Bayesian Econometrics for Business & Economics: Assignment 1

By Group 15

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Model with normal distribution and Gibbs sampling

a. With a non-informative prior (flat prior) for μ

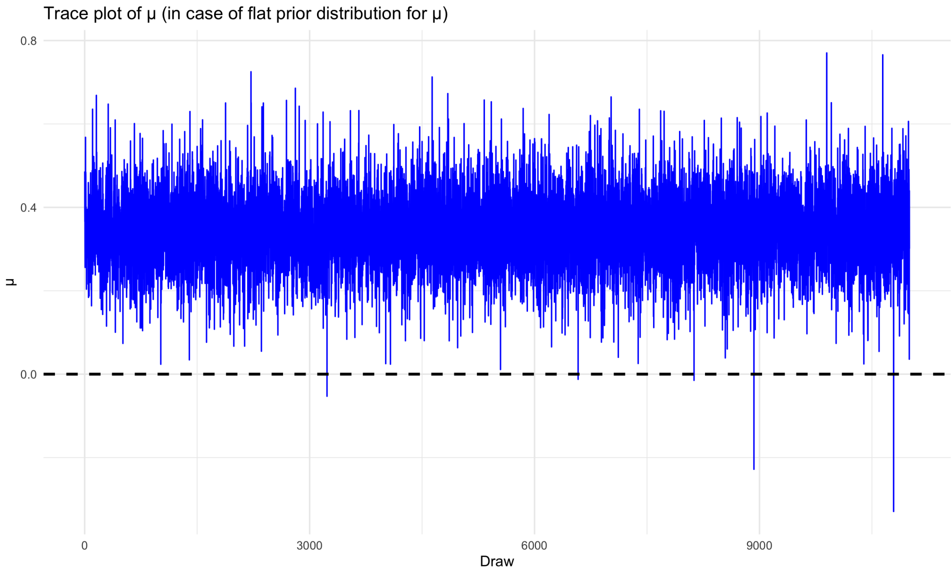

Figure 1.1 Trace plot of μ (in case of flat prior distribution for μ)

Figure 1.1 reveals the trace plot of the simulation of μ using the *Gibbs sampling method* with a **Non-informative prior (flat prior)**. The posterior probabilities are $\Pr(\mu > 0|y) = 0.9993$ and $\Pr(\mu < 0|y) = 0.0008$. The two-sided p-value is 0.0016. This indicates strong posterior evidence that the true mean is positive even without any prior information. The result suggests that the data alone provide substantial support for $\mu > 0$.

b. With a normal prior density for μ: $m_{prior} = 0$, $v_{prior} = 10000$

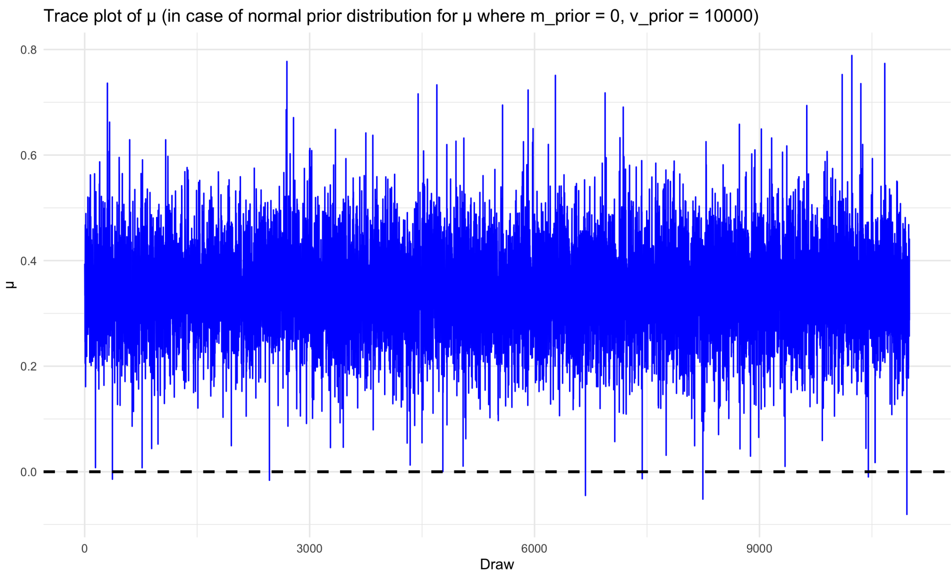

Figure 1.2 Trace plot of μ (in case of normal prior distribution for μ where $m_{prior} = 0$, $v_{prior} = 10000$)

