Bayesian Econometrics for Business & Economics: Assignment 2

By Group 15

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Random walk Metropolis(-Hastings) method

a. Daily returns of the S&P 500

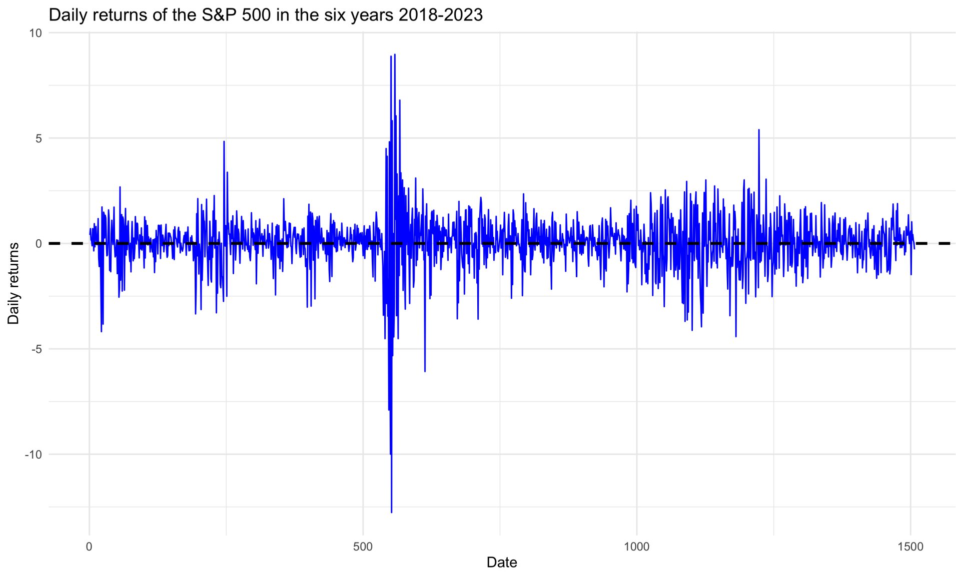


Figure 1.1.1 The daily returns of the S&P 500 in the six years 2018-2023

The dataset contains daily S&P 500 returns over the period 2018-2023. Figure 1.1.1 shows the time series plot of the returns y_t The series exhibits the characteristics of financial return data:

- returns fluctuate around zero,
- large variation occur during high-volatility periods like COVID-19,
- volatility cluster can be observed, indicates further analysis using ARCH or GARCH models.

b. Normal candidate distribution $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.0275^2)$

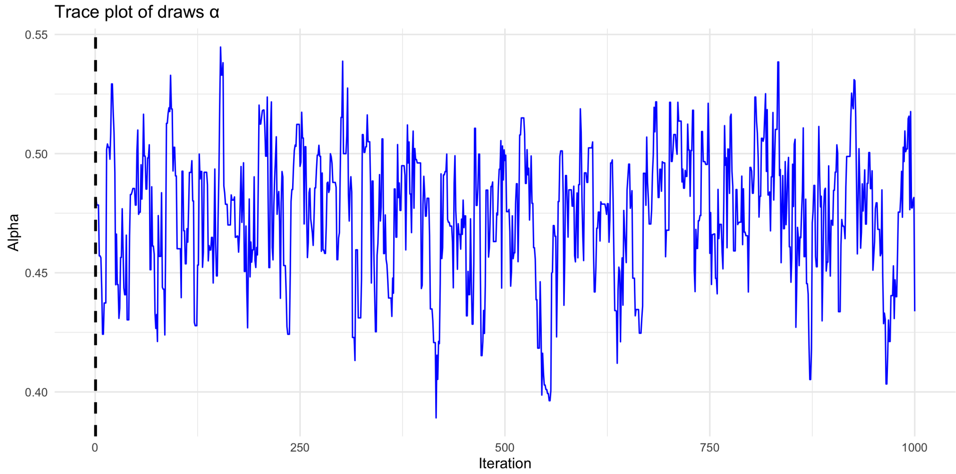


Figure 1.2.1 Trace plot of α using the $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.0275^2)$

