1 Introduction

- Energy classification: **Primary** (Wind, tide, solar, unprocessed fossils, uranium), **Secondary** (ready to use, electricity, petrol, gas (methane is both primary and secondary).
- Energy Types: Kinetic, Gravitational, Electrical, Nuclear, Electrical, Nuclear
- 1 'tonne of oil equivalent' (toe) = 12MWh. UK mean consumption is 4 toe per person per year. 1kW for 1 year = 0.75 toe.
- 1KWh = 3.6MJ = 8.6×10^{-5} toe = 1.3×10^{-4} tce (tonne of coal equivalent).
- For an energy system to be sustainable: needs to last a long time (essentially indefinitely), be benign to people, society and ecosystems, not prejudice, in an adverse way, the prospects of future generations.
- Oil and gas will soon reach 'Hubbert peaks' (production trends move like bell curve). 10 years left for oil before peak they say... At current world consumption coal has 210 years of reserve, oil has a 60 year reserve, natural gas 160 years. Because fossil fuels will be around for quite a while, so will heat engines.

2 Heat Engines

- **Key thermodynamics:**
  - 1st Law: $dU = dQ + dW$, the sum of heating Q and work W is equal to the internal change in energy U. Conservation of energy.
  - 2nd Law: Essentially not all energy has the same usefulness (because of entropy). We can convert all work into heat, but cannot convert all heat into work.
  - The upper limit of efficiency in regards to converting heat into work is called the Carnot Cycle efficiency limit. Not engine can be more efficient than the Carnot Engine.
- Isothermal process: The temperature T of the working fluid does not change. This implies that there is no change in internal energy ($dU = 0$) and thus $dW = -dQ \Rightarrow \Delta Q = RTln\frac{V_2}{V_1}$. Image a fluid that has to remain at a fixed temperature. If you apply work to it, then it needs to lose heat in order to remain at the same temperature, and if you apply heat to it, it needs to do work, i.e. expand, in order to remain at the same temperature. We use the isothermal process to effectively transfer heat into work (however, given the 2nd law a total conversion is not possible).
- Adiabatic process: This is a simplifying assumption (it's not really possible to have a perfectly adiabatic system), and it assumes that we can add work to a system/fluid so quickly that there will be no time for the fluid to release heat, i.e. $dQ = 0$. This means that $dU = dW$ and leads to the equations $pV^\gamma = const.$ and $\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ [Go over Thermo notes again!].
- **Carnot Cycle:**
  - Isothermal expansion (adding Q_h to fluid) (top left corner to top right corner, A to B): $Q_h = RT_h ln\left(\frac{V_B}{V_A}\right)$.
  - Adiabatic Process 1 (top right to bottom right, B to C) (extracting work): $\frac{T_h}{T_c} = \left(\frac{V_C}{V_B}\right)^{\gamma-1}$
  - Isothermal compression (bottom right to bottom left, C to D), heat Q_c is released: $Q_c = -RTln\frac{V_D}{V_C}$.
  - Adiabatic Process 2 (D to A): $\frac{T_c}{T_h} = \left(\frac{V_A}{V_D}\right)^{\gamma-1}$
  - Other equations we thus get are: $\frac{V_B}{V_A} = \frac{V_C}{V_D}$ (from the adiabatic processes) and $\frac{Q_h}{Q_c} = \frac{T_h}{T_c}$.
  - The efficiency η is defined as the work done divided by the heat input: $\eta = \frac{W}{Q_h} = 1 - \frac{T_c}{T_h}$. So the hotter T_h (hot) is relative to T_c (cold) the more efficient the system.
  - All reversible engines (i.e. where no energy is lost) are Carnot efficient - which is impossible.
  - Best efficiency for an externally heated engine: Max power of the engine occurs when $\eta = 1 - \sqrt{\frac{T_c}{T_h}}$.
- **Co-generation**: The idea is that otherwise-wasted heat from a heat engine can be used as low-grade for other energy purposes. The relative saving, S, in total energy in going from one system (s-system) to another system (c-system) is $S = \frac{E_s - E_c}{E_s}$

