Time Series and Dynamics Econometrics

Assignment 2

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Part I: Forecasting, Parameter Estimation, and Model Selection

Question 1

Figure 1.1 illustrates the Dutch GDP quarterly growth rates and Figure 1.2 reveals the sample autocorrelation function up to 12 lags. The plot of dutch quarterly GDP growth rates from 1987Q2 to 2009Q1 shows substantial fluctuations, including the sharp fall during the global financial crisis in 2008-2009. The periods of both positive and negative growth strongly suggest there is cyclical behavior regarding time series.
The sample autocorrelation function(SACF) up to 12 lags indicates significant correlation ar the first few lags($0-4$), then it decays generally. The pattern is consist with autoregressive dynamics. In fact, we can see from the pattern that GDP growth series is persistent and thus an AR model could provide a representation of its dynamics. Based on SACF, we could probably choose a low-order AR(p) model.

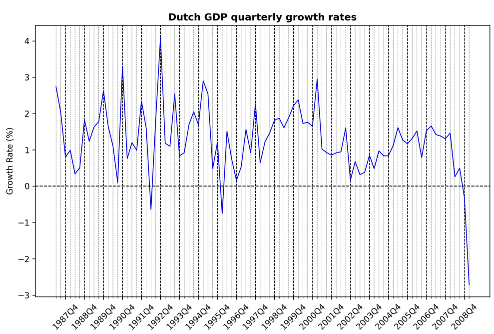

Figure 1.1 GDP quarterly growth rates

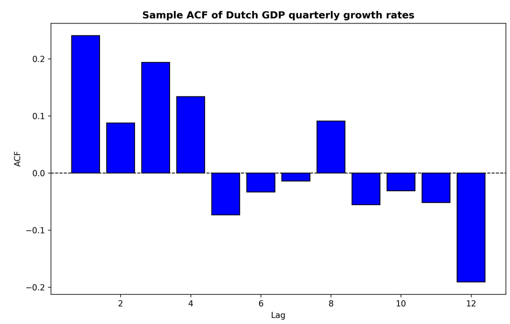

Figure 1.2 The Sample autocorrelation function up to 12 lags

Question 2

Estimate the AR(p) model with a maximum $p = 4$ lags. Based on the Bayesian information criterion (BIC), select the model with the lowest value of the information criterion. Table 2.1 shows the values of BIC for corresponding estimated models.

To Be revised

The AR(p) model with up to 4 lags are used to estimated and evaluated using the Bayesian Information Criterion(BIC). The results in Table 2.1 indicates that AR(1) is the best fit model, with the lowest BIC value of -13.593. The estimated coefficient on the first lag is 0.896, which is close to 1, and the intercept is 0.267.
This parameter indicates that GDP growth is autocorrelated, that is, a positive shock in past time could cause above-average growth in the following quarters, while a negative shock can lead to presistently low growth. The coefficient implies that deviations from the mean are corrected with a slow change. The selection of AR(1) model seems consistent, as the first lag has the most significant and explanatory power are not provided by higher-order lags.
As a result, using BIC, AR(1) model should be the best model to describe the dynamics of the Dutch GDP quarterly growth rate. The estimated coefficients are shown in Table 2.2.
The final estimated AR(1) model is:

$$X_t = 0.896 + 0.267 X_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$$

AR(p)	BIC
1	-13.592538
2	-9.570916
3	-8.761758
4	-5.198308

Table 2.1 Bayesian Information Criterions for different AR(p) models

Coefficient	Value
α	0.896161
ϕ_1	0.267446

Table 2.2 The estimated coefficients for estimated AR(1) model

Question 3

The forecast up to 8 quarters (2 years) ahead show that the AR(1) model predicts GDP growth towards the intercept term. After the sharp fall in 2009Q1, the model shows a slowly adjustment to recover. From the plot we can see the short-term movements are influenced by the most recent shocks, but the long-run tendency seems to become stable towards the unconditional mean of the series.

