UK Population Growth Model

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Abstract

In the last century the UK population has increased from 38 to 59 million and is projected to increase by 9.7 million over the next 25 years [BBC22a]. This growth in population has raised many negative issues and will continue to exacerbate existing problems, such as the long standing housing crises, leading to increased housing prices and a rise in wealth inequality. Other issues include increased building on greenbelt land and continued strain on the transport network. However a growth in population also brings positive connotations to various professions, such as economic stakeholders looking to exploit the increase in the productive capacity of the economy. With population growth being evident in having such a widespread affect the prediction and analysis of future population figures would be of high importance to all parties affected by it. Here we propose a method using a system of differential equations that factor in the variables affecting population growth; Birth rates, death rates and immigration. This initial simplification of the problem into three separate differential equations allowed us to investigate further into these factors giving us three separate figures for each variable that would then be used within the final differential equation. Furthermore using the euler method we are able to graph and predict population figures for the next 10 decades. The main result shows that the population will continue to grow at a fairly steady rate, approximately by 500,000 a year, for the next ten years.

1 Introduction

This report is an investigation into the population growth of the United Kingdom. In the last century the number of people living in the UK has increased from 38 to 59 million people, despite decreasing growth rates [Sim17]. In order to predict what the figures for the UK population will look like in the future, a differential equation will be set up and then numerically integrated between the time of the current population (now) and the next 10 decades. The determining factors of the differential equation are birth rates, death rates and immigration. Each of these factors will be investigated further in their own sections of this report. The solution to this differential equation is of high relevance to various economic stakeholders and professions, especially in the public sector. For example, businesses might consider drastic increases in population as an opportunity to plan expansions in resources in order capitalise on consequent consumption advances, while public servants, like city planners, need this vital information in order to confirm or cancel various planned building procedures.

The following approach will be taken in order set up a meaningful differential equation that will allow one to predict future UK population figures. Each of the determining factors, birth, death and immigration will be set up separately as their own differential equations and then combined together to form the final differential equation for the total population. In other words, the final differential equation consists of a system of differential equations.

As the first step in the modelling process, we identify the dependent and independent variables. The independent variable will be time t measured in years and the following will be in the dependent variables (birth, death, immigration and population), each of which are functions of time (and in most cases of population too):

$$B = B(t, P) \quad \text{The number of people born each year} \tag{1}$$
$$D = D(t, P) \quad \text{The number of people that die each year} \tag{2}$$
$$I = I(t) \quad \text{The number of people that immigrate to the UK each year} \tag{3}$$
$$P = P(t, P) \quad \text{The number of people in the UK each year} \tag{4}$$

Since we are looking to model the *change* in population over time (in years) and each of the factors that effect these changes in population are functions of time, we propose to the set up the final differential equation as follows:

$$\frac{dP(t,P)}{dt} = \frac{dB(t,P)}{dt} + \frac{dD(t,P)}{dt} + \frac{dI(t)}{dt} \tag{5}$$

Finally, this differential equation will be solved by numerical integration using the Euler Method and if it is linear, by analytical methods as well. The programme that will be used to implement the Euler Method is ipython and the code can be found in the appendix.

1.1 Birth rates

The birth rate of the population is directly dependant on the reproducing population of the United Kingdom (ages 19-35). The factors that affect the rate of change of the reproducing populous will be considered to derive a differential relationship for the birth rate.

The UK birth rate is very fluctuant, but trends show the birth rate generally dropping from 1930 to the early 2000s and generally increasing after 2001. While this is the general trends birth rate fluctuates largely year to year. Such as there was approximately 28000 less births in 2014 compared to 2011 censuses, but there was an increase in birth rate from 2014 to 2015. This is of course due to the multitude of factors that can affect birth rate. [ONS22c]

Death rate

As with any demographic sector, a proportion of the populous will be reduced by fatality. While this is relatively low for the reproducing population, this will still need to be taken into account. A multitude of factors can affect death rate in this early stage such as disease or fatal accidents. The office of national statics states that 6.1% of the reproducing population was lost to fatality in 2014.[ONS22b]

Migration

The UK is a very popular location for individuals and families to immigrate to, this results in a relatively high rate of immigration. Emigration will also need to be considered, this will be significantly lower than the rate of immigration. Within this equation a simplier migration model is used to consider the change in the reproducing population. A more extensive model is used to consider migration as a whole in section 3.

