

# Thermodynamics 1

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——— Conversions ———

- 1 inch  $\approx$  2.54 cm
- 1 bar = 100 kNm<sup>-2</sup>
- 1 litre = 1 dm<sup>3</sup>

## 1 Systems and Energy

**State properties** -Temperature(T), Pressure(p), Density( $\rho$ ), Internal Energy(u,U), Enthalpy(h, H), Entropy(s, S)

- These are properties that are **independent of path** and define equilibrium (only need two to do so).

### 1.1 Essential definitions

#### SYSTEMS

- Closed System - A space containing a fluid with a boundary between the fluid and surroundings. **No exchange of matter. Can be a theoretical or physical system. W and Q can cross the boundary.**
- Stationary System - No change in kinetic or gravitational potential energy. **System can still be 'stationary' even if it is moving at a constant velocity.**
- Isolated System - No exchange of anything. **No mass or heat transfer.**

#### POSTULATE

- State Postulate - In a 'simple' equilibrium system, the entire system is specified for two independent state properties. In other words, one only needs to measure two properties to know the others exactly.

#### WORK, HEAT AND INTERNAL ENERGY

- Heat transfer (Q) - Energy in transit from one place to another. Can occur by conduction, convection, radiation (and combustion  $Q = m_f q_{cal}$ ,  $m_f$  = mass of fuel,  $q_{cal}$  = calorific value).
- Work - Product of force and displacement ( $W = \pm \int_1^2 F(s)ds$  or for a piston cylinder  $W = \pm \int_1^2 p dv$ )
- Internal energy (U) - combination of all forms of **microscopic** energy in a fluid. It is **independent** of path and is therefore a state property.
- Together they form the NON FLOW ENERGY EQUATION (NFEE):  $\Delta U = W + Q$ . Note, this equation is only valid for **closed and stationary systems**

#### PREAMBLE TO THE 1ST LAW

- "When any closed system is taken through a cycle the **net work done** by the system upon the surroundings is **EQUAL to the net heat** supplied to the system from the surroundings."  $\sum \delta Q + \sum \delta W = 0$

#### The First Law of Thermodynamics

- Corollary 1 - "There exists a property (internal energy,  $U$ ), such that a change in its value is equal to the difference between the heat supplied and the work done during any change of state." ( $\Delta U = W + Q$ )
- Corollary 2 - The internal energy of an isolated system remains unchanged.
- Corollary 3 - A perpetual motion machine of the first kind is impossible. (First kind means that the machine produces work without use of heat - an impossibility)
- Note, the first law is an axiom and also shows that energy can only be transferred from hot bodies to cold bodies.
- Furthermore it says "Energy cannot be created or destroyed, only transformed"

### Other definitions

- Process - Transforms system from one equilibrium state to another.
- Path - transition between start and end state.
- Quasi-equilibrium process - A very slow changing process. System is very close to equilibrium as it moves between start and end points.
- Equilibrium - All driving forces acting on the system are balanced.
- The zeroth Law - two objects at the same temperature are at thermal equilibrium. In other words, if two systems are in equilibrium then the third will be in equilibrium too, if it is connected to one of the other two systems.

### 1.2 Other little bits of info

- Gauge pressure has a 'datum' of standard atmospheric pressure  $1 \text{ atm} = 1.013 \text{ bar}$ . In other words it is absolute pressure shifted by one atmospheric pressure.  $0 \text{ bar (g)} = 0 \text{ barg} = 1 \text{ atm} \Rightarrow 1 \text{ bar (g)} = 1 \text{ barg} = 2 \text{ atm}$ .
- Isotropic - equal in all directions. A solid is said to be isotropic if the expansion of solid is equal in all directions when thermal energy is provided to the solid (wiki).

## 2 Gas Properties

### Definition of Property

- Extensive - depends on size of system, i.e. how much substance is in it. Represented by CAPITAL LETTERS for certain properties ( $H, S, U...$ ), other extensive properties are (Volume ( $V$ ), mass ( $m$ )...)
- Intensive - does not depend in size of the system in question. Instead they are intrinsic to the material in question and refer to a unit mass for some specific properties. An intensive property can always be found by dividing two extensive properties, e.g.  $\rho = m/V$  Shown by lower case letters for some properties ( $u, h, s$ ). Other intensive properties are ( $T, p, v (= \frac{V}{m} = \frac{1}{\rho})$ )

### Assumptions for the ideal gas law

- Gas acts as a 'perfect gas'.  $\Rightarrow$  **Molecule-to-molecule** forces are very small. Perfectly elastic collisions. Molecules have negligible volume.

