Mathematical and Data Modelling 3

EMAT30005

Mathematical Model of a Park Lock System

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Abstract

Park lock systems are one of several critical safety mechanisms in modern automatic transmission vehicles. Legally, the parking gear must be able to hold the weight of the vehicle on its own under certain conditions. These conditions outline the performance requirements; the parking system must not engage at vehicle speeds of over 5km/h and must be able to hold the car on a gradient of up to 30%. Both of these performance requirements are investigated in this project. An ordinary differential equation is derived using Lagrangian principles to model the behaviour of the components and then solved to find the time needed to engage. The equations that describe the motion of these components incorporate certain parameters which have been found to affect the successful engagement of the system the most - referred to as 'critical design features'. We found that the design of the parking gear has the most profound effect on the range of values that the other parameters can take. Final sets of parameter values that were tuned to satisfy both performance requirements were found. A Graphical User Interface was also created to allow ease in varying parameter values and generating engagement plots - we used this as our primary testing method for parameter values.

Contents

NOMENCLATURE TABLE

Parameters	Function
m_r	Mass of actuator rod and compression spring
I_p	Moment of inertia of pawl
k_p	Pawl return spring constant
l_0	Natural length of compression spring
l_c	Compressed length of actuator rod spring
ν_0	Default twist of pawl return spring
$v_r(0)$	Initial velocity of actuator rod
ρ	Final drive gear ratio
R	Radius of vehicle's wheel
Variables	Function
x	Position of actuator rod
v_r	Velocity of actuator rod
ν	Angular position of pawl
γ	Offset between gear tooth and pawl tooth
ϕ	Angular position of parking gear
θ	Angle of slope on which vehicle is parked
Critical Design Features	Function
α	Angular size of tooth gap
β	Parking pawl tooth size, measured as an angle
k_r	Compression spring constant
e	Pawl-gear coefficient of restitution

1 Introduction

In order for vehicles to successfully remain stationary when parked, they must be equipped with a system to stop the wheels rolling. The hand brake installed in vehicles is typically enough, however standard safety requirements call for a supporting mechanism to lock the car in place.

Manual transmission vehicles mechanically connect to the engine via the clutch; hence the large resistance set by engine compression must be overcome for the car to roll. This is considered enough to meet safety specifications. Automatic transmissions, however, connect to the engine via a torque converter - this is a fluid coupling device and its resistance to rolling is provided by the viscosity of its hydraulic fluid. The reason torque converters are used is that, by design of automatic vehicles, some slippage between the engine and transmission is necessary to curb stalling each time the wheels temporarily stop. The fluid must have a low viscosity for this reason and therefore the torque converter cannot be used in the same way as a clutch - i.e. for 'engine braking'. In these vehicles a separate parking gear is necessary.

The objective of this report is to create a mathematical model of a parking gear lock system to be examined and modified in order to meet the performance requirements laid out by law. The most common method used in industry for meeting these conditions is currently trial and error. This, however, involves potentially wasting dozens of parking gear lock system prototypes and is a petty drain of time and money for companies. Additionally, every company that decides to start producing a new car must go through this wasteful process. Our report outlines a more systematic approach to their designs.

The legal performance requirements (PRs) of a parking gear system are to inhibit the park lock engaging at speeds of more than 5km/h (PR1) and to prevent the car rolling down gradients of up to 30% (PR2) [1]. Other performance guidelines of the park lock are that it should be able to hold for a lifetime of usage and the park lock must not be able to self-disengage [2], however, these will not be explored in detail in this project. The critical design features which we found in our preparatory research - such as spring stiffness, tooth gap, and component shape design - are included in the modelling procedure and are explored so as to meet the PRs. The park lock system contains two sets of independent components that become dependent after the parking lock is engaged. The first set of components, the actuating set (rod and pawl) are modelled using Lagrangian principles, and the second set, consisting of the parking gear, is modelled both as a function of the velocity of the car (PR1) and as a function of slope gradient (PR2). The resultant equations of motion are built into a custom-made program with a graphical user interface (GUI) that allows the user to adjust critical design features and parameters.

2 Theory

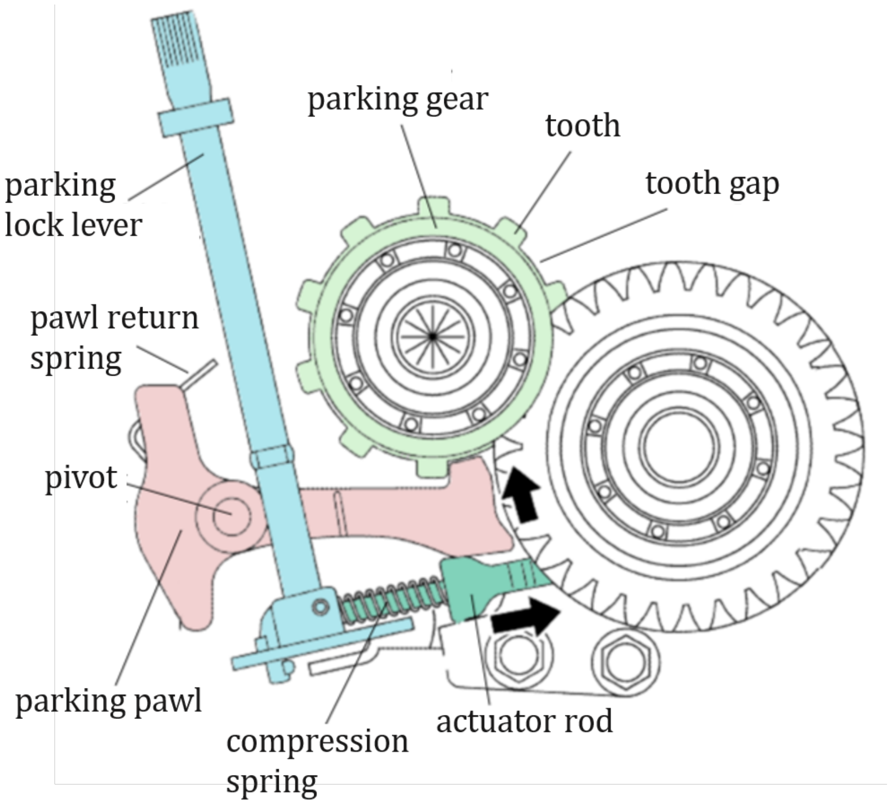


Figure 1: This schematic shows all essential components of the parking lock system in the engaged setting, with arrows tracing their path to engagement.

2.1 Engagement

Moving the park lock lever into the park position compresses the compression spring, eventually pushing the actuator rod forward. Due to the sloped profile of the actuator rod, this movement forces the parking pawl to rotate about its pivot pin (shown by the dark arrows in Figure 1) - here it attempts to engage in a tooth gap on the parking gear. The rotation of the parking pawl about the pivot is dependent only on the displacement of the actuator rod; this is because the pawl return spring is twisted such that it always exerts a downward force on the pawl, ensuring that it and the actuator rod are always in contact.
When the vehicle is travelling slower than 5km/h on a gradient shallower than 30% and is then geared to park, the parking pawl engages into the parking gear (linked to the drive shaft through a series of gears) and prevents the wheel axle of the car from turning. It should be noted that at this point the compression spring has returned to its natural length and the actuator rod is translated to a new position.
When the vehicle is driving faster than 5km/h the parking pawl attempts to engage in the tooth gap but should fail because the parking gear is spinning too fast. If the pawl hits the top of a parking gear tooth it will ratchet and jitter (bounce against the teeth of the parking gear) until the system is disengaged. Alternatively, if the pawl tooth does happen to fall into a gap in the parking gear, the momentum of the parking gear should end up pushing the parking pawl back out again (as per the legal requirements outlined in Section 1). In this case, the pawl could break or the compression spring would be forced into compression until the system is disengaged by means of the lever or if the car is slowed [3].

