Bayesian Econometrics for Business & Economics: Assignment 4

By Group 15

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Forecasting US real GDP growth using Autoregressive (AR) models

a. US quarterly real GDP growth

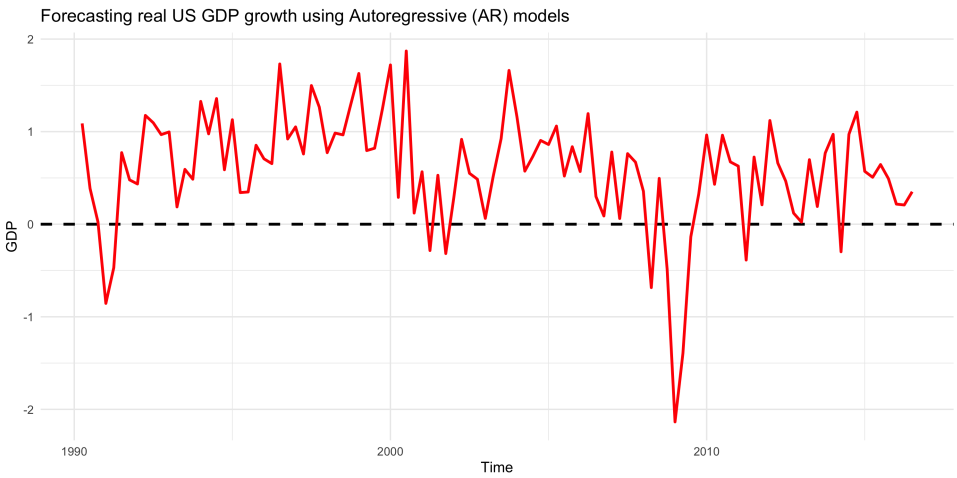

Figure 1.1.1 Distribution of dataset

b. Frequentist/Classical prediction for y_{T+1} and 95% prediction interval for AR(2) model using Ordinary Least Squares (OLS)

Approach	y_{T+1}	**95% Interval**
Frequentist/Classical	0.4361	[-0.6484, 1.5205]

Table 1.2.1 The prediction of y_{T+1} and 95% Interval using Frequentist/Classical approach

c. Bayesian prediction for y_{T+1} and 95% prediction interval for AR(2) model using Bayesian approach

Approach	y_{T+1}	95% Interval
Frequentist/Classical	0.4361	[-0.6484, 1.5205]
Bayesian (Non-Informative Prior)	0.4400	[-0.6773, 1.5747]

Table 1.3.1 The prediction of y_{T+1} and 95% Interval

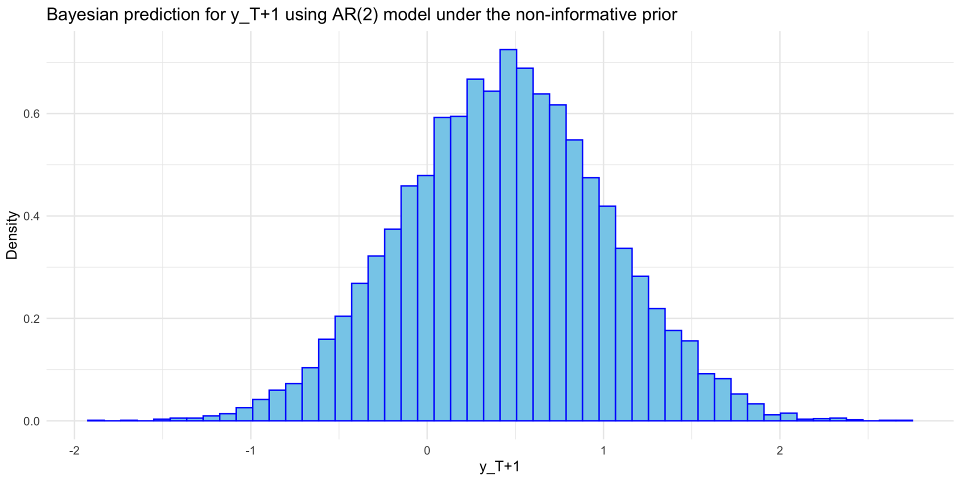

Figure 1.3.1 Distribution of simulation results for y_{T+1} using Bayesian approach (Non-Informative Prior)