ODE for Reproducing popluation

One major factor that needs to be considered is the number of females of in reproducing demographic (19-35). Data from the office of national statistics show that $731 * 10^6$ woman are of reproducing age in 2014 this would be 11.3% of the uk population.[ONS22c]

$$\text{Ratio of females in reproducing demographic to total population} = \alpha \tag{6}$$
$$\alpha = \frac{\text{Females in reproducing poplation}}{\text{Total population}} = \frac{7313853}{64596752} \approx 0.11 \tag{7}$$

The death rate of the reproducing population will also need to be considered. The office of national statistic sates that 6.1% of the reproducing population died in 2014, this is multiplied by the percentage of females in the reproducing population to give a death rate of 0.0069% of the total population. [ONS22b]

$$\text{deathrate} = 0.061 * 0.11 = -0.0069P \tag{8}$$

The rate of people immigrating to uk within the reproducing demographic also needs to be considered. In 2014 $631 * 10^3$ people immigrated to the uk, if approximately 50% is taken to consider females immigrated to the UK, this would be 0.49% of the total uk population. $319 * 10^3$ people emigrated out of the country this equates to 0.25% of the population. [ONS22a]

$$\text{Immigration} = 0.0049 \tag{9}$$

$$\text{Emmigration} = -0.0025 \tag{10}$$

To consider the birth rate the number of births per female needs to be considered. In 2014 there was $7.3 * 10^6$ females in the reproducing population and $6.9 * 10^5$ births. This gives the number of births per female as 0.095. [ONS22b]

$$\text{Total Female poplous in the reproducing demographic 2014} = 7313853 \tag{11}$$

$$\text{Total births in 2014} = 695233 \tag{12}$$

$$\frac{\text{Birthrate}}{\text{Female Population}} = \frac{695233}{7313853} = 0.095 \tag{13}$$

The components derived above are used to construct the differential equation for the birth rate of the UK population.

$$\frac{dB}{dt} = \tag{14}$$

$$\text{Births per Female} * P * (\text{Reproducing Female Ratio} + \text{Immigration} - \text{Emmigration} - \text{Death}) \tag{15}$$

$$\frac{dB}{dt} = 0.095(0.11P + 0.0049P - 0.025P - 0.0069P) = 0.0101P \tag{16}$$

This predicts that the number of births each year are increasing, which keeps in line with the most recent trends. The figure below of historic data shows that while the birth rate is increasing it fluctuates between increasing and decreasing. This is due to a multitude of external factors that cannot be factored in within the limits of this model, with a number of factors being almost unpredictable.

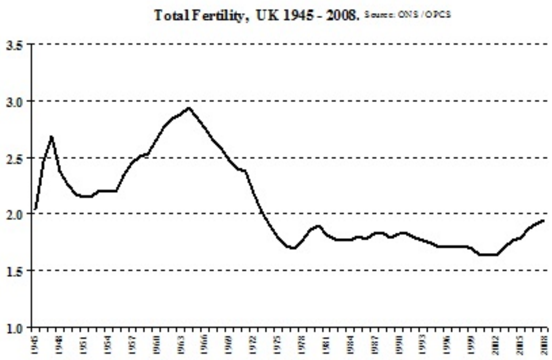

Figure 1: UK Fertility (Source: ONS)

1.2 Death rates

According to the office of National Statistics death rates on average have stayed constant since 2012 meaning that the average life expectancy of a member of the UK population stays the same at 81.5 years.