2.2 Disengagement

The gear stick must be moved out of the parking position. This pulls the compression spring into extension and naturally, the spring reverts back to its original length by retracting the actuator rod. This allows the pawl to decouple from the parking gear as it rotates into the space freed by the rod by the aid of gravity and (much more significantly) the pawl return spring's torsion.

3 System Mechanics

This section describes our fundamental work used to model the parking lock system. For readability key variables and terms can be found again listed on Page 1.

3.1 Model Assumptions

1 The actuator rod and pawl are always in contact - this is a result of the pawl return spring's sustained torsion. This allows us to model the two components as one system.

2 We ignore friction as its effects are largely insignificant for smooth metallic components. Head of Engineering Services for Romax Technology Ltd. and University of Bristol alumnus Chris Halse indicated the most likely material to be an alloy of steel - dry surfaces of this kind tend to have $\mu_{static} \lesssim 0.35$ [4]. Combined with the small time scale our models consider, the force of friction is considered negligible.

3 The effects of gravity on the rod-pawl system are ignored as vertical displacements are negligible within the scope that the equations consider.

4 The car is assumed to be at a constant velocity when the parking lock is engaged - this can be any rational number input when the model considers flat ground, but is set to zero when testing the second PR. The assumption is necessary to validate our calculations of the engagement window. Adding inertia to the model would introduce redundant complexity as it's generally only applied in low-momentum scenarios (around 5km/h speeds).

5 Our use of Lagrangian principles includes the assumption that energy is totally conserved within the rod-pawl system - i.e. no heat or noise is produced and the initial rod-pawl impact is elastic.

6 The pawl's angular momentum is insignificant compared to the parking gear's. This is so that when ratcheting the collisions don't affect the parking gear's dynamics and our equations still hold.

7 The compression spring obeys Hooke's law. This is done, firstly, so that we may relate the spring's restorative force and the position of the rod. Secondly this allows us to couple the compression spring with the actuator rod and model them as a spring-mass oscillator under simple harmonic motion (SHM). This is a common and safe assumption, and is applied when the rod position or velocity is needed.

3.2 Rod-Pawl Mechanics

To begin with we model a one-degree-of-freedom Lagrangian system using a generalised coordinate, x, which describes the position of the actuator rod in relation to where it would be if the parking pawl was fully engaged with the parking gear. We can include the parking pawl in this system without adding any more co-ordinates; its rotation ν is solely dependent on x. This is a corollary to Assumption 1, and is expressed as $\nu = f(x)$. Thus, the kinetic and potential energies are given by

$$T = \frac{1}{2}m_r\dot{x}^2 + \frac{1}{2}I_p\dot{\nu}^2 \tag{1}$$

and

$$U = \frac{1}{2}k_r x^2 + \frac{1}{2}k_p(\nu - \nu_0)^2 \tag{2}$$

respectively. The subscripts r and p denote the actuator rod and parking pawl respectively, and m_r is the mass of the actuator rod and compression spring. $\dot{x}$ and $\dot{\nu}$ are the derivatives of x and ν with respect to time (linear & angular velocities). The constant ν_0 is the twist of the pawl return spring measured from equilibrium. Using equations (1) and (2), we apply Lagrange's formula to find the equation of motion describing the rod-pawl system:

$$L = T - U = \frac{1}{2}m_r\dot{x}^2 + \frac{1}{2}I_p\dot{\nu}^2 - \frac{1}{2}k_r x^2 - \frac{1}{2}k_p(\nu - \nu_0)^2 \tag{3}$$

and given

$$\frac{d}{dt}\frac{dL}{d\dot{x}} - \frac{dL}{dx} = 0 \tag{4}$$

then, using $\nu = f(x)$, we obtain

$$\ddot{x}[m_r + I_p f'(x)^2] + \dot{x}^2[2I_p f'(x)f''(x)] - \left(\frac{1}{2}\dot{x}^2[2I_p f'(x)f''(x)] - k_r x - k_p f'(x)[f(x) - \nu_0]\right) = 0 \quad (5)$$

Equation (5) can be simplified if we define $f(x)$ to be linear, i.e. of the form $f(x) = Ax + B$. All second derivatives are now 0 and the equation of motion simplifies to a non-homogeneous, second order, linear ordinary differential equation (ODE):

$$\ddot{x}[m_r + I_p f'(x)^2] + k_r x + k_p f'(x)[f(x) - \nu_0] = 0 \quad (6)$$

The linearisation above arises from the assumption that the surfaces of the parking pawl and actuator rod are always in contact; they can be considered, in more abstract terms, as a point on a surface instead of a surface on a surface. Although this neglects some of the friction present between the two components, we are interested here in the general behaviour of the parking pawl in relation to the actuator rod more than the details of their dynamics.

3.3 Gear Mechanics

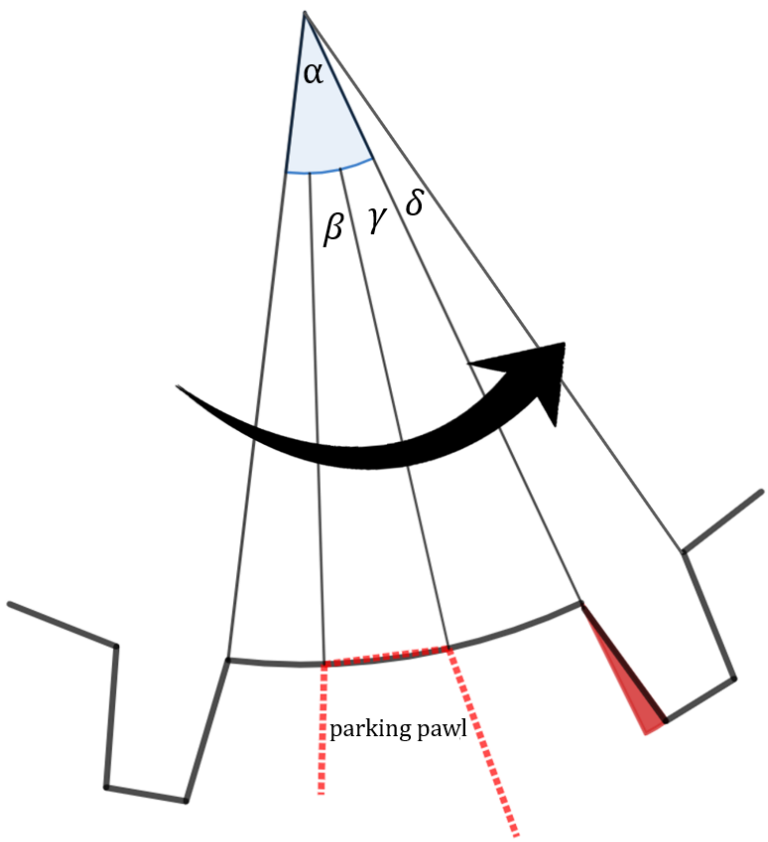


Figure 2: Diagram of the parking gear component. α defines the angle between opposite sides of adjacent teeth, β defines the angular size of the pawl tooth, γ is the offset between the bordering sides of the pawl and the prior tooth, and δ (used in Section 3.5) is the angular size of the prior tooth.

