VU Minor Applied Econometrics

Bayesian Econometrics for Business & Economics (Bayesian statistics & simulation methods)

Period 2, 2025-2026 (E_MFAE_BEBE)

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Assignments (for $(30 - \epsilon)\%$ of the grade)

There are two alternatives:

(I) Four assignments that involve programming the simulation methods (e.g. in Python/Matlab/R).

(II) Write an essay to summarize (and give feedback on – what could be done better/more?) four Bayesian articles.

About the assignments:

- The assignments should be made with 'groups' of 1/2/3 students.
- A .pdf file with answers must be submitted.
  For alternative (I) also the code must be submitted.
  (For example, the Python/Matlab/R files).
- Deadline for assignment 1: **Wednesday November 12, 23:59**.
- Deadline for assignment 2: **Wednesday November 19, 23:59**.

Alternative (II):

Assignment 1: summarize (and give feedback on – what could be done better/more?) the following article:

George Casella, Edward I. George (1992).
Explaining the Gibbs Sampler.
The American Statistician 46 (3), 167-174.

Assignment 2: summarize (and give feedback on – what could be done better/more?) the following article:

Lennart Hoogerheide, David Ardia, Nienke Corré (2012).
Stock Index Returns' Density Prediction using GARCH Models: Frequentist or Bayesian Estimation?
Economics Letters 116 (3), 322-325.

Alternative (I): Assignment 1: Model with normal distribution and Gibbs sampling

A company that has multiple shops in different cities wants to analyze whether shop A and shop B have a different expected profit.
The company has computed the difference y_j (= profit of shop A - profit of shop B, in 10000 euros) for 10 periods:

1.6088
-1.4579
0.4502
1.0701
-0.3803
1.1201
0.4199
0.3998
-0.1904
0.3932

In this exercise we assume that $y_j \sim N(\mu, \frac{1}{h})$, i.i.d. where mean μ and precision h are unknown parameters.

(a) First we assume that we specify the (improper) non-informative prior

$$p(\mu, h) \propto \begin{cases} \frac{1}{h} & \text{if } h > 0 \\ 0 & \text{else} \end{cases}$$

Simulate from the posterior distribution of μ and h by applying the **Gibbs sampling method (from slide 52 of lecture 2)** with 11000 draws (with a *burn-in* of 1000 draws) and initial value $\mu = \bar{y}$.

Please first read the hints/warnings on slides 6-7 of this assignment!

Inspect the *trace plot* of the draws of μ (i.e., the graph with μ_i on the vertical axis and $i = 1, 2, \ldots, 11000$ on the horizontal axis): does the Gibbs sampling method keep moving through the parameter space or does it 'get stuck' in a certain point?

Use the resulting 10000 draws to estimate the posterior probabilities $\Pr(\mu < 0|y)$ and $\Pr(\mu > 0|y)$. Interpret the results.

Hints/warnings:

(1) There exist two versions of the Gamma distribution with parameters:

- *shape* a and *scale* b (or *shape* k and *scale* θ), used during this course.
- *shape* a and *rate* $1/b$ (or *shape* α and *rate* β)

For example, see also
`https://en.wikipedia.org/wiki/Gamma_distribution`.
Check which version of the Gamma distribution your computer package uses. If necessary, use arguments a and $\frac{1}{b}$.

(2) On the lecture slides the second parameter of the normal distribution $N(\mu, \sigma^2)$ is the **variance** σ^2, whereas computer packages often require the **standard deviation** $\sigma = \sqrt{\sigma^2}$.

Hint: So, the pseudocode is given by:

- Compute $\mu_0 = \bar{y}$, the sample mean of the $n = 10$ observations.
- Set $n_{draws} = 11000$ (and please notice the difference between $n_{draws} = 11000$, which is the number of iterations of the for-loop that we can choose ourselves, and $n = 10$, the number of available observations, where $n = 10$ appears in the conditional posterior distributions)
- Do for draw $i = 1, \ldots, n_{draws}$:
  - Simulate h_i from Gamma($a = n/2, b = (\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu_{i-1})^2)^{-1}$) distribution.
  - Simulate μ_i from normal distribution with mean $\bar{y}$ and standard deviation $\frac{1}{\sqrt{h_i n}}$.
- Estimate $\Pr(\mu < 0|y)$ as the fraction of the 10000 draws μ_i $(i = 1001, 1002, \ldots, 11000)$ that are negative, and $\Pr(\mu > 0|y)$ as the fraction of the 10000 draws μ_i $(i = 1001, 1002, \ldots, 11000)$ that are positive.