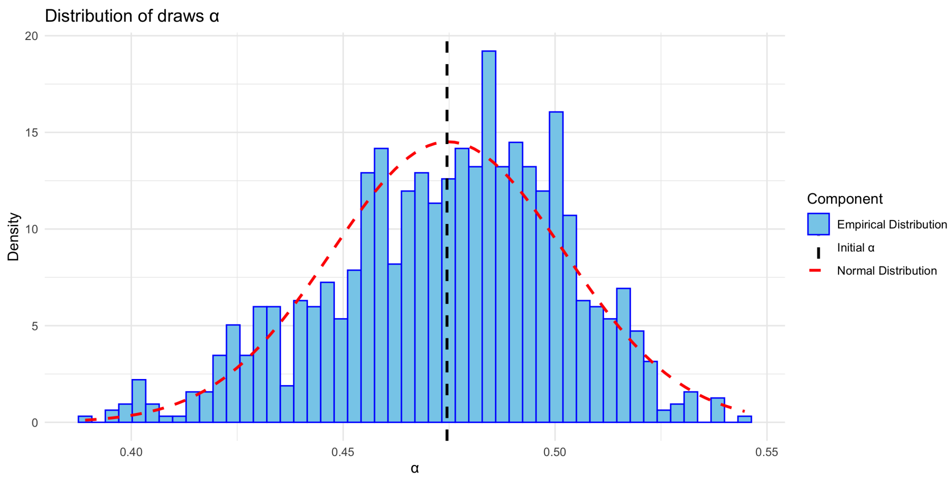


Figure 1.2.2 Distribution of α using the $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.0275^2)$

Parameter	Posterior Mean	Posterior Stdev	Serial Correlation
α	0.4716168	0.03119988	0.8142619

Acceptance percentage: 72.9%

Table 1.2.1: Posterior summary for α with candidate distribution $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.0275^2)$

We estimate the ARCH(1) model with a uniform prior on [0, 1).The random walk Metropolis uses $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.0275^2)$. Using 1100 draws and a burn-in of 100, we obtain the posterior summary of α. The observations as below:

- The posterior of mean of α is approximately insert value.
- The posterior standard deviation is approximately insert value. The acceptance rate is around insert value.
- The first-order autocorrelation of the post burn-in draws is insert value. indicating moderate persistence in the chain.

c. Normal candidate distribution $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.002^2)$

Figure 1.3.1 Trace plot of α using the $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.002^2)$

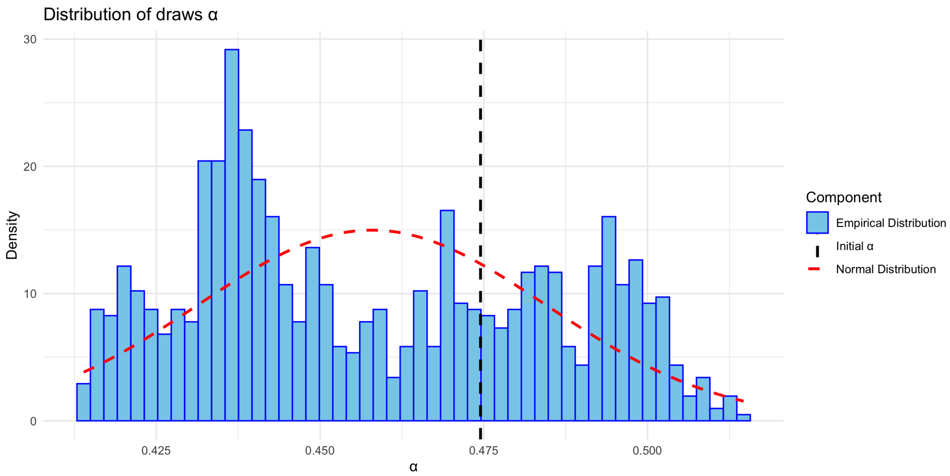


Figure 1.3.2 Distribution of α using the $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.002^2)$

Parameter	Posterior Mean	Posterior Stdev	Serial Correlation
α	0.4643195	0.01205968	0.9852297

Acceptance percentage: 98.8%

Table 1.3.1: Posterior summary for α with candidate distribution $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.002^2)$

Using a candidate distribution with standard deviation 0.002 results in extremely small proposal steps. The results as below:

- The acceptance rate becomes very high.
- The posterior mean and SD are similar to (b), yet the distribution is far from approximately normal distribution.

