TSDE Group Submission Assignment 1

Group 19

Lanlan Hou (2801069) and Alfred Sou (2796840)

1.

a. $\phi = 0$,

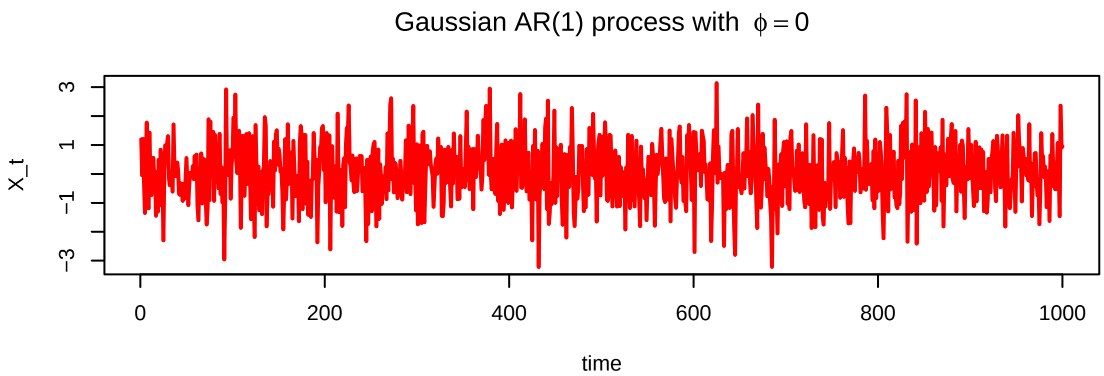
b. $\phi = 0.9$,

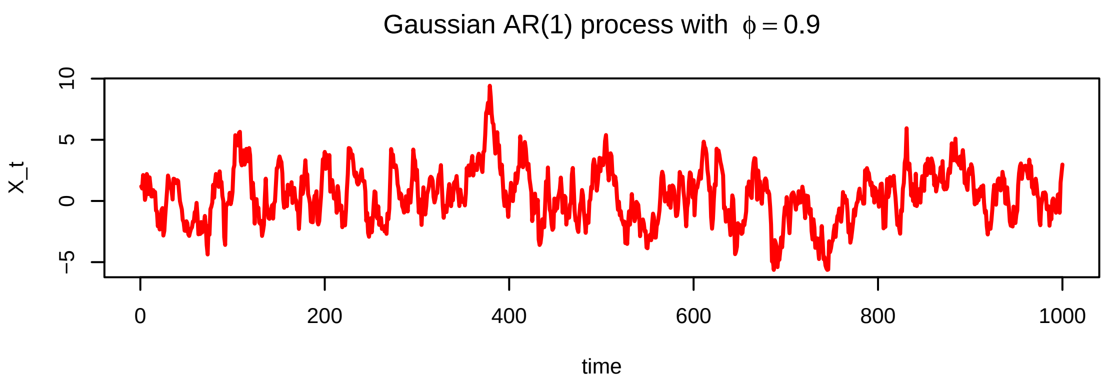
c. $\phi = 1$,

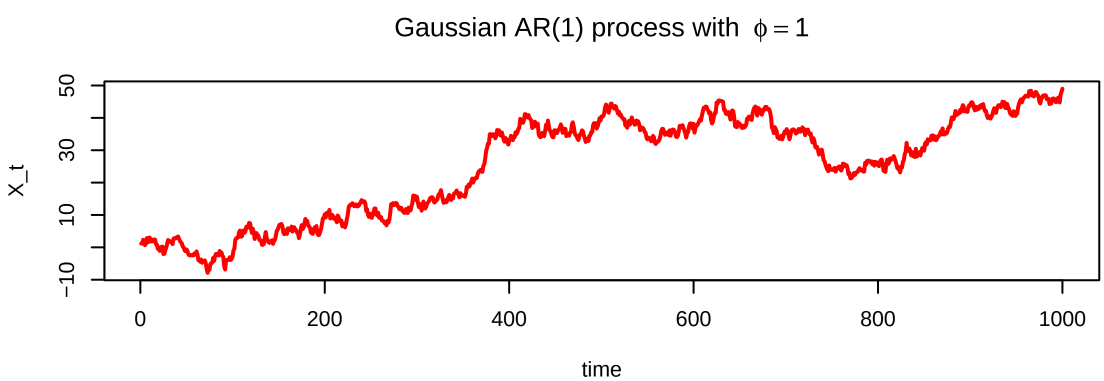
d. $\phi = 1.01$

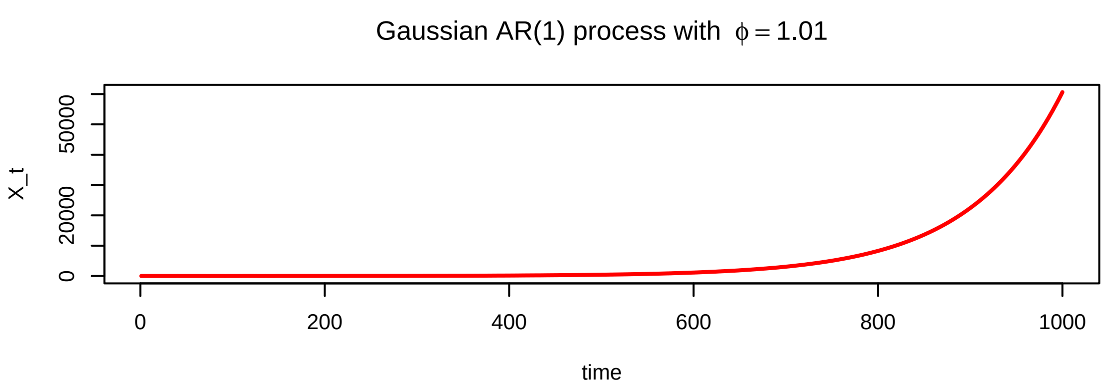
```r
par(mfcol = c(4, 1))
t <- 1000
phi <- 0

wn_var <- 1
wn <- rnorm(t, mean = 0, sd = wn_var)

x_t <- numeric(t)
x_0 <- 0
x_t[1] <- phi * x_0 + wn[1]

for(i in (2:t)){
  x_t[i] <- phi * x_t[i-1] + wn[i]
}

title_text <- bquote("Gaussian AR(1) process with " ~ phi == .(phi))

plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)

phi <- 0.9

x_t[1] <- phi * x_0 + wn[1]

for(i in (2:t)){
  x_t[i] <- phi * x_t[i-1] + wn[i]
}

title_text <- bquote("Gaussian AR(1) process with " ~ phi == .(phi))
```

```r
plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)