(b) Now suppose we have prior

$$p(\mu, h) = p(\mu) \times p(h)$$

with a normal prior density for μ:

$$\mu \sim N(m_{prior}, v_{prior})$$

with $m_{prior} = 0, v_{prior} = 100^2$. That is, a prior with mean 0 and large variance.
And the same prior $p(h)$ for h:

$$p(h) \propto \begin{cases} \frac{1}{h} & \text{if } h > 0 \\ 0 & \text{else} \end{cases}$$

Simulate from the posterior distribution of μ and h by applying the **Gibbs sampling method (from slide 54 of lecture 2)** with 11000 draws (with a *burn-in* of 1000 draws) and initial value $\mu = \bar{y}$. See the hint on slide 10 of this assignment.

Use the resulting 10000 draws to estimate $\Pr(\mu < 0|y)$ and $\Pr(\mu > 0|y)$. Compare the results with part (a).

Hint: So, the pseudocode is given by:

- Compute $\mu_0 = \bar{y}$, the sample mean of the $n = 10$ observations.
- Set $n_{draws} = 11000$, $m_{prior} = 0$, $v_{prior} = 100^2 = 10000$
- Do for draw $i = 1, \ldots, n_{draws}$:
  - Simulate h_i from Gamma($a = n/2, b = (\frac{1}{2}\sum_{j=1}^{n}(y_j - \mu_{i-1})^2)^{-1}$) distribution.
  - Simulate μ_i from normal distribution with mean $\frac{\frac{m_{prior}}{v_{prior}} + h_i n \bar{y}}{\frac{1}{v_{prior}} + h_i n}$ and standard deviation $\frac{1}{\sqrt{\frac{1}{v_{prior}} + h_i n}}$.
- Estimate $\Pr(\mu < 0|y)$ as the fraction of the 10000 draws μ_i ($i = 1001, 1002, \ldots, 11000$) that are negative, and $\Pr(\mu > 0|y)$ as the fraction of the 10000 draws μ_i ($i = 1001, 1002, \ldots, 11000$) that are positive.

(c) Now suppose we have prior

$$p(\mu, h) = p(\mu) \times p(h)$$

with a normal prior density for μ:

$$\mu \sim N(m_{prior}, v_{prior})$$

with $m_{prior} = 0.5, v_{prior} = 0.25^2$. That is, a prior with mean 0.5 and small variance.
And the same prior $p(h)$ for h:

$$p(h) \propto \begin{cases} \frac{1}{h} & \text{if } h > 0 \\ 0 & \text{else} \end{cases}$$

Simulate from the posterior distribution of μ and h by applying the **Gibbs sampling method (from slide 54 of lecture 2)** with 11000 draws (with a *burn-in* of 1000 draws) and initial value $\mu = \bar{y}$.

Use the resulting 10000 draws to estimate $\Pr(\mu < 0|y)$ and $\Pr(\mu > 0|y)$. Compare the results with part (a) and (b).

(d) Perform a classical/frequentist two-sided test of

$H_0 : \mu = 0$ versus
$H_1 : \mu \neq 0$

at 5% significance, comparing the t-statistic $\frac{\bar{y}}{\sqrt{s^2/n}}$ with the 2.5% and 97.5% percentiles of the t_9 distribution (for which you can simply use the values -2.2622 and 2.2622). What is the conclusion?

Compare the conclusion with parts (a), (b) and (c).

Assignment 2: random walk Metropolis(-Hastings) method

(a) The file `Assignment2Dataset.csv` contains the daily returns of the S&P 500 in the six years 2018-2023. Show a graph of the daily returns y_t.