Consider a car wheel on an axle linked to the parking gear axle via a gear train (see Figure 8). We know the ratio of their angular velocities is constant and so we expand accordingly:

$$\rho = \frac{\dot{\phi}_{gear}}{\dot{\phi}_{wheel}} = \frac{(\alpha - \beta - \gamma)/T}{v_{car}/R} = \frac{R(\alpha - \beta - \gamma)}{T v_{car}} \quad (7)$$

Here R is the vehicle's wheel radius, α, β, and γ are defined pictorially in Figure 2, T is the time taken by the gear to sweep the unlabelled angle $\alpha - \beta - \gamma$, and ρ is the gear ratio between the final drive gear and the car's wheel. The gear ratio can be defined as the ratio of angular velocities, although practically it's

also the ratio between torque outputs, number of teeth, and radii.
We defined T the way we did so that it also represents the time in which the pawl may engage in the gear; it measures how long until the pawl will overshoot the current tooth gap. Re-formulating Equation (7) so that T is the subject gives us the engagement window:

$$T = \frac{R(\alpha - \beta - \gamma)}{\rho v_{car}} \tag{8}$$

This derivation follows from Assumption 6 as we presume the instantaneous angular velocity, $\dot{\phi}_{gear}$ normally derived using calculus and limits, can also be calculated by $\frac{\text{angle swept}}{\text{time taken}}$.
Separate to modelling the gear mechanics, we must also meet legal and industry requirements. For this we say the parking gear should have symmetric teeth. The symmetry in each tooth will allow the car to park facing both up and down a slope as the pawl will behave the same in both instances. Second, the tooth flank angle (the small angle in the red triangle in Figure 2) should be kept small. This is to help reduce the occurrence of ratcheting, as the pawl tooth is unlikely to slide up the flank and disengage inappropriately with perfectly radial teeth.

3.4 Ratcheting

The effect of ratcheting is an expected by-product of the system design. Our program implements the ratcheting effect very simply, although this can be developed easily with the use of loops. In order to model the pawl-gear impact we look at the actuator rod's velocity at the instant before the impact - Assumption 1 allows us to use this value as the parking pawl's pre-collision velocity. The model takes the pawl-gear coefficient of restitution (CoR, e) as an input and uses it to calculate the post-collision velocity (which will face opposite to the inbound velocity). A CoR approaching $e = 1$ translates to a more elastic collision, where a perfectly elastic collision (at $e = 1$) loses no kinetic energy and the objects rebound with the same relative speeds. Using the Newtonian formula for e we now again know the parking pawl's position (on the gear's circumference) and angular velocity (about the pivot pin). Ideally we would use both of these as new initial conditions for the ODE - this may be done by adding a loop structure, but it's been omitted from the model as the ratcheting phenomenon doesn't align with this project's aims and time constraints. The program does make use of the outbound pawl velocity, however, the pawl's position is not stored but instead set to the start of the next tooth gap (i.e. $\gamma = 0$). This spares a lot of computing power but is not thoroughly realistic as the pawl is guaranteed to lock successfully when $\gamma = 0$.

3.5 Parking on a Slope

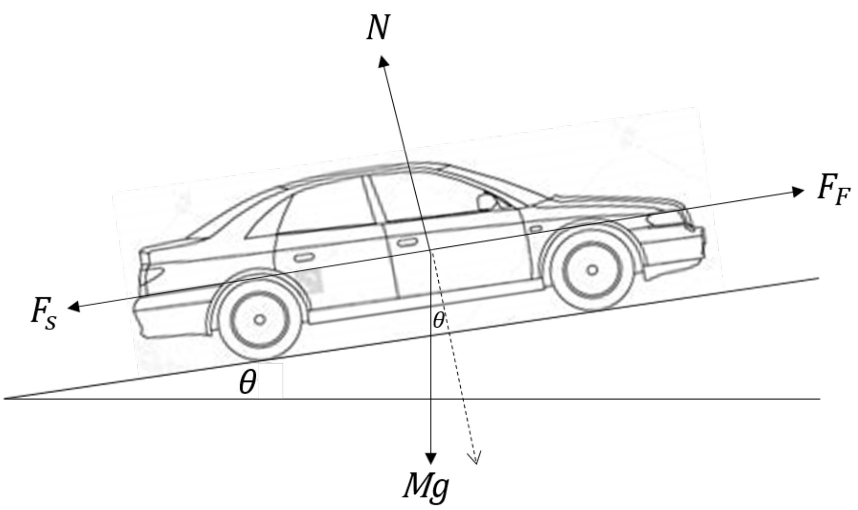


Figure 3: Diagram showing the forces acting on a vehicle on a slope of angle θ. All forces are modelled as acting on the vehicle's centre of mass.

The main difference between implementing PR1 and PR2 is that the vehicle is no longer considered to be at a constant velocity when applying PR2, but instead to be accelerating from stationary down a slope. Figure

3 shows the forces acting on a vehicle when this is the case; Mg is the gravitational force on the vehicle and F_S is its component parallel to the slope. This is opposed by the friction force F_F and the normal force N - both actually act on the contact area between the tyres and the road but are modelled as originating from the vehicle's centre of mass.

$$F_S = Mg\sin(\theta) \qquad N = Mg\cos(\theta) \qquad F_F = \mu Mg\cos(\theta)$$

The equations above describe what these forces represent; M is the mass of the vehicle, g is the acceleration due to gravity, θ is the slope angle in radians, and μ is the coefficient of static friction. To find out if the parking pawl can engage or not, the angular velocity of the parking gear is required. The angular velocities of the parking gear and the wheels are related by the final drive gear ratio ρ, thus if the distance the wheel has moved can be calculated then the angular velocities can be calculated and the time constraint on the parking pawl's engagement can be found. The studied case is when a vehicle comes to stop on a slope and the driver engages the park lock system with the parking pawl in the least favourable position, whether the pawl will be able to engage or not by the time the gear has rotated. The 'worst case scenario' is when the pawl narrowly misses a tooth gap and instead hits the next tooth. Here the pawl has not engaged and the vehicle will roll and lock into the next gap given PR1 is not violated. Since the vehicle is initially stationary, the distance the vehicle's wheel can roll is

$$d_{\max} = R\psi_w \tag{9}$$

where

$$\psi_w = \frac{\psi_g}{\rho} \tag{10}$$

and

$$\psi_g = \frac{2\pi}{n} = \alpha + \delta \tag{11}$$

Here n is the number of teeth on the parking gear, R is the radius of the wheel and ρ is the final drive gear ratio (see Figure 8). Equation (11) is the maximum angle the parking gear can rotate if the pawl were to engage in the next tooth gap, thus ψ_g denotes the combined angle of the tooth gap, α, and the angular size of one tooth, δ. From Equations (9) and (10) we can compute the angular velocity of both the wheel and the parking gear:

$$\dot{\phi}_{wheel} = \frac{v_{\max}}{R} = \frac{\sqrt{2a_{\text{slope}}d_{\max}}}{R} = \frac{\sqrt{2g\sin(\theta)d_{\max}}}{R} \tag{12}$$

$$\dot{\phi}_{gear} = \rho\dot{\phi}_{wheel} \tag{13}$$

In Equation (12), $v_{\max}$ is the maximum velocity gained from the car rolling and a_{slope} is the acceleration of the car down the slope, assuming gravity is the only factor. This can be found by balancing forces in Figure 3. $v_{\max}$ is calculated from the equation $v^2 = u^2 + 2as$, with initial velocity $u = 0m/s$ and $s = d_{\max}$. Equation (13) links the wheel's angular velocity with the parking gear's through the final drive gear ratio relationship. Derived similarly to Equation (8), the engagement window is

$$T = \frac{R(\alpha + \delta - \beta - \gamma)}{\rho v_{\max}} \tag{14}$$

with the only difference being the added tooth angle, δ, resulting in a slightly larger window.