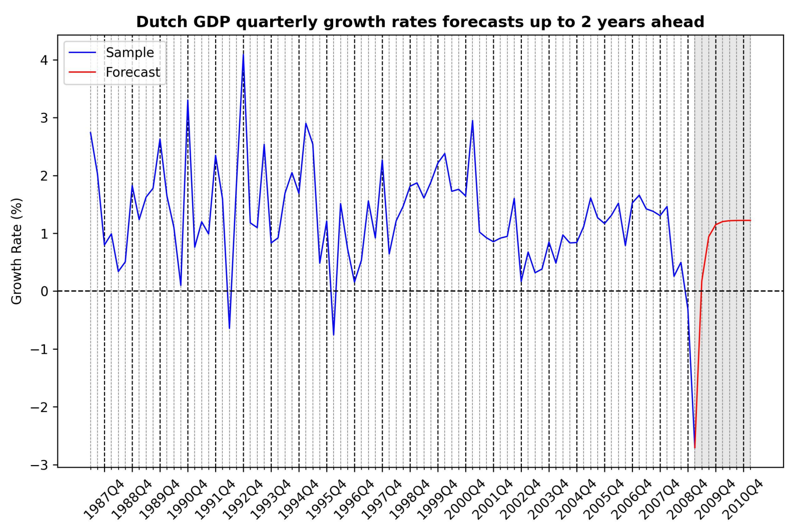

Fig 3.1 Forecast up to 2 years ahead for the Dutch GDP quarterly growth rate until the first quarter of 2011

Question 4

The 95% confidence interval for forecast is plotted in Figure 4.1. Using the Jarque-Bera test to test for the normality of the residuals of the AR(p) model, the normality assumption for ε_t can be checked. Based on the result shown in Table 4.1, since the JB test statistic is greater than the 95% critical value of the χ^2_2 under the null, therefore the null hypothesis is rejected as the normality of assumption the residuals is rejected.
Using the Breusch-Godfrey test to test for autocorrelation up to lag k of the residuals of the AR(p) model, the white noise assumption for ε_t can be checked. Based on the result shown in Table 4.2, since the BG test statistic is smaller than the 95% critical value of the χ^2_4 under the null, therefore do not reject the null hypothesis as the residuals has no autocorrelation.

JB Test Statistic	Critical Value (5% at χ^2_2)
27.33	5.991

Table 4.1 Jarque-Bera (JB) test statistic and the critical value of χ^2_2 distribution under the null

BG Test Statistic	Critical Value (5% at χ^2_4)
0.0	9.488

Table 4.2 Breusch-Godfrey (BG) test statistic and the critical value of χ^2_4 distribution under the null

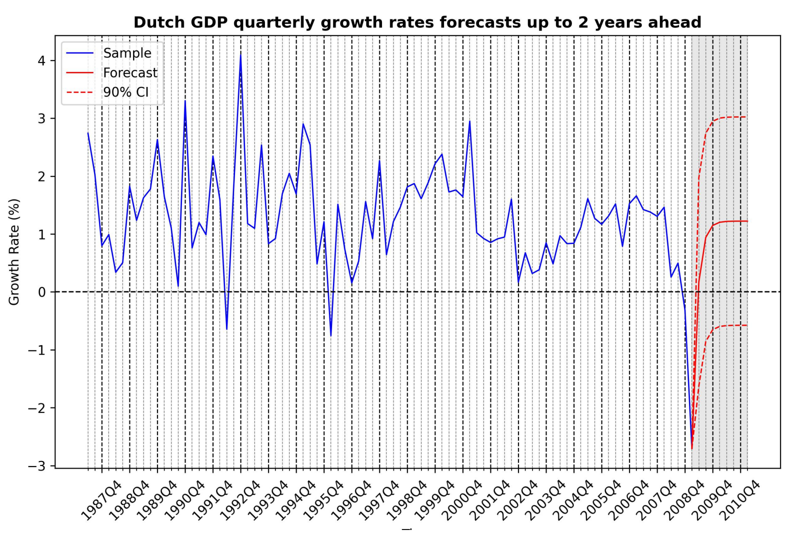

Fig 4.1 95% confidence interval of the forecast

Part II: Impulse Response Functions, Autoregressive Distributed Lag Models, and Granger Causality

Question 1

Figure 5.1 reveals the sample of *Dutch quarterly unemployment rates* and *Dutch GDP quarterly growth rates* respectively.