(b) We consider the ARCH(1) model (with variance targeting, so that α is the only unknown parameter). For α we specify the uniform prior on [0,1). In other words, we consider the ARCH(1) model and prior of lecture 3.

Use the **random walk Metropolis(-Hastings) method** (given on the next two slides) to estimate the posterior mean and posterior standard deviation of the parameter α. Use 1100 draws and a burn-in of 100 draws. Also show a histogram of the 1000 draws after the burn-in. Use the normal distribution $\tilde{\alpha} \sim N(\alpha_{i-1}, 0.0275^2)$ as the candidate distribution.

Also inspect the trace plot of the draws α and compute the acceptance percentage and the first order serial correlation in the sequence of accepted (and repeated) draws α. Interpret the results.

Hint: The following pseudocode can be used:

- Choose feasible initial value $\alpha_0 = 0.4745$ and set $n_{draws} = 1100$.
- Do for draw $i = 1, \ldots, n_{draws}$:
  - Simulate candidate draw $\tilde{\alpha}$ from normal distribution with mean α_{i-1} and standard deviation 0.0275.
  - If $\tilde{\alpha} < 0$ or $\tilde{\alpha} \geq 1$, then set acceptance probability $a = 0$ (since the prior density $p(\tilde{\alpha})$ and posterior density $p(\tilde{\alpha}|y)$ are equal to 0 for $\tilde{\alpha} < 0$ and for $\tilde{\alpha} \geq 1$). Else compute acceptance probability

    $$a = \min \left\{ \exp \left[\ln p(y|\tilde{\alpha}) - \ln p(y|\alpha_{i-1}) \right], 1 \right\}$$

    with loglikelihood $\ln p(y|\alpha)$ given on the next slide.
  - Simulate U from uniform distribution on $[0, 1]$.
  - If $U \leq a$, then accept: $\alpha_i = \tilde{\alpha}$ (accept candidate draw).
    If $U > a$, then reject: $\alpha_i = \alpha_{i-1}$ (repeat previous draw).
- Estimate the posterior mean and posterior standard deviation of α as the sample mean and sample standard deviation of the 1000 draws α_i $(i = 101, 102, \ldots, 1100)$.

In our ARCH(1) model: loglikelihood[1]

$$\ln p(y|\alpha) = \sum_{t=2}^{n} \left\{ -\frac{1}{2} \ln(\boxed{2\pi} [s^2(1-\alpha) + \alpha y_{t-1}^2]) - \frac{y_t^2}{2[s^2(1-\alpha) + \alpha y_{t-1}^2]} \right\}$$

[1] The factor 2π is part of the constant scaling factor of the posterior density kernel and can be left out in this assignment.

(c) Repeat part (b) where the normal candidate distribution has the **very small** standard deviation 0.002 instead of 0.0275, and compare the results with part (b).

(d) Repeat part (b) where the normal candidate distribution has the **very large** standard deviation 6.0 instead of 0.0275, and compare the results with part (b).

(e) Estimate the GARCH(1,1) model of lecture 3 using the random walk Metropolis(-Hastings) method (again using 1100 draws with a burn-in of 100 draws) using a bivariate normal candidate distribution with mean $\theta_{i-1} = (\alpha_{i-1}, \beta_{i-1})'$ and variance-covariance matrix

$$\begin{pmatrix} 0.000430 & -0.000510 \\ -0.000510 & 0.000626 \end{pmatrix}$$

Compute the posterior mean and standard deviation of α and β.

Also inspect the trace plots of the (accepted and repeated) draws of α and β and compute the acceptance percentage and the two first order serial correlations in the sequences of (accepted and repeated) draws of α and β. Interpret the results.

Also make a scatter plot of (accepted and repeated) draws of α and β and a scatter plot of the *candidate* draws of α and β. Compare these two scatter plots.

(f) Repeat part (e) where the candidate distribution has variance-covariance matrix

$$\begin{pmatrix} 0.000430 & \mathbf{0} \\ \mathbf{0} & 0.000626 \end{pmatrix}$$

Compare the results with part (e).