Table 1.3.1 reveals the results of prediction of y_{T+1} using frequentist approach and Bayesian approach with non-informative prior. Figure 1.3.1 demonstrates the distribution of simulation result for the prediction. Using the non-informative prior $p(\beta, h) \propto 1/h$, the Bayesian prediction $\hat{y}_{T+1}$ is 0.4400 with a 95% prediction interval equal to $[-0.6773, 1.5747]$. These values are very close to the result using frequentist approach. Since the prior is flat, the posterior is dominated by the likelihood and results a Bayesian forecast almost identical to the classical one. The Bayesian interval is slightly wider since it takes into account uncertainty on parameters β and h.

d. Bayesian prediction for y_{T+1} and 95% prediction interval for Bayesian Model Averaging (BMA)

Parameter	**AR(1)**	**AR(2)**
Prior Model Probability	$\frac{1}{2}$	$\frac{1}{2}$
Restriction Notation	$p(y\|\beta_2 = 0, \text{AR(2) model})$	$p(y\|\text{AR(2) model})$
Marginal Density	0.8263351	0.5458401
Model Probability	0.3977919	0.6022081
Prior Odds Ratio	1	
Bayes Factor	0.6605554	
Savage-Dickey density ratio (SDDR)	0.6605554	

Table 1.4.1: Bayesian model comparison for AR(1) and AR(2)

Approach	y_{T+1}	**95% Interval**
Frequentist/Classical	0.4361	[-0.6484, 1.5205]
Bayesian (Non-Informative Prior)	0.4400	[-0.6773, 1.5747]
Bayesian Model Averaging (BMA)	0.4442	[-0.9666, 1.8762]

Table 1.4.2 The prediction of y_{T+1} and 95% Interval using different approaches

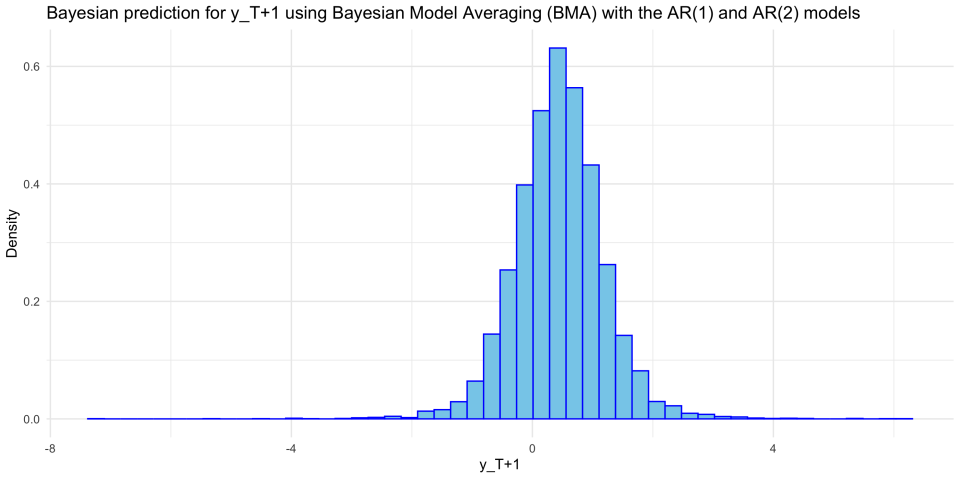

Figure 1.4.1 Distribution of simulation results for y_{T+1} using Bayesian Model Averaging (BMA)

Table 1.4.1 illustrates the prior model probabilities and corresponding posterior model probabilities for AR(1) and AR(2) models. Table 1.4.2 reveals the results of prediction of y_{T+1} using the previous two approaches and Bayesian Moving Average. Figure 1.4.1 demonstrates the combined distribution of simulation results from both AR(1) and AR(2) models for the prediction. Using Bayesian Moving Average (BMA), the Bayes factor is equal to the Savage-Dickey density ratio (SDDR), which is 0.6606 in favour of the AR(1) restriction $\beta_2 = 0$. It reflects that unrestricted AR(2) is preferred over the restricted AR(1). With equal prior model probabilities, the posterior model probabilities are computed as $\Pr(\text{AR}(1) \mid y) = 0.3978$ and $\Pr(\text{AR}(2) \mid y) = 0.6022$ respectively. The BMA prediction for y_{T+1} is 0.4442 and the 95% prediction interval is $[-0.9666, 1.8762]$. Consequently, the BMA prediction is closer to the prediction using Bayesian approach with non-informative prior than the prediction using the frequentist approach. Moreover, the 95% prediction interval is wider than in part (c), since BMA incorporates with more parameter uncertainty and model uncertainty from both AR(1) and AR(2) models.