phi <- 1

x_t[1] <- phi * x_0 + wn[1]

for(i in (2:t)){
  x_t[i] <- phi * x_t[i-1] + wn[i]
}

title_text <- bquote("Gaussian AR(1) process with " ~ phi == .(phi))

plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)

phi <- 1.01

x_t[1] <- phi * x_0 + wn[1]

for(i in (2:t)){
  x_t[i] <- phi * x_t[i-1] + wn[i]
}

title_text <- bquote("Gaussian AR(1) process with " ~ phi == .(phi))

plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)
```


Gaussian AR(1) process with $\phi = 0$
X_t
−3
−1
1
3
0
200
400
600
800
1000
time


Gaussian AR(1) process with $\phi = 0.9$
X_t
−5
0
5
10
0
200
400
600
800
1000
time

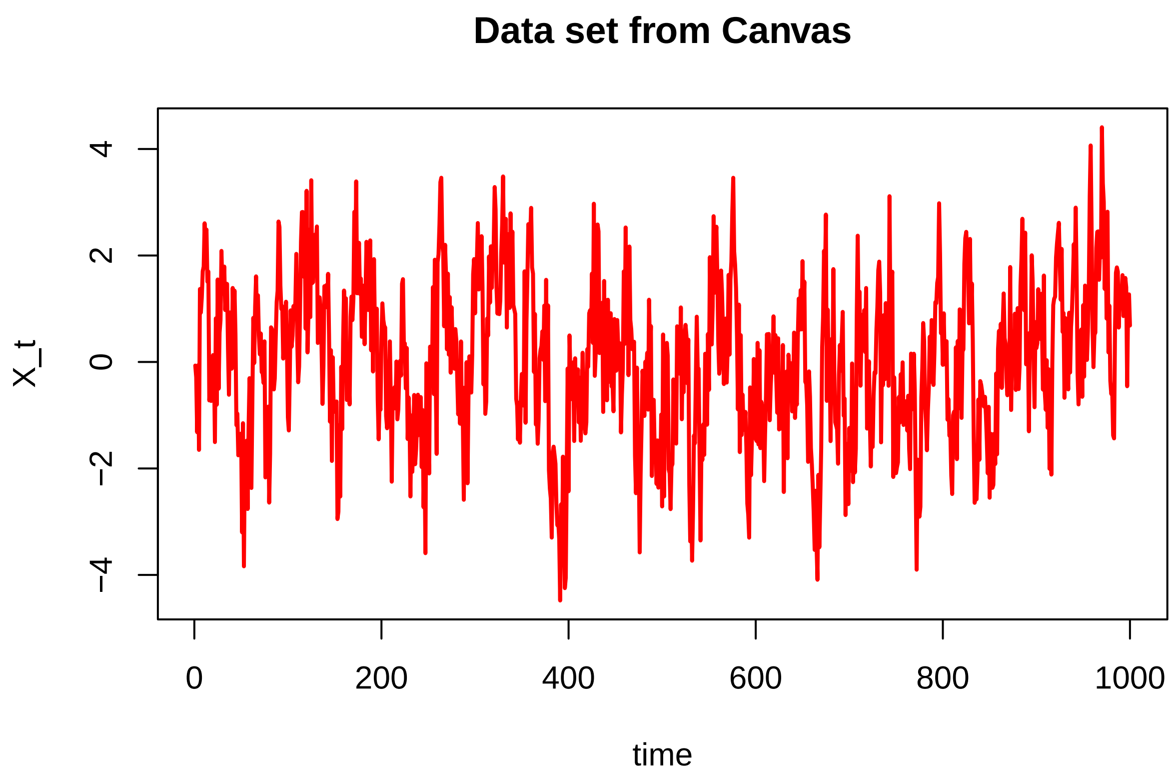

Gaussian AR(1) process with $\phi = 1$
X_t
−10
10
30
50
0
200
400
600
800
1000
time


Gaussian AR(1) process with $\phi = 1.01$
X_t
0
20000
50000
0
200
400
600
800
1000
time

Comment on the differences between the plots.