- **Stirling Engines** - approach Carnot Cycle efficiency. Works by combining a displacer with a piston (90 degrees out of phase from each-other), momentum and a hot and cold temperature difference. Lots of different ideas being worked on with Sterling engines, as they are quiet and efficient.
- CHP (Combined Heat and Power) units is a co-generation system that produces heat and electrical power from gas simultaneously.
- Mechanical work deduced from Carnot, $W = Q_{in} \times (T_h - T_c)/T_h$

3 Renewables Introduction

- Better for the planet, climate change and carbon footprint, less harmful by-products.
- Of the renewable only tidal and geothermal are non-solar driven energy sources. Solar heating, solar direct conversion, wind, wave, hydro, and biomass are all dependent on the sun.
- Of the 1.3 kW/m^2 of sun energy is split 23% into evaporation, 30% reflected and 47% into warming earth up. Means we have vast amounts of low grade thermal energy
- 5 time increase renewable energy sources production in past ten years than past, a rapidly growing industry.

4 Renewables: Solar Thermal

- Focus sun to create hot part on a fluid in a working heat engine and environment to provide the cold portion.
- Uses parabolic dish to focus sunlight - can either have 'line focus' tracking (max temperatures of 200 to 400 degrees, tracks sun by elevation, usually used on pipes that connected to a heat engine, see Noor 1 Morocco) or 'point focus' tracking (max temperatures of 1500C, tracks sun by elevation and azimuthal orientation, usually fired sunlight straight onto heat engine). Heat is then used to generate vapour and then a vapour cycle or directly onto a Stirling-like engine.
  - Some 'sums' as Joey boi likes to say:
    * Radiation absorbed by focal point $Q_a = \alpha Q_s$, where α is the absorptivity coefficient, and $Q_s = \eta_{optics} ICA$ is the energy reflected by the mirrors onto the focal point, where C is the concentration (Don't know what I is!).
    * Radiation Emitted (loss) by focal point (where the engine is) is $Q_l = A\epsilon\sigma T_H^4$, where A is the csa of the focal point, ϵ is the emiisivity, σ is the Stephan Boltzman constant and T is the temperature of the hot surface.
    * Efficiency $\eta_{total} = \eta_{receiver} \times \eta_{carnot} = (1 - \frac{\sigma T_H^4}{IC})(\frac{T_H - T_C}{T_H}$ and $\eta_{receiver} = \frac{Q_a - Q_l}{Q_s}$.
- Other solar thermal ideas and projects are 'solar ponds' or OTEC (Ocean Thermal Energy Conversion). Or heat harvesting on peoples housing using Thermomax Evacuated Tubes.

5 Renewables: Solar Direct Conversion

- Sun light is converted directly into electrical energy, via solar panels.
- The solar panels work by having two thin layers of silicon. The first silicon (4 valence electrons) layer is 'poisoned' or 'doped' with phosphorus (5 valence electrons), thereby making the doped silicon layer almost 3000 times more conductive. By adding a phosphorus atom, you add one 'free electron' to the doped sample thereby making it more conductive. If that electron leaves the phosphorus atom, then the phosphorus atom becomes positively charged (because it now has one more proton). Phosphorus doping creates mobile negative charges. In the other silicon layer, you dope it with boron (3 valence electrons) and this layer now has 'holes', i.e. where an electron should be put isn't. If that 'hole' (h+) moves away from the boron atom, it becomes more negatively charged. Boron doping creates mobile positive charges (holes). (1 less electron per silicon atom,

acceptor impurity, p-type). When a photon hit the upper layer (n-type) electrons get displaced and can be used in a circuit feeding back into the p-type layer, which pulls the electrons in the wire towards it, only for the electrons to then displace back into the n-type layer.

- The net current produced is $I_D = I_0 e^{\left(\frac{eV}{kT} - I_0\right)}$. Voltage also given in notes.
- Efficiency around 12 to 22% at best. Complex and expensive manufacturing process.
- Monocrystalline solar cells are more efficient and expensive than Polycrystalline solar cells, which are more expensive and more efficient than Amorphous solar cells. Some hybrid solar cells seem to be very effective too.
- Fastest growing energy source.