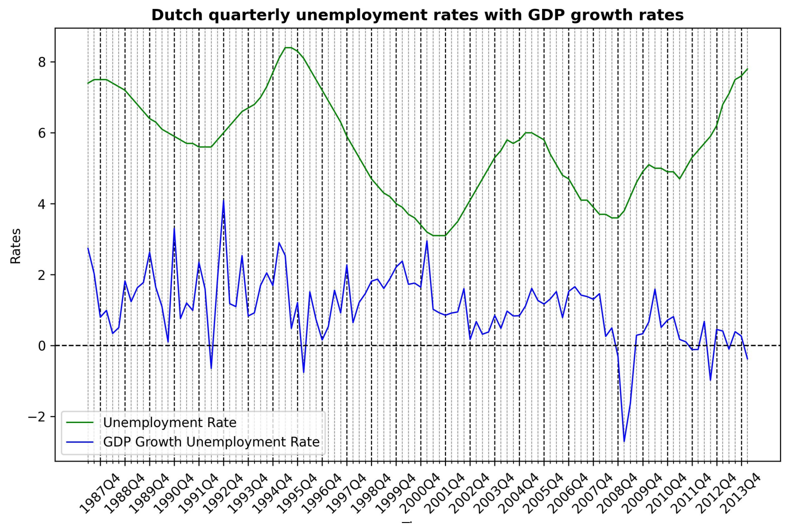

Figure 5.1 Dutch quarterly unemployment rates and Dutch GDP quarterly growth rates

Estimate the AR(p) model with a maximum $p = 4$ lags. Based on the Akaike information criterion (AIC), select the model with the lowest value of the information criterion. Table 5.1 shows the values of AIC for corresponding estimated models.

AR(p)	AIC
1	-27.402526
2	-27.596039
3	-32.063605
4	-30.546780

Table 5.1 Akaike Information Criterions for different AR(p) models

As a result, using AIC, AR(3) model should be the best model to describe the dynamics of the Dutch GDP quarterly growth rate. The estimated coefficients are shown in Table 5.2.

Coefficient	Value
α	0.374459
ϕ_1	0.334066
ϕ_2	0.070056
ϕ_3	0.221246

Table 5.2 The estimated coefficients for estimated AR(3) model

The estimated AR(3) model is:

$$X_t = 0.374 + 0.334X_{t-1} + 0.07X_{t-2} + 0.221X_{t-3} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$$

The coefficients are $0.374, 0.334$, and 0.070, indicating significant persistence and the influence of past growth rates on the outcomes. The dynamics shows that GDP growth is impacted not just be the most recent quarter but also by the history of the past two or three quarters.
Estimate the ADL(p, q) model with maximum $p = 4$ and $q = 4$ lags. Based on the Akaike information criterion (AIC), select the model with the lowest value of the information criterion. Table 5.3 shows the values of AIC for corresponding estimated models.

ADL(p, q)	AIC
(1,1)	-331.081207
(1,2)	-336.554599
(1,3)	-337.821944
(1,4)	-339.463030
(2,1)	-450.037393
(2,2)	-450.122618
(2,3)	-451.089826
(2,4)	-451.902496
(3,1)	-455.483881
(3,2)	-455.605025
(3,3)	-457.198646
(3,4)	-457.353364
(4,1)	-454.496211
(4,2)	-454.576435
(4,3)	-456.364361
(4,4)	-456.621331

Table 5.3 Akaike Information Criterions for different ADL(p,q) models

As a result, using AIC, ADL(3,4) model is the lowest with $AIC = -457.353364$, which should be the best model to describe the dynamics of the Dutch unemployment rate. The estimated coefficients are shown in Table 5.4.

Coefficient	Value
α	0.183940
ϕ_1	1.575612
ϕ_1	-0.343417
ϕ_1	-0.259779
β_0	-0.010593
β_1	-0.020603
β_2	-0.009545
β_3	0.018781
β_4	-0.005376

Table 5.4 The estimated coefficients for estimated ADL(3,4) model

The estimated ADL(3,4) model is:

$$Y_t = 0.184 + 1.576Y_{t-1} - 0.343Y_{t-2} - 0.26Y_{t-3} - 0.011X_t - 0.021X_{t-1} - 0.01X_{t-2} + 0.019X_{t-3} - 0.005X_{t-4} + \varepsilon_t$$

where $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$.