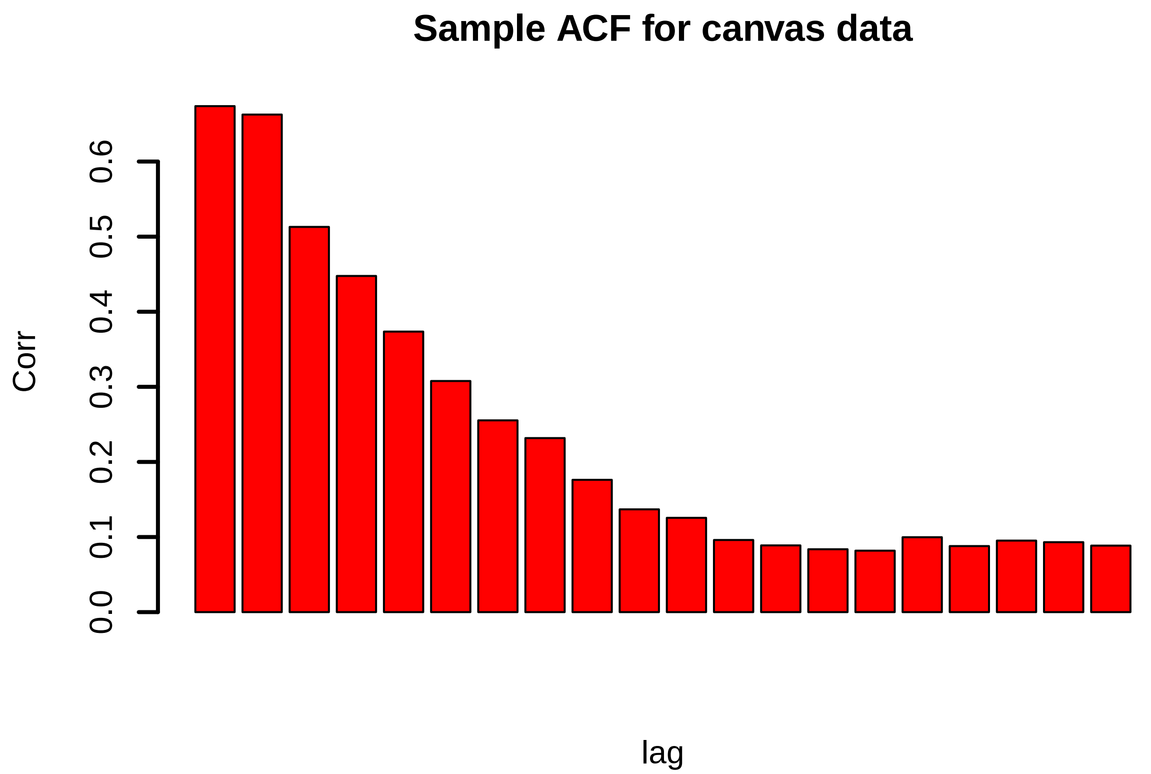
2.

```r
canvas_data_df <- read.csv("data_tsde_assignment_1.csv", header = FALSE)
canvas_data <- canvas_data_df$V1

title_text <- 'Data set from Canvas'

plot(canvas_data, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)
```




```r
lag <- 20
n <- length(canvas_data)
sacf_x <- numeric(lag)

for (i in (1:lag)) {
  x <- canvas_data[1:(n-i)]
  y <- canvas_data[(1+i):n]
  sacf_x[i] <- cor(x, y)
}

barplot(sacf_x,
```

```
        main = 'Sample ACF for canvas data',
        xlab = 'lag', ylab = 'Corr', col = "red", lwd = 2)
```



Explain your answer.

3.

```
est_phi <- numeric(lag)

for (i in (1:lag)) {
  est_phi[i] <- sacf_x[i] ^(1/i)
}

mean_est_phi <- mean(est_phi)
print(paste('The estimated phi is', mean(est_phi)))
```

```
## [1] "The estimated phi is 0.829617535898157"
```

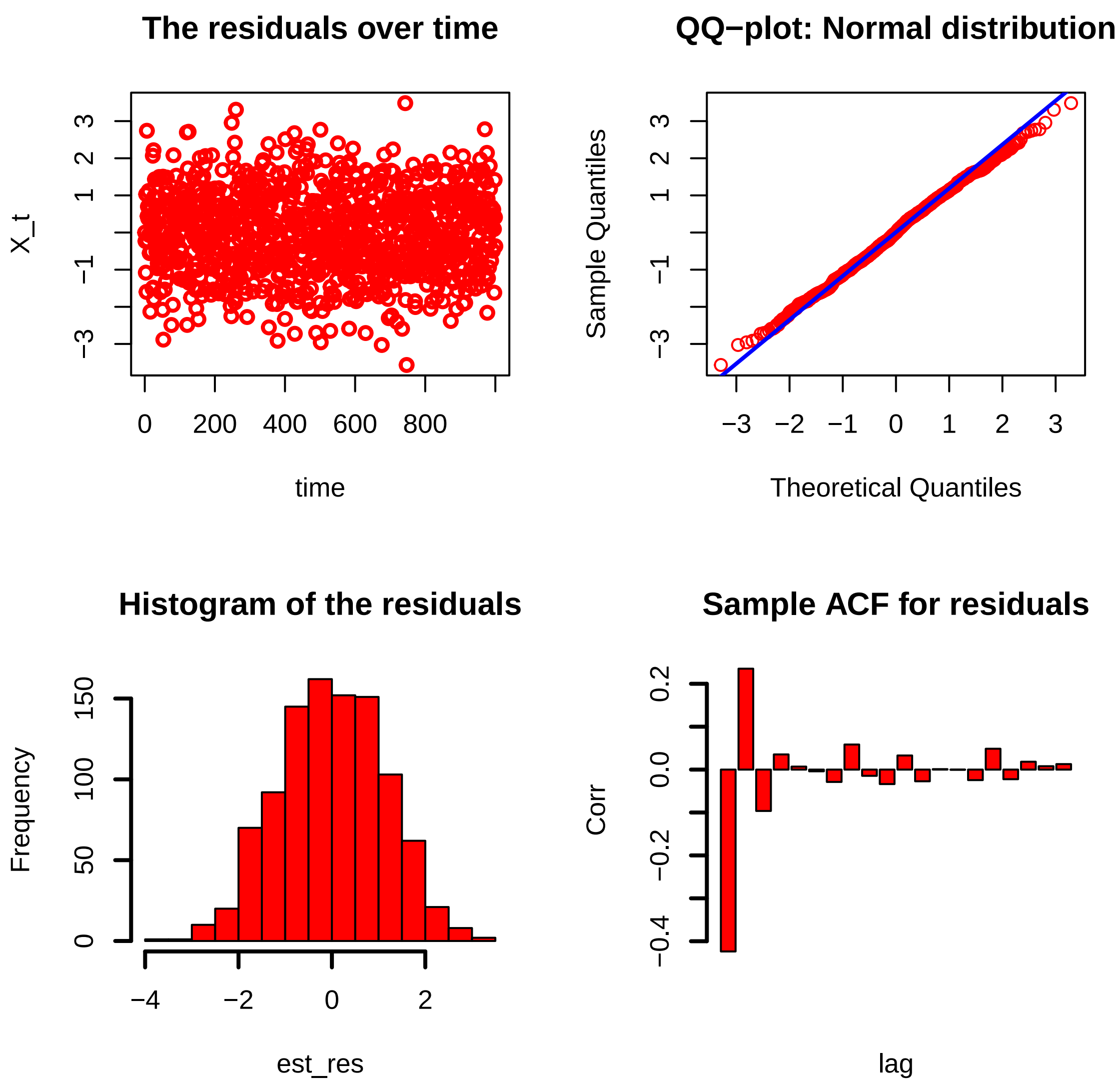
```
est_res <- numeric(n)

for (i in (2:n)) {
```

```r
  est_res[i] <- canvas_data[i] - (mean_est_phi * canvas_data[i-1])
}

mean_est_res <- (1/(n-1)) * (sum(est_res[2:n]))

temp <- 0
for (i in (2:n)) {
  temp <- temp + (est_res[i] - mean_est_res)^2
}

est_var <- (1/(n-1)) * temp

print(paste('The estimated sigma is', est_var))
```

```
## [1] "The estimated sigma is 1.24160244906456"
```