6 Renewables: Wind Energy

- Kinetic Energy per unit volume of air is $KE = \frac{1}{2}\rho u^2$.
- Power available in fluid (air) of velocity u passing through an area A is $P_A = AuKE = \frac{1}{2}(\rho A u)u^2$.
- **Betz limit** (the max efficiency of a wind turbine) is $\frac{16}{27} = 59.3\%$.
- Power generated is proportional to the area swept by the turbine blades and how high it is off the ground.
- For optimum use of blades (maximise lift, minimise drag), one decreases the twist and thickness (width) of the blade as the radius increases.
- Designed to stall if wind speeds get too high. Typical rating are $200 - 500W/m^2$ of swept area.
- Haliade-X 12MW turbine coming soon.

7 Renewables: Biomass

- Biomass captures 5 to 8 times the whole of the world' primary energy consumption.
- When biomass fuel is burnt, it delivers the same heat as would have been lost by natural decay of the material.
- Carbon-dioxide released is exactly equal to the amount absorbed during the growth and creation of the biomass. This is a **sustainable short carbon cycle**.
- The simplest way of liberating energy from biomass is burning it $CH_20 + 0_2 \to CO_2 + H_2O + Energy$. This process is basically reversed during photosynthesis.
- Great if it is used to turn waste into energy.
- **Pyrolysis**: Heat up biomass at high temperatures and pressures without air, to get a more dense form of the biomass.
- Anaerobic digestion: let bacteria eat away at the biomass and produce methane or other gas to then use for power generation. Fermentation is an example of anaerobic digestion.
- Some plants can be compressed to produce vegetable oils with properties not too dissimilar to diesel.

8 Renewables: Wave Energy

Extracting energy from waves. Waves are caused by wind.

- Shallow water velocity (D ¡ L/4) $V = \sqrt{gD}$, where D is the water depth.
- Otherwise, $v = \sqrt{\frac{gL}{2\pi}}$
- Power in wave = $\frac{\rho g^2 H^T}{32\pi}$

8.1 Fixed Devices

- Various different devices around. Most are oscillating water column (OWC) arrangement, i.e. they use the wave to push air through at turbine.

8.2 Floating Devices

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8.3 Wave Energy Theoretical Power generation

- Power $P = \frac{\rho g^2 H^2 T}{32\pi} \approx H^2 T$, where H is the height between the trough and peak of the wave, T is the period of the wave.
- Average Power $P_s = \alpha_s H_s^2 T_s$, where H_s is the significant wave height, which corresponds to the average height of the highest one third of waves, and T_s is the zero-up-crossing period, defined as the average time between upward movements of the surface of the wave through the mean level, $\alpha_s = 0.49 KW/(sm^3)$.
- 95% of the energy in a wave is contained within a layer between the water's surface and depth $L/4$ under the mean water level (L is the wavelength).

9 Renewables: Tidal Power

Essentially the moon and suns' gravitational forces cause water particles to squeeze water away from surface of the earth towards and away from the sun and moon.

Pros	**Cons**
Very predictable form of Energy	Ecologically damaging
Could look attractive and bring wealth and power to an area	Civil engineering very expensive
Clean and Renewable Energy	

9.1 General Knowledge

- Period for 2 tides is 24.8 hours $\Rightarrow$ 12 hours and 25 min per tide. Extra 50 min occurs because of lunar cycle (29.5 days) and earth and moon both move anti-clockwise together.
- Spring tides $\Rightarrow$ sun and moon aligned, giving high tides, Neap tides $\Rightarrow$ sun and moon at 90 degrees, giving lower tides.
- Tidal bulges occur 12 min before moon is overhead, due to shear of rotating earth on water
- Tidal range needs to be over 5 meters for energy to be economically viable.
- Severn Estuary could generate around 10% of UK's electrical needs, however delayed because of environmental impact on marine life. Ideas for lagoons (separating water into separate sections so you can always generate energy), or 'watermills' are potential solutions.

9.2 Physics

- Only computation given is for displacement of a particle (bulge) but a lot more in appendix - not sure how much I need to learn[DO THIS WHEN YOU'VE DONE QUESTIONS]
- $\delta r = \frac{m_m}{m_e}\left(\frac{r}{d}\right)^3 r$ where r is the radius of the earth, d is the distance between the earth and the moon, and δr is the bulge $\Rightarrow$ assumes $d >> r >> \delta r$. Otherwise, $\delta r = \frac{m_m}{m_e}\left(\frac{r}{d}\right)^3 r \left(1 \pm \frac{3}{2}\left(\frac{r}{d}\right) \pm O\left(\frac{r}{d}\right)^2\right)$, if $r >> \delta r$ but $d > r$.