**IDEAL GAS LAW** (Ideal gas is a perfect gas and the internal energy is function of temperature only ( $U = f(T)$ ))

$$PV = n\tilde{R}T$$

where,  $R = \frac{\tilde{R}}{m}$  and  $n = \frac{m}{m}$ . Thus,  $\boxed{pV = mRT}$  and

$$pv = RT$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

For air at  $25^\circ\text{C}$ ,  $\tilde{m} = 29\text{kgkmol}^{-1}$ .  $\tilde{R} = 8.314\text{Jkmol}^{-1}\text{K}^{-1}$  always. Thus the specific gas constant  $R$  for air  $= \frac{8.314\text{Jkmol}^{-1}\text{K}^{-1}}{29\text{kgkmol}^{-1}} = 0.287\text{kJkg}^{-1}\text{K}^{-1}$  Need to know the following molar masses too: Argon (40), Nitrogen (28), Oxygen (32), Steam (18), Hydrogen (2)  $[\text{kgkmol}^{-1}]$ .

### Internal Energy

- $U = mc_v(T - T_0)$ , or  $u = c_v(T - T_0)$ .  $T_0$  is the datum temperature e.g.  $0^\circ\text{C}$ . Because  $U$  is state property, these equations are always true, even if the volume is changing.  $c_v$  is an indicator of the internal energy.

### Heat transfer $Q$

- $Q_v = mc_v(T_2 - T_1)$  ONLY IF: **Volume is constant** and **Work = 0**.

### Specific heat capacities

- Definition: The specific heat capacity is the quantity of energy required to raise the temperature of one unit mass by one degree of temperature.  $c_v$  refers to such energy addition at constant volume and  $c_p$  refers to such energy addition at constant pressure for a fluid.
- For air  $c_v = 0.718\text{kJkg}^{-1}\text{K}^{-1} = 2.5R_A = 2.5 \times 0.287\text{kJ/kg/K}$ . ( $R_A$  is the specific gas constant  $R$  for air).
- For air,  $c_p = 1.005\text{kJkg}^{-1}\text{K}^{-1} = c_v + R = 3.5R$

### Enthalpy

- A measure of the total energy of a thermodynamic system. It is the sum of the internal energy and 'a term with a sense of work',  $pV$ .  $H = U + pV$
- Enthalpy is also the **reversible** heat addition at **constant pressure**.

$$Q_p = \Delta U - W_{flow} = U_2 - U_1 - W_{flow} = U_2 - U_1 - (-p(V_2 - V_1)) = (U_2 + p_2V_2) - (U_1 + p_1V_1) = \Delta H$$

Note, pressure is constant so  $p = p_1 = p_2$  and thus  $W = -\int_1^2 p dV = -p(V_2 - V_1)$

- Enthalpy of an ideal gas only depends to temperature:

$$\begin{aligned} U &= mc_v(T - T_0), pV = mRT \\ \text{let } T_0 &= 0 \Rightarrow H = U + pV = mc_vT + mRT = m(c_v + R)T \\ \Rightarrow H &= mc_pT \Rightarrow H = mc_p(T - T_0) \Rightarrow \Delta H = mc_p(T_2 - T_1) \end{aligned}$$

The key thing here are the subscripts: if it is  $(T - T_0)$  then it is just  $U$  or  $H$ , because you are just determining the internal energy or enthalpy at a certain temperature relative to a datum temperature.  $\Delta H, \Delta U$ , however, describes a change so there must be a change in temperature  $(T_2 - T_1)$ , which is essentially  $(T_2 - T_0 - (T_1 - T_0)) = T_2 - T_1$

- Essentially -  $\Delta H = mc_p\Delta T = Q_p$
- Therefore, also, specific enthalpy  $h = c_pT$  and  $\Delta h = c_p\Delta T$