Figure 1.2 reveals the trace plot of the simulation of μ using the *Gibbs sampling method* with a **Normal prior density for μ:** $\mu \sim N(0, 100^2)$. The posterior probabilities are $\Pr(\mu > 0|y) = 0.9994$ and $\Pr(\mu < 0|y) = 0.0007$. The two-sided p-value is 0.0014. The large variance of prior reflects almost no prior belief, hence the posterior is still dominated by the data. The Bayesian posterior probabilities and two-sided p-value are almost identical to part (a). It suggests that it may be a proper prior but not useful at all.

c. With a normal prior density for μ: $m_{prior} = 0.5$, $v_{prior} = 0.0625$

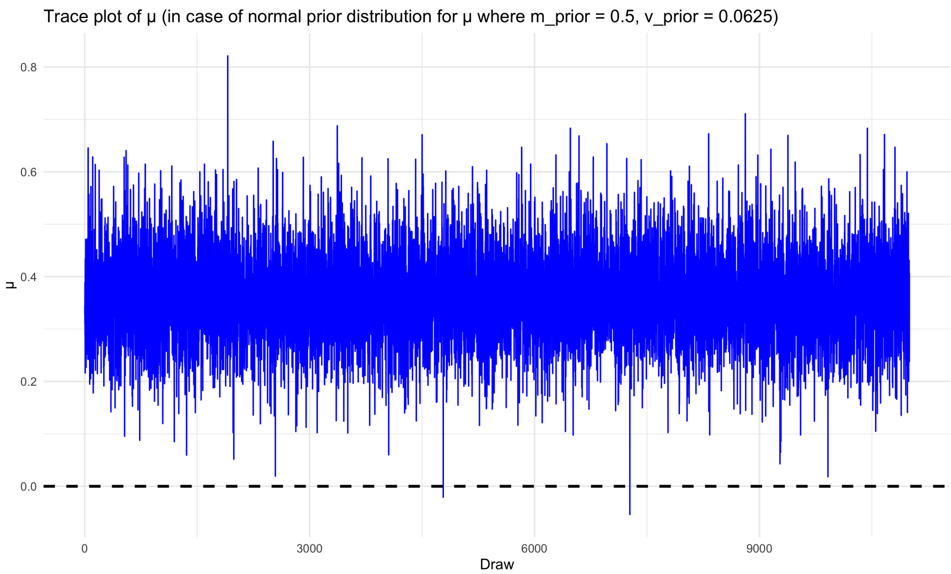

Figure 1.3 Trace plot of μ (in case of normal prior distribution for μ where $m_{prior} = 0.5$, $v_{prior} = 0.0625$)

Figure 1.3 reveals the trace plot of the simulation of μ using the *Gibbs sampling method* with a **Normal prior density for μ:** $\mu \sim N(0.5, 0.25^2)$. The posterior probabilities are: $\Pr(\mu > 0|y) = 1.00$ and $\Pr(\mu < 0|y) = 0.00$. The two-sided p-value is 0. The result gives a strong evidence that with an informative prior, the posterior is completely concentrated above zero. With the two-sided p-value approximates to 0, both the prior assumption and the sample data implies a positive mean that $\mu > 0$.

d. A classical/frequentist two-sided test

t-Statistic	Degree of Freedom	Acceptance Region (5%)	p-Value	Decision
1.2483	9	[-2.2622, 2.2622]	0.2434	Do not reject H_0

Table 1.4.1: The classical/frequentist two-sided t-test

Table 1.4.1 reveals the result of the classical/frequentist two-sided test. Since the t-test statistic lies within the acceptance region, also with a p-value greater than 0.05, thus do not reject the null H_0 that $\mu = 0$.

In parts (a) to (c), the posterior probabilities $\Pr(\mu > 0 \mid y)$ and $\Pr(\mu < 0 \mid y)$ are obtained from the Gibbs sampling results. To compute the two-sided p-value, it can be defined as:

$$p_{two.sided} = 2 \times \min\left\{ \Pr(\mu > 0 \mid y),\ \Pr(\mu < 0 \mid y) \right\}$$

It is directly comparable to a classical two-sided p-value. Table 1.4.2 compares the results with different approaches.

Part	Approach	p-value (two-sided)	Decision ($\alpha = 5\%$)
(a)	(Improper) Non-informative prior	0.0016	Significant
(b)	$\mu \sim N(0, 10000)$	0.0014	Significant
(c)	$\mu \sim N(0.5, 0.0625)$	0.0000	Significant
(d)	classical/frequentist	0.2434	Not significant

Table 1.4.2: Comparison of results Bayesian posterior approach with different prior distribution and the classical/ frequentist approach

In contrast, with the Bayesian results, all p-values indicate strong evidence to reject the null hypothesis. The difference conclusions may because the Bayesian approach directly measures the probability that μ is positive given the observed data, while the frequentist test evaluates how extreme the sample mean would be if $\mu = 0$. With a small sample, Bayesian updating can lead to sharper conclusions than the t-test.