VU Minor Applied Econometrics

Bayesian Econometrics for Business & Economics (Bayesian statistics & simulation methods)

Period 2, 2025-2026 (E_MFAE_BEBE)

Lennart Hoogerheide

Vrije Universiteit Amsterdam

E-mail: l.f.hoogerheide@vu.nl

Assignments (for $(30 - \epsilon)\%$ of the grade)

There are two alternatives:

(I) Five assignments that involve programming the simulation methods (e.g. in Python/Matlab/R).

(II) Write an essay to summarize (and give feedback on – what could be done better/more?) five Bayesian articles.

About the assignments:

- The assignments should be made with 'groups' of 1/2/3 students.
- A .pdf file with answers must be submitted.
  For alternative (I) also the code must be submitted.
  (For example, the Python/Matlab/R files).
- Deadline for assignment 1: **Wednesday November 12, 23:59**.

Alternative (II):

Assignment 1: summarize (and give feedback on – what could be done better/more?) the following article:

George Casella, Edward I. George (1992).
Explaining the Gibbs Sampler.
The American Statistician 46 (3), 167-174.

Alternative (I): Assignment 1: Model with normal distribution and Gibbs sampling

A company that has multiple shops in different cities wants to analyze whether shop A and shop B have a different expected profit.
The company has computed the difference y_j (= profit of shop A - profit of shop B, in 10000 euros) for 10 periods:

1.6088
-1.4579
0.4502
1.0701
-0.3803
1.1201
0.4199
0.3998
-0.1904
0.3932

In this exercise we assume that $y_j \sim N(\mu, \frac{1}{h})$, i.i.d. where mean μ and precision h are unknown parameters.

(a) First we assume that we specify the (improper) non-informative prior

$$p(\mu, h) \propto \begin{cases} \frac{1}{h} & \text{if } h > 0 \\ 0 & \text{else} \end{cases}$$

Simulate from the posterior distribution of μ and h by applying the **Gibbs sampling method (from slide 52 of lecture 2)** with 11000 draws (with a *burn-in* of 1000 draws) and initial value $\mu = \bar{y}$.

Please first read the hints/warnings on slides 6-7 of this assignment!

Inspect the *trace plot* of the draws of μ (i.e., the graph with μ_i on the vertical axis and $i = 1, 2, \ldots, 11000$ on the horizontal axis): does the Gibbs sampling method keep moving through the parameter space or does it 'get stuck' in a certain point?

Use the resulting 10000 draws to estimate the posterior probabilities $\Pr(\mu < 0|y)$ and $\Pr(\mu > 0|y)$. Interpret the results.

Hints/warnings:

(1) There exist two versions of the Gamma distribution with parameters:

- *shape* a and *scale* b (or *shape* k and *scale* θ), used during this course.
- *shape* a and *rate* $1/b$ (or *shape* α and *rate* β)

For example, see also
`https://en.wikipedia.org/wiki/Gamma_distribution`.
Check which version of the Gamma distribution your computer package uses. If necessary, use arguments a and $\frac{1}{b}$.

(2) On the lecture slides the second parameter of the normal distribution $N(\mu, \sigma^2)$ is the **variance** σ^2, whereas computer packages often require the **standard deviation** $\sigma = \sqrt{\sigma^2}$.

Hint: So, the pseudocode is given by:

- Compute $\mu_0 = \bar{y}$, the sample mean of the $n = 10$ observations.
- Set $n_{draws} = 11000$ (and please notice the difference between $n_{draws} = 11000$, which is the number of iterations of the for-loop that we can choose ourselves, and $n = 10$, the number of available observations, where $n = 10$ appears in the conditional posterior distributions)
- Do for draw $i = 1, \ldots, n_{draws}$:
  - Simulate h_i from Gamma($a = n/2, b = (\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu_{i-1})^2)^{-1}$) distribution.
  - Simulate μ_i from normal distribution with mean $\bar{y}$ and standard deviation $\frac{1}{\sqrt{h_i n}}$.
- Estimate $\Pr(\mu < 0|y)$ as the fraction of the 10000 draws μ_i ($i = 1001, 1002, \ldots, 11000$) that are negative, and $\Pr(\mu > 0|y)$ as the fraction of the 10000 draws μ_i ($i = 1001, 1002, \ldots, 11000$) that are positive.

(b) Now suppose we have prior

$$p(\mu, h) = p(\mu) \times p(h)$$

with a normal prior density for μ:

$$\mu \sim N(m_{prior}, v_{prior})$$

with $m_{prior} = 0, v_{prior} = 100^2$. That is, a prior with mean 0 and large variance.
And the same prior $p(h)$ for h:

$$p(h) \propto \begin{cases} \frac{1}{h} & \text{if } h > 0 \\ 0 & \text{else} \end{cases}$$

Simulate from the posterior distribution of μ and h by applying the **Gibbs sampling method (from slide 54 of lecture 2)** with 11000 draws (with a *burn-in* of 1000 draws) and initial value $\mu = \bar{y}$. See the hint on slide 10 of this assignment.

Use the resulting 10000 draws to estimate $\Pr(\mu < 0|y)$ and $\Pr(\mu > 0|y)$. Compare the results with part (a).

Hint: So, the pseudocode is given by:

- Compute $\mu_0 = \bar{y}$, the sample mean of the $n = 10$ observations.
- Set $n_{draws} = 11000$, $m_{prior} = 0$, $v_{prior} = 100^2 = 10000$
- Do for draw $i = 1, \ldots, n_{draws}$:
    - Simulate h_i from Gamma($a = n/2, b = (\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu_{i-1})^2)^{-1}$) distribution.
    - Simulate μ_i from normal distribution with mean $\frac{\frac{m_{prior}}{v_{prior}} + h_i n \bar{y}}{\frac{1}{v_{prior}} + h_i n}$ and standard deviation $\frac{1}{\sqrt{\frac{1}{v_{prior}} + h_i n}}$.
- Estimate $\Pr(\mu < 0|y)$ as the fraction of the 10000 draws μ_i $(i = 1001, 1002, \ldots, 11000)$ that are negative, and $\Pr(\mu > 0|y)$ as the fraction of the 10000 draws μ_i $(i = 1001, 1002, \ldots, 11000)$ that are positive.

(c) Now suppose we have prior

$$p(\mu, h) = p(\mu) \times p(h)$$

with a normal prior density for μ:

$$\mu \sim N(m_{prior}, v_{prior})$$

with $m_{prior} = 0.5, v_{prior} = 0.25^2$. That is, a prior with mean 0.5 and small variance.
And the same prior $p(h)$ for h:

$$p(h) \propto \begin{cases} \frac{1}{h} & \text{if } h > 0 \\ 0 & \text{else} \end{cases}$$

Simulate from the posterior distribution of μ and h by applying the **Gibbs sampling method (from slide 54 of lecture 2)** with 11000 draws (with a *burn-in* of 1000 draws) and initial value $\mu = \bar{y}$.

Use the resulting 10000 draws to estimate $\Pr(\mu < 0|y)$ and $\Pr(\mu > 0|y)$. Compare the results with part (a) and (b).

(d) Perform a classical/frequentist two-sided test of

$H_0 : \mu = 0$ versus
$H_1 : \mu \neq 0$

at 5% significance, comparing the t-statistic $\frac{\bar{y}}{\sqrt{s^2/n}}$ with the 2.5% and 97.5% percentiles of the t_9 distribution (for which you can simply use the values -2.2622 and 2.2622). What is the conclusion?

Compare the conclusion with parts (a), (b) and (c).