Time Series and Dynamic Econometrics
Assignment 1

Last updated: September 1, 2025

Instructions:

1. The assignment is made in groups of **exactly two students**. The same group is maintained for Assignments 1, 2, and 3.
2. The assignment is made using a programming language of choice, such as `python`/`R`/`matlab`/`Ox`. Software packages such as `Eviews` or `STATA` can **not** be used.
3. Within your chosen program language, you may use packages for computing statistical estimates (such as sample autocorrelations). However, simulations should be done manually (using for loops).[1]
4. The assignment must be **uploaded in PDF format** on Canvas before the **deadline**:

   Monday, September 15, 2025, at 23:59.

   The code must be uploaded as separate file(s) from the PDF. Do **not** upload the data.
5. The assignment is graded as either "complete" or "incomplete"; in the latter case the assignment must be revised. To pass the TSDE course, all three assignments must be graded as "complete."
6. On the first page of the assignment state the names and student numbers of the group members.
7. Do **not** copy the questions in your assignment (this invokes the plagiarism detection software). Simply write:
   1. <YOUR ANSWER TO SUBQUESTION 1>.
   2. <YOUR ANSWER TO SUBQUESTION 2>.
   3. ...

Remark (Plagiarism). *The VU has a* ***zero tolerance*** *policy for plagiarism, which can lead to exclusion from the Minor "Applied Econometrics: A Big Data Experience for All" without graduating. All assignments are checked for plagiarism using dedicated software, and cases that raise suspicion are automatically sent to the examination board for review.*

[1] You may of course use a package for drawing normal random variables.

Part I: Introduction to Time Series

This part of the assignment starts with a brief analysis of your own simulated data. Next, we consider the use of the data set `data_tsde_assignment_1.csv` that you can find on Canvas.

Simulations

1. Consider the following Gaussian AR(1) process,

$$X_t = \phi X_{t-1} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim NID(0, \sigma_\varepsilon^2),$$

for $t = 1, \ldots, T$. Using initial value $X_0 = 0$, simulate and plot a path of length $T = 1000$ for $\sigma_\varepsilon = 1$ and each of the following parameter values:

   (a) $\phi = 0$

   (b) $\phi = 0.9$

   (c) $\phi = 1$

   (d) $\phi = 1.01$

   To facilitate comparison, use the same draws for $\{\varepsilon_t\}_{t=1}^T$ in each case. Comment on the differences between the plots.

Canvas data set

We continue with the data set `data_tsde_assignment_1.csv` from Canvas.

2. Plot the data. Plot the sample autocorrelation function (SACF) up to twenty lags. Which AR(1) process from Question 1 do you consider to be most suitable for this data set? Explain your answer.

3. Estimate the parameter ϕ using the SACF, and use the estimate to compute the corresponding model residuals. Next, use the residuals to obtain an estimate of σ_ε and report both estimates. What does the estimate of ϕ tell you about the dynamic properties of the data?

4. Investigate the residuals obtained in the previous question by plotting:

   - the residuals over time;
   - a histogram of the residuals;
   - a QQ-plot of the residuals against the normal distribution;
   - their SACF up to twenty lags.

   Do you find the AR(1) model assumptions reasonable? Explain your answer.

Part II: ARMA Models: Definitions and Stochastic Properties

Autoregressive processes: simulations

1. Consider the following Gaussian AR(2) process,

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t, \qquad \{\varepsilon_t\} \sim NID(0, \sigma_\varepsilon^2),$$

   for $t = 1, \ldots, T$. Using initial values $X_{-1} = X_0 = 0$, simulate and plot a path of length $T = 1000$ for $\sigma_\varepsilon = 1$ and each of the following parameter values:

   (a) $\phi_1 = 0.4$ and $\phi_2 = 0.45$

   (b) $\phi_1 = \phi_2 = 0.5$

   To facilitate comparison, use the same draws for $\{\varepsilon_t\}_{t=1}^{T}$ in each case. Comment on the differences between the plots.

2. For both AR processes from the previous question, analytically derive the roots of the autoregressive polynomial

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2$$

   and explain how they were obtained. Relate the results to your plots from the previous question.

Autoregressive Processes: Canvas data set

3. Reconsider the Canvas data set `data_tsde_assignment_1.csv`. Use the given parameter values $\hat{\phi}_1 = 0.4$ and $\hat{\phi}_2 = 0.45$ to compute the residuals

$$\hat{\varepsilon}_t = x_t - \hat{\phi}_1 x_{t-1} - \hat{\phi}_2 x_{t-2}, \qquad t = 3, \ldots, T.$$

   Investigate these residuals by creating the same plots as in Question 4 of Part 1 of the assignment. Based on the given parameters, do you find the AR(2) model assumptions reasonable? If your conclusion is different than in Question 4 of Part 1 of the assignment, then explain what might be the cause of this difference.

Moving average processes: simulations

Consider the MA(1) process

$$X_t = \varepsilon_t + \theta_1 \varepsilon_{t-1}, \qquad \{\varepsilon_t\} \sim NID(0, \sigma_\varepsilon^2),$$

for $t = 1, \ldots, T$.

4. Using initial value $\varepsilon_0 = 0$, simulate and plot a path of length $T = 1000$ for $\sigma_\varepsilon = 1$ and each of the following parameter values:

   (a) $\theta_1 = 0.5$

   (b) $\theta_1 = 2$

   To facilitate comparison, use the same draws for $\{\varepsilon_t\}_{t=1}^{T}$ in each case. In addition, plot the SACF up to twenty lags for both MA time series. Discuss and explain the differences and similarities between your results.

5. Discuss an advantage of the ARMA(1,1) model over the MA(1) and AR(1) models. In your answer, comment specifically on the ACF of the models.