4.

```r
par(mfcol = c(2, 2))
plot(est_res, type ='p',
     main = 'The residuals over time',
     xlab = "time", ylab = "X_t", col = "red", lwd = 2
     )

hist(est_res,
     main = 'Histogram of the residuals',
    col = "red", lwd = 2)

qqnorm(est_res, main = "QQ-plot: Normal distribution", col= "red")
qqline(est_res, col = "blue", lwd = 2)

sacf_res <- numeric(lag)

for (i in (1:lag)) {
  x <- est_res[2:(n-i)]
  y <- est_res[(2+i):n]
  sacf_res[i] <- cor(x, y)
}

barplot(sacf_res,
        main = 'Sample ACF for residuals',
        xlab = 'lag', ylab = 'Corr', col = "red", lwd = 2)
```



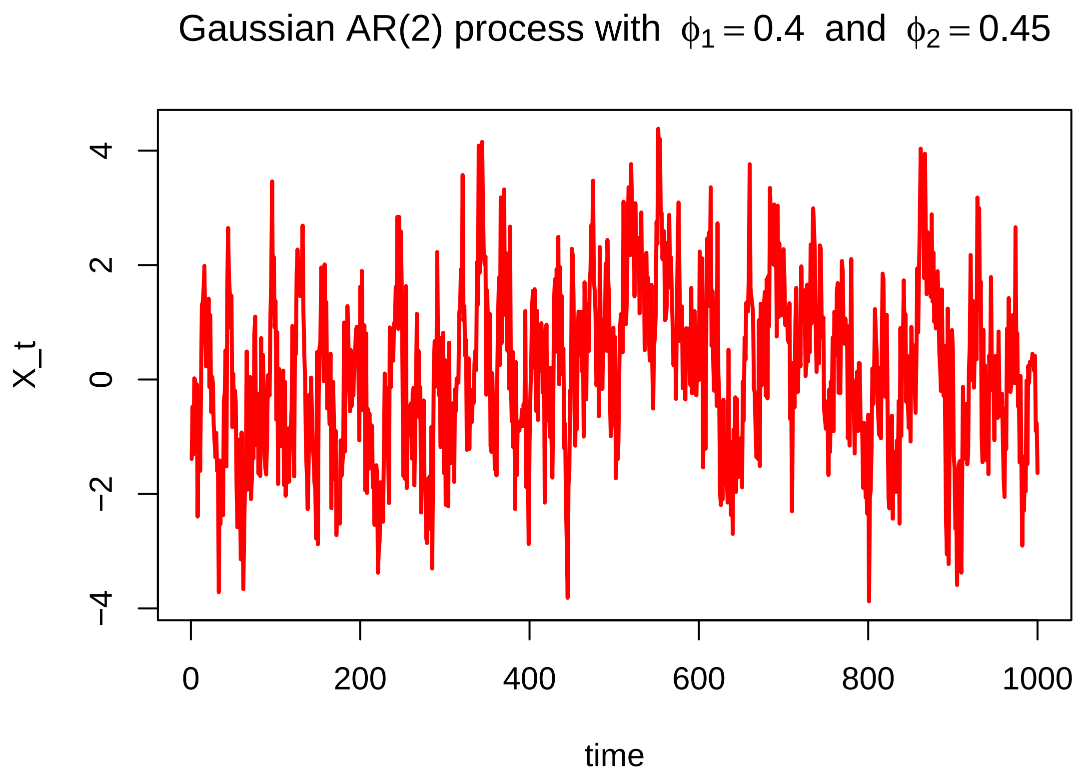
5.