- Tide proportional $1/d^3$.
- Sidereal and Sinodic [LOOK THEM UP]
- Sun 25% pull of moon's pull

9.3 Focusing and Resonance

- Tidal resonance occurs when the period for surface waves is a multiple of the forcing period of the tide. Shallow water wave speed is $c = (gy)^{0.5}$, where y is the tidal range (height difference between high and low tide).
- For perfect resonance matching $\frac{2L}{(gy)^{0.5}} = \frac{1}{2}(12.4 \times 3600)$, where L is the distance between two boundaries.
- Very complex, depends on full topology of the environment. Other complication include moon's path being elliptical and its inclination relative to the earth and sun. Result in 10% variation in tide height.
- Atlantic ocean good basin for resonance, wave speed proportional to period of waves, results in high tides on european and american coasts.

9.4 Tidal Power Generation

- For tidal barrages: $Power = P.E./T = (\rho V g h_{av})/T = (\rho y A g y/2)/T$, where A is the constant area of the basin behind the barrage, y is the tidal range, and ρ is the density of water.
  - For tidal barrages you can use 'flood generation' by passing the incoming tide through turbines, or 'ebb generation' which uses sluice gates to trap the tide and then generate energy by releasing the tide again. Or one can use two way generation.

10 Renewables: Hydroelectricity

Water trapped by a dam is fed via turbine from one reservoir with higher potential energy to another reservoir with lower potential energy. 20% of the worlds energy source. Three gorges with 18.2 GW as powerful as 18 nuclear reactors.

Pros	**Cons**
Very efficient form of Energy, achieving +90% efficiency	Impact on environment
Renewable	Location may not be good
40% load factor (ratio of average energy consumption over max energy consumption)	Expensive to build
18.2 GW of Energy (Three Gorges China)	Once built, essentially irreversible

10.1 Types of Turbines

- **Francis Turbines**: Medium head, medium flow rate. Most used, can work between head range of 2 to 200m. Can be 95%+ efficient. Not suited for very small heads (propeller better). They're most efficient when their blades are moving nearly as fast as the water. For very high water speeds this may not be desirable, so a Pelton turbine is better suited). Mixed fluid flow.
- Pelton turbines: Good for high head, low flow rate. Impulse turbines. Work by firing highspeed jets onto blades (that's why they are better suited for high speeds, yet used for low flow rate reservoirs). Tangential fluid flow.
  - Power output: $P = \rho Q g H = A\sqrt{2g^3}\sqrt{H^3}$, where $Q = Av = A\sqrt{2gH}$, and $v = \sqrt{2gH}$ from (K.E. = P.E), and A is the jet cross-sectional area. Note that $P = \frac{A}{2}v^3 gH$ too.
  - Efficiency greatest when speed of cups are half the speed of the water (because the cups reflect the water).
- **Propeller (Kaplan turbines)**: Good for low head, high flow rate. Axial fluid flow.

- Francis turbines are the most efficient of the three. But the most suitable turbine is dependent on the environment:
  - Compute the specific speed factor N_s, defined as $N_s = n\sqrt{\frac{P}{H^{2.5}}}$, where n is the number of revolutions per minute, P is the power and H is the head. $N_s \approx \frac{d}{D}\frac{v_B}{v_W}$, where d is the incoming flow diameter, and D is the turbine diameter, while v_B and v_W are blade and water speeds, respectively.

10.2 Hydroelectric Power generation

- Theoretical $Power = \rho Q g H$, where Q is the volumetric flow rate (m^3/s) and H is the Head (m) (height difference between upper and lower reservoir).
- Actual $Power = \eta\rho Q g H_e$, where H_e is the effective Head (5 to 10% less than the true Head, due to water being delivered to rotating machine, and the machine is not 100% efficient either, hence η.
- Very predictable form of energy. Good for places like Iceland. Not very efficient though, because its expensive to go so far into the earth

11 Geothermal (not renewable extraction rates are higher than replenishing rates)

Generating energy from heat coming from the earth's core. Temperature gradient about 20 to 30 degrees per km. So need to dig quite deep.