## 3 Energy Balances

### Types of processes

- Isochoric - Constant volume process  $\Rightarrow W = 0 \Rightarrow \Delta U = Q_v$
- Isobaric - Constant pressure process  $\Rightarrow \Delta U = Q_p + W \Rightarrow Q_p = \Delta U - W = \Delta U + p\Delta V = \Delta H$
- Isothermal - Constant temperature process  $\Rightarrow \Delta U = 0 \Rightarrow W = -Q_T$ , also  $pV = mRT = \text{const.} \Rightarrow W = -\int_1^2 p dV = -mRT \int_1^2 \frac{dV}{V} = -mRT \ln\left(\frac{V_2}{V_1}\right)$

- Adiabatic - No heat transfer, so  $Q = 0$  which means  $\Delta U = W$ , i.e. all energy is transferred as work. This process is done very quickly so as to ensure no heat transfer. Also, it's done in a closed system, so there is no transfer of mass either.
- Isentropic - A reversible (frictionless, ideal) and adiabatic process. This means there is no heat transfer ( $Q = 0$ ) and no change in entropy  $\Delta S = 0$ . It's like an ideal adiabatic process.

### Polytropic (many routes) processes

- A polytropic process is a thermodynamic process that satisfies the following relation: Def. -  $pV^n = p_1V_1^n = p_2V_2^n = \text{const.}$ , where  $n$  is the so-called index of expansion or compression.
- For isobaric process -  $n = 0$
- For isochoric process -  $n = \infty$
- For isothermal process -  $n = 1$
- For isentropic process (ideal adiabatic process) -  $n = \gamma = \frac{c_p}{c_v} = \frac{3.5R}{2.5R} = 1.4$  (for air). THIS MEANS:  
 $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{2/7}$  and  $P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{7/2}$  and since  $Q = 0$ ,  $W = \Delta U = mc_v\Delta T (= m \frac{R}{1.4-1}\Delta T)$
- Derivative  $\frac{dp}{dV}$  of a polytropic process:

$$pV^n = c \Rightarrow p = cV^{-n} \Rightarrow \frac{dp}{dV} = c(-n)V^{-n-1} = \frac{-ncV^{-n}}{V} = \frac{-np}{V}$$

- General formula for work. Note, for  $n \neq 1$  no gas laws are used so integration is possible. When  $n = 1$ , then the isothermal process integration must be used.  $W_b$  is the reversible boundary work.

$$W_b = - \int_1^2 p dV = - \int_1^2 cV^{-n} dV = - \left[ \frac{cV^{-n+1}}{-n+1} \right]_1^2 = \left[ \frac{(cV^{-n})V}{n-1} \right]_1^2 = \left[ \frac{(p)V}{n-1} \right]_1^2 = \frac{p_2V_2 - p_1V_1}{n-1} = \frac{mR(T_2 - T_1)}{n-1}$$

### Steady flow

- Open system - a system in which fluid can move in and out continuously.
- Derivation of STEADY FLOW ENERGY EQUATION (SFEE). Note,  $u$ ,  $p$ ,  $T$ ,  $h$  remain constant:
  - Energy added to system:  $\delta W$  (shaft work),  $\delta Q$  (heat addition),  $\delta mp_1v_1$  (Flow work) (comes from  $\delta V = \delta mv$ ),  $\delta mu_1$  (specific internal energy),  $\delta mc_1^2/2$  (kinetic energy),  $\delta mgz_1$  (potential energy)
  - Energy rejected from the system:  $\delta mp_2V_2$  (flow work),  $\delta mu_2$  (specific internal energy),  $\delta mc_2^2/2$  (kinetic energy),  $\delta mgz_2$  (potential energy).
  - Energy in = Energy out and power in = power out, so (taking time derivative of all terms):

$$\begin{aligned} \delta \dot{Q} + \delta \dot{W} &= \dot{m} ((u_2 + p_2V_2 + 0.5c_2^2 + gz_2) - (u_1 + p_1V_1 + 0.5c_1^2 + gz_1)) \\ &\Rightarrow \dot{Q} + \dot{W} = \dot{m} ((h_2 + 0.5c_2^2 + gz_2) - (h_1 + 0.5c_1^2 + gz_1)) \end{aligned}$$