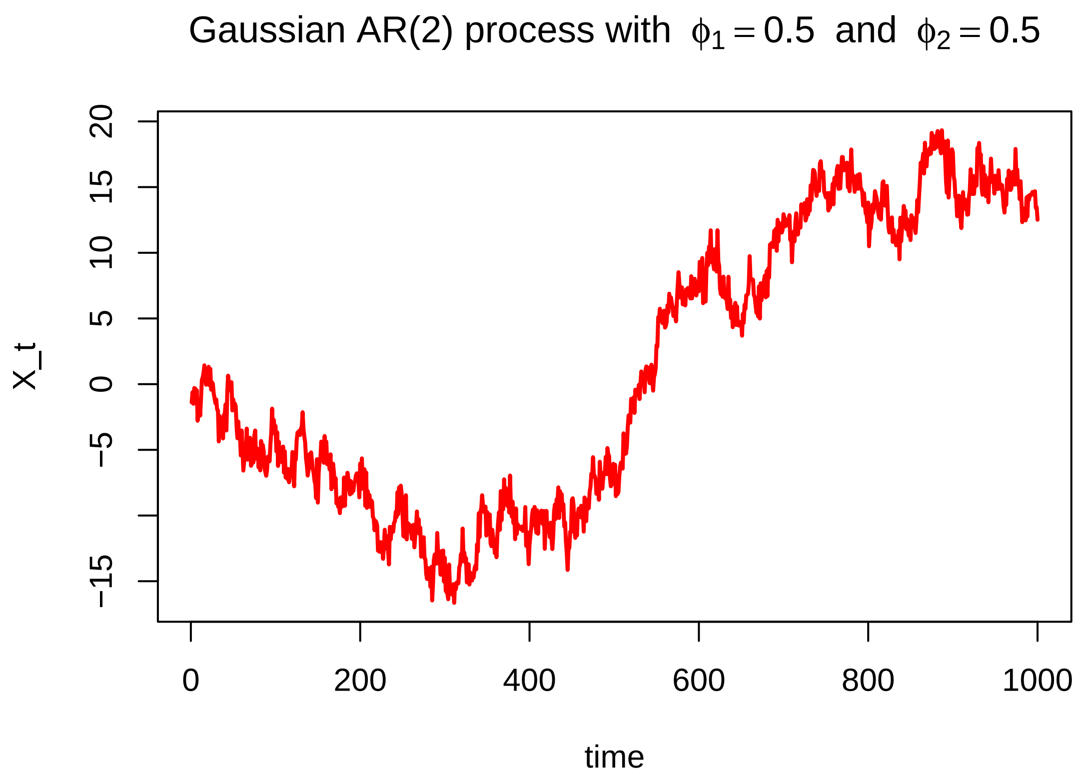
a. $\phi_1 = 0.4$, $\phi_2 = 0.45$

b. $\phi_1 = \phi_2 = 0.5$

```r
par(mfcol = c(2, 1))
t <- 1000
phi_1 <- 0.4
phi_2 <- 0.45

wn_var <- 1
wn <- rnorm(t, mean = 0, sd = wn_var)
```

```r
x_t <- numeric(t)
x_0 <- 0
x_t[1] <- phi_1 * x_0 + phi_2 * x_0 + wn[1]
x_t[2] <- phi_1 * x_t[1] + phi_2 * x_0 + wn[2]

for(i in (3:t)){
  x_t[i] <- phi_1 * x_t[i-1] + phi_2 * x_t[i-2] + wn[i]
}

title_text <- bquote("Gaussian AR(2) process with " ~
                     phi[1] == .(phi_1) ~ " and " ~
                     phi[2] == .(phi_2))

plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)

phi_1 <- 0.5
phi_2 <- 0.5

x_t <- numeric(t)
x_0 <- 0
x_t[1] <- phi_1 * x_0 + phi_2 * x_0 + wn[1]
x_t[2] <- phi_1 * x_t[1] + phi_2 * x_0 + wn[2]

for(i in (3:t)){
  x_t[i] <- phi_1 * x_t[i-1] + phi_2 * x_t[i-2] + wn[i]
}

title_text <- bquote("Gaussian AR(2) process with " ~
                     phi[1] == .(phi_1) ~ " and " ~
                     phi[2] == .(phi_2))

plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)
```


Gaussian AR(2) process with $\phi_1 = 0.4$ and $\phi_2 = 0.45$
X_t
−4
−2
0
2
4
0
200
400
600
800
1000
time


Gaussian AR(2) process with $\phi_1 = 0.5$ and $\phi_2 = 0.5$
X_t
−15
−5
0
5
10
15
20
0
200
400
600
800
1000
time

Comment on the differences between the plots.

