Bayesian Econometrics for Business & Economics: Assignment 3

By Group 15

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26th November 2025

Posterior model probabilities & Importance Sampling

a. Histogram of dataset

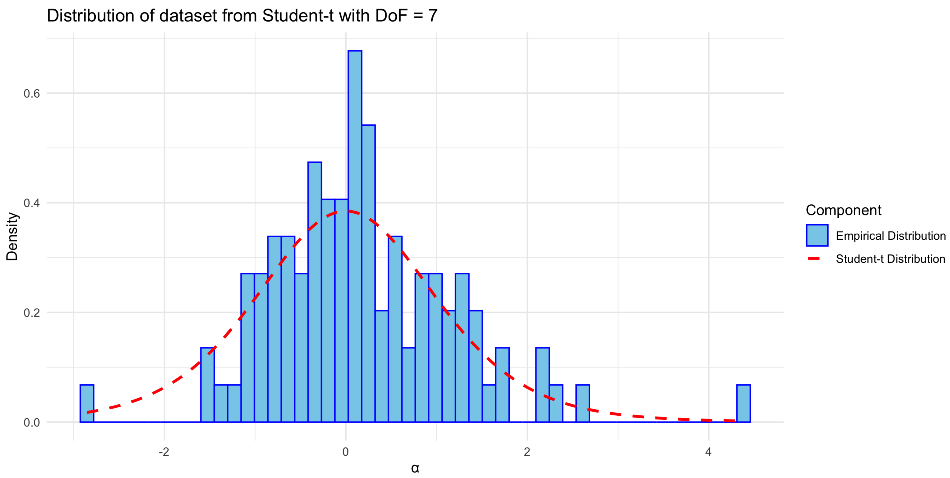

Figure 1.1.1 Distribution of dataset

b. Importance sampling with a uniform prior for DoF on [4.1, 50]

Parameter	**Student-t**	**GED**
Prior Distribution	$DoF \sim Unif(4.1, 50)$	$\beta \sim Unif(0.38, 1.88)$
Prior Model Probability	$\frac{1}{2}$	$\frac{1}{2}$
Marginal Likelihood	3.76×10^{-62}	2.68×10^{-62}
Posterior Model Probability	0.58362446	0.41637554
Model Comparison (Student-t vs GED)		
Prior Odds Ratio	1	
Bayes Factor	1.40167806	
Posterior Odds Ratio	1.40167806	

Table 1.2.1: Bayesian model comparison for Student-t distribution and GED

c. Importance sampling with a uniform prior for DoF on [4.1, 1000]

Parameter	**Student-t**	**GED**
Prior Distribution	$DoF \sim Unif(4.1, 1000)$	$\beta \sim Unif(0.38, \approx 1.99)$
Prior Model Probability	$\frac{1}{2}$	$\frac{1}{2}$
Marginal Likelihood	7.71×10^{-63}	2.58×10^{-62}
Posterior Model Probability	0.23024883	0.76975117
Model Comparison (Student-t vs GED)		
Prior Odds Ratio	1	
Bayes Factor	0.29912112	
Posterior Odds Ratio	0.29912112	

Table 1.3.1: Bayesian model comparison for Student-t distribution and GED

Table 1.2.1 and 1.3.1 shows the comparison between different choices of prior distribution. When the Student-t degrees of freedom are restricted to the interval $[4.1, 50]$, the prior mass lies on the distribution gradually and the data fits the distribution well. As a result, the Student-t model obtains a slightly higher marginal likelihood than the GED model and receives a posterior probability of about 58%. However, when the Student-t degrees of freedom are expanded to the interval $[4.1, 1000]$, the Student-t model becomes far more diffuse. The prior mass are placed on very large DoF values that correspond to approximately Gaussian tails but the data seems do not support. Consequently, it decreases the marginal likelihood substantially, while the GED model remains largely unaffected. As a result, the posterior probability shifts strongly in favor of the GED model (about 77%). It illustrates that a spread prior that include poorly fitting parameter regions may reduce the marginal likelihood and the posterior of the corresponding model.

d. With a different prior odds ratio

Parameter	Student-t	GED
Prior Distribution	$DoF \sim Unif(4.1, 50)$	$\beta \sim Unif(0.38, 1.88)$
Prior Model Probability	$\frac{1}{10}$	$\frac{9}{10}$
Marginal Likelihood	3.76×10^{-62}	2.68×10^{-62}
Posterior Model Probability	0.13475499	0.86524501
Model Comparison (Student-t vs GED)		
Prior Odds Ratio	0.9	
Bayes Factor	1.40167806	
Posterior Odds Ratio	0.15574201	

Table 1.4.1: Bayesian model comparison for Student-t distribution and GED

Parameter	Notation	Equal Priors	Unequal Priors
Prior Model Probability	Pr(Student-t)	1/2	1/10
Prior Model Probability	Pr(GED)	1/2	9/10
Prior Odds Ratio	$\frac{\Pr(\text{Student}-t)}{\Pr(\text{GED})}$	1	$\frac{1}{9}$
Marginal Likelihood	$p(y \mid \text{Student-}t)$	3.76×10^{-62}	3.76×10^{-62}
Marginal Likelihood	$p(y \mid \text{GED})$	2.68×10^{-62}	2.68×10^{-62}
Bayes Factor	$\frac{p(y \mid \text{Student}-t)}{p(y \mid \text{GED})}$	1.40167806	1.40167806
Posterior Odds Ratio	$\frac{\Pr(\text{Student}-t)}{\Pr(\text{GED})} \times \frac{p(y \mid \text{Student}-t)}{p(y \mid \text{GED})}$	1.40167806	0.15574201
Posterior Model Probability	$\Pr(\text{Student-}t \mid y)$	0.58362446	0.13475499
Posterior Model Probability	$\Pr(\text{GED} \mid y)$	0.41637554	0.86524501

Table 1.4.2: Comparison of Bayesian model results under two prior specifications

Table 1.4.1 illustrate how posterior model probabilities varies with different prior model probabilities. Table 1.4.2 reveals the comparison in detail. Compare to the sampling method in part (b), both sampling methods use identical parameter priors, result in the same marginal likelihoods and the Bayes factor in both cases. However, when equal prior model probabilities are used, the posterior probability of the Student-t model is about 0.58, which is higher preference over the GED model. When the prior probability of the GED model is increased to 9/10, the posterior probability of the Student-t model falls sharply to about 0.14. This demonstrates the Bayesian principle that posterior model support is the product of prior beliefs and evidence. The choice of prior model weights can significantly change the posterior conclusions even when the data-driven Bayes factor remains unchanged.