### Steady flow applications

- Nozzles (used to accelerate gas) [ $Q = 0$ ,  $W = 0$ ,  $c_1 = 0$ ,  $z_1, z_2 = 0$ ]  $\Rightarrow$  SFEE becomes  $h_1 = h_2 + 0.5c_2^2$  or  $T_1 = T_2 + \frac{c_2^2}{2c_p}$ , if you substitute  $\Delta h = c_p\Delta T$
- Diffusers (used to decelerate gas) [ $Q = 0$ ,  $W = 0$ ,  $c_2 = 0$ ,  $z_1, z_2 = 0$ ]  $\Rightarrow$  SFEE becomes  $h_2 = h_1 + 0.5c_1^2$  or  $T_2 = T_1 + \frac{c_1^2}{2c_p}$ , if you substitute  $\Delta h = c_p\Delta T$
- Turbine (expander, used to produce work) and Compressor (used to increase (using work) pressure) [ $Q = 0$ ,  $z_1, z_2 = 0$ ]  $\Rightarrow \dot{W} = \dot{m} ((h_2 + 0.5c_2^2) - (h_1 + 0.5c_1^2)) \Rightarrow \dot{W} = \dot{m} (c_p\Delta T + 0.5(c_2^2 - c_1^2))$ 
  - Assumptions:

- Limitations: Friction causes heating, resulting in LOWER (mistake in notes i think) value of exiting enthalpy  $h_2$  and less work being available.
- *Turbine Isentropic Efficiency*  $\eta_t = \frac{W}{W'} = \frac{\text{realpower}(/work)}{\text{reversiblepower}(/work)} = \frac{h_2 - h_1}{h'_2 - h_1}$
- *Compressor Isentropic efficiency*  $\eta_c = \frac{W'}{W} = \frac{\text{reversiblepower}}{\text{realpower}} = \frac{h'_2 - h_1}{h_2 - h_1}$
- If frictionless then  $\Delta S = \int \frac{dQ}{T} = 0$
- Throttling valves (used to reduce pressure without work or heat transfer)  $[W, Q, c_1, c_2, z_1, z_2 = 0] \Rightarrow h_1 = h_2$  (isenthalpic)

## 4 The Second Law

### Reversible heat engines

- Heat engine - Abstract theoretical concept that allows us to try and achieve maximum efficiency. A hot reservoir (source) at a temperate  $T_1$  gives heat energy  $+Q_1$  to a heat engine to produce work. However, some of the energy  $-Q_2$  is lost/captured by a another cold reservoir (sink), so the actual produced work is  $W = -(Q_1 + Q_2)$ . The efficiency is work out divided by energy in  $\Rightarrow \eta = \frac{|W|}{Q_1} = \frac{|Q_1| - |Q_2|}{|Q_1|}$  (using absolute values for simplicity)
- Reversible heat engine - Has a heat pump that transfers the lost energy ( $Q_2$ ) in the cold reservoir back to the hot reservoir without any work - which is of course completely unrealistic. Some work needs to be added to run the heat pump.
- **Reversible process** def. - When a fluid undergoes a reversible process, both the fluid and the surroundings can always be restored to their original states. Furthermore it implies that the process is quasi-static, i.e. always in quasi-equilibrium and that process is frictionless, i.e. all energy transformations are ideal.

