# Bayesian Econometrics for Business & Economics: Assignment 1


**By Group 15**

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# Model with normal distribution and Gibbs sampling

## a. With a non-informative prior (flat prior) for $\mu$

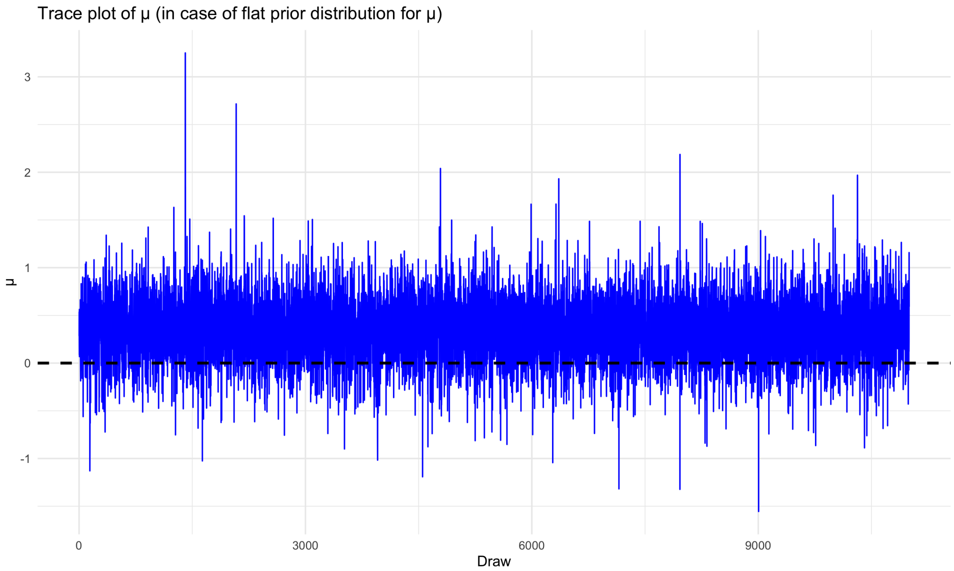


Figure 1.1 Trace plot of $\mu$ (in case of flat prior distribution for $\mu$)

Figure 1.1 reveals the trace plot of the simulation of $\mu$ using the *Gibbs sampling method* with a **Non-informative prior (flat prior)**. The posterior probabilities are $\Pr(\mu > 0|y) = 0.8796$ and $\Pr(\mu < 0|y) = 0.1205$. The two-sided $p$-value is 0.241. This indicates strong posterior evidence that the true mean is positive even without any prior information. The result suggests that the data alone provide substantial support for $\mu > 0$.

## b. With a normal prior density for $\mu$: $m_{prior} = 0$, $v_{prior} = 10000$

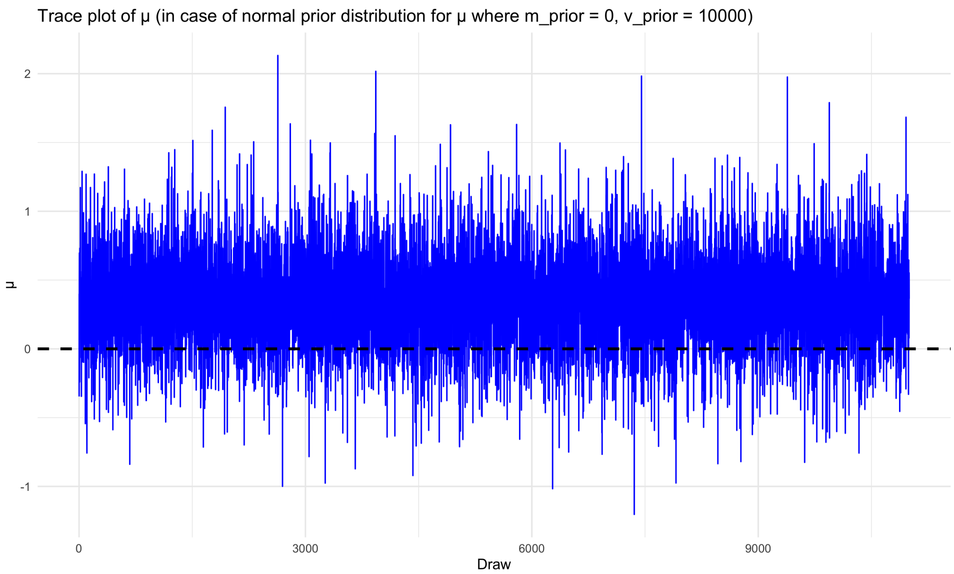


Figure 1.2 Trace plot of $\mu$ (in case of normal prior distribution for $\mu$ where $m_{prior} = 0$, $v_{prior} = 10000$)

Figure 1.2 reveals the trace plot of the simulation of $\mu$ using the *Gibbs sampling method* with a **Normal prior density for $\mu$:** $\mu \sim N(0, 100^2)$. The posterior probabilities are $\Pr(\mu > 0|y) = 0.8812$ and $\Pr(\mu < 0|y) = 0.1189$. The two-sided $p$-value is 0.2378. The large variance of prior reflects almost no prior belief, hence the posterior is still dominated by the data. The Bayesian posterior probabilities and two-sided $p$-value are almost identical to part (a). It suggests that it may be a proper prior but not useful at all.

