PHYSICS EQUATIONS

STATICS

- Basic statics
  - $\sum F_x = 0$
  - $\sum F_y = 0$
  - $\sum M = 0$
  - Friction
    - If the object is not sliding (or before it slides) the frictional force can be anything from zero to $F_{max} = \mu_s N$, so $|F| \leq \mu_s N$. This is static friction and F_{max} is sometimes referred to as limiting friction.
    - Dynamic friction or Kinetic friction or sliding friction is when one object is sliding/moving relative to another surface. Here $F = \mu_k N$.
    - The coefficient for kinetic friction μ_k is usually less than the coefficient for static friction μ_s.
  - Strain is equal to the stretch over the natural length. $\epsilon = \frac{stretchlength}{naturallength} = \frac{x}{l}$
  - Stress is the force on the cross-sectional area of rod. $\sigma = \frac{F}{A}$.
  - Young's modulus is the stress over strain. $E = \frac{\sigma}{\epsilon}$. It is also equal the ratio of the elastic yield strength to the elastic yield strength: $E = \frac{Elasticyieldstrength}{Elasticyieldstrain}$.
  - Poisson's ratio is simply the ratio of the negation of the transverse stain to the axial strain. Basically the ratio of how much the rod is being pulled by how much its cross-sectional area is shrinking. $v = -\frac{\epsilon_{tran}}{\epsilon_{axial}}$
  - Shear stress is the force acting on the cross-sectional area of a rod. F/A
  - Shear strain is the length by which the rod is being deformed divided by length to which the shearing is occurring. $\Delta x/l$
  - The shear modulus is the ratio of the shear stress to the shear strain.
- Trusses
  - Step 1: Find all reaction forces on the truss. There will always be one end of the truss that has both a vertical and horizontal force, while the other just has one vertical reaction force. Use force balance and moments about a convenient point to find all the reaction forces.
  - Method of Sections

    * Cut the truss wherever you want and then equate all the external forces using a free body diagram.
    * If there are too many unknowns **balance moments** as well.
  - Method of joints
    * Look at a joint and draw a free body diagram with all the tension and compression forces acting on it. Don't forget to add the external forces acting on the joint as well. Then equate all the forces in the x and y directions.
    * If the internal force of a rod is negative then the rod is under compression. If it is positive it is in tension.
    * Note, $T_{AB} = T_{BA}$. If you know which forces are in tension and in compression you can draw them correspondingly on your diagrams. Tension goes away. Compression goes towards.
  - Don't consider gravitational attraction.
  - The reason you need one reaction force on a roller is that there would be two horizontal forces and no unique solution for them. Thus you would not be able to compute the horizontal forces in the truss.

- Beams
  - For distributed loads:
    * The total downward (distributed) force is $\displaystyle\int_{x_1}^{x_2} f(x)dx$
    * The total clockwise moment is $\displaystyle\int_{x_1}^{x_2} xf(x)dx$ or its the distributed force times its position of centre of mass which is given by $\hat{x} = \frac{\int xf(x)dx}{\int f(x)dx}$
  - Finding shear forces:
    * Split the beam up into different sections where the forces are acting.
    * Then simply do a force balance in the y direction. The $\sum F_y = 0$.
    * If you have a distributed load, simply integrate the function that describes the distributed load. $V'(x) = -p(x)$. When you integrate don't forget to find the initial condition $V(0)$.
  - Finding bending moments:
    * Split the beam up like for the shear forces and then draw two moments going in the same direction. Then do a moment balance eliminating one of the moments.
    * For a distributed load, you can integrate the shear function: $M'(x) = V(x)$