This indicates that too small steps leads to high acceptance but poor performance.

d. Normal candidate distribution $\tilde{\alpha} \sim N(\alpha_{i-1}, 6^2)$

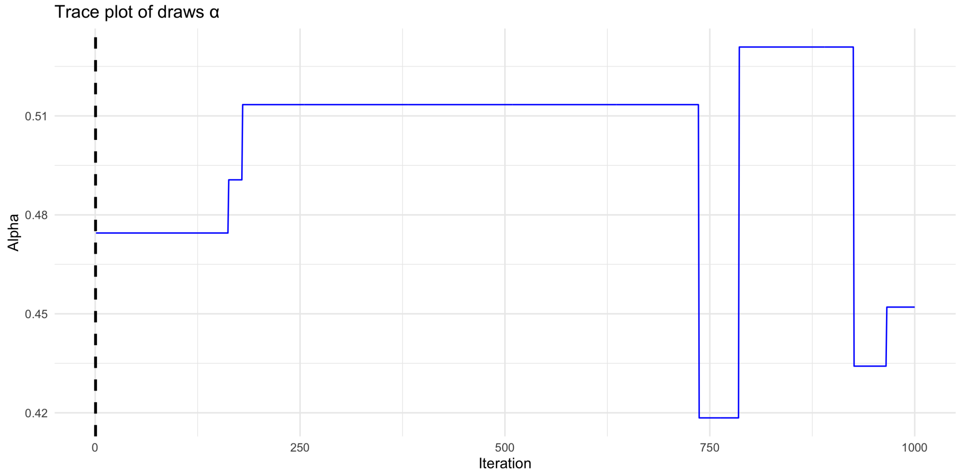


Figure 1.4.1 Trace plot of α using the $\tilde{\alpha} \sim N(\alpha_{i-1}, 6^2)$

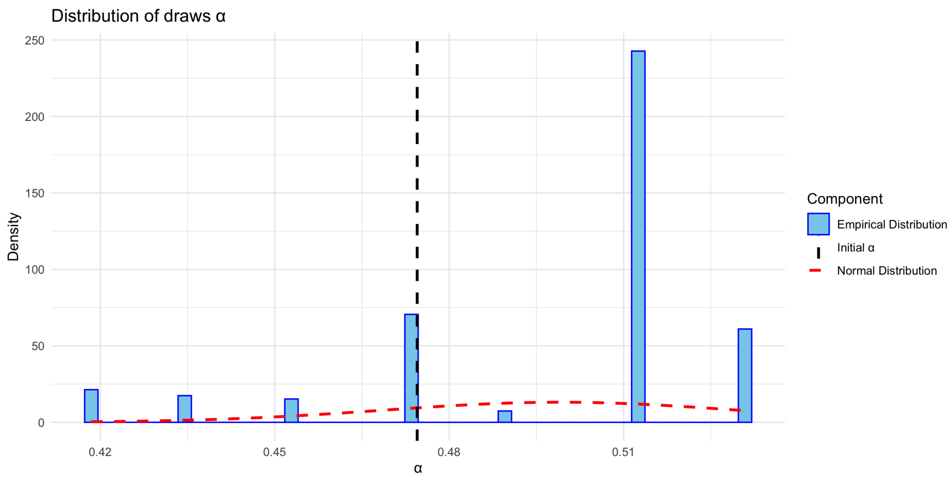


Figure 1.4.2 Distribution of α using the $\tilde{\alpha} \sim N(\alpha_{i-1}, 6^2)$

Parameter	**Posterior Mean**	**Posterior Stdev**	**Serial Correlation**
α	0.4690121	0.02038376	0.9832660

Acceptance percentage: 0.9%

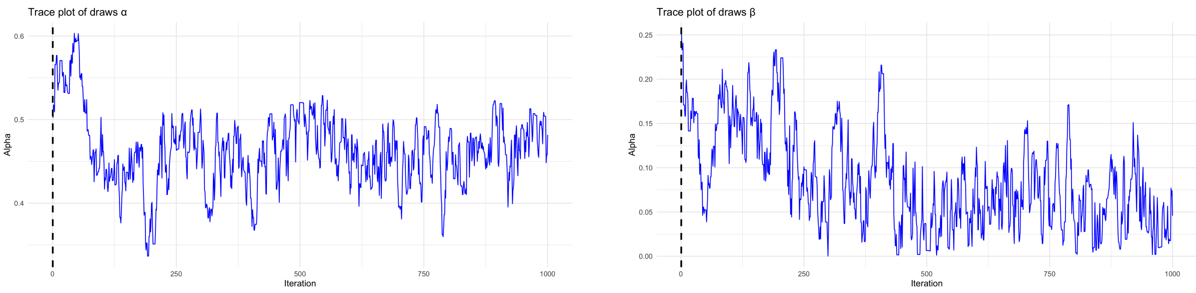
Table 1.4.1: Posterior summary for α with candidate distribution $\tilde{\alpha} \sim N(\alpha_{i-1}, 6^2)$

When we applied a extremely high standards deviation(6.0), the results as below:

- Acceptance rate is close to 0.
- Posterior mean and SD are meaningless because it failed to analysis any related parameters.
- The trace plot is almost flat.