Death rates are effected by several different factors:

Birth Rates:

Of course, the amount of people dying is directly proportional to the amount of people being born. The higher the number of human births, the higher the amount of humans that have the opportunity to die. The people born in the large 'baby booms' in the mid 60's in the UK, for example, will result in an increase in death rates in the next 10 to 20 years from now, because in these years the people born in the 60's will start to die. Furthermore, the large number of births in the 60's led to a larger reproducing population in the late 80's, resulting in another increase in birth rates for that decade, which in turn will lead to another increase in death rates 20 to 40 years from now [bbc22b]. These knock-on effects and shifts in dying and reproducing population will be accounted for in this death rate model by including the total population term, P in the equation. This means that the death rate will be dependent on the population and it will allow the model to scale with rising birth rates as population will also be scaled with birth rates.

This is also a limitation of the model, since the death rate should be delayed by the life expectancy of the people who are born each year and not simply increase with increasing birth rates. For example, the death rate should increase by the same amount as the birth rate 81.5 years later, if we take this years life expectancy.

Infant mortality

According to the office of national statistics, 2.721 thousand infant deaths were registered in 2015 [OoN20]. To account for this in our model, we divide this number by the total population of 2015 and multiply it by P, since we are trying to scale it to increases in population. This shown below.

$$\text{For now,} \quad \frac{dD}{dt} = \frac{2721}{65429048}P = 0.0000416P \tag{17}$$

Life expectancy:

The big trend over the last few centuries, since the middle ages, is that the UK life expectancy age is forever increasing. This can be summed up to the huge advancement in health care and living conditions due to modern technology. Although now increasing at a much slower rate with periods of plateau in more recent years, a positive increasing trend can be seen in regards to the average life span of a member of the UK population. This increase, however, is very minimal. The office for national statistics estimates it is increasing by around 11.3 weeks per person per year [OoN21].
To determine the yearly increase in population, one has to multiply this by the fraction of the population that are born into the population each year, because it is their lives that are expected to increase. Since this is a very small portion of the total population and the increase of only a few weeks per year is a very minimal increase, the net increase of population will be very small each year. Nevertheless, the dying population will become slightly older and the number of deaths each year will decrease by a small amount. To account for this in our model we multiply the increase in life expectancy of the fractional population by the total population P, as shown below:

$$11.3 \quad \text{weeks} = 0.217 \quad \text{years} \tag{18}$$

$$0.217 \times \ \text{fraction of newborns in population} = 0.217 \times \frac{695233 \text{ newborns}}{65429048 \text{ population}} = 0.0023 \tag{19}$$

$$\text{For now,} \quad \frac{dD}{dt} = -0.0023P \tag{20}$$

Officially, the fraction of newborns in the population is dependent on the birth rate differential equation from section 1.1. However, this would have been too complex, so it was decided to take statistical data from 2015. To conclude this model, we add the results:

$$\frac{dD}{dt} = (0.000416 - 0.0023)P = -0.0019P \tag{21}$$

This is the final model for the death rate differential equation. We can now numerically integrate this with respect to time, implementing the Euler Method. Using a step-size of 0.05 and changing P to represent not the total population but rather the dying population, we can generate the graph shown in Figure 2.

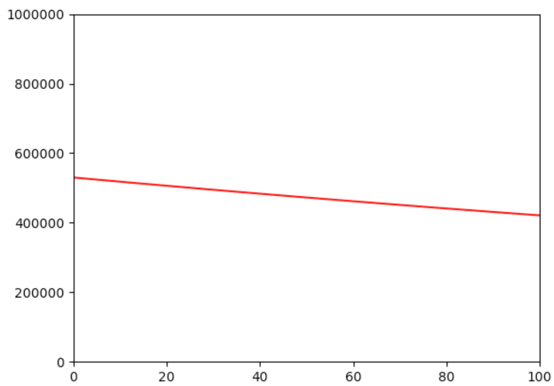

Figure 2: Euler Method solution to death rate ODE for the next century

This differential equation predicts that the death rate will have decreased by approximately 100 thousand people in 100 years from now. This is a very small decrease and could be considered as following the general trend of a consistent death rate of the past few years.