4 Generating Data

The complexity of our governing ODE and the numerous vehicle-specific parameters it requires meant that purely maths-based approaches would not achieve the depth of analysis we need. In order to generate useful, representative graphs and investigate the effects of different design features our project calls for an integrated GUI that allows us to easily adjust any parameter or critical design feature. A GUI was created in the MATLAB App Designer space, seen in Figure 11. The GUI includes a toggle that switches between testing for PR1 and PR2, the predominant difference being the addition of acceleration in PR2 tests. Other small differences are mostly program-related, such as setting the car velocity to zero and the slope angle to non-zero. Although a program could theoretically test both PRs in conjunction, this capability wasn't included to eliminate redundant code complexity and due to time constraints. The program also includes a back-end conversion between user-friendly degrees and computer-friendly radians.

Another point of interest is the way we indicate engagement: using initial conditions $x(0) = l_c - l_0$ where l_c is the compressed spring length and $v(0) = v_r$, we defined engagement to be at the actuator rod's point of zero displacement. Therefore with this information, and following on from Assumption 1, we may conclude that a zero-displacement actuator rod indicates an engaged parking pawl.

4.1 Parameter Research

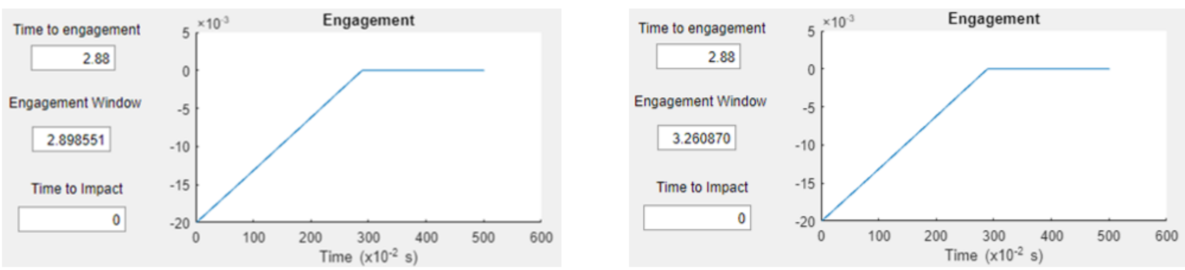
In order to accurately explore the critical design features, the parameters for different components of the park lock system must be approximated. This means doing research on what the values will be in real life for a standard car.
Firstly looking at the mass of the actuator rod we found it to be in the region of 0.7kg to 1kg [5]. Since we have only found this range from one source, this will not be a strict range. Looking at what happens at 0.7kg and above was investigated since we think the range we found is lower than in reality. The article used to find the actuator rod mass [5] also tells us that the force exerted by the spring needs to be about 25N. Therefore we can say the constant k is in the region $25/0.02 = 1250N/m$. Again, we will investigate a range surrounding this, $1250N/m \pm 250N/m$. In Equation (5) you can see there is a term $f(x)$, where $f(x) = ax$, we have assumed a=1 since this is most simple. The moment of inertia of the pawl we have assumed to be and the pawl return spring constant we have assumed to be. This is because we have not been able to find any data on them so have assumed them to be the same throughout the whole thing. The natural length and compressed length of the actuator rod spring are approximated 0.05m and 0.03m respectively, and have been approximated from a video [6]. Looking at the wheel radius, we must take into account not only the hub cap but the tyre as well. So we must look at the tyre radius, which is average about 20 inches [7], which roughly equates to 0.5m. There is also the initial velocity of the rod, which is very difficult to find out without actually doing an experiment with an existing system. Since we cannot do that, we are going to allow this parameter.

5 Model Analysis

5.1 Performance Requirement 1

This scenario will be our basis for implementing PR1. Here we numerically solve Equation (6) to give the engagement time and use Equation (8) to give the engagement window. The scenario also models the pawl-gear collision and subsequent ratcheting of the pawl when it fails to engage initially, as outlined in Section 3.4. With regards to our model this is equivalent to setting $\alpha < \beta + \gamma$.
A critical design feature central to meeting PR1 is the difference of angles $\alpha - \beta$ (γ is a random offset). Increasing α (sub-angles kept proportional) or decreasing β increases the engagement time; fundamentally this equates to larger tooth gaps, and physically this is achieved by decreasing the number of teeth (directly increases α) or decreasing the angular size of the pawl tooth (decreases β). The quantity $\alpha - \beta$ can therefore also be used to tune the maximum car speed at which the system will lock - a larger value for $\alpha - \beta$ will allow for a higher maximum speed as the increase in both of them will divide, as seen in Equation (8). This will keep the engagement window constant if they're increased by the same factor.

Figure 4: A plot of the actuator rod's displacement over time - note again the actuator rod's displacement is interchangeable with the pawl's as an application of Assumption 1. This demonstrates how $\alpha - \beta$ affects the engagement window; reducing β by 1° adds 0.36 seconds. The pawl's dynamics and engagement time are unaffected.



(a) Engagement plot where the pawl tooth angle $\beta = 4^\circ$. *(b) Engagement plot where the pawl tooth angle $\beta = 3^\circ$.*

A second critical design feature is the compression spring constant; by using a stiffer spring (one with a higher spring constant) the pawl actually takes longer to lock with the gear. In the confines of an automatic transmission the parking break is engaged by means of the parking lever - when put in the parking position there's an initial force on the spring (originating at the parking lever) starting the chain of events that'll lock the car in place. Hooke's Law can relate this force and the compression spring's deformation by $F = k_r x$. Now looking at the restoration dynamics for this spring, we assume all potential energy imparted by the parking lever is transformed into kinetic energy

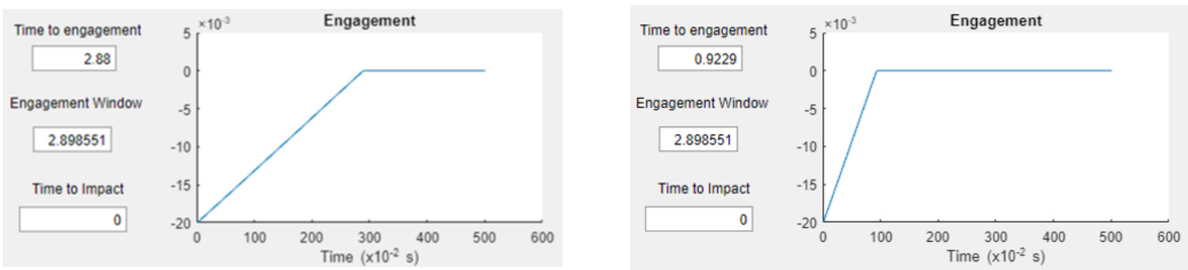
$$\frac{1}{2}k_r x^2 = \frac{1}{2}m_r v_r^2 \tag{15}$$

and rearrange to give

$$v_r = x\sqrt{\frac{k_r}{m_r}} \tag{16}$$

where x is the deformation (identical to its definition in Section 3.2), m_r is the rod-spring mass, k_r is the compression spring constant, and v_r is the rod-spring velocity whilst extending to equilibrium. Hooke's relation, when F is fixed, shows an increased spring constant will decrease the deformation by the same factor. Equation (16) then uses this smaller deformation linearly whereas the larger spring constant appears under a radical sign. Specifically, increasing k_r by a factor $A > 1$ will scale v_r by a factor $\frac{1}{\sqrt{A}} < 1$. This is an inverse relationship between k_r and v_r. Citing Assumption 5 and the assumption that the initialising force on the compression spring is kept roughly constant when gearing to park, we can say a park lock system with a stiffer compression spring will take longer to engage.