DYNAMICS

- Integrating with vectors: Integrate each vector individually for each vector direction. Then plug in initial conditions for each vector as well.
- For projectile motion, find y in terms of x and then find unknowns in terms of V^2 so that you can then substitute it into most equations because V^2 turns up a lot.
- Lagrangian $L = T - V$
- A force is conservative if its work done does not depend on the path it take, but only on its initial and final positions. It is basically all forces whose work done is the difference in potential energy. $W(P_1, P_2) = V(P_2) - V(P_1)$. If forces are conservative, total energy is conserved.
- Relationship between force and potential energy. $F_x = -\frac{dV}{dx}$
- Work done is $W = \displaystyle\int \underline{F} \cdot \underline{ds}$. Make sure F and ds are in the same coordinate system (either polar or Cartesian) and that the sign is correct. If the force and the displacement vector are orthogonal then the work done is zero, because of the dot product. If the force is opposing the motion then add a negative sign. Friction always opposes the motion so it should always be negative. If you trying to convert arc length ds into polar coordinates it becomes $ds = Re_r d\theta$.
- You can differentiate the total energy of a system and then set it equal to zero to find the acceleration: $\frac{dE(t)}{dt} = 0$.
- For circular motion especially you can set $\dot{H} = M (= I_{CM}\ddot{\theta})$ for rigid bodies to find the acceleration too.
- The kinetic energy of a particle $T = \frac{1}{2}mv^2$ is given by $v^2 = v_x^2 + v_y^2 = (\dot{x}^2 + \dot{y}^2)$
- If you are using Newton's second law for rotational bodies then using polar coordinates you can just use $ma = m(R\ddot{\theta}e_\theta - R(\dot{\theta}^2)e_r)$ and then set $m\ddot{\theta}$ equal to all forces that are acting in the same e_θ direction.
- Work Energy Theorem $W^{res}_{x(t_0),x(t_1)} = \displaystyle\int \underline{F} \cdot \underline{ds} = T(t_1) - T(t_0)$. Basically the resultant work over a certain distance between two times is the same as the difference in kinetic energy at those two points.
- **Rigid bodies**
  - Parallel Axis Theorem. Used to find the moment of inertia about a different point to the centre of mass. $I_A = I_{CM} + md^2$.
  - Angular Momentum of a rigid body $H_O = I_{CM}\omega + |\underline{r} \times m\underline{v}_{CM}|$

- If you want to find the moment of inertia at the centre of a disc and there is particle on its outer edge then you can simply sum the inertia's up, like so: $I_O = I_O^{particle} + I_O^{disc}$
- The position of the centre of mass of a disk and a particle on a disc from the centre is $\frac{m}{M+m}R$. Simply put the smaller mass on the nominator to find the distance from the larger mass.
- The moment of inertia of a particle is zero. But using the parallel axis theorem you can show that the inertia caused by that particle at another point is $I_A = mR^2$
- The total kinetic energy of a rigid body under translational and rotational movement is given by $T = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$
- Rigid bodies can also have potential energy, which is simply mgh_{CM}, so total energy $E = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 + mgh_{CM}$

Linear Motion

$$\underline{F_{res}} = \frac{d\underline{p}}{dt} = m\frac{d\underline{v}}{dt} = m\underline{a} \tag{1}$$

$$\underline{J} = \int_{t_0}^{t_1} \underline{F}\mathrm{d}t = \underline{p}(t_1) - \underline{p}(t_0) \tag{2}$$

$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k} \tag{3}$$

$$\dot{\underline{r}} = \underline{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k} \tag{4}$$

$$\ddot{r} = \underline{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k} \tag{5}$$

$$\tag{6}$$

Angular Motion

$$\underline{\mathcal{T}_{res}} = \underline{r} \times \underline{F_{res}} = \frac{dL}{dt} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\underline{r} \times m\underline{v}\right) \tag{7}$$

$$\underline{r} = r\hat{e}_r \tag{8}$$

$$\dot{\underline{r}} = \underline{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta \tag{9}$$

$$\ddot{\underline{r}} = \underline{a} = R\ddot{\theta}\hat{e}_\theta - R\dot{\theta}^2\hat{e}_r \tag{10}$$

Energy

$$E = T + V = const. \tag{11}$$

$$T = \frac{1}{2}mv^2 \tag{12}$$

$$V_{pot} = mgh \tag{13}$$

$$V_{elastic} = \frac{1}{2}kx^2 \tag{14}$$

$$\dot{E} = 0 \tag{15}$$

$$\omega = \frac{2\pi}{T} \tag{16}$$