Outside factors:

It has to be noted that our model for death rates is not robust and simply cannot support this very delicate and ever changing statistic. There are many factors that make the death rates non linear and unpredictable. Our mathematical model simply cannot support all of these outside factors and therefore we have only focused on the two main ones as listen above.

However our model will still give us a good prediction of the trend we should expect in regards to death rates among the UK population but we will just lose some accuracy due to not being able to model unpredictable population altering events. Outside events that aren't factored in the model are; wars, waves of illness and natural disasters.

Events like this obviously cannot be predicted and do not have consistent statistics, therefore it is very hard to include in a mathematical model but that does not mean that they should not be noted.

With modern healthcare and technology combined with the fact that the UK is considered a very stable country politically and geographically, there will be a very very minimal amount of deaths due to outside factors in this modern day. These numbers were too minimal to include in a model dealing with a starting population of over 60 million.

1.3 Migration rates

Migration rates have shown a general increase in the last 40 years [Mig22], with an increase of 232,000 in yearly net migration. However there have been fluctuations in the past ten years with it decreasing by 79,000 from 2010 to 2012 and then increasing by 155,000 in 2015 for it to go back down again in 2016 by 59,000, with the net migration of 2016 being 273,000. There are various factors which affect the inflow and outflow of people within the UK and must be looked into in order to derive a differential equation.

Looking at past statistical data it shows that net migration has a general increase of 600 per year for the last 10 years[Mig22], however if looking at data for over 40 years it is shown that there is a net migration increase of 5800 per year. Therefore giving us a short term and long term net migration differential. However other factors are still to be included.

$$\frac{dI}{dt} = 600 \tag{22}$$

$$\frac{dI}{dt} = 5800 \tag{23}$$

With the UK being a country which has a relatively low rate of natural disasters when compared to other countries, shown by its world risk index score of 3.54% [Wik22], it assures a sense of safety which would encourage migration to the UK. However with this being dependent on environmental factors such as its location away from plate boundaries and hot spots, this is expected to not change a substantial amount in the next 10 decades so current migration rates still hold and this does not need to be factored in further.

The stable government within the UK when compared to the political instability in other countries offers a security and again is a positive incentive for migration into the UK. However there seems to be no drastic changes happening within the government system in the foreseeable future and so again does not need to be factored in.

With the country being a war free zone that also brings about positive incentives to migrate to the UK, however again this does not look likely to change and so the current net increase can still be used.

It is evident that the strength of the British economy is a huge factor in migration rates. This is clear to see due to the fact that during the UK recession of 2008-2009 the net migration dropped by 44,000 from 2007 and then increased by 27,000 in 2010 coming out of the recession[Mig22]. Therefore this posses the question of will there be another recession, with Brexit happening it has risen some concern of another recession being on its way, with a recent Bloomberg survey that stated 71% of financial sector economist think a recession will happen[Ind22]. With the statistical data already factoring in a recession that happened in 2008-2009 the differential equation again does not change.

With Brexit being imposed over the next 2 years, it is expected that this will cause a decrease in the amount of net migration due to the fact that one of the policies of brexit is to restrict immigration flow. Looking at past statistical data[Mig22] when the UK first joined the EU in 2004 immigration from EU citizens rose significantly by approximately 88,000 from 2003-2007 making that an increase rate of 8800 per year within 1 decade and an increase of 2200 per year for the past 4 decades. With the brexit act imposing an opposing act, it would be expected that migration would follow suit and decrease by the same amount.However in order for the UK to keep some of the same prevelidges it had when it was in the EU it may have to show leancy towards inwards migration and so is not so black and white. Therefore an aproximation of 60% of the 88,000 will be used. When looking at a differential equation for the next 10 years factoring in this value shows us that the rate of change of net migration would show to decrease in the next ten years.