11.1 Temperatures and Power extraction

- Temperatures depend on location: radio-active hot rocks (like granite)
- Let cold water percolate into hot soil/rocks and then extract hot steam to generate power.
- Iceland perfect for using georthermal energy.

11.2 Geothermal store

- Idea is to use ground as both a heat source and heat sink. You pump in heat during the summer and get heat out during the winter.

12 Nuclear Fission

12.1 General Stuff:

- Atoms with more than around 60 neutrons start to become more unstable.
- Fission of heavy atoms into smaller lighter and more stable atoms results in a small amount of energy (1MeV).
- A nucleon allows to fall for 10 meters in a gravity field produces approximately $10^{-6}eV$ and combustion only results in $1eV$ per electron of energy. Moving from gravitational to chemical energy results in an increased energy density magnitude of 10^6 and another further 10^6 is added when going from chemical to nuclear.
- Mass is NOT conserved as one goes through a fission or fusion process.
- Neutrons fired with energy of around $2MeV$. Reactors usually try to increase the temperature, because 'thermal neutrons' have a higher probability of causing fission.
- 11% of world's power generation. (Coal 40%, gas 22%).
- $1eV = 1.6022 \times 10^{-19}J$. Power density extremely high for fission.

- Whole fission cycle produces around 200 MeV which is enormous.
- For a reactor you want to keep the number of neurons firing into other uranium atoms around 1 (don't want exponential growth - otherwise you get a bomb).

12.2 Background:

- Rate of decay of N nuclei given by $\frac{dN}{dt} = -\lambda N \Rightarrow N = N_0 e^{-\lambda t}$.
- Half-life $T_{1/2} = \frac{\log_e 2}{\lambda}$ occurs when $N = N_0/2$.
- The expected lifetime of nuclei $T_{av} = \frac{1}{N_0}\int t \frac{dN}{dt} dt = \frac{1}{\lambda}$.
- Cross-sectional area σ is defined as the number of reactions per second divided by the number of incident neutrons per m^2 per second for a single nucleus. Units called 'barn', where 1 barn $= 10^{-28} m^2$. 'The probability that something will happen'.
- Macroscopic cross-section $\sum = N\sigma$, where N is the number of nuclei per unit volume.
- Mean free path ξ is the average distance travelled by neutrons before collision.
- The absorption rate with respect to positions is given by $\frac{dI}{dx} = -N\sigma_a I = -\sum_a I \to I = I_0 e^{-\sum_a x}$. 'Intensity decreases'.
- The mean free path until absorption $\xi_a = \frac{1}{\sum_a}$.
- The moderator slows down the fast neutrons to be 'thermal' (if they are too fast then fission is less likely apparently) and does not absorb the neutrons but reflects them (either use light-water, heavy water (deuterium) or graphite).
- Absorber - material that absorbs the neutrons, boron is good for this.

12.3 Four Factor Formula:

- Typically Uranium-235 is used. The reaction is as follows:

$$^{235}U +^1 n(slow) \to^{236} U \to X + Y + (2.43)^1 n(fast) + energy$$

  the $2.43 = \nu$ is the average number of neutrons that come out after the reaction (usually always shown to be three), but it depends on what atoms are made. Approximately 200MeV of energy is released. The released neutrons then fire into other uranium atoms causing a chain reaction and lots of energy. The energy is then used to heat water and turn a turbine to turn an alternator to produce energy.
- Fast neutron fission yield $\eta = \nu \frac{N_{235}\sigma_{f235}}{N_{235}\sigma_{a235} + N_{238}\sigma_{a238}}$ if ^{235}U and ^{238}U are being used together.
- The four factor formula: $k = \epsilon\eta p f$, where ϵ is the fast fission factor (0 none get absorbed by structure, 1 all get absorbed), p is the resonance escape factor, and $f = \frac{\sum_a (fuel)}{\sum_a (core)}$ is the thermal utilization factor. k is the multiplication factor for the system, i.e. one thermal neutron will produce k thermal neutrons during one complete fission cycle.
- Joe gives the sizes of these variables, like $\eta < \nu$ I believe, see around $50 - 60$ min 1.05. lecture.