Figure 5: These figures contrast the effects of a lesser compression spring stiffness k_r. The effects are slight for our given values hence our exaggerated comparison; reducing the order of magnitude by one yields an engagement time over three times smaller.

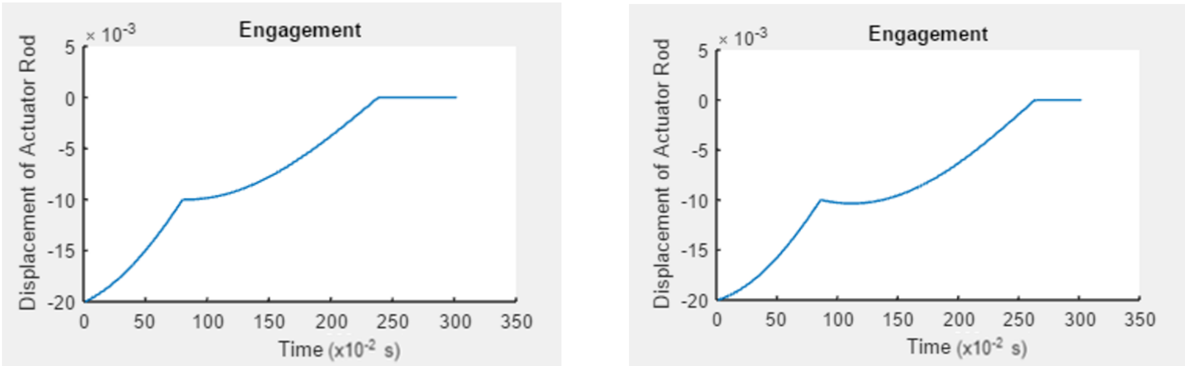


(a) Engagement plot where the compression spring stiffness $k_r = 1,000$N/m.

(b) Engagement plot where the compression spring stiffness $k_r = 100$N/m.

Another important factor in the ratcheting dynamics is the value of e. This is inherent to the combination of materials we use and - as mentioned in Section 3.4 - effectively controls the speed at which the pawl bounces off the parking gear after impact. As expected increasing the CoR causes the pawl's rebound curve to dip more, and the pawl takes longer to lock. This is evident when comparing Figure 6a, where the dip is shallow, and Figure 6b, where the engagement time is longer and the effect is more pronounced even though the two look identical until the recoil. Industrial considerations would favour a smaller CoR so as to raise the likelihood of re-engagement in case the pawl ratchets by shortening the time spent recovering from the bounce.

Figure 6: A comparison of different CoR values. The dynamics have been slowed and amplified for clearer analysis of the ratcheting effect. This was done by lowering the initial rod velocity and the compression spring stiffness accordingly.



(a) Showcase of the ratcheting effect where $e = 0.01$.

(b) Showcase of a stronger ratcheting effect where $e = 0.15$.

The final set of parameter values which we recommend for the parking gear lock system to meet PR1 are visible in our GUI in Figure 11.

5.2 Performance Requirement 2

In this model, the performance requirement being explored is the fact the vehicle must engage safely on a slope of up to 30%. Unlike the other models, the vehicle's velocity is no longer considered to be constant and is instead calculated from the vehicle's acceleration and distance travelled down the slope. Hence, the acceleration would be of greater magnitude on a steeper slope - resulting in a more restrictive engagement window. The model must work for slopes of up to 30% grade however, this requirement is not a strict limit like PR1 and therefore engagement at slightly higher slopes grades is not an issue. This case can also be considered as an optimal engagement even in the event that the pawl initially makes contact with a tooth since we model the engagement as the pawl sliding into position as the gear rotates meaning there is no impact and that $\gamma = 0$.

The first critical design feature is the tooth gap and size. A smaller tooth gap and tooth size equate to a smaller engagement window. In the simulations below, the default settings for parameter values were used (see Appendix, Table 1. The rod, spring and car parameters are varied for certain values of n, α, and β to find a range of possible values. These are only chosen values for demonstration and there is a (limited) number of configurations possible for each n. In each instance, only one parameter value was varied to find its possible range of values for that given configuration. These ranges can be seen in full in the Appendix in Table 2.

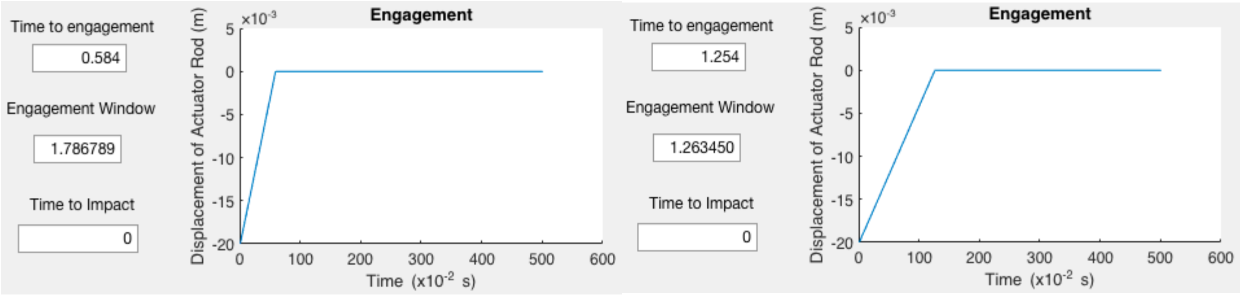
All simulations were used with a slope angle of $\theta = 16.7°$, since this is the critical value for PR2. However, note that a change in this value will result in a change in slope acceleration, with larger angles giving a larger acceleration. For this value of θ, the time it would take for the car to reach 5km/h would be

$$t = \frac{v-u}{a} = \frac{5-0}{\sqrt{2g\sin(16.7°)d_{\max}}} = 3.757\text{s},$$

which is longer than our engagement time. It is found that for any given value of rod mass the system will engage with small variations in the engagement time. Increasing the mass will decrease the engagement time slightly. The system also requires a minimum initial rod velocity of 0.163m/s (for default setting) to engage, with this value increasing as n increases (also, α & β decreases). For $n = 25$, $\alpha = 12°$, $\beta = 10°$, this velocity needs a minimum value of 0.26m/s.

The next critical design feature is the spring stiffness which yields a larger engagement time as it increases. For realistic values of k_r, the system will always engage for the given default parameter values. The same can be said for the wheel radius, although this increases the engagement window as opposed to engagement time. The length of the rod (compressed and uncompressed) doesn't have an effect on the system, only the difference between them. The range of the difference decreases as n increases, with a maximum value of 0.064m for the default settings. A smaller difference in the rod lengths yields a smaller engagement time.

Figure 7: These graphs show the rod (and thus the pawl) engaging after rolling down the slope.



(a) Simulation with default parameter values. For details, see Table 1.