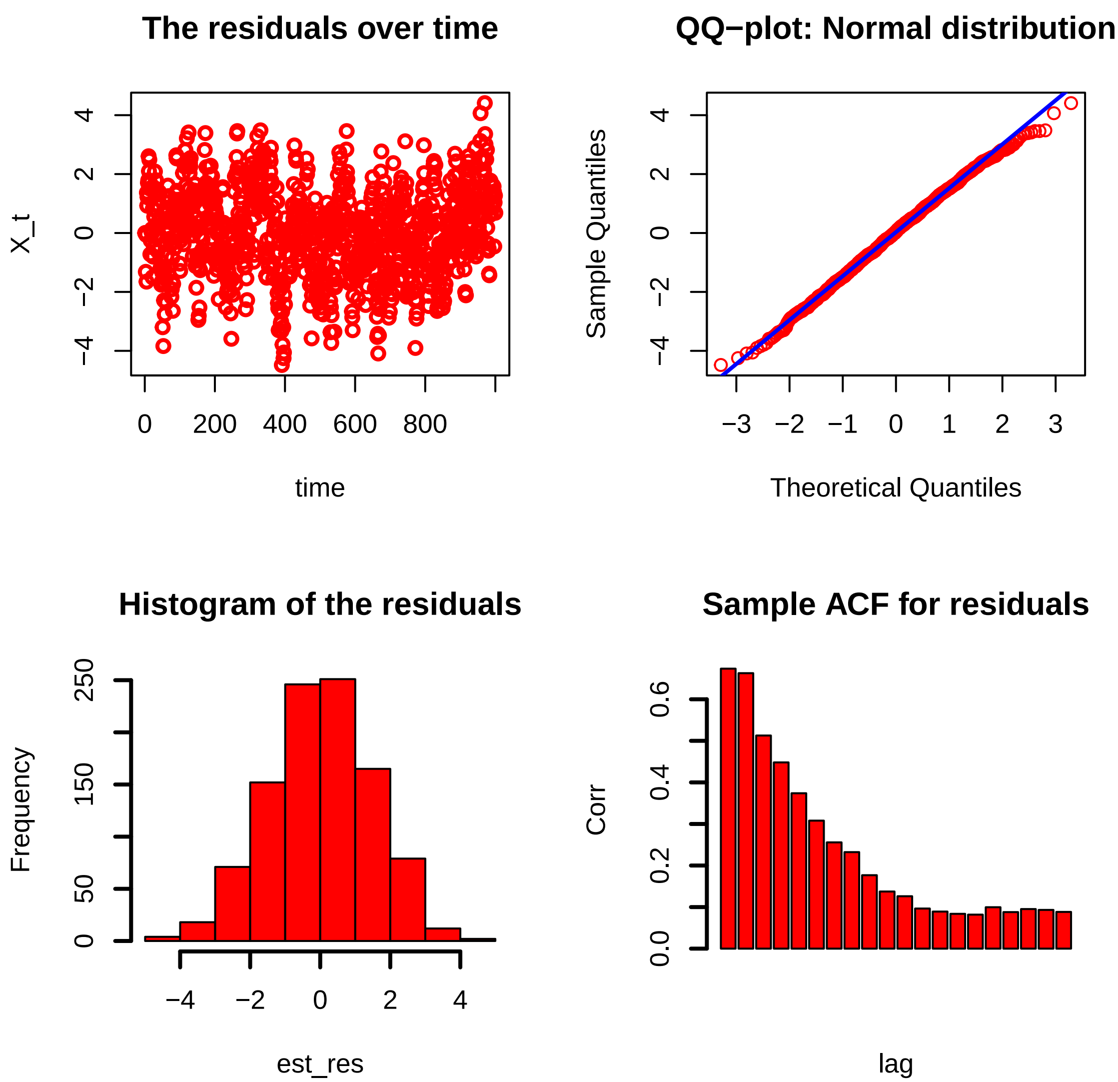
6.

For $\phi_1 = 0.4, \phi_2 = 0.45$,
$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 = 1 - 0.4L - 0.45L^2 = (1 - 0.9L)(1 + 0.5L)$
Therefore, the roots are $|z_1^*| = 1/0.9 = 1.11 > 1$ and $|z_2^*| = 1/0.5 = 2 > 1$

For $\phi_1 = 0.5, \phi_2 = 0.5$,
$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 = 1 - 0.5L - 0.5L^2 = (1 - L)(1 + 0.5L)$
Therefore, the roots are $|z_1^*| = 1/1 = 1$ and $|z_2^*| = 1/0.5 = 2 > 1$

7.

```r
phi_1 <- 0.4
phi_2 <- 0.45

est_res <- numeric(n)

for (i in (3:n)) {
  est_res[i] <- canvas_data[i]
                - phi_1 * canvas_data[i-1]
                - phi_2 * canvas_data[i-2]
}


par(mfcol = c(2, 2))
plot(est_res, type ='p',
     main = 'The residuals over time',
     xlab = "time", ylab = "X_t", col = "red", lwd = 2
     )

hist(est_res,
     main = 'Histogram of the residuals',
    col = "red", lwd = 2)

qqnorm(est_res, main = "QQ-plot: Normal distribution", col= "red")
qqline(est_res, col = "blue", lwd = 2)

sacf_res <- numeric(lag)

for (i in (1:lag)) {
  x <- est_res[2:(n-i)]
  y <- est_res[(2+i):n]
  sacf_res[i] <- cor(x, y)
}

barplot(sacf_res,
        main = 'Sample ACF for residuals',
        xlab = 'lag', ylab = 'Corr', col = "red", lwd = 2)
```



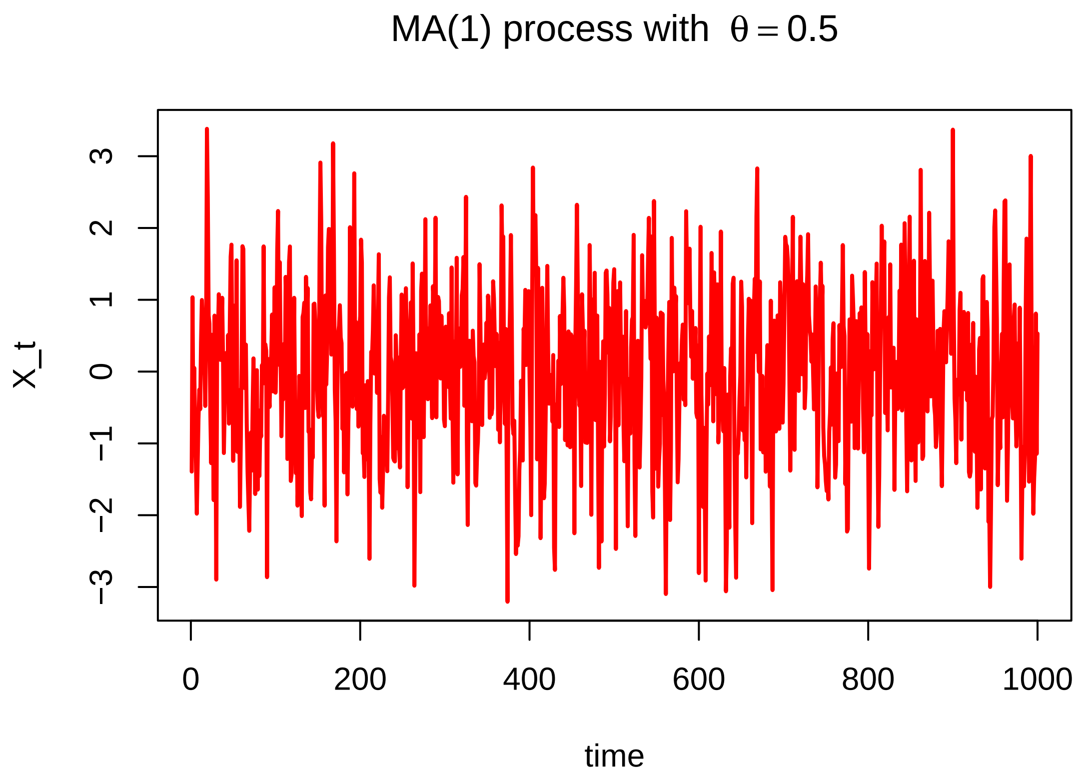
8.