13 Nucluear Fusion

- Most common reaction $^2D +^3 T \to^4 He + n + 17.6MeV$ (neutron takes 80% of energy). Deuterium is naturally occurring isotope of hydrogen; Tritium, another isotope of hydrogen, is not naturally occuring (half-life 12.3 years) and so its made by bombarding lithium-7 isotope (93% of naturally occurring lithium is Li-7) with neutrons $n + 7^L i \to^4 He + 3^T + n - 2.87MeV$.
- D and T repel, so require extreme temperatures $T \geq 10keV$ and pressure to fuse together.
- A well designed reactor will have lithium blankets around the reactor to absort neutrons and produce more tritium for further power.

- **Lawson condition** states that for a fusion reactor to be self-sustaining, the nuclei density n, the length of time the nuclei density is held τ_E and the temperature T of the nuclei must be above $3 \times 10^{21}\ m^{-3} s^1 keV$, i.e. $n\tau_E T \geq 3 \times 10^{21}$.
- Lots of energy, 2kg of deuterium and 3kg of tritium would produce $1.69 \times 10^{15} J$ of energy - mind bogglingly huge.
- Current methods being tested: Tokamak and Laser beams.

14 Fuel Cells:

- These essentially just convert a fuel into electrical energy. Unlike batteries a supply of fuel needs to be present, e.g. hydrogen and oxygen for a redox reaction.
- Advantages are that there are no moving parts. Not as efficient as gas, but great if hydrogen is sourced cleanly.
- There are different types: PEM (Proton exchange membrane), PA (Phosphoric acid), SO (solid oxide), MC (Molton carbonate).

15 Economics

- The future value F of P amount of money invested at an interest rate r per year, and an inflation rate of i per year, will be worth
$$F = P\frac{(1+r)^n}{(1+i)^n}$$
after n years. Alternatively $P = F(1+r_e)^{-n}$, where $r_e = \frac{r-i}{1+i}$ is the *effective* interest rate.
- Say you need to borrow C amount of money to get your project started. You want to pay back your 'capital' C over n year **in equal amounts each year**. You need to take interest (considering only interest here, not inflation) into account, as the C will grow with interest every year. Therefore you pay $A + rC$ amount of money every year which will satisfy both interest and capital repayments. A is called the *amortisation payement.* Instead of just dividing C by n, the numbers of years, to find A (as this would result in paying too much), we solve for A, given that
$$C = A + A(1+r) + A(1+r)^2 + ... + A(1+r)^{n-1} = A\frac{(1+r)^n - 1}{r} \Rightarrow A = C\frac{r}{(1+r)^n - 1} = Cf_{AF}^n$$
which tells us how much A should be in order to pay back C plus the interest it acrews in the n years you need in order to pay it back. Cf_{AF}^n is called the 'annuity future worth factor' over n years. So we then pay back $C_A = rC + A = Cf_{AP}^n$ annually. f_{AP}^n is called the 'annuity present worth factor' over n years.
- **Value of savings**: Net Present Value, NPV, is $NPV = \frac{S_A}{f_{AP}^n} - C$ where S_A is your annual saving.
- **Present Value & Expenditure**:
  - If expenditure is incurred before savings begin: If capital cost at the reference date is $PV = C_A\frac{1}{f_{AF}^N}$, which means the net present value is $NPV = \frac{S_A}{f_{AF}^N}$
  - If savings are built up over N years, with each year providing an extra S_A/N of saving, makes the Net Present Value: $NPV = \frac{S_A}{N}\frac{1}{f_{AP}^n}\frac{1}{f_{AP}^N} - \frac{C}{N}\frac{1}{f_{AF}^N}$.
- Goes into more complex examples, which I can't be bothered to type out.
- **Levelised Cost Of Electricity** (LCOE) is the reation of the total cost over the total energy produced of the complete (lifetime) generation cycle (£$/MWh$): $LOCE = \frac{Cost}{Energy} = \frac{\sum_{t=1}^n I_t + M_t + F_t}{\sum_{t=1}^n E_t}$, where I_t is the investment in year t, M_t is the operation and maintenance in year t, F_t is the fuel expenditure in year t and E_t is the electrical energy generated in year t, and n is the life time of the asset.