(b) Simulation with $n = 20$, $\alpha = 15°$, $\beta = 10°$, spring stiffness 1,250N/m & initial velocity of the rod 0.26m/s.

The figures above are example simulations for parking on a slope. Figure 7a shows engagement for the default parameter values. As mentioned previously, the increase in spring stiffness and a decrease in initial rod velocity cause an increase in engagement time, while decreasing the tooth gap (increase in the number of teeth) cause a decrease in the engagement window. Both of these can be seen in Figure 7b. See figures 9 and 10 in the Appendix for full list of values used. The conclusive set of parameter values to meet this PR can also be found in the Appendix, see Figure 12.

6 Limitations

Research for this project was restricted as we had limited time and only public-access websites at our disposal. The main reason this was a problem is that our equations require many parameters specific to the vehicle in question and so finding information about niche components such as the pawl return spring, in particular standard materials and sizes, was difficult. For this reason our results come with a degree of inaccuracy, despite our sound assumptions and equations. We also only model the engagement of the system without investigating the dynamics of its disengagement - in analysing this we might find further complications to the design of the system. As of yet we have also not fully explored the geometry of the actuator rod and parking pawl and just assumed the horizontal position of the rod and the rotation of the pawl are linearly related. In future works more time should be spent on exploring the geometry here as it's vital for effective and cheap mass production.

Although we found configurations valid for each of the PR's, we don't look into finding a valid set of parameter values that allows us to engage both on a 30% incline *and* when travelling at 5km/h. This would either require a revamp of the underlying maths in our GUI program or a complex non-linear optimisation task. Due to time constraints, we cannot integrate the PR's into our system totally. Another area our project lacks in is friction. Friction has been observed to be the cause of self-locking and self-unlocking [2] so, if added, it could be treated as another critical design feature. Although the mathematics behind friction over these small timescales and distances can be made simple, its addition to the model would include materials science research and an overhaul of our GUI program, hence its omission.

A simplification our model makes is not considering ratcheting as a repeated effect - the program overrides the position of the pawl, setting $\gamma = 0$ and forcing it to automatically engage after a single bounce. This can be amended by methods discussed in Section 3.4. Correcting this is not paramount as the typical number of teeth on a parking gear and the final drive gear ratio mean the car would only roll a few centimetres if the pawl were to ratchet along a few more teeth.

Lastly, we were limited by computing power and programming experience. As the compression spring extends back to its natural length, we know the adjoined actuator rod begins with a velocity of 0 and accelerates under the laws of Equation (6). However with realistic inputs for most parameters MATLAB flags a memory error as it attempts to use gigabytes of RAM to compute the trajectory of the rod. We suspect this is due to inefficient code structure and choice of numerical integration technique. To avoid this issue we limit the time the program considers and we give the rod a non-zero initial velocity where necessary, essentially 'kicking' the rod as to reduce the number of repeat calculations necessary to complete the integration. The effects are shown in the figures below, where the 'kicked' system with a higher given initial rod velocity takes significantly less time to lock.

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Appendix

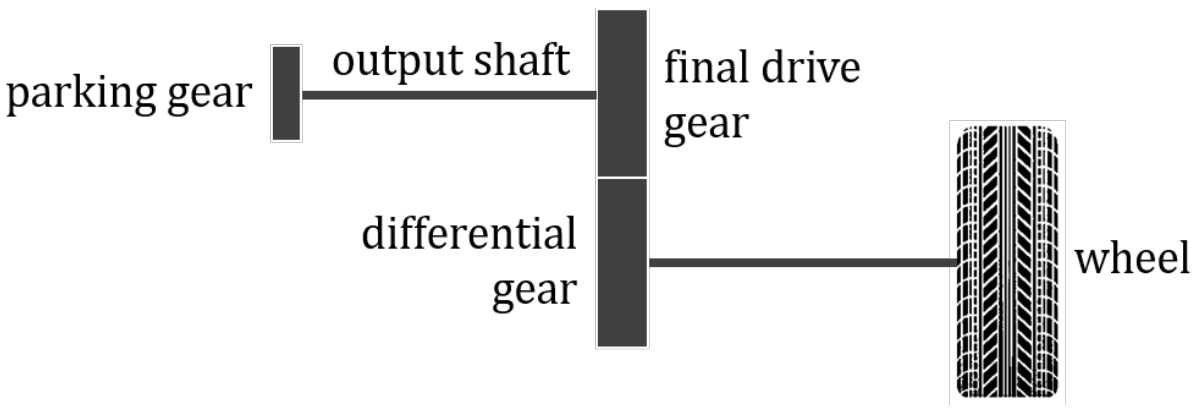

Figure 8: A depiction of how the wheels link to the parking gear via the differential and final drive gears.

Gear Parameters	
Tooth Gap, α	$30°$
Pawl Tooth, β	$25°$
Offset, γ	$0°$
Tooth, δ	$6°$
Actuator Rod Parameters	
Rod Mass, m	$0.7kg$
Spring Stiffness, k_r	$1000N/m$
Initial Velocity, $v_r(0)$	$0.5m/s$
Surface Curvature Coefficient, A	1
Spring Parameters	
Natural Length, l_0	$0.05m$
Compressed Length, l_c	$0.03m$
General Parameters	
Slope Angle, θ	$16.7°$
Car Wheel Radius, R	$0.5m$

Table 1: Default parameter values for the incline simulations. Research into value ranges were conducted by varying a certain parameter & keeping the others at these default values.

	Configuration				
Parameter	**Default**	**1**	**2**	**3**	**4**
Number of Teeth , n	10	15	20	25	30
Tooth Gap, α (°)	30	20	15	12	10
Pawl Tooth, β (°)	25	15	10	10	8
Tooth, δ (°)	6	4	3	2.4	2
Spring Stiffness, k_r (N/m)	< 9350	< 6250	< 4650	< 3750	< 3100
Initial Velocity, $v_r(0)$ (m/s)	> 01.63	> 0.2	> 0.23	> 0.26	> 0.285
Spring Deformation, $l_0 - l_c$ (m)	< 0.064	< 0.05	< 0.043	< 0.038	< 0.0353
Car Wheel Radius, R (m)	any	any	> 0.11	> 0.14	0.1605

Table 2: Range of possible values for spring stiffness, initial rod velocity, spring deformation, and wheel radius for the slope case where $\theta = 16.7°$. Values for rod mass m_r and the rod surface curvature coefficient A were kept constant at 0.7kg and 1 respectively.