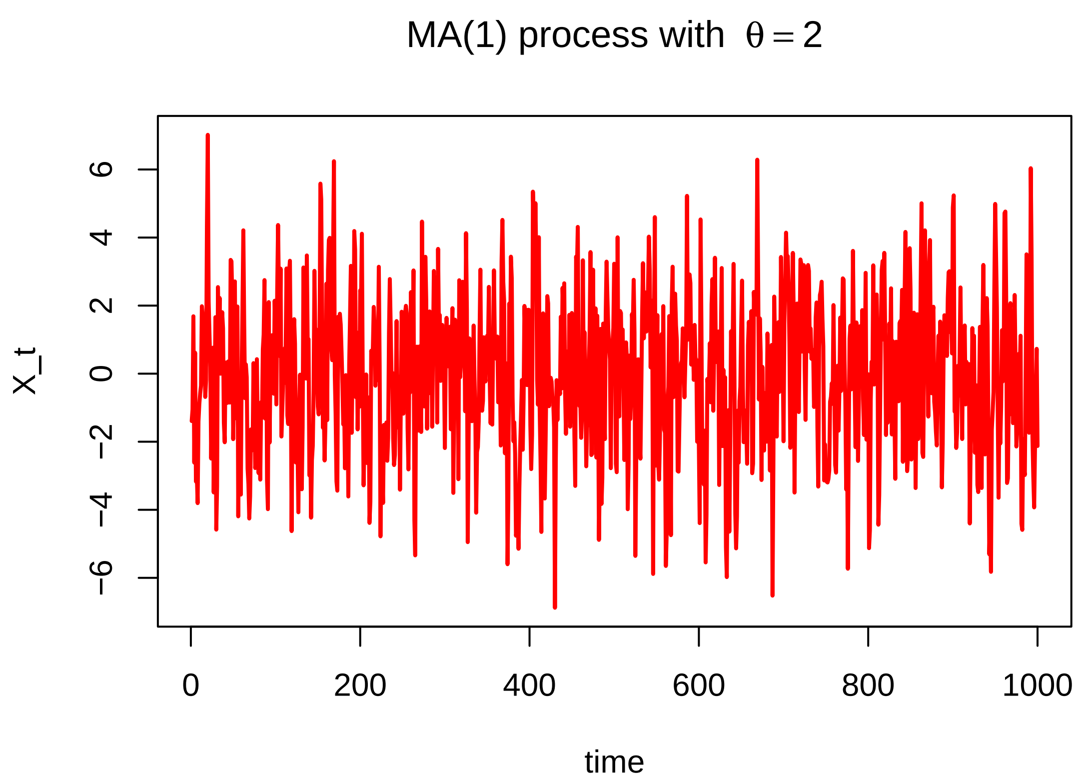
a. $\theta_1 = 0.5$

b. $\theta_1 = 2$

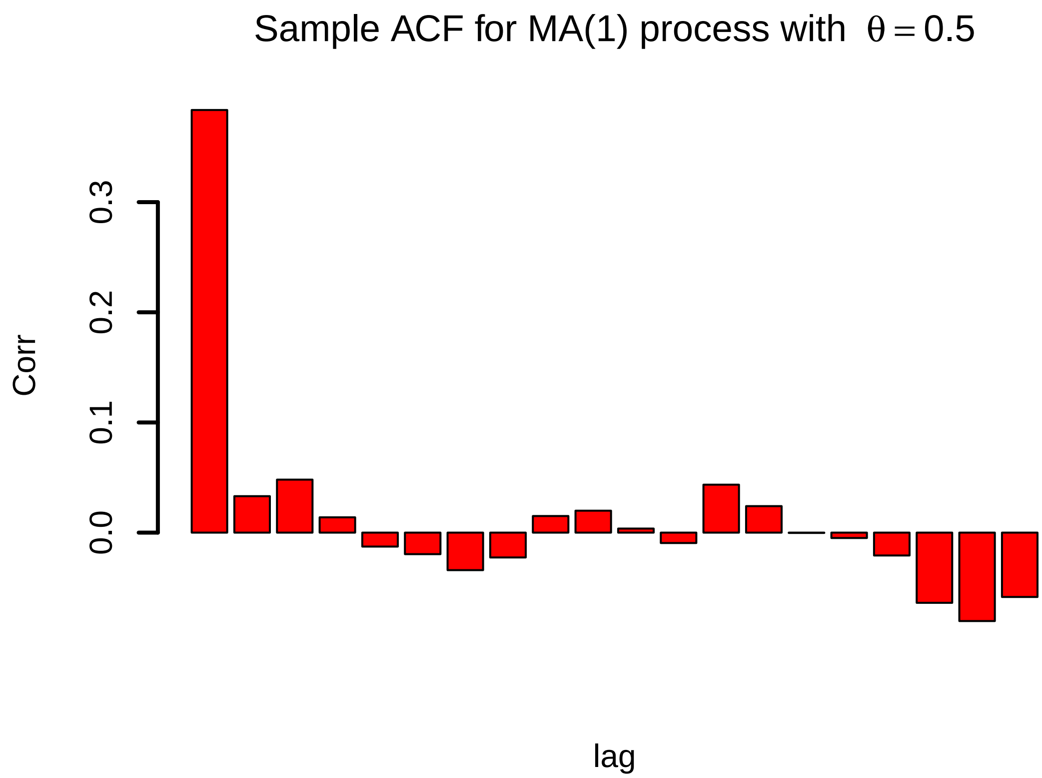
```
par(mfcol = c(2, 1))
t <- 1000
theta <- 0.5

wn_var <- 1
wn <- rnorm(t, mean = 0, sd = wn_var)

x_t <- numeric(t)
```

