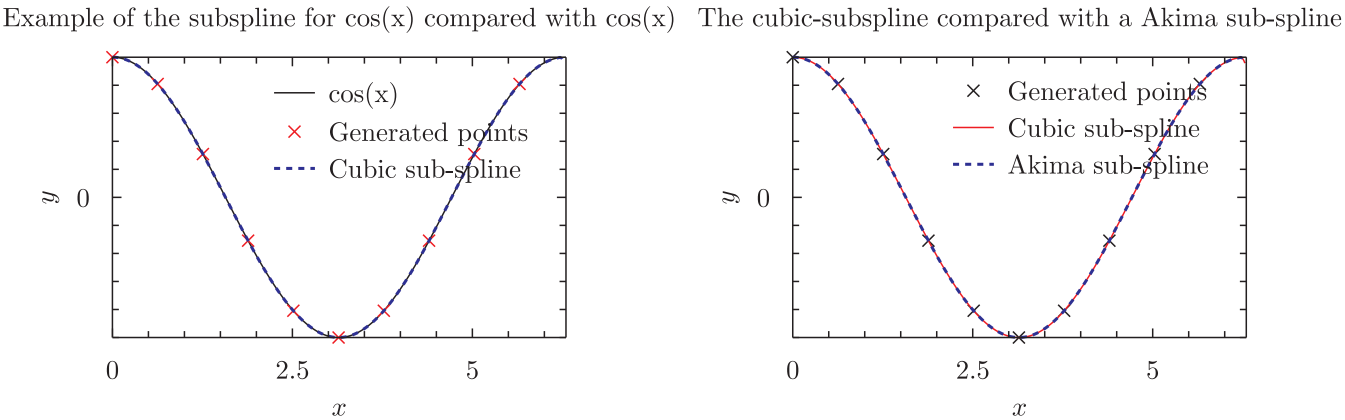

Figure 1: A plot of a cos function and its derivitve. There is two sub-splines for the cos(x) points, a cubic sub-spline and Akima sub-spline.

As can be seen here both my cubic sub-spline and the Akima sub-spline fit the simple cos(x) points well and do a good job as splines without any wiggles.

4 derivitive and integral

Once this was done I found the derivitive and integral of the spline.THis was done using similar methods to the interpolation homework. Binsearch was used to get an interval and then either the differentiated equation (8) or intergrated equation (9) was returned. This method worked and produced figure (4). As you would expext for a sub-spline that should be showing a cos(x) line the derivitive and integral are both sin(x) functions and show that the derivitive and integral of the spline work.

$$derivitive = b_i + 2c_i(x - x_i) + 3d_i(x - x_i)^2 \ . \tag{8}$$

$$integral = y_i(x - x_i) + \frac{b_i(x - x_i)^2}{2} + \frac{c_i(x - x_i)^3}{3} + \frac{d_i(x - x_i)^4}{4} \ . \tag{9}$$

References

[1] Fedorov, D.V. (2021), "Introduction to numerical methods",

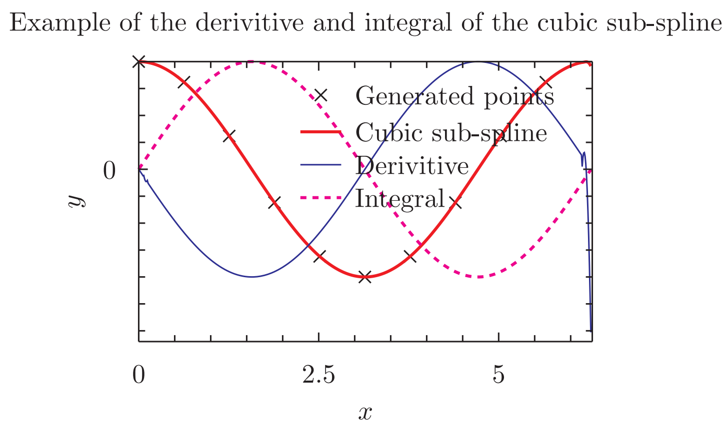

Figure 2: A plot of the cubic sub-spline, its derivitive and its integral.