### Statements and Corollaries

- Definitions:
  - Reversible process. When a fluid undergoes a reversible process, both the fluid and the surroundings can always be restored to their original states. [Impossible in reality: Can be achieved theoretically by transfer by infinitesimally small temperature differences and forces on a moving boundary (to produce work) only infinitesimally different from external].
- Kelvin-Planck Statement - 'It is impossible for any device (engine) that operates on a **cycle** to receive heat from a **single** reservoir and produce A NET AMOUNT (100% conversion) of work.' This tells us:
  - Heat engines must exchange heat with  $\geq 2$  reservoirs.
  - An engine (or system) exchanging heat with 1 reservoir does **zero** work or **accepts** positive work from surroundings.
  - Work is more valuable than heat.
- **Corollary 1 Clausius Statement of the second law** - 'It is impossible to construct a device that operates in a cycle and transfers heat from a cooler to a hotter body without work being done on the system (device) by the surroundings.'
  - Essentially says that a heat pump will require work in order to transfer energy from a colder to a hotter body (like a fridge).
- **Corollary 2** - 'It is impossible to construct an engine (Heat engine, H.E.) operating between only two reservoirs which will have a higher efficiency than a reversible engine operating between the same two reservoirs.'
- **Corollary 3** - 'All reversible engines operating between the same two reservoirs have the same efficiency.'
- **Corollary 4 - Thermodynamic temperature scale** 'A scale of temperature can be defined which is independent of any particular thermometric substance (like a lake say or water), and which provides an absolute zero of temperature.'

- The efficiency of a reversible heat engine can be described as a function of temperature only.  $\eta_{RHE} = \frac{|W|}{Q_1} = \frac{|Q_1| - |Q_2|}{|Q_1|} = 1 - \frac{T_2}{T_1} = 1 - \frac{T_L}{T_H}$  (which is also **Carnot Efficiency**)
- **Corollary 5** - 'The efficiency of any reversible engine operating between more than two reservoirs must be less than that of a reversible engine operating between two reservoirs which have temperatures equal to the highest and lowest temperatures of the fluid in the original engine.'
- **Corollary 6 The Clausius Inequality** - 'Whenever an engine undergoes a cycle,  $\oint \frac{dQ}{T} \leq 0$ ' (where T refers to fluid that supplies heat). Note, only for a reversible engine does it equal zero, otherwise (for irreversible engines) it is LESS than zero.
- **Corollary 7 Entropy** - 'There exists a property of equilibrium state of a closed system such that a change in its value is equal to  $\Delta S = \int_1^2 \frac{dQ_{rev}}{T}$  for any reversible process undergone by the system between state 1 and 2.'
- $\Delta S$  is independent of path and is thus a state property.
- **Corollary 8** - 'The entropy of any closed system which is THERMALLY ISOLATED from the surroundings either increases or, if the process undergone is reversible, remains constant'.  $\Rightarrow \Delta S_1 + \Delta S_2 + \Delta S_3 \dots \geq 0$

## Measures

- Coefficient of performance  $\Rightarrow COP_{HP} = \frac{|Q_1|}{|W|} = \frac{|Q_1|}{|Q_1| - |Q_2|}$  (for a heat pump). Notice it is the reciprocal to the efficiency of a heat engine.

## ENTROPY in practice

- Only for pure substances, like ideal gases. From NFEE  $\Delta U = Q + W \Rightarrow dU = dQ_{rev} + dW$  now dividing by  $mT$ ,  $\Rightarrow \frac{du}{T} = \frac{dQ_{rev}}{mT} + \frac{dW}{mT} = ds - \frac{pdv}{T} \Rightarrow ds = \frac{du}{T} + p\frac{dv}{T}$  which is also known as **Gibbs equation** (also sometimes given this form:  $dU = Tds - pdV$ )
- Integrate Gibbs equation using  $du = c_v dT$  and  $p = \frac{RT}{v}$  to get  $\Rightarrow s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$ 
  - If isentropic, which means  $s_1 = s_2$  then you can show that  $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ , where  $\gamma = \frac{c_p}{c_v}$
  - For incompressible fluids  $dV = 0$  so  $\Rightarrow ds = \frac{du}{T} = c \frac{dT}{T}$  (ignore distinction between  $c_v$  and  $c_p$ ). On integration  $s_2 - s_1 = c \ln\left(\frac{T_2}{T_1}\right)$
  - Anisentropic is when a process is not isentropic. All irreversible compressions and expansions will be anisentropic.