Revise For unemployment, the ADL$(3, 4)$ model we chose with the lowest AIC(-457.353). The estimated coefficient reveal a strong autoregressive component that unemployment depends much on its past values. Negative coefficients on GDP growth lags indicates that higher economic growth reduces unemployment.

Question 2

Coefficient	Estimate	t-Statistic	p-Value
α	0.183940	3.350503	0.000807
ϕ_1	1.575612	15.848529	0.000000
ϕ_2	-0.343417	-1.833577	0.066717
ϕ_3	-0.259779	-2.604875	0.009191
β_0	-0.010593	-0.733466	0.463274
β_1	-0.020603	-1.355751	0.175178
β_2	-0.009545	-0.634061	0.526041
β_3	0.018781	1.228312	0.219330
β_4	-0.005376	-0.369037	0.712100

Table 5.4: Estimated coefficients with t-statistics and p-values for the ADL(3,4) model

Based on the p-values for the estimated coefficients shown in Table 5.4, at a 10% level of significance, since none of the lagged GDP growth terms are significant, it suggests that do not reject the null hypothesis, which yields that GDP growth does not *Granger-cause* unemployment.

Question 3

The long-run equilibrium of a time series $\{Y_t\}$ is generated by an ADL process is the value $\bar{Y}$ to which $\{Y_t\}$ converges if $\{X_t\}$ is fixed at $\bar{X}$ $\forall$ t and there are no errors. Therefore, For the ADL(3,4) model,

$$Y_t = \alpha + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 X_{t-4} + \varepsilon_t$$

where $\bar{Y}$ is found by setting $X_t = \bar{X}$ and $\varepsilon_t = 0$ for all times t, which is defined by:

$$\begin{aligned}
\bar{Y} &= \alpha + \phi_1\bar{Y} + \phi_2\bar{Y} + \phi_3\bar{Y} + \beta_0\bar{X} + \beta_1\bar{X} + \beta_2\bar{X} + \beta_3\bar{X} + \beta_4\bar{X} + 0 \\
(1 - \phi_1 - \phi_2 - \phi_3)\bar{Y} &= \alpha + (\beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4)\bar{X} \\
\bar{Y} &= \frac{\alpha}{1 - \phi_1 - \phi_2 - \phi_3} + \frac{\beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4}{1 - \phi_1 - \phi_2 - \phi_3}\bar{X}
\end{aligned}$$

By substituting the values into the long-run equilibrium relation, the long-run equilibrium relation between the unemployment rate and the GDP growth rate is:

$$\bar{Y} = \frac{\alpha}{1 - \phi_1 - \phi_2 - \phi_3} + \frac{\beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4}{1 - \phi_1 - \phi_2 - \phi_3}\bar{X}$$

Question 4

The IRFs analysis two scenarios: a positive GDP shock of $+2\%$(good scenario) and a negative GDP shock of -2%(bad scenario).
In the good scenario, GDP growth has a sudden increase immediately, the gradually decays back to the baseline. The unemployment rate falls in a short term and then converges back to equilibrium. The curve shows that the positive shock can reduce unemployment but not in the long term.
In the bad scenario, GDP growth declines sharply and then recover back. Unemployment rises significantly after the negative shock and remains for a short period then goes down.
In both scenarios, the curves of unemployment rate adjustment seem to be asymmetric.