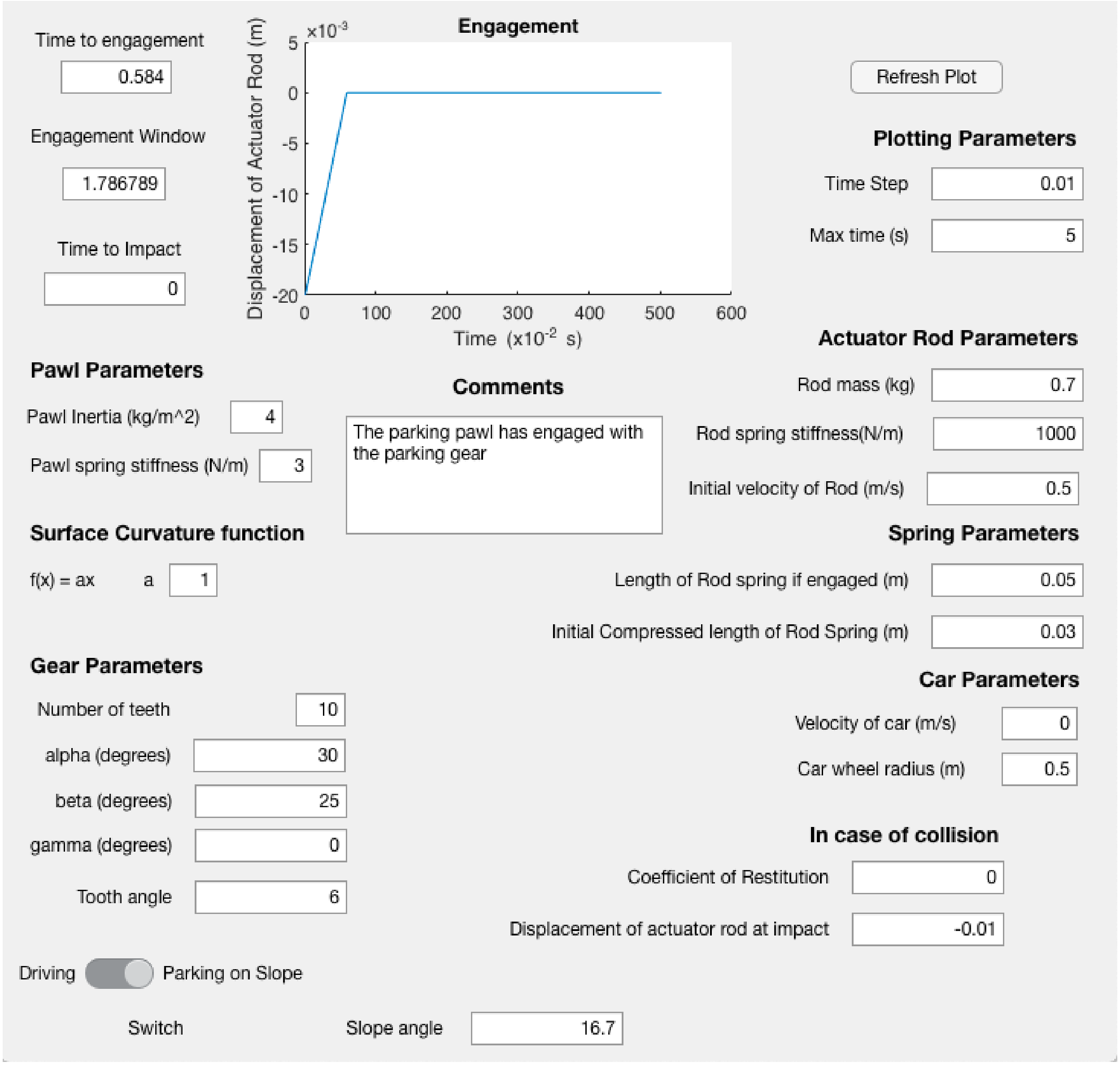

Figure 9: Our GUI showing the engagement plot on a 16.7° *slope for the default parameter values stated in Table 1. Note that the GUI ignores the velocity of the car when we consider an incline.*

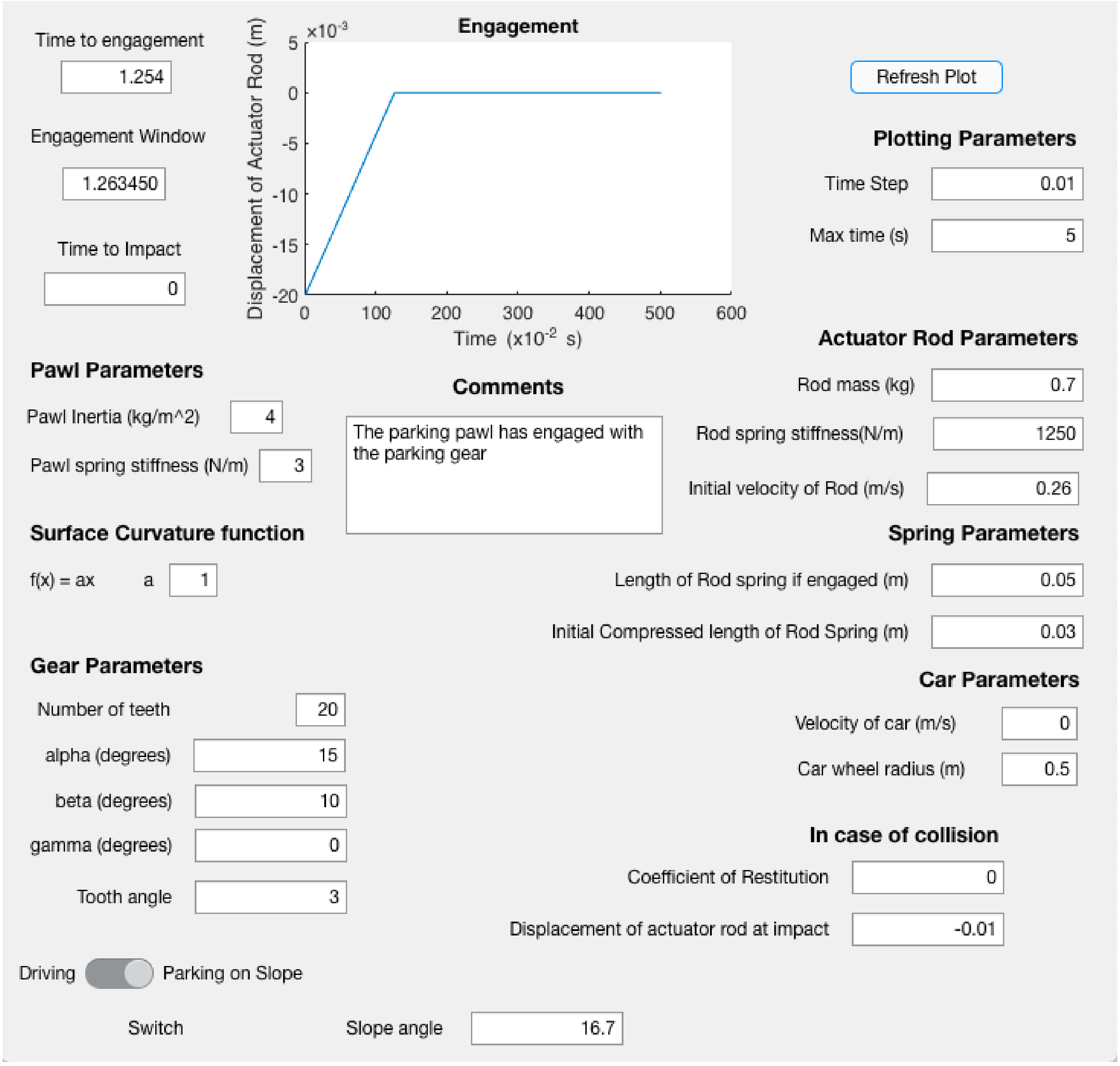

Figure 10: Our GUI showing the engagement plot on a 16.7° slope for the second configuration in Table 2.