$$\frac{dI}{dt} = 600 - 0.6 * 8800 = -4680 \tag{24}$$

However looking at a bigger picture of 40 plus decades we must factor it in twice to negate the affect of the increased migration from 2003-2007. We observe that it only hinders the overall trend of population growth from net migration. This would be due to the positive migration factors being of more importance over a longer period of time as seen in the statistical data.

$$\frac{dI}{dt} = 5800 - 0.6 * 2200 * 2 = 3160 \tag{25}$$

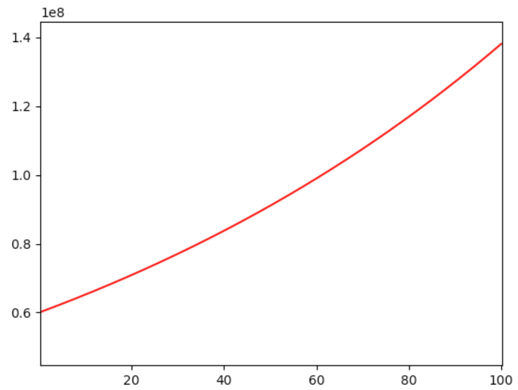

Figure 3: Euler Method solution to population ODE for the next century

1.4 The final model

As set out to do in the introduction, the final model can now be found by summing the individual differential equations of the previous sections.

$$\frac{dP}{dt} = \frac{dB}{dt} + \frac{dD}{dt} + \frac{dI}{dt} \tag{26}$$

$$\frac{dP}{dt} = 0.0101P - 0.0018P + 3160 \tag{27}$$

$$\frac{dP}{dt} = 0.0083P + 3160 \tag{28}$$

Using the Euler Method (see Appendix for code) and a step size of 0.05 the final population differential equation can be numerically integrated and plotted on a graph. This is shown above. Since this is first-order, linear, non-homogeneous ODE, the differential equation can also be solved analytically. This is done below, so as to be able to precisely compare our results with those found in literature.

$$\frac{dP}{dt} - 0.0083P = 3160 \tag{29}$$

$$\implies \frac{d}{dt}(e^{-0.0083t}P) = 3160e^{-0.0083t} \tag{30}$$

$$\implies P = \frac{3160}{-0.0083} + Ce^{0.0083t} \tag{31}$$

$$\text{Since } P(0) = 65429048 \tag{32}$$

$$\implies P(t) = 65809770e^{0.0083t} - 380723 \tag{33}$$

Analysis

According to the 2014 National Bulletin by the office for national statistics the UK population is expected to increase by approximately 9.7 million over the next 25 years [Nat22]. Plugging in $t = 22$ (since this bulletin is now 3 years old) into the population ODE, we get a prediction that the population will increase by 13.1 million.

Does this prediction digress too much from the 'real' value? The truth is no one really knows. Even the top statisticians who have predicted the population growth of the entire planet over the next century cannot be sure about 1 billion people. Nevertheless, a deviation of approximately 3.5 million people when dealing in the millions is perhaps arguably too high. Therefore, we will only look at the predictions for the next 10 years and see how they compare instead.

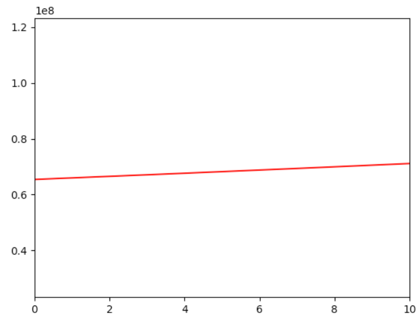

Figure 4: Euler Method solution to population ODE for the next decade

The National Bulletin predicted that the UK population will rise to 70 million by 2027 [Nat22], which now, is only ten years away. In this year, our model predicts that the population will rise to approximately 71 million, which although off by a million, is far more realistic. For the next year, our model predicts that the population will grow by 550,000 people. Although the bulletin does not have any prognosticated figures for 2018, this value seems significantly closer to the general trends of the past few years, which have been increases of around 500,000 per year [BBC21].