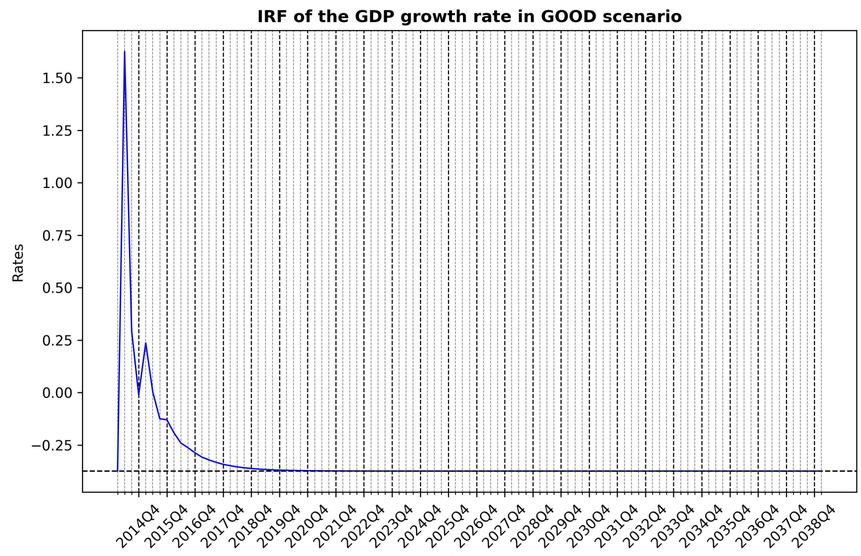

Figure 8.1 The IRF for the GDP growth rate in 'good' scenario

Figure 8.2 The IRF for the unemployment rate in 'good' scenario

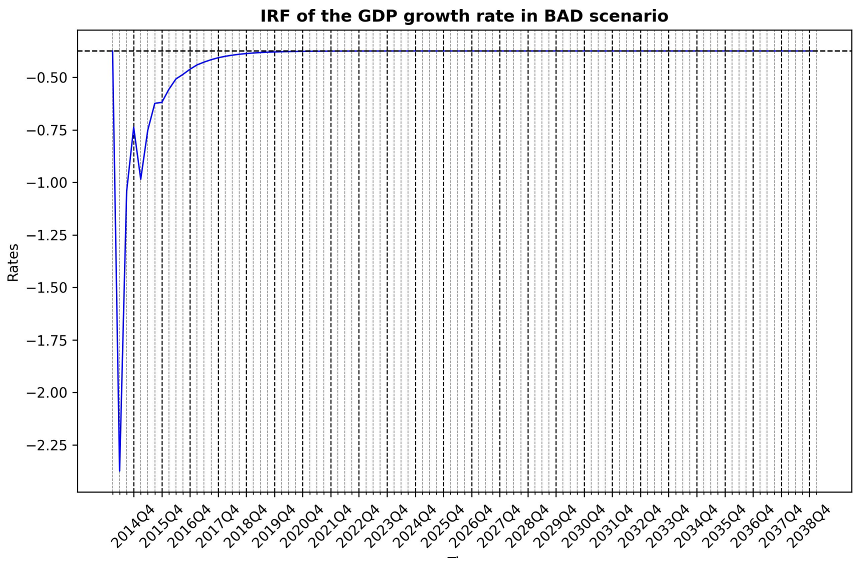

Figure 8.3 The IRF for the GDP growth rate in 'bad' scenario

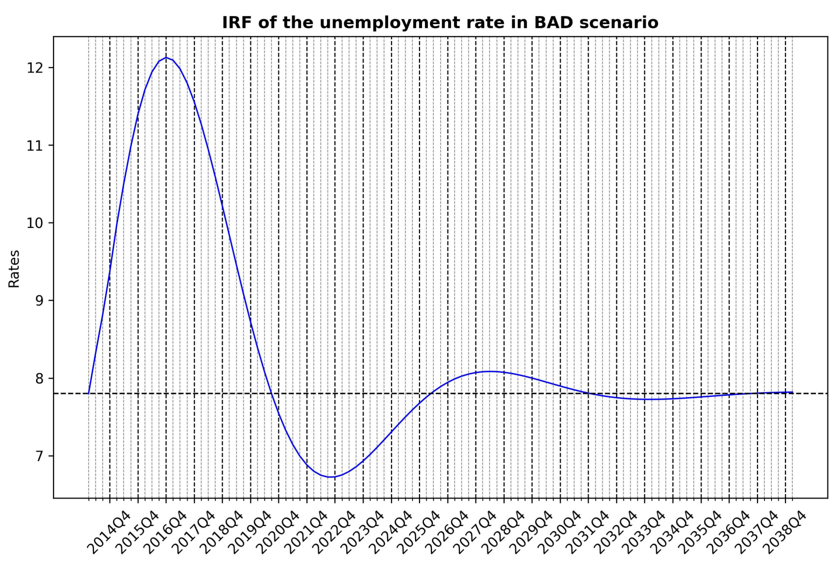

Figure 8.4 The IRF for the unemployment rate in 'bad' scenario