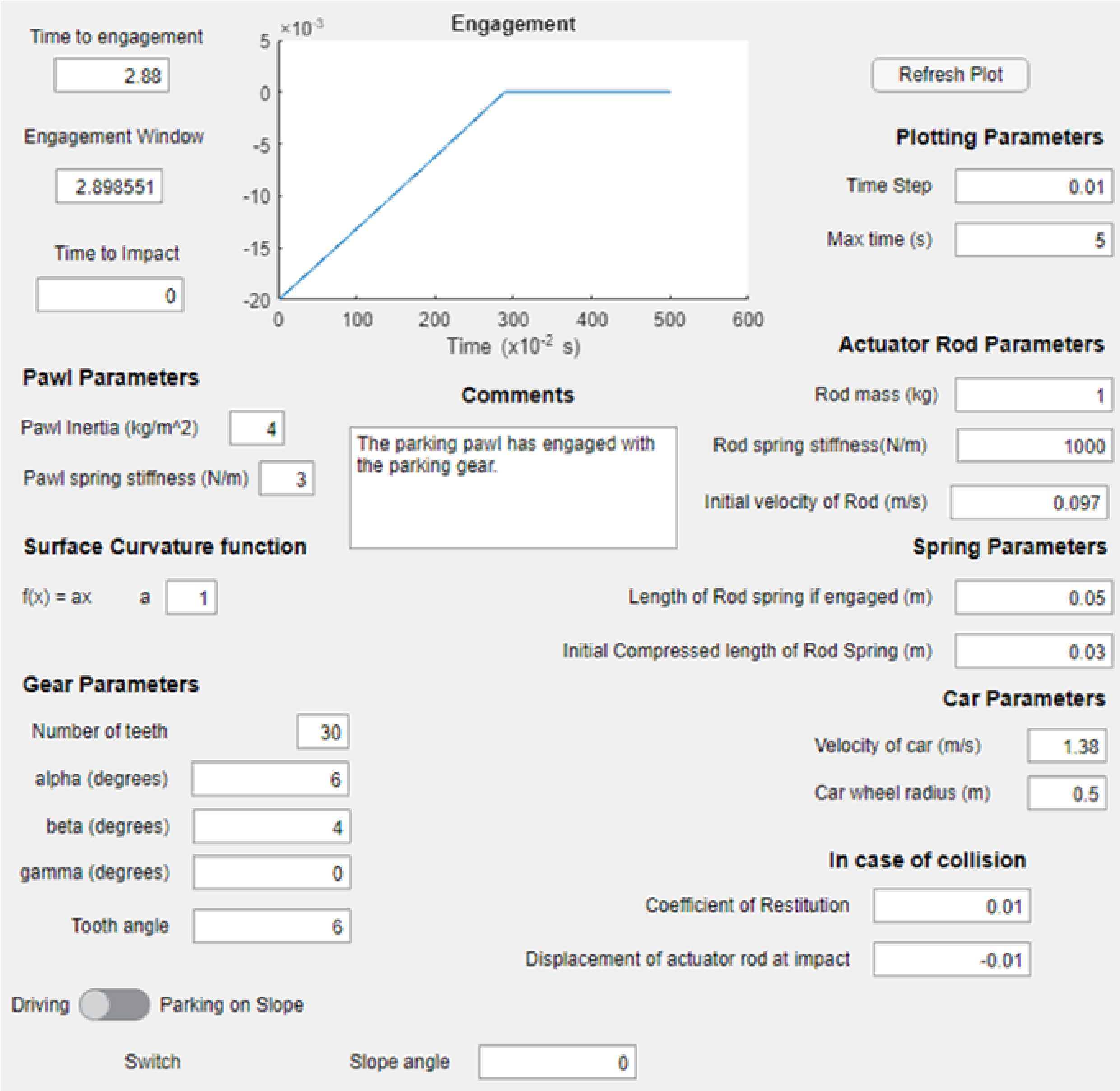

Figure 11: These are our conclusive choices of parameter values that meet PR1.

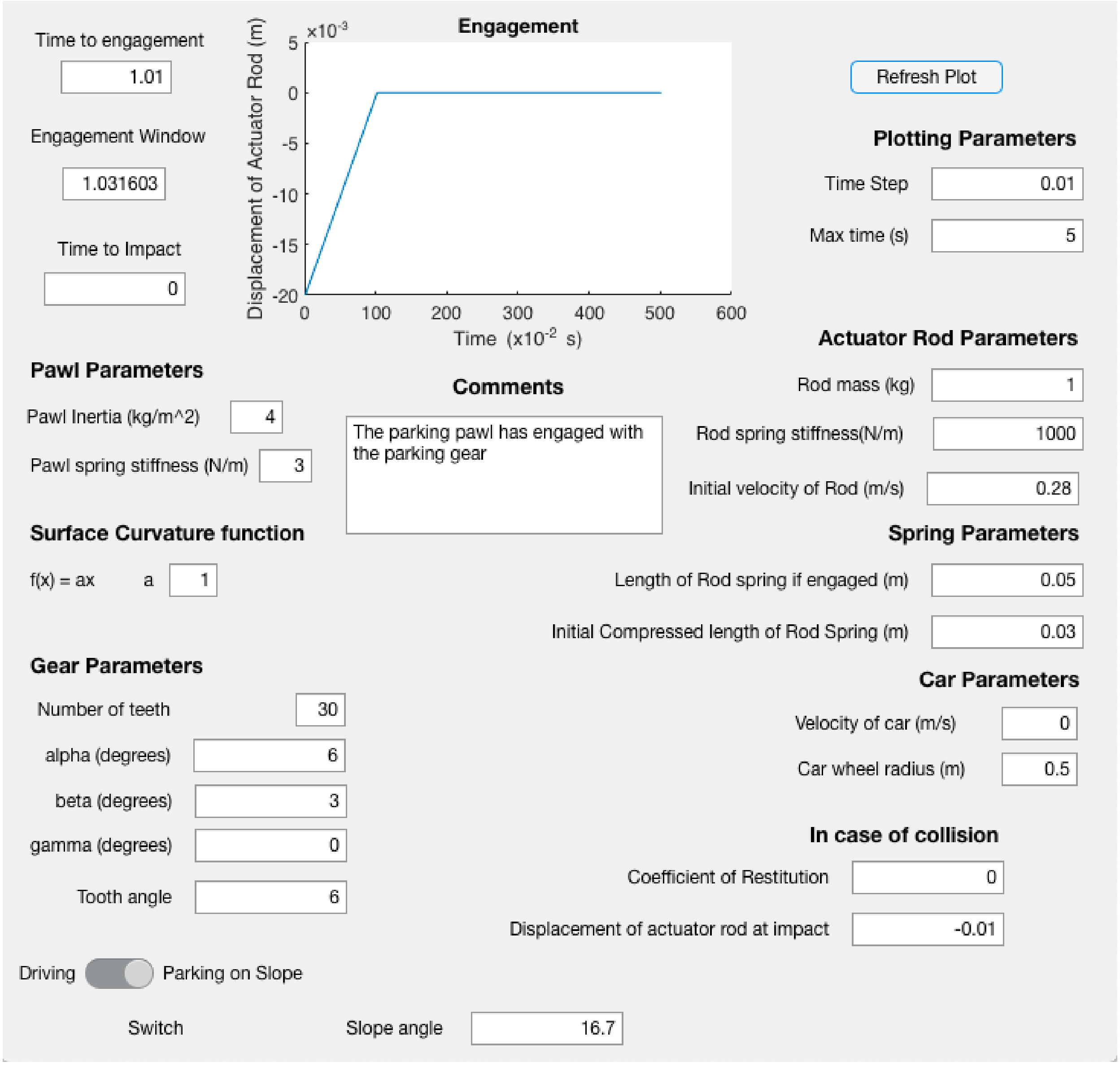

Figure 12: These are our conclusive choices of parameter values that meet PR2. The initial rod velocity is the minimum value needed for a successful engagement in this given configuration.