This shows the opposite result as part (c), too large steps leads to no acceptance and no significant change of the chain.

e. Estimation of GARCH(1,1) model



(a) α (b) β

Figure 1.5.1: Trace plots of accepted draws of α and β

Parameter	Posterior Mean	Posterior Stdev	Serial Correlation
Accepted Draws			
α	0.45635789	0.04177004	0.9151522
β	0.07328584	0.05390924	0.9307200
Candidate Draws			
α	0.45735223	0.04577661	0.8525184
β	0.07228235	0.05997977	0.8784868

Acceptance percentage: 78.5%

Table 1.5.1: Posterior summary of accepted and candidate draws for α and β

We estimate the GARCH(1, 1) model:

$$\sigma_t^2 = s^2(1-\alpha-\beta) + \alpha\, y_{t-1}^2 + \beta\, \sigma_{t-1}^2.$$

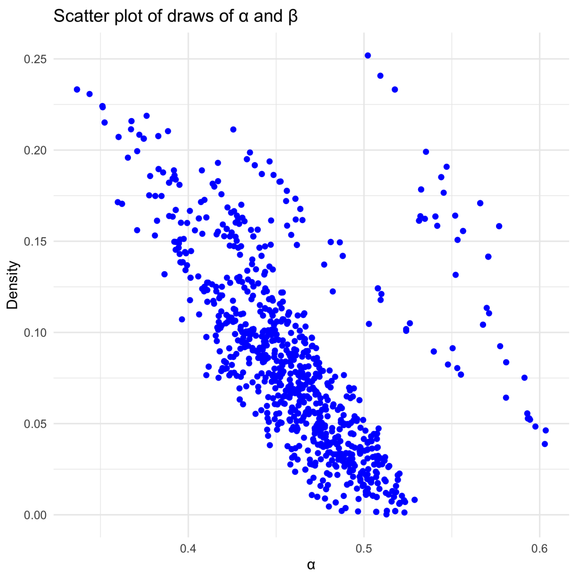
using a bivariate normal proposal with mean $\theta_{i-1} = (\alpha_{i-1}, \beta_{i-1})$, the covariance matrix is:

$$\Sigma = \begin{pmatrix} 0.000430 & -0.000510 \\ -0.000510 & 0.000626 \end{pmatrix}$$

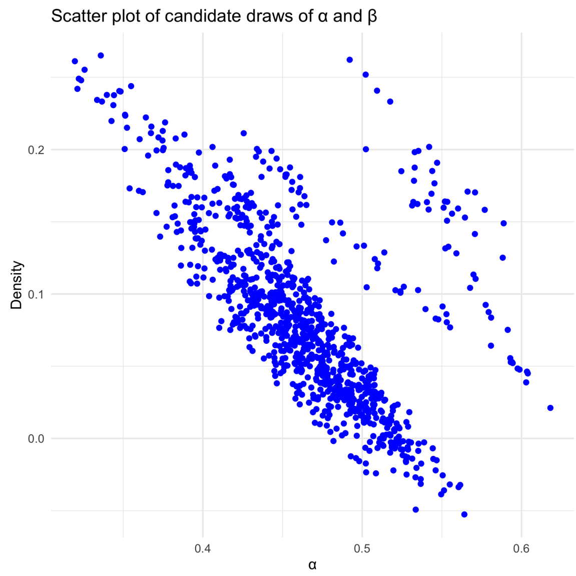
Posterior results as below:

- Posterior mean of α is insert value
- Posterior mean of β is insert value
- Posterior SDs is insert value
- Acceptance rate is insert value

From Figure, we notice that the trace plots show reasonable mixing for both α and β. The scatter plot of accepted draws shows the typical positive relationship between α and β, also indicates the correlation structure in the covariance matrix. Since it satisfies $\alpha + \beta < 1$, it is consistent with a covariance-stationary GARCH process.