## 5 Gas Cycles

- Assumptions
  - ideal gas with air properties
  - gas circulates in a closed loop
  - heat inputs approximate combustion (air standard cycle)
  - heat rejection approximates exhaust
  - (constant heat capacities)

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————— NFEE applies —————
- **Otto Cycle** [Four stroke petroleum engine] [Usually 14x more air than fuel] [r usually equals around 8]
  - 1-to-2 Isentropic Compression (BDC to TDC)  $\Rightarrow P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow \frac{mRT_1}{V_1} V_1^\gamma = \frac{mRT_2}{V_2} V_2^\gamma \Rightarrow T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ . Here  $r = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} \Rightarrow T_2 = T_1 r^{\gamma-1}$

- 2-to-3 Isochoric heat addition **Powerstroke (1)** (ignition)  $\Rightarrow Q_{in} = \Delta U - W (= 0) = mc_v \Delta T = mc_v(T_3 - T_2)$
- 3-to-4 Isentropic expansion **Powerstroke (2)** (TDC to BDC)  $\Rightarrow T_3 = T_4 r^{\gamma-1}$
- 4-to-1 Isochoric heat rejection **Exhaust** (BDC to TDC) and **Induction** (TDC to BDC) [both strokes equivalent to a cooling process]  $\Rightarrow Q_{out} = mc_v(T_4 - T_1)$
- Note,  $Q_{3-4} = 0, Q_{1-2} = 0$  because those paths are isentropic.
- **Efficiency**  $\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_4 - T_1}{T_4 r^{\gamma-1} - T_1 r^{\gamma-1}} = 1 - \frac{1}{r^{\gamma-1}}$ . Note, in reality is far less because too many simplifications are being made. Also, if compression ratio is too large ( $r > 10$ ) (which is desired because it increases efficiency) 'knocking' (2 almost simultaneous ignitions are caused - because the gas is too compressed - and this produces large shock-waves when they hit into each-other, causing a 'pinging/knocking' noise, which damages the piston over time) might occur.

• **Diesel Cycle** [assumes constant pressure]

- 1-to-2 Isentropic compression  $\Rightarrow T_2 = T_1 r^{\gamma-1}$ , where the compression ratio  $r = \frac{V_1}{V_2}$  (Remember, it is the ratio of max volume to min volume, so  $r \neq \frac{V_4}{V_3}$ .  $W_{in} = \Delta U = mc_v(T_2 - T_1)$ )
- 2-to-3 isobaric expansion **Fuel injection (1)**  $\Rightarrow Q_{in} = \Delta U - W = \Delta U + p\Delta V = \Delta H = mc_p \Delta T$  and since  $p_2 = p_3 \Rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2} = r_c$  the cut off ratio. So  $Q_{in} = mc_p(T_2 r_c - T_2)$ . Work is also done  $Q_{out,1} = -(pV_3 - pV_2) = -mR(T_2 r_c - T_2)$  (or can also get this from  $Q_{out,1} = \Delta U (= mc_v \Delta T) - Q_{in}$ )
- 3-to-4 isentropic expansion **Fuel injection (2)**  $\Rightarrow T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3 \left(\frac{r_c}{r}\right)^{\gamma-1}$ .  $W_{out,2} = \Delta U = mc_v(T_4 - T_3)$
- 4-to-1 isochoric heat rejection **Exhaust** and **Induction**  $\Rightarrow Q_{out} = \Delta U - W (= 0) = mc_v \Delta T = mc_v(T_1 - T_4)$  (should be negative answer)
- **Efficiency**  $\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{mc_v(T_4 - T_1)}{mc_p(T_2 r_c - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$ . Now, since  $T_3 = r_c T_2 = r_c r^{\gamma-1} T_1 \Rightarrow T_4 = r_c^\gamma T_1$  then  $\eta = 1 - \frac{(r_c^\gamma - 1)}{\gamma r^{\gamma-1}(r_c - 1)}$
- Assumptions: Standard air cycle assumptions - model combustion as heat addition, model exhaust/intake as heat rejection, constant pressure heat addition, constant heat capacity throughout cycle, perfect gas, ignore pressure losses in manifold and throttle, and at exhaust.