## c. With a normal prior density for $\mu$: $m_{prior} = 0.5$, $v_{prior} = 0.0625$

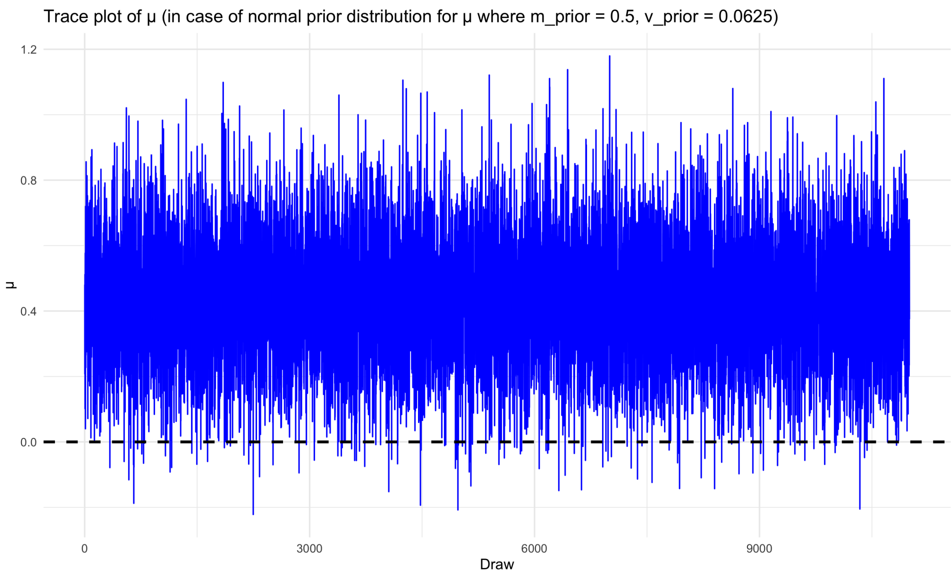


Figure 1.3 Trace plot of $\mu$ (in case of normal prior distribution for $\mu$ where $m_{prior} = 0.5$, $v_{prior} = 0.0625$)

Figure 1.3 reveals the trace plot of the simulation of $\mu$ using the *Gibbs sampling method* with a **Normal prior density for $\mu$:** $\mu \sim N(0.5, 0.25^2)$. The posterior probabilities are: $\Pr(\mu > 0|y) = 0.9894$ and $\Pr(\mu < 0|y) = 0.0107$. The two-sided $p$-value is 0.0214. The result gives a strong evidence that with an informative prior, the posterior is completely concentrated above zero. With the two-sided $p$-value approximates to 0, both the prior assumption and the sample data implies a positive mean that $\mu > 0$ .

## d. A classical/frequentist two-sided test

| $t$-Statistic | Degree of Freedom | Acceptance Region (5%) | $p$-Value | Decision |
|---|---|---|---|---|
| 1.2483 | 9 | [-2.2622, 2.2622] | 0.2434 | Do not reject $H_0$ |

Table 1.4.1: The classical/frequentist two-sided $t$-test

Table 1.4.1 reveals the result of the classical/frequentist two-sided test. Since the $t$-test statistic lies within the acceptance region, also with a $p$-value greater than 0.05, thus do not reject the null $H_0$ that $\mu = 0$.

In parts (a) to (c), the posterior probabilities $\Pr(\mu > 0 \mid y)$ and $\Pr(\mu < 0 \mid y)$ are obtained from the Gibbs sampling results. To compute the two-sided $p$-value, it can be defined as:

$$p_{two.sided} = 2 \times \min \left\{ \Pr(\mu > 0 \mid y),\ \Pr(\mu < 0 \mid y) \right\}$$

It is directly comparable to a classical two-sided $p$-value. Table 1.4.2 compares the results with different approaches.

| Part | Approach | $p$-value (two-sided) | Decision ($\alpha = 5\%$) |
|---|---|---|---|
| (a) | (Improper) Non-informative prior | 0.241 | Not significant |
| (b) | $\mu \sim N(0, 10000)$ | 0.2378 | Not significant |
| (c) | $\mu \sim N(0.5, 0.0625)$ | 0.0214 | Significant |
| (d) | classical/frequentist | 0.2434 | Not significant |

Table 1.4.2: Comparison of results Bayesian posterior approach with different prior distribution and the classical/ frequentist approach

In contrast, with the Bayesian results, all $p$-values indicate strong evidence to reject the null hypothesis. The difference conclusions may because the Bayesian approach directly measures the probability that $\mu$ is positive given the observed data, while the frequentist test evaluates how extreme the sample mean would be if $\mu = 0$. With a small sample, Bayesian updating can lead to sharper conclusions than the $t$-test.