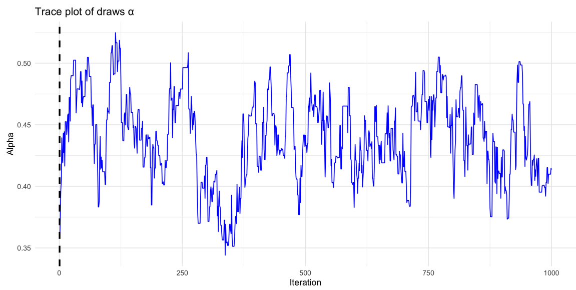
(a) Accepted



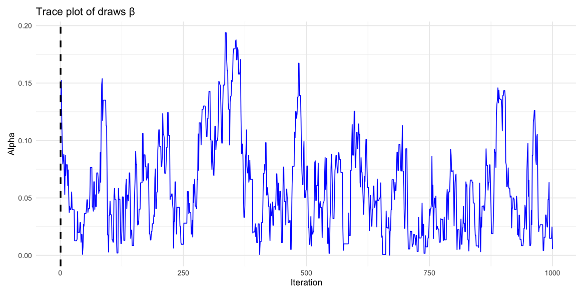
(b) Candidate

Figure 1.5.2: Scatter plots of accepted and candidate draws of α and β

f. Estimation of GARCH(1,1) model



(a) α



(b) β

Figure 1.6.1: Trace plots of accepted draws of α and β

We repeat part(e) using a diagonal covariance matrix here:

$$\Sigma = \begin{pmatrix} 0.000430 & 0 \\ 0 & 0.000626 \end{pmatrix}$$

The results as below:

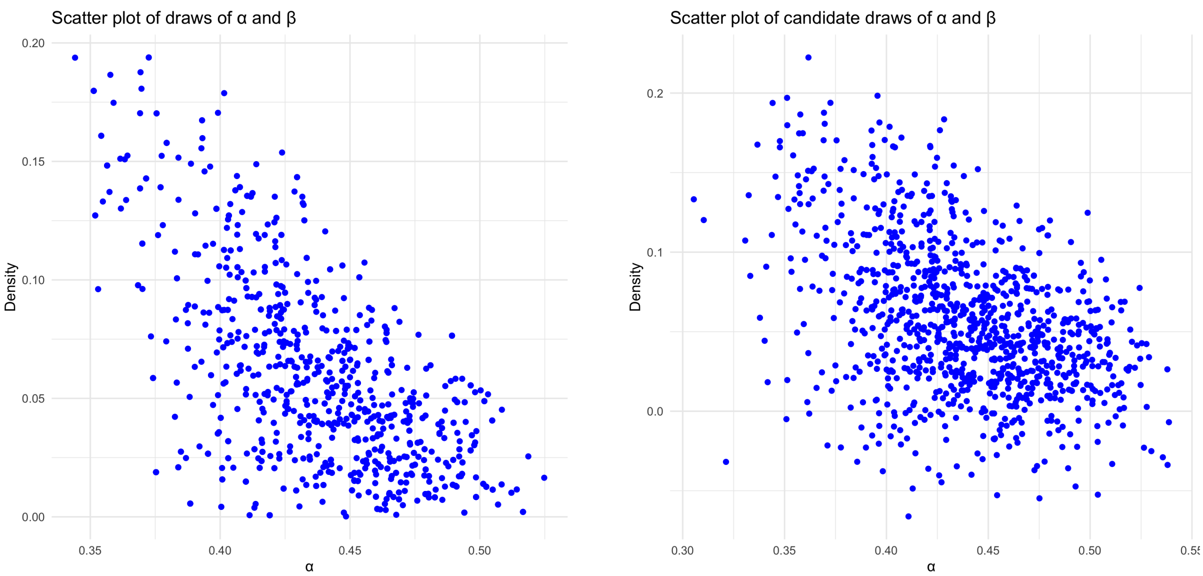
- Posterior means and SDs are similar to part (e).
- Acceptance rate differs.
- The autocorrelations of α and β are higher.

The scatter plot of α and β is also similar, indicates the posterior correlation between α and β. From the figure, it is clear that the diagonal proposal ignore the natural posterior correlation between α and β, leading to less efficient sampling.

Parameter	Posterior Mean	Posterior Stdev	Serial Correlation
Accepted Draws			
α	0.43276374	0.03340731	0.8983115
β	0.07055411	0.04465984	0.9058978
Candidate Draws			
α	0.43251695	0.03758393	0.7885335
β	0.07095002	0.05053973	0.8237490

Acceptance percentage: 65.9%

Table 1.6.1: Posterior summary of accepted and candidate draws for α and β



(a) Accepted

(b) Candidate

Figure 1.6.2: Scatter plots of accepted and candidate draws of α and β