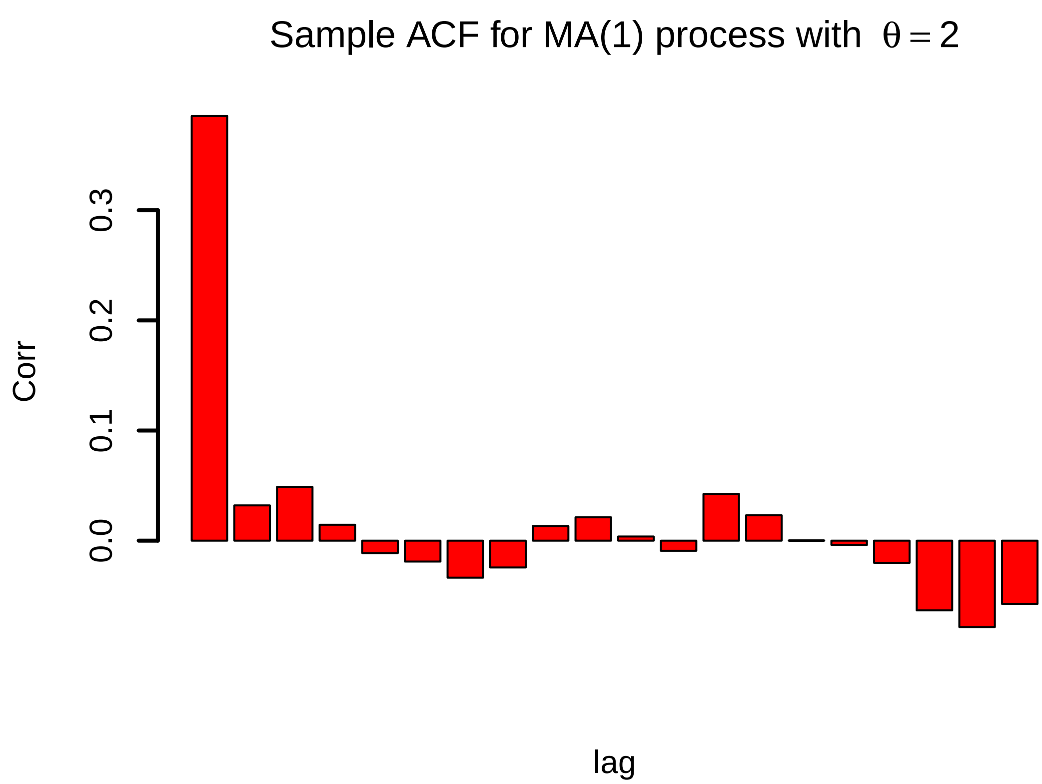
```r
wn_0 <- 0
x_t[1] <- wn[1] + theta * wn_0

for(i in (2:t)){
  x_t[i] <- wn[i] + theta * wn[i-1]
}

title_text <- bquote("MA(1) process with " ~ theta == .(theta))

plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)

lag <- 20
n <- t
sacf_a <- numeric(lag)

for (i in (1:lag)) {
  x <- x_t[1:(n-i)]
  y <- x_t[(1+i):n]
  sacf_a[i] <- cor(x, y)
}

theta <- 2

x_t <- numeric(t)
wn_0 <- 0
x_t[1] <- wn[1] + theta * wn_0

for(i in (2:t)){
  x_t[i] <- wn[i] + theta * wn[i-1]
}

title_text <- bquote("MA(1) process with " ~ theta == .(theta))

plot(x_t, type = 'l', main = title_text,
     xlab = "time", ylab = "X_t", col = "red", lwd = 2)
```


MA(1) process with θ = 0.5
X_t
3
2
1
0
−1
−2
−3
0
200
400
600
800
1000
time


MA(1) process with θ = 2
X_t
6
4
2
0
−2
−4
−6
0
200
400
600
800
1000
time

```r
sacf_b <- numeric(lag)

for (i in (1:lag)) {
  x <- x_t[1:(n-i)]
  y <- x_t[(1+i):n]
  sacf_b[i] <- cor(x, y)
}

par(mfcol = c(2, 1))

theta <- 0.5

title_text <- bquote("Sample ACF for MA(1) process with " ~ theta == .(theta))

barplot(sacf_a,
        main = title_text,
        xlab = 'lag', ylab = 'Corr', col = "red", lwd = 2)

theta <- 2

title_text <- bquote("Sample ACF for MA(1) process with " ~ theta == .(theta))

barplot(sacf_b,
        main = title_text,
        xlab = 'lag', ylab = 'Corr', col = "red", lwd = 2)
```


Sample ACF for MA(1) process with θ = 0.5
Corr
0.0
0.1
0.2
0.3
lag


Sample ACF for MA(1) process with θ = 2
Corr
0.0
0.1
0.2
0.3
lag

Discuss and explain the differences and similarities between your results.

9.