Limitations and Discussion

Since our model follows an exponential curve, and despite it having a low increasing gradient, it is clear that our model will become increasingly inaccurate the further into the future we look. This was also evident by the data analysed above. The predictions for the near future (1 to 10) years from now, are more accurate (in the sense that more statisticians seem to agree on these values) than the predictions made for the next 25 years and beyond. Interestingly, however, an article by the telegraph suggests that if the 2012 bulletin trends were to be kept on the same projection the population could rise to 132 million [Tel21] in the next century, which is surprisingly close to the 140 million our model predicts (see figure 2). Nevertheless, a forever increasing population is simply not possible. Logically speaking, at some point in the future, space and resources will run out and the population will cease to grow. This is why various models found in literature include a negative kP^2 term in their differential equation, so as to incorporate a 'maxing-out' of the population. When the United Kingdom will reach this stage is very much up for debate, but we can assume that it is still a few decades away. Therefore, we can consider our model as valid, but limited to the next few decades and accurate to the next few years.

1.5 Conclusion - Improving results

The main issue with trying to model population growth is deciding which data is worth including and which is not. For each of the smaller models (birth rates, death rates and immigration) of the previous sections, a concious decision had to me made to limit their models to two or three factors. Consequently the final population ODE is a very simple model, so simple in fact that it can be solved analytically. While this is advantageous to making things more understandable it has also led to a model that is unreliable for predictions beyond ten years. Therefore, to improve on this model and for general study into population growth, it is recommended to consider significantly more factors. For death rates, for example, further investigation into factors, such as increased pollution levels, trends in increasing deaths to cancer and respiratory diseases could be analysed. Of course, it is impossible to include every factor and some things are very difficult to find data for, but it is still recommended to consider as many things as possible.

In conclusion, the key aspect that should be taken away from this report is the following: Every model, whether detailed or simple, will always have a large margin of error, because there are so many outside factors that cannot be accounted for. As mentioned afore, even the current best models for world population growth cannot be certain of 1 billion a people. With Brexit and the passing of Article 50, global warming, increases in migration due to war torn areas, etc... current models for UK population growth are becoming ever more unstable and inaccurate. Nevertheless, as this report has shown, we expect the general trends to continue and population to rise.

2 Appendix

2.1 Integration methods

The following python code was used to numerically solve the differential equations of the previous sections. The Euler method is applied using a function that takes a differential equation as a function (f), the initial condition ($y0$), the lower boundary (a), upper boundary (b) and the step size (h). For our model we set f to our various differential equations (shown in code below as well), $y0$ is set to the current population (65429048), a to zero, b to 100 and the step size h to 0.1.

```
def euler_method(f,y0,a,b,h):
    y_lst = [] #Creating two empty lists
    t_lst = []
    t,y = a,y0 #Setting t and y to initial values
    while t<= b: #Until we reach the upper bound
        y_lst.append(y) #Add y and t to the empty lists
        t_lst.append(t)
        t += h #Increment t by step size
        y += h * f(t,y) #Increment y by step size times derivative function
    return t_lst,y_lst #Return lists to plot
```

Below are the functions used to plot the differential equations set up in the various sections:

```
def birth_ode(t, pop):
    dBdt = 0.0101*pop
    return dBdt

def death_ode(t,pop):
    dDdt = -0.0023*pop
    return dDdt

def immigration_ode(t,pop):
    dIdt = 3160
    return dIdt

def pop_ode(t,pop):
        dPdt = 0.0083P + 3160
    return dPdt
```

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