————— SFEE applies —————

————— Pressure, Temperature, Velocity

• **Brayton Joule Cycle**

- 1-to-2 (isentropic compression) **Compression**  $\Rightarrow p_1 V_1^\gamma = p_2 V_2^\gamma \Rightarrow V_1^\gamma = \left(\frac{mRT_1}{p_1}\right)^\gamma \Rightarrow \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} T_1 = T_2 \Rightarrow r_p^{\frac{\gamma-1}{\gamma}} T_1 = T_2$  (Also  $W_{12} = h_2 - h_1 = c_p(T_2 - T_1)$ )
- 2-to-3 (isobaric expansion) **Fuel Injection and Combustion (1)** From SFEE  $\Rightarrow \dot{Q}_{in} + \dot{W} (= 0) = \dot{m}[(h_3 - h_2) + 0 + 0] = \dot{m}c_p(T_3 - T_2)$
- 3-to-4 (isentropic expansion) **Fuel Injection and Combustion (2)** Note  $\boxed{r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4}} \Rightarrow T_3 = T_4 r_p^{\frac{\gamma-1}{\gamma}}$ . (Also,  $W_{34} = c_p(T_4 - T_3)$ )
- 4-to-1 (Isobaric compression) **Exhaust + Induction**  $\dot{Q}_{out} = \dot{m}c_p(T_4 - T_1)$
- **Efficiency**  $\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}$
- In the cycle the efficiency for the compressor and turbine are  $\eta_{comp} = \frac{T'_2 - T_1}{T_2 - T_1}$  and  $\eta_{turb} = \frac{T_4 - T'_3}{T'_4 - T_3}$ , where T' is the temperature if the process were isentropic (reversible).
- Work ratio = net work/turbine work  $W_{net} = \frac{W_{turb} - W_{comp}}{W_{turbine}}$
- In reality Brayton cycle is irreversible and thus not isentropic nor isobaric. Entropy increases from 1-2 and 3-4 and pressure drops from 2-3.

- If  $T_4 > T_2$  (which it will be for the cycle to work) we can use a re-generator which takes the exhaust heat and puts it into the heat exchanger (which allows for less energy needed to be added to the system  $Q_{in} \downarrow$  for it to run well). The **effectiveness** of this can be measured like so:  $e = \frac{Heattransfer}{Maxheattransfer} = \frac{mc_p(T_5 - T_2)}{mc_p(T_4 - T_2)} = \frac{T_5 - T_2}{T_4 - T_2}$ , where  $T_5 \leq T_4$  (Max effectiveness occurs when  $T_5 = T_4$ ).
- Other options: Compressors with inter-cooling (demands less work) and turbines with reheating (produces more work). Both approach Ericsson Cycle.

• **Turbojet cycle**

- 1-to-2 **Diffuser** [ $T_2 = T_1 + \frac{c_1^2}{2c_p}$ ] [From  $T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$ ]  
—Brayton cycle starts—
- 2-to-3 **Compressor** [ $V \approx 0$ ] [ $T_3 = \left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} T_2$ ] [Note, here  $\boxed{r_p = \frac{p_3}{p_2}}$  because it is the compression ratio.]
- 3-to-4 **Combustion** [ $V \approx 0$ ] [ $p_4 = p_3 = r_p p_2$ ]
- 4-to-5 **Turbine** [ $V = 0$ ] [Same work as compressor, constant  $c_p$ ] [ $T_3 - T_2 = T_4 - T_5$ ] [ $p_5 = p_4 \left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{\gamma-1}}$ ]  
—Brayton cycle ends—
- 5-to-6 **Nozzle** [ $p_6 = p_1$ ] [ $T_6 = \frac{T_5}{\left(\frac{p_5}{p_6}\right)^{\frac{\gamma-1}{\gamma}}}$ ] [Apply SFEE  $T_5 = T_6 + \frac{c_5^2}{2c_p} \Rightarrow v_6 = \sqrt{2c_p(T_5 - T_6)}$ ]
- **Thrust**  $\Rightarrow F = \dot{m}(v_6 - v_1)$  [Velocities are taken relative air frame  $\rightarrow v_1 =$  airframe velocity]
- Also,  $\dot{W} = \dot{Q} = \dot{m}c_p(T_6 - T_1)$  and from SFEE  $0 = c_p(T_6 - T_1) + 0.5(v_6^2 - v_1^2)$

**Key things: Combustion is modeled as constant pressure heat addition** Also Turbofan is simply a jet engine (turbojet cycle) with a fan that drives by-pass air around the engine which